Computer Algebra Independent Integration Tests

Summer 2023 edition

5-Inverse-trig-functions/5.3-Inverse-tangent/148-5.3.2-d-x-^m-a+b-arctan-c-x^n-^p

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CHAPTER 1

INTRODUCTION

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This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [166]. This is test number [148].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (166)	0.00 (0)
Mathematica	98.19 (163)	1.81 (3)
Maple	90.96 (151)	9.04 (15)
Mupad	65.06 (108)	34.94 (58)
Sympy	57.83 (96)	42.17 (70)
Maxima	56.02 (93)	43.98 (73)
Fricas	55.42 (92)	44.58 (74)
Giac	51.20 (85)	48.80 (81)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

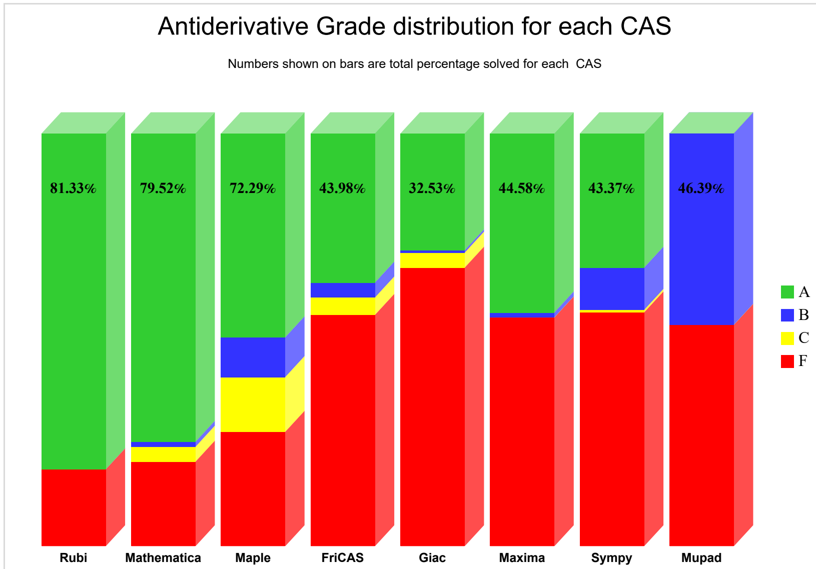
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

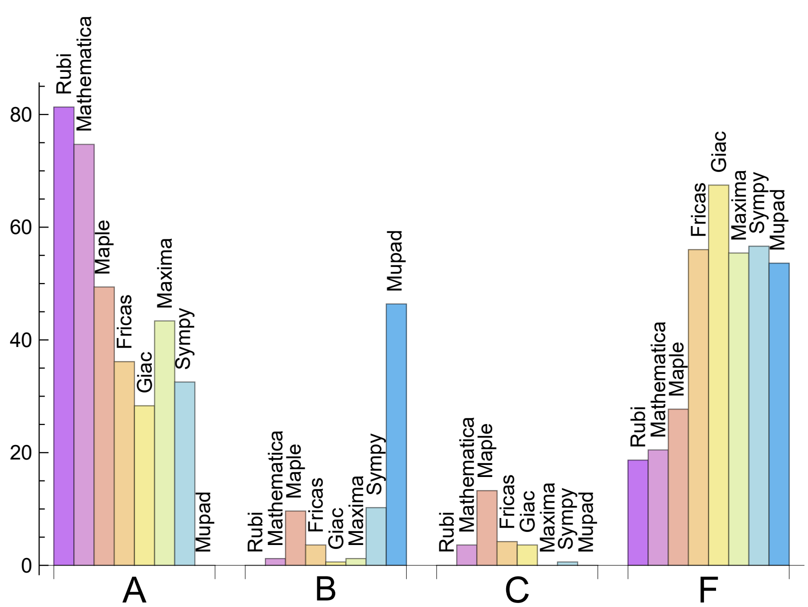
System	% A grade	% B grade	% C grade	% F grade
Rubi	81.325	0.000	0.000	18.675
Mathematica	74.699	1.205	3.614	20.482
Maple	49.398	9.639	13.253	27.711
Maxima	43.373	1.205	0.000	55.422
Fricas	36.145	3.614	4.217	56.024
Sympy	32.530	10.241	0.602	56.627
Giac	28.313	0.602	3.614	67.470
Mupad	0.000	46.386	0.000	53.614

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	3	100.00	0.00	0.00
Maple	15	100.00	0.00	0.00
Mupad	58	0.00	100.00	0.00
Maxima	73	82.19	1.37	16.44
Fricas	74	83.78	0.00	16.22
Sympy	70	88.57	11.43	0.00
Giac	81	93.83	6.17	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Rubi	0.14
Fricas	0.25
Maxima	0.45
Mupad	0.56
Mathematica	0.83
Maple	3.02
Sympy	17.36
Giac	24.28

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	50.70	0.97	38.00	0.99
Giac	54.19	0.88	39.00	0.96
Fricas	73.03	1.16	45.50	1.08
Maxima	88.04	2.54	51.00	1.00
Sympy	101.60	1.48	58.00	1.09
Rubi	123.03	1.00	75.50	1.00
Mathematica	161.93	1.20	65.00	1.12
Maple	517.07	4.06	60.00	1.00

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

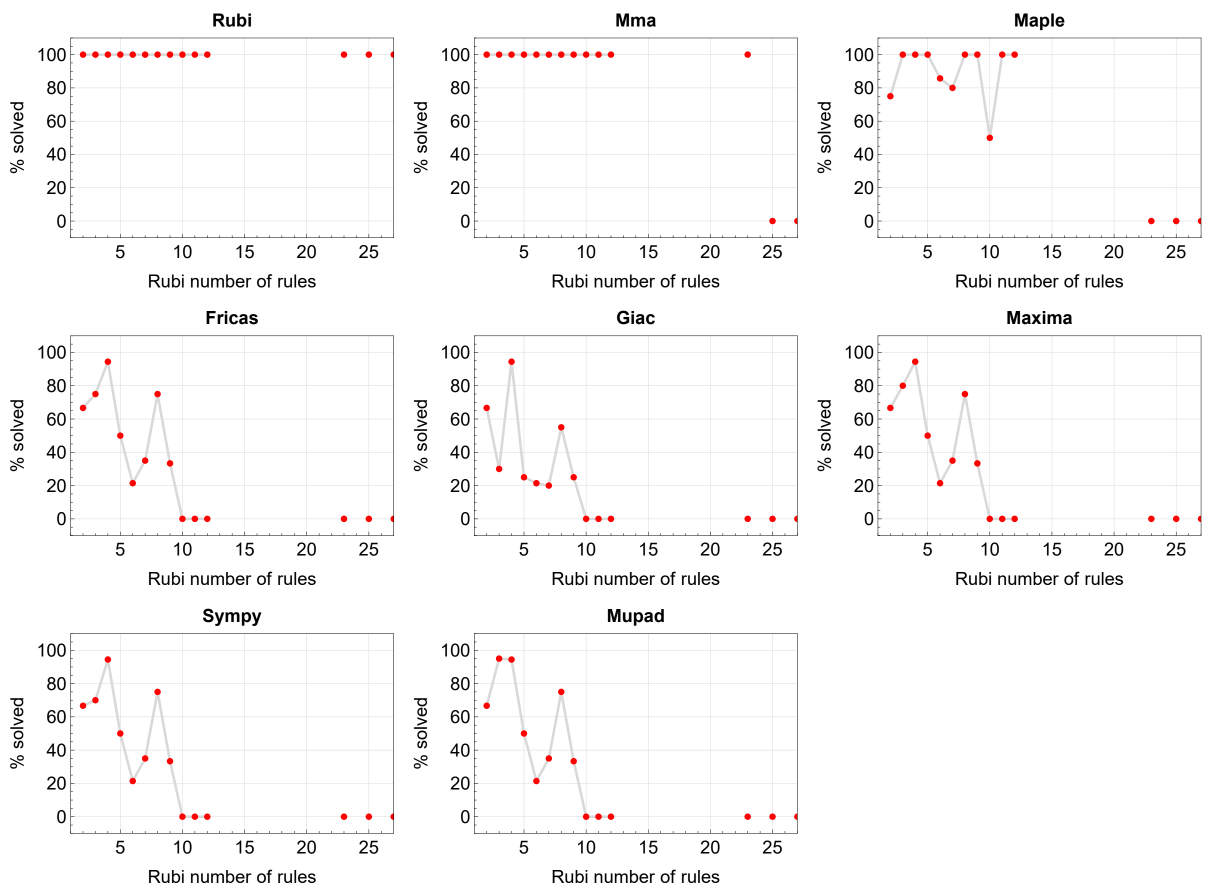


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

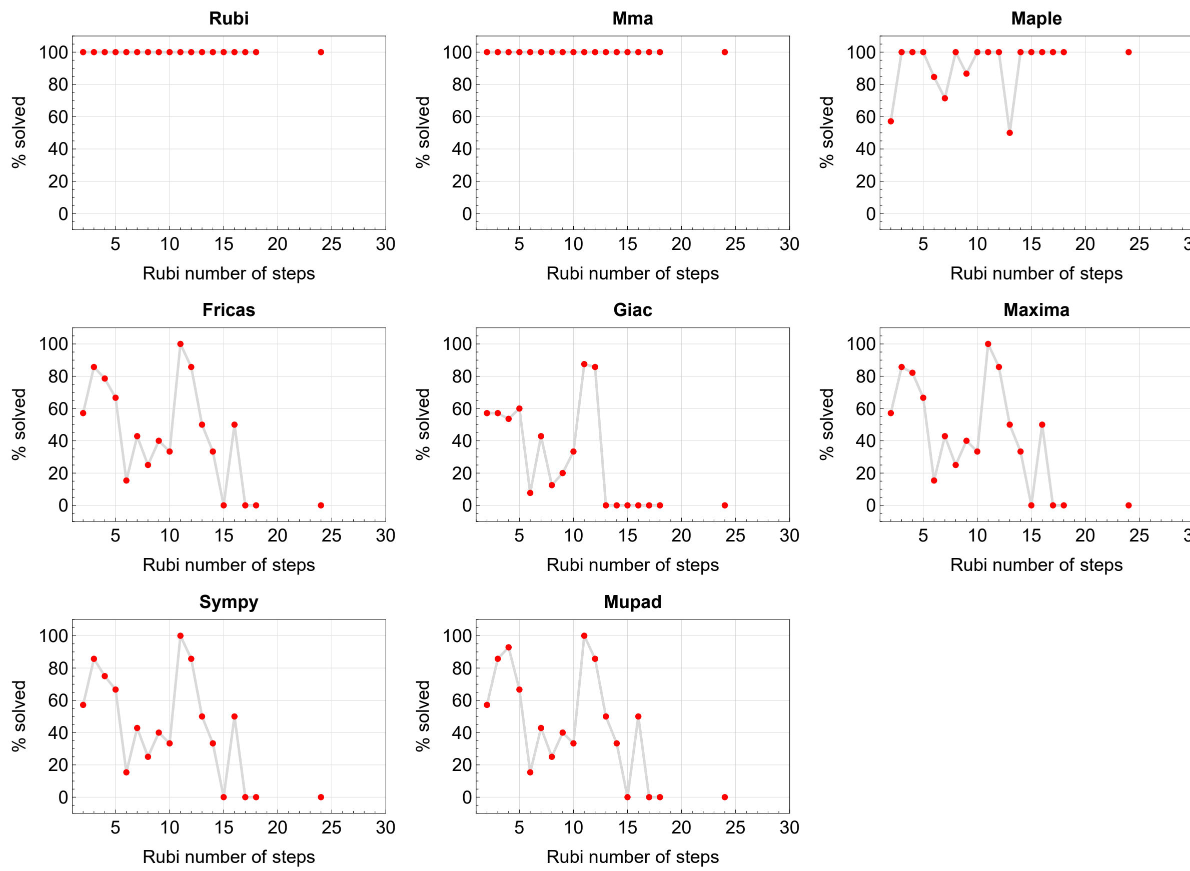


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

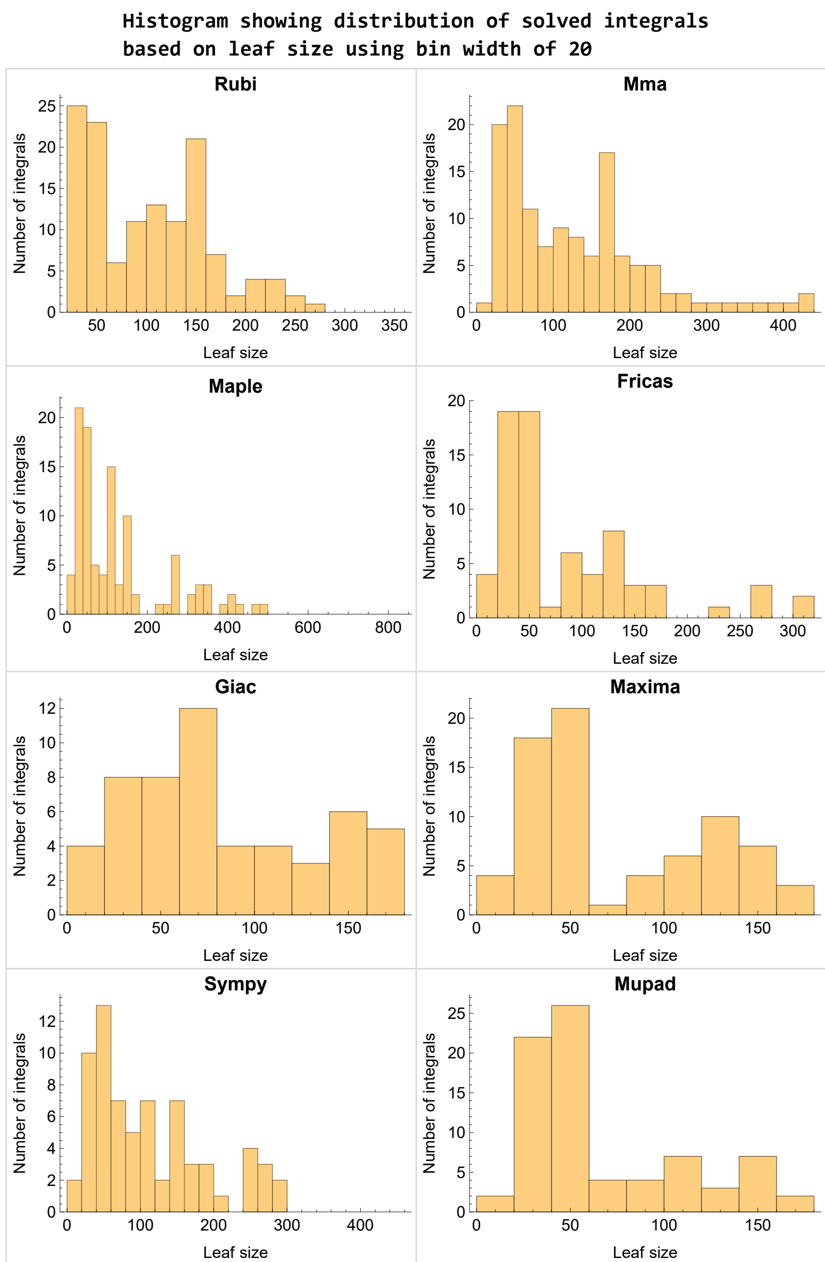


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

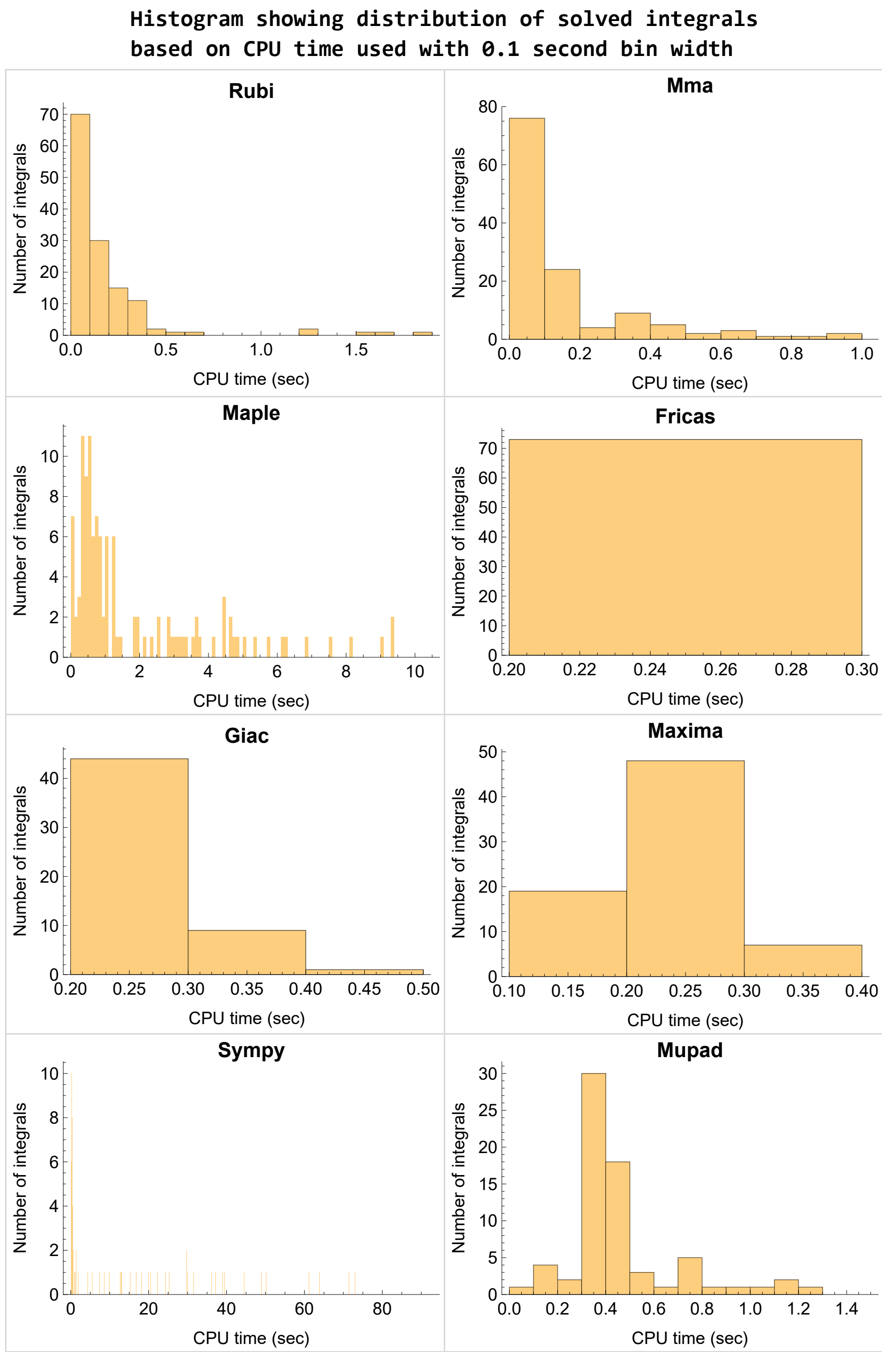


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

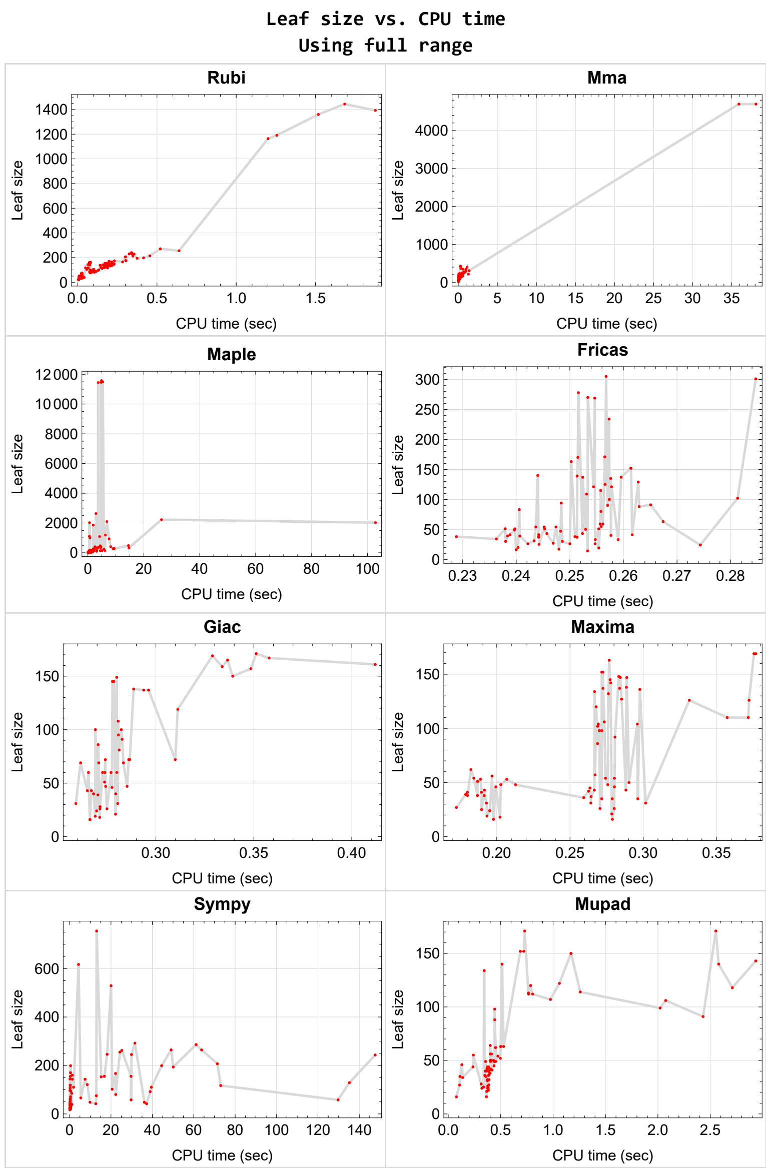


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 55, 57, 58, 59, 91, 92, 94, 95, 127, 128, 130, 131}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {82, 83}

Maple {19, 25, 27, 30, 31, 33, 75, 79, 86, 90, 114, 118, 120, 122, 126, 144, 148, 150, 151}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

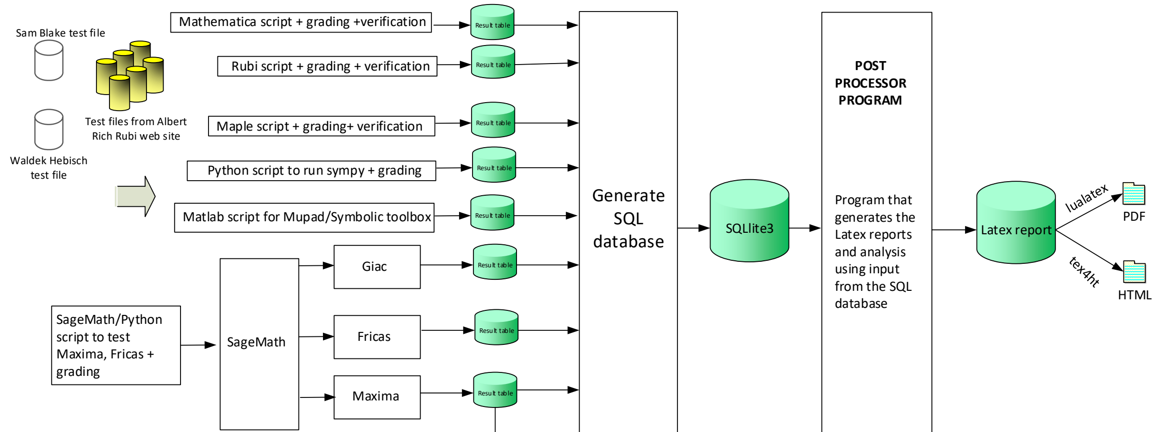
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



High level overview of the CAS independent integration test build system

One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form "{n,n,..}" which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

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June 27, 2023
Design.vsdx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

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2.1 List of integrals sorted by grade for each CAS

Rubi 22
Mma 22
Maple 23
Fricas 23
Maxima 23
Giac 24
Mupad 24
Sympy 24

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 53, 56, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 93, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 129, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 10, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 53, 56, 60, 61, 62, 63, 64, 65, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 86, 87, 88, 89, 90, 93, 96, 97, 98, 99, 100, 101, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 129, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 160, 161, 162, 163, 164, 165, 166 }

B grade { 82, 83 }

C grade { 9, 11, 66, 102, 158, 159 }

F normal fail { 81, 84, 85 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 21, 23, 24, 26, 53, 60, 61, 62, 63, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 76, 77, 80, 96, 97, 98, 99, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 115, 116, 119, 132, 133, 134, 135, 137, 138, 139, 140, 142, 145, 146, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164 }

B grade { 20, 22, 28, 29, 32, 34, 87, 123, 136, 141, 143, 147, 149, 152, 153, 166 }

C grade { 19, 25, 27, 30, 31, 33, 64, 75, 79, 86, 90, 100, 114, 118, 120, 122, 126, 144, 148, 150, 151, 165 }

F normal fail { 56, 78, 81, 82, 83, 84, 85, 88, 89, 93, 117, 121, 124, 125, 129 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 15, 17, 21, 23, 60, 61, 62, 63, 65, 66, 67, 74, 76, 80, 96, 97, 98, 99, 101, 102, 103, 105, 107, 109, 111, 113, 115, 119, 132, 133, 134, 135, 137, 138, 139, 140, 142, 146, 154, 155, 156, 158, 159, 160, 161, 162, 163, 164 }

B grade { 104, 106, 108, 110, 112, 166 }

C grade { 53, 68, 69, 70, 71, 72, 73 }

F normal fail { 7, 14, 16, 18, 19, 20, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 56, 64, 75, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 93, 100, 114, 116, 117, 118, 120, 121, 122, 123, 124, 125, 126, 129, 136, 141, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 157, 165 }

F(-1) timedout fail { }

F(-2) exception fail { 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52 }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 15, 17, 21, 23, 53, 60, 61, 62, 63, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 76, 80, 96, 97, 98, 99, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 115, 119, 132, 133, 134, 135, 137, 138, 139, 140, 142, 146, 154, 155, 156, 158, 159, 160, 161, 162, 163, 164 }

B grade { 157, 165 }

C grade { }

F normal fail { 7, 14, 16, 18, 19, 20, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 56, 64, 75, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 93, 100, 114, 116, 117, 118, 120, 121, 122, 123, 124, 125, 126, 129, 136, 141, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 166 }

F(-1) timedout fail { 34 }

F(-2) exception fail { 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52 }

Giac

A grade { 6, 53, 60, 61, 62, 63, 65, 67, 68, 69, 70, 71, 72, 73, 74, 76, 96, 97, 98, 99, 101, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 115, 133, 135, 137, 139, 154, 155, 156, 158, 159, 160, 161, 162, 163, 164 }

B grade { 136 }

C grade { 66, 102, 132, 134, 138, 146 }

F normal fail { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 56, 64, 75, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 93, 100, 114, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 129, 140, 141, 142, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 157, 165, 166 }

F(-1) timedout fail { 30, 31, 32, 33, 34 }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 15, 17, 21, 23, 53, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 76, 80, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 115, 119, 132, 133, 134, 135, 136, 137, 138, 139, 140, 142, 146, 154, 155, 156, 157, 158, 159, 160, 162, 163, 164, 165 }

C grade { }

F normal fail { }

F(-1) timedout fail { 14, 16, 18, 19, 20, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 56, 75, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 93, 114, 116, 117, 118, 120, 121, 122, 123, 124, 125, 126, 129, 141, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 161, 166 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 15, 17, 21, 23, 53, 60, 62, 65, 66, 67, 68, 69, 70, 71, 73, 74, 76, 96, 98, 102, 104, 105, 106, 108, 110, 112, 132, 133, 134, 135, 137, 138, 139, 140, 142, 146, 154, 155, 156, 161, 162, 163 }

B grade { 61, 63, 80, 97, 99, 101, 103, 107, 109, 111, 113, 115, 119, 158, 159, 160, 164 }

C grade { 72 }

F normal fail { 7, 14, 16, 18, 19, 20, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 56, 64, 75, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 93, 100, 114, 116, 117, 118, 120, 121, 122, 123, 124, 125, 126, 136, 141, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 157, 165, 166 }

F(-1) timedout fail { 91, 94, 95, 127, 128, 129, 130, 131 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name **`Mathematica`** was abbreviated to **`MMA`**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	64	52	57	54	63	0	52
N.S.	1	1.00	1.08	0.88	0.97	0.92	1.07	0.00	0.88
time (sec)	N/A	0.021	0.006	0.559	0.267	0.244	0.354	0.000	0.497

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	61	54	56	59	60	0	54
N.S.	1	1.00	1.09	0.96	1.00	1.05	1.07	0.00	0.96
time (sec)	N/A	0.029	0.019	0.388	0.197	0.256	0.303	0.000	0.474

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	53	44	48	47	53	0	44
N.S.	1	1.00	1.10	0.92	1.00	0.98	1.10	0.00	0.92
time (sec)	N/A	0.019	0.006	0.557	0.276	0.248	0.274	0.000	0.236

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	50	46	46	49	49	0	42
N.S.	1	1.00	1.11	1.02	1.02	1.09	1.09	0.00	0.93
time (sec)	N/A	0.024	0.012	0.383	0.199	0.240	0.251	0.000	0.404

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	42	36	37	34	42	0	34
N.S.	1	1.00	1.14	0.97	1.00	0.92	1.14	0.00	0.92
time (sec)	N/A	0.011	0.005	0.526	0.264	0.236	0.229	0.000	0.137

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	28	31	33	26	31	27
N.S.	1	1.00	1.00	0.97	1.07	1.14	0.90	1.07	0.93
time (sec)	N/A	0.008	0.004	0.313	0.193	0.255	0.108	0.259	0.107

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	34	0	0	0	0	28
N.S.	1	1.00	1.00	0.97	0.00	0.00	0.00	0.00	0.80
time (sec)	N/A	0.022	0.005	0.480	0.000	0.000	0.000	0.000	0.314

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	38	39	39	37	37	0	36
N.S.	1	1.00	1.09	1.11	1.11	1.06	1.06	0.00	1.03
time (sec)	N/A	0.018	0.005	0.303	0.179	0.251	0.270	0.000	0.349

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	46	39	31	26	37	0	42
N.S.	1	1.00	1.24	1.05	0.84	0.70	1.00	0.00	1.14
time (sec)	N/A	0.015	0.006	0.481	0.264	0.242	0.229	0.000	0.370

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	54	52	51	50	61	0	46
N.S.	1	1.00	1.02	0.98	0.96	0.94	1.15	0.00	0.87
time (sec)	N/A	0.024	0.019	0.324	0.187	0.253	0.359	0.000	0.130

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	46	44	46	41	46	0	42
N.S.	1	1.00	0.96	0.92	0.96	0.85	0.96	0.00	0.88
time (sec)	N/A	0.018	0.005	0.500	0.280	0.239	0.289	0.000	0.396

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	69	60	62	59	71	0	56
N.S.	1	1.00	1.08	0.94	0.97	0.92	1.11	0.00	0.88
time (sec)	N/A	0.026	0.013	0.340	0.182	0.256	0.470	0.000	0.412

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	138	146	163	152	199	0	171
N.S.	1	1.00	0.96	1.01	1.13	1.06	1.38	0.00	1.19
time (sec)	N/A	0.214	0.137	1.002	0.277	0.261	0.478	0.000	0.728

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	169	266	0	0	0	0	0
N.S.	1	1.00	0.99	1.56	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.198	0.602	1.991	0.000	0.000	0.000	0.000	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	111	118	136	121	155	0	134
N.S.	1	1.00	0.99	1.05	1.21	1.08	1.38	0.00	1.20
time (sec)	N/A	0.154	0.100	1.024	0.298	0.254	0.383	0.000	0.343

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	131	238	0	0	0	0	0
N.S.	1	1.00	0.95	1.72	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.142	0.339	1.833	0.000	0.000	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	75	88	104	83	107	0	88
N.S.	1	1.00	0.99	1.16	1.37	1.09	1.41	0.00	1.16
time (sec)	N/A	0.078	0.161	0.973	0.296	0.241	0.274	0.000	0.444

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	90	123	0	0	0	0	0
N.S.	1	1.00	1.08	1.48	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.075	0.144	2.193	0.000	0.000	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	132	132	179	1002	0	0	0	0	0
N.S.	1	1.00	1.36	7.59	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.207	0.189	0.743	0.000	0.000	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	102	270	0	0	0	0	0
N.S.	1	1.00	1.24	3.29	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.104	0.197	2.348	0.000	0.000	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	90	98	98	94	119	0	140
N.S.	1	1.00	1.14	1.24	1.24	1.19	1.51	0.00	1.77
time (sec)	N/A	0.086	0.081	0.725	0.272	0.248	0.326	0.000	2.580

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	153	310	0	0	0	0	0
N.S.	1	1.00	1.09	2.21	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.158	0.473	2.908	0.000	0.000	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	128	132	152	135	170	0	171
N.S.	1	1.00	1.10	1.14	1.31	1.16	1.47	0.00	1.47
time (sec)	N/A	0.150	0.114	0.974	0.272	0.258	0.437	0.000	2.553

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	291	402	0	0	0	0	0
N.S.	1	1.00	1.14	1.58	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.639	0.953	2.524	0.000	0.000	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	271	271	396	1185	0	0	0	0	0
N.S.	1	1.00	1.46	4.37	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.521	1.124	6.276	0.000	0.000	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	225	340	0	0	0	0	0
N.S.	1	1.00	1.16	1.75	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.375	0.626	2.563	0.000	0.000	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	206	206	269	1088	0	0	0	0	0
N.S.	1	1.00	1.31	5.28	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.300	0.680	4.188	0.000	0.000	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	152	276	0	0	0	0	0
N.S.	1	1.00	1.16	2.11	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.177	0.427	2.888	0.000	0.000	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	192	240	0	0	0	0	0
N.S.	1	1.00	1.61	2.02	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.146	0.157	5.707	0.000	0.000	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	206	206	368	2026	0	0	0	0	0
N.S.	1	1.00	1.79	9.83	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.302	0.321	0.575	0.000	0.000	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	116	116	214	1862	0	0	0	0	0
N.S.	1	1.00	1.84	16.05	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.188	1.296	1.912	0.000	0.000	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	176	354	0	0	0	0	0
N.S.	1	1.00	1.32	2.66	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.202	0.376	4.621	0.000	0.000	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	213	213	305	2097	0	0	0	0	0
N.S.	1	1.00	1.43	9.85	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.348	1.383	6.803	0.000	0.000	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	265	423	0	0	0	0	0
N.S.	1	1.00	1.34	2.14	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.415	0.864	4.497	0.000	0.000	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	8	8	10	8	10	10	7	3	10
N.S.	1	1.00	1.25	1.00	1.25	1.25	0.88	0.38	1.25
time (sec)	N/A	0.006	0.742	10.581	0.255	0.231	0.269	23.919	0.293

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	6	6	8	6	8	8	7	3	8
N.S.	1	1.00	1.33	1.00	1.33	1.33	1.17	0.50	1.33
time (sec)	N/A	0.003	0.020	4.118	0.231	0.250	0.257	21.736	0.291

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	8	3	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	0.80	0.30	1.20
time (sec)	N/A	0.010	0.514	4.589	0.238	0.233	0.368	24.442	0.274

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	8	8	10	8	45	10	8	3	10
N.S.	1	1.00	1.25	1.00	5.62	1.25	1.00	0.38	1.25
time (sec)	N/A	0.006	0.715	10.509	0.258	0.234	0.310	49.107	0.294

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	6	6	8	6	39	8	8	3	8
N.S.	1	1.00	1.33	1.00	6.50	1.33	1.33	0.50	1.33
time (sec)	N/A	0.003	1.095	5.540	0.234	0.251	0.298	47.802	0.300

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	51	12	10	3	12
N.S.	1	1.00	1.20	1.00	5.10	1.20	1.00	0.30	1.20
time (sec)	N/A	0.009	1.238	4.234	0.265	0.245	0.375	49.411	0.299

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	8	0	0	10	3	10
N.S.	1	1.00	1.20	0.80	0.00	0.00	1.00	0.30	1.00
time (sec)	N/A	0.005	1.483	2.086	0.000	0.000	0.400	53.543	0.307

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	8	8	10	6	0	0	8	3	8
N.S.	1	1.00	1.25	0.75	0.00	0.00	1.00	0.38	1.00
time (sec)	N/A	0.003	1.812	1.641	0.000	0.000	0.315	53.952	0.326

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	10	0	0	10	3	12
N.S.	1	1.00	1.17	0.83	0.00	0.00	0.83	0.25	1.00
time (sec)	N/A	0.009	1.102	1.947	0.000	0.000	0.361	148.199	0.293

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	8	0	0	10	3	10
N.S.	1	1.00	1.20	0.80	0.00	0.00	1.00	0.30	1.00
time (sec)	N/A	0.005	0.839	1.792	0.000	0.000	1.406	81.002	0.301

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	8	8	10	6	0	0	8	3	8
N.S.	1	1.00	1.25	0.75	0.00	0.00	1.00	0.38	1.00
time (sec)	N/A	0.002	1.711	1.527	0.000	0.000	0.852	81.253	0.323

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	10	0	0	10	3	12
N.S.	1	1.00	1.17	0.83	0.00	0.00	0.83	0.25	1.00
time (sec)	N/A	0.009	0.957	2.726	0.000	0.000	0.858	142.203	0.316

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	8	0	0	10	3	10
N.S.	1	1.00	1.20	0.80	0.00	0.00	1.00	0.30	1.00
time (sec)	N/A	0.005	0.935	1.751	0.000	0.000	0.364	70.975	0.292

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	8	8	10	6	0	0	10	3	8
N.S.	1	1.00	1.25	0.75	0.00	0.00	1.25	0.38	1.00
time (sec)	N/A	0.002	0.006	1.588	0.000	0.000	0.324	64.900	0.313

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	10	0	0	12	3	12
N.S.	1	1.00	1.17	0.83	0.00	0.00	1.00	0.25	1.00
time (sec)	N/A	0.009	1.304	2.352	0.000	0.000	0.476	68.146	0.296

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	8	0	0	10	3	10
N.S.	1	1.00	1.20	0.80	0.00	0.00	1.00	0.30	1.00
time (sec)	N/A	0.005	1.035	1.943	0.000	0.000	0.670	249.366	0.329

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	8	8	10	6	0	0	10	3	8
N.S.	1	1.00	1.25	0.75	0.00	0.00	1.25	0.38	1.00
time (sec)	N/A	0.002	1.735	1.520	0.000	0.000	0.638	239.105	0.351

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	10	0	0	12	3	12
N.S.	1	1.00	1.17	0.83	0.00	0.00	1.00	0.25	1.00
time (sec)	N/A	0.009	2.338	2.635	0.000	0.000	0.909	184.339	0.354

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	108	69	86	80	110	86	49
N.S.	1	1.00	0.92	0.59	0.74	0.68	0.94	0.74	0.42
time (sec)	N/A	0.049	0.035	0.253	0.269	0.256	1.953	0.271	0.357

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	387	44	15	3	18
N.S.	1	1.00	1.12	1.00	24.19	2.75	0.94	0.19	1.12
time (sec)	N/A	0.016	4.832	2.559	3.735	0.261	7.445	72.748	0.650

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	295	30	15	3	18
N.S.	1	1.00	1.12	1.00	18.44	1.88	0.94	0.19	1.12
time (sec)	N/A	0.016	3.147	3.820	2.202	0.248	4.351	72.162	0.622

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	73	73	60	0	0	0	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.024	0.041	0.000	0.000	0.000	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	14	3	18
N.S.	1	1.00	1.12	1.00	1.12	1.12	0.88	0.19	1.12
time (sec)	N/A	0.018	0.346	3.526	0.268	0.245	1.260	87.381	0.273

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	10	3	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	1.00	0.30	1.20
time (sec)	N/A	0.005	0.499	1.693	0.371	0.239	1.814	80.207	0.406

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	15	3	18
N.S.	1	1.00	1.12	1.00	1.12	1.12	0.94	0.19	1.12
time (sec)	N/A	0.016	0.418	4.359	0.538	0.250	155.449	79.272	0.406

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	59	50	54	51	58	60	49
N.S.	1	1.00	1.09	0.93	1.00	0.94	1.07	1.11	0.91
time (sec)	N/A	0.026	0.012	0.583	0.280	0.245	29.725	0.274	0.403

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	52	45	48	51	80	47	44
N.S.	1	1.00	1.11	0.96	1.02	1.09	1.70	1.00	0.94
time (sec)	N/A	0.024	0.017	0.661	0.213	0.255	22.305	0.274	0.374

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	48	41	43	38	48	43	40
N.S.	1	1.00	1.12	0.95	1.00	0.88	1.12	1.00	0.93
time (sec)	N/A	0.020	0.008	0.734	0.288	0.229	9.849	0.267	0.353

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	41	36	38	39	66	40	35
N.S.	1	1.00	1.14	1.00	1.06	1.08	1.83	1.11	0.97
time (sec)	N/A	0.011	0.024	0.536	0.191	0.238	5.410	0.279	0.357

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	63	0	0	0	0	32
N.S.	1	1.00	1.00	1.62	0.00	0.00	0.00	0.00	0.82
time (sec)	N/A	0.041	0.008	0.747	0.000	0.000	0.000	0.000	0.389

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	44	39	41	43	75	60	38
N.S.	1	1.00	1.13	1.00	1.05	1.10	1.92	1.54	0.97
time (sec)	N/A	0.018	0.010	0.266	0.190	0.252	12.925	0.273	0.395

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	48	39	35	30	42	72	41
N.S.	1	1.00	1.17	0.95	0.85	0.73	1.02	1.76	1.00
time (sec)	N/A	0.019	0.009	0.375	0.272	0.238	12.686	0.286	0.406

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	60	53	53	54	92	69	50
N.S.	1	1.00	1.09	0.96	0.96	0.98	1.67	1.25	0.91
time (sec)	N/A	0.028	0.016	0.303	0.189	0.247	38.959	0.262	0.408

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	179	121	147	171	153	169	64
N.S.	1	1.00	1.11	0.75	0.91	1.06	0.95	1.05	0.40
time (sec)	N/A	0.081	0.068	0.869	0.289	0.257	15.270	0.329	0.400

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	177	118	145	152	143	165	62
N.S.	1	1.00	1.11	0.74	0.91	0.96	0.90	1.04	0.39
time (sec)	N/A	0.079	0.056	0.535	0.277	0.261	7.490	0.337	0.452

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	107	103	127	140	617	149	49
N.S.	1	1.00	0.76	0.74	0.91	1.00	4.41	1.06	0.35
time (sec)	N/A	0.076	0.061	0.358	0.285	0.244	4.336	0.280	0.453

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	158	107	132	139	121	138	55
N.S.	1	1.00	1.10	0.75	0.92	0.97	0.85	0.97	0.38
time (sec)	N/A	0.064	0.074	0.349	0.276	0.251	8.534	0.289	0.240

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	177	115	142	170	529	159	63
N.S.	1	1.00	1.11	0.72	0.89	1.07	3.33	1.00	0.40
time (sec)	N/A	0.074	0.085	0.417	0.278	0.252	20.038	0.334	0.500

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	177	118	138	163	155	150	63
N.S.	1	1.00	1.11	0.74	0.87	1.03	0.97	0.94	0.40
time (sec)	N/A	0.073	0.086	0.468	0.288	0.250	29.701	0.339	0.528

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	121	151	169	137	199	145	150
N.S.	1	1.00	0.98	1.22	1.36	1.10	1.60	1.17	1.21
time (sec)	N/A	0.179	0.120	0.855	0.377	0.252	44.486	0.279	1.170

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	154	154	141	333	0	0	0	0	0
N.S.	1	1.00	0.92	2.16	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.171	0.364	3.561	0.000	0.000	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	85	110	126	100	155	100	112
N.S.	1	1.00	0.94	1.22	1.40	1.11	1.72	1.11	1.24
time (sec)	N/A	0.108	0.083	1.290	0.331	0.257	16.832	0.269	0.805

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	107	140	0	0	0	0	0
N.S.	1	1.00	1.06	1.39	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.097	0.152	4.438	0.000	0.000	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	151	151	201	0	0	0	0	0	0
N.S.	1	1.00	1.33	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.227	0.172	0.000	0.000	0.000	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	97	97	127	339	0	0	0	0	0
N.S.	1	1.00	1.31	3.49	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.129	0.194	3.328	0.000	0.000	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	98	118	110	115	167	0	152
N.S.	1	1.00	1.13	1.36	1.26	1.32	1.92	0.00	1.75
time (sec)	N/A	0.116	0.107	0.651	0.357	0.256	22.225	0.000	0.688

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	1393	1393	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.878	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	1191	1191	4697	0	0	0	0	0	0
N.S.	1	1.00	3.94	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.256	38.101	0.000	0.000	0.000	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	1164	1164	4697	0	0	0	0	0	0
N.S.	1	1.00	4.04	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.201	35.915	0.000	0.000	0.000	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	1360	1360	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.518	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	1444	1444	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.684	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	149	149	170	399	0	0	0	0	0
N.S.	1	1.00	1.14	2.68	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.213	0.512	4.665	0.000	0.000	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	224	275	0	0	0	0	0
N.S.	1	1.00	1.56	1.91	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.182	0.134	9.388	0.000	0.000	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	229	229	417	0	0	0	0	0	0
N.S.	1	1.00	1.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.355	0.301	0.000	0.000	0.000	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	138	138	239	0	0	0	0	0	0
N.S.	1	1.00	1.73	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.218	0.469	0.000	0.000	0.000	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	149	149	196	465	0	0	0	0	0
N.S.	1	1.00	1.32	3.12	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.230	0.369	4.421	0.000	0.000	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	406	50	0	20	20
N.S.	1	1.00	1.11	1.00	22.56	2.78	0.00	1.11	1.11
time (sec)	N/A	0.017	2.263	0.171	3.974	0.244	0.000	0.423	0.336

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	303	34	17	20	20
N.S.	1	1.00	1.11	1.00	16.83	1.89	0.94	1.11	1.11
time (sec)	N/A	0.017	1.467	0.176	2.349	0.247	111.972	0.521	0.320

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	75	75	65	0	0	0	0	0	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.027	0.087	0.000	0.000	0.000	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	20	0	20	20
N.S.	1	1.00	1.11	1.00	1.11	1.11	0.00	1.11	1.11
time (sec)	N/A	0.018	0.457	0.188	0.240	0.247	0.000	0.278	0.283

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	124	36	0	20	20
N.S.	1	1.00	1.11	1.00	6.89	2.00	0.00	1.11	1.11
time (sec)	N/A	0.019	0.478	0.164	0.394	0.238	0.000	0.300	0.351

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	59	50	54	51	58	60	49
N.S.	1	1.00	1.09	0.93	1.00	0.94	1.07	1.11	0.91
time (sec)	N/A	0.025	0.012	0.750	0.274	0.238	129.726	0.266	0.439

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	52	45	48	51	117	47	44
N.S.	1	1.00	1.11	0.96	1.02	1.09	2.49	1.00	0.94
time (sec)	N/A	0.023	0.019	0.688	0.203	0.240	73.035	0.285	0.384

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	48	41	43	38	48	43	40
N.S.	1	1.00	1.12	0.95	1.00	0.88	1.12	1.00	0.93
time (sec)	N/A	0.020	0.010	0.816	0.267	0.251	36.158	0.265	0.384

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	41	36	38	39	102	40	35
N.S.	1	1.00	1.14	1.00	1.06	1.08	2.83	1.11	0.97
time (sec)	N/A	0.014	0.023	0.589	0.180	0.241	20.583	0.268	0.111

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	63	0	0	0	0	32
N.S.	1	1.00	1.00	1.62	0.00	0.00	0.00	0.00	0.82
time (sec)	N/A	0.037	0.009	0.517	0.000	0.000	0.000	0.000	0.375

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	44	39	41	43	110	60	38
N.S.	1	1.00	1.13	1.00	1.05	1.10	2.82	1.54	0.97
time (sec)	N/A	0.017	0.010	0.260	0.180	0.246	39.528	0.280	0.399

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	48	39	35	30	42	72	41
N.S.	1	1.00	1.17	0.95	0.85	0.73	1.02	1.76	1.00
time (sec)	N/A	0.020	0.010	0.437	0.296	0.249	37.249	0.287	0.418

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	60	53	53	54	129	69	50
N.S.	1	1.00	1.09	0.96	0.96	0.98	2.35	1.25	0.91
time (sec)	N/A	0.023	0.019	0.410	0.207	0.245	135.233	0.283	0.433

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	179	155	148	269	255	167	114
N.S.	1	1.00	1.03	0.89	0.85	1.55	1.47	0.96	0.66
time (sec)	N/A	0.232	0.080	0.804	0.283	0.255	24.304	0.358	1.259

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	131	98	92	234	755	95	91
N.S.	1	1.00	1.30	0.97	0.91	2.32	7.48	0.94	0.90
time (sec)	N/A	0.078	0.047	0.418	0.281	0.257	13.066	0.281	2.432

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	170	144	137	270	245	137	107
N.S.	1	1.00	1.03	0.87	0.83	1.64	1.48	0.83	0.65
time (sec)	N/A	0.215	0.087	0.485	0.284	0.253	29.934	0.296	0.975

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	183	105	102	121	286	108	118
N.S.	1	1.00	1.59	0.91	0.89	1.05	2.49	0.94	1.03
time (sec)	N/A	0.063	0.067	0.765	0.269	0.258	61.135	0.281	2.710

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	181	167	152	305	264	171	122
N.S.	1	1.00	1.03	0.95	0.86	1.73	1.50	0.97	0.69
time (sec)	N/A	0.303	0.115	1.270	0.273	0.257	63.808	0.351	1.060

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	185	113	106	137	292	119	106
N.S.	1	1.00	1.58	0.97	0.91	1.17	2.50	1.02	0.91
time (sec)	N/A	0.066	0.050	0.844	0.273	0.260	31.576	0.311	2.075

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	170	153	137	278	246	157	113
N.S.	1	1.00	1.03	0.93	0.83	1.68	1.49	0.95	0.68
time (sec)	N/A	0.283	0.056	0.661	0.273	0.252	18.140	0.349	0.764

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	170	104	98	90	262	91	99
N.S.	1	1.00	1.63	1.00	0.94	0.87	2.52	0.88	0.95
time (sec)	N/A	0.057	0.074	0.494	0.270	0.257	25.230	0.283	2.022

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	179	164	147	301	264	161	120
N.S.	1	1.00	1.03	0.94	0.84	1.73	1.52	0.93	0.69
time (sec)	N/A	0.304	0.075	0.638	0.284	0.285	49.046	0.412	0.786

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	121	151	169	129	243	145	150
N.S.	1	1.00	0.98	1.22	1.36	1.04	1.96	1.17	1.21
time (sec)	N/A	0.187	0.111	1.043	0.376	0.263	147.668	0.278	1.172

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	154	154	141	11449	0	0	0	0	0
N.S.	1	1.00	0.92	74.34	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.182	0.117	3.690	0.000	0.000	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	85	110	126	91	194	100	112
N.S.	1	1.00	0.94	1.22	1.40	1.01	2.16	1.11	1.24
time (sec)	N/A	0.105	0.073	1.474	0.372	0.265	50.123	0.282	0.766

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	107	140	0	0	0	0	0
N.S.	1	1.00	1.03	1.35	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.101	0.092	5.020	0.000	0.000	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	154	154	201	0	0	0	0	0	0
N.S.	1	1.00	1.31	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.215	0.182	0.000	0.000	0.000	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	100	100	125	11455	0	0	0	0	0
N.S.	1	1.00	1.25	114.55	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.133	0.194	4.777	0.000	0.000	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	98	118	110	102	207	0	152
N.S.	1	1.00	1.13	1.36	1.26	1.17	2.38	0.00	1.75
time (sec)	N/A	0.112	0.101	0.785	0.372	0.281	71.423	0.000	0.721

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	154	154	167	11496	0	0	0	0	0
N.S.	1	1.00	1.08	74.65	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.190	0.458	5.340	0.000	0.000	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	240	240	346	0	0	0	0	0	0
N.S.	1	1.00	1.44	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.339	0.591	0.000	0.000	0.000	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	147	147	170	935	0	0	0	0	0
N.S.	1	1.00	1.16	6.36	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.199	0.145	7.574	0.000	0.000	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	224	275	0	0	0	0	0
N.S.	1	1.00	1.61	1.98	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.186	0.077	9.008	0.000	0.000	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	232	232	421	0	0	0	0	0	0
N.S.	1	1.00	1.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.341	0.288	0.000	0.000	0.000	0.000	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	133	133	240	0	0	0	0	0	0
N.S.	1	1.00	1.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.208	0.447	0.000	0.000	0.000	0.000	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	146	146	196	11581	0	0	0	0	0
N.S.	1	1.00	1.34	79.32	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.229	0.372	4.884	0.000	0.000	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	407	50	0	20	20
N.S.	1	1.00	1.11	1.00	22.61	2.78	0.00	1.11	1.11
time (sec)	N/A	0.016	2.233	0.194	4.058	0.261	0.000	0.435	0.354

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	304	34	0	20	20
N.S.	1	1.00	1.11	1.00	16.89	1.89	0.00	1.11	1.11
time (sec)	N/A	0.016	1.528	0.186	2.368	0.271	0.000	0.532	0.338

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	75	75	65	0	0	0	0	0	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.026	0.093	0.000	0.000	0.000	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	20	0	20	20
N.S.	1	1.00	1.11	1.00	1.11	1.11	0.00	1.11	1.11
time (sec)	N/A	0.018	0.483	0.178	0.248	0.259	0.000	0.288	0.303

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	132	36	0	20	20
N.S.	1	1.00	1.11	1.00	7.33	2.00	0.00	1.11	1.11
time (sec)	N/A	0.017	0.495	0.187	0.385	0.240	0.000	0.282	0.375

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	55	46	45	41	46	81	45
N.S.	1	1.00	1.10	0.92	0.90	0.82	0.92	1.62	0.90
time (sec)	N/A	0.020	0.028	1.278	0.264	0.262	0.142	0.281	0.436

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	48	47	43	40	41	69	40
N.S.	1	1.00	1.12	1.09	1.00	0.93	0.95	1.60	0.93
time (sec)	N/A	0.021	0.022	1.010	0.192	0.258	0.121	0.271	0.368

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	44	37	36	31	36	72	36
N.S.	1	1.00	1.13	0.95	0.92	0.79	0.92	1.85	0.92
time (sec)	N/A	0.015	0.022	1.250	0.259	0.243	0.108	0.274	0.384

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	27	27	25	22	46	25
N.S.	1	1.00	1.00	1.00	1.00	0.93	0.81	1.70	0.93
time (sec)	N/A	0.009	0.004	1.223	0.172	0.244	0.081	0.278	0.337

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	87	0	0	0	72	32
N.S.	1	1.00	1.00	2.23	0.00	0.00	0.00	1.85	0.82
time (sec)	N/A	0.031	0.010	1.332	0.000	0.000	0.000	0.310	0.372

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	37	36	38	41	36	39	43
N.S.	1	1.00	1.09	1.06	1.12	1.21	1.06	1.15	1.26
time (sec)	N/A	0.013	0.025	0.694	0.187	0.244	0.278	0.270	0.375

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	48	41	42	37	44	60	50
N.S.	1	1.00	1.12	0.95	0.98	0.86	1.02	1.40	1.16
time (sec)	N/A	0.017	0.025	1.233	0.263	0.244	0.298	0.277	0.400

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	60	45	54	55	60	51	56
N.S.	1	1.00	1.09	0.82	0.98	1.00	1.09	0.93	1.02
time (sec)	N/A	0.027	0.026	1.057	0.184	0.256	0.355	0.274	0.401

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	111	141	134	125	144	0	140
N.S.	1	1.00	0.91	1.16	1.10	1.02	1.18	0.00	1.15
time (sec)	N/A	0.173	0.098	3.218	0.267	0.257	0.202	0.000	0.513

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	152	358	0	0	0	0	0
N.S.	1	1.00	1.00	2.36	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.180	0.399	3.776	0.000	0.000	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	73	105	104	88	97	0	98
N.S.	1	1.00	0.89	1.28	1.27	1.07	1.18	0.00	1.20
time (sec)	N/A	0.105	0.059	3.170	0.269	0.263	0.156	0.000	0.444

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	105	308	0	0	0	0	0
N.S.	1	1.00	1.27	3.71	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.080	0.128	3.619	0.000	0.000	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	148	148	203	1106	0	0	0	0	0
N.S.	1	1.00	1.37	7.47	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.208	0.166	0.549	0.000	0.000	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	107	142	0	0	0	0	0
N.S.	1	1.00	1.11	1.48	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.091	0.115	6.113	0.000	0.000	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	99	103	120	109	117	137	143
N.S.	1	1.00	1.18	1.23	1.43	1.30	1.39	1.63	1.70
time (sec)	N/A	0.095	0.091	3.085	0.268	0.253	0.337	0.294	2.934

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	253	480	0	0	0	0	0
N.S.	1	1.00	1.18	2.24	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.455	0.726	14.518	0.000	0.000	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	229	229	336	2634	0	0	0	0	0
N.S.	1	1.00	1.47	11.50	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.343	0.961	2.888	0.000	0.000	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	174	405	0	0	0	0	0
N.S.	1	1.00	1.20	2.79	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.212	0.293	8.159	0.000	0.000	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	119	119	215	2028	0	0	0	0	0
N.S.	1	1.00	1.81	17.04	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.166	0.277	102.875	0.000	0.000	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	230	230	422	2225	0	0	0	0	0
N.S.	1	1.00	1.83	9.67	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.325	0.274	26.327	0.000	0.000	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	222	275	0	0	0	0	0
N.S.	1	1.00	1.63	2.02	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.171	0.126	9.309	0.000	0.000	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	178	321	0	0	0	0	0
N.S.	1	1.00	1.21	2.18	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.188	0.319	14.681	0.000	0.000	0.000	0.000	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	34	30	31	27	39	31	31
N.S.	1	1.00	0.67	0.59	0.61	0.53	0.76	0.61	0.61
time (sec)	N/A	0.009	0.017	0.162	0.302	0.247	1.131	0.281	0.373

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	28	25	26	20	32	26	26
N.S.	1	1.00	0.67	0.60	0.62	0.48	0.76	0.62	0.62
time (sec)	N/A	0.007	0.014	0.039	0.280	0.240	0.632	0.275	0.377

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	18	17	16	14	19	16	16
N.S.	1	1.00	0.82	0.77	0.73	0.64	0.86	0.73	0.73
time (sec)	N/A	0.004	0.021	0.031	0.279	0.253	0.396	0.266	0.078

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	24	35	0	0	0	24
N.S.	1	1.00	1.00	0.77	1.13	0.00	0.00	0.00	0.77
time (sec)	N/A	0.024	0.007	1.046	0.279	0.000	0.000	0.000	0.321

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	30	19	21	17	94	21	21
N.S.	1	1.00	1.11	0.70	0.78	0.63	3.48	0.78	0.78
time (sec)	N/A	0.008	0.012	0.037	0.279	0.248	0.557	0.279	0.366

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	34	27	26	26	160	26	24
N.S.	1	1.00	0.81	0.64	0.62	0.62	3.81	0.62	0.57
time (sec)	N/A	0.009	0.014	0.045	0.270	0.250	1.346	0.271	0.375

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	30	25	24	24	85	24	24
N.S.	1	1.00	0.83	0.69	0.67	0.67	2.36	0.67	0.67
time (sec)	N/A	0.010	0.019	0.036	0.195	0.274	1.010	0.270	0.381

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	25	20	19	19	24	19	0
N.S.	1	1.00	0.86	0.69	0.66	0.66	0.83	0.66	0.00
time (sec)	N/A	0.008	0.013	0.036	0.193	0.255	0.606	0.269	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	16	16	17	16	16
N.S.	1	1.00	1.00	0.85	0.80	0.80	0.85	0.80	0.80
time (sec)	N/A	0.006	0.009	0.055	0.198	0.240	0.136	0.266	0.364

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	18	26	20	18	22
N.S.	1	1.00	1.00	0.86	0.82	1.18	0.91	0.82	1.00
time (sec)	N/A	0.006	0.015	0.196	0.202	0.255	0.404	0.271	0.383

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	31	26	25	33	143	28	27
N.S.	1	1.00	0.84	0.70	0.68	0.89	3.86	0.76	0.73
time (sec)	N/A	0.010	0.023	0.310	0.190	0.259	1.355	0.271	0.387

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	57	50	0	0	0	25
N.S.	1	1.00	1.00	1.73	1.52	0.00	0.00	0.00	0.76
time (sec)	N/A	0.028	0.010	0.882	0.290	0.000	0.000	0.000	0.334

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	32	75	0	63	0	0	0
N.S.	1	1.00	0.82	1.92	0.00	1.62	0.00	0.00	0.00
time (sec)	N/A	0.025	0.016	1.899	0.000	0.267	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [82] had the largest ratio of [1.9170000000000004]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	3	1.00	12	0.250
2	A	4	3	1.00	12	0.250
3	A	4	3	1.00	12	0.250
4	A	4	3	1.00	12	0.250
5	A	3	3	1.00	10	0.300
6	A	3	2	1.00	8	0.250
7	A	3	2	1.00	12	0.167
8	A	5	5	1.00	12	0.417
9	A	3	3	1.00	12	0.250
10	A	4	3	1.00	12	0.250
11	A	4	3	1.00	12	0.250
12	A	4	3	1.00	12	0.250
13	A	16	7	1.00	14	0.500
14	A	14	9	1.00	14	0.643
15	A	11	7	1.00	14	0.500
16	A	9	8	1.00	14	0.571
17	A	6	5	1.00	12	0.417
18	A	5	5	1.00	10	0.500
19	A	6	5	1.00	14	0.357
20	A	4	4	1.00	14	0.286
21	A	8	7	1.00	14	0.500
22	A	8	7	1.00	14	0.500
23	A	13	8	1.00	14	0.571
24	A	33	11	1.00	14	0.786

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	24	11	1.00	14	0.786
26	A	18	10	1.00	14	0.714
27	A	12	9	1.00	14	0.643
28	A	8	8	1.00	12	0.667
29	A	5	6	1.00	10	0.600
30	A	8	6	1.00	14	0.429
31	A	5	6	1.00	14	0.429
32	A	7	6	1.00	14	0.429
33	A	14	11	1.00	14	0.786
34	A	16	8	1.00	14	0.571
35	N/A	0	0	1.00	8	0.000
36	N/A	0	0	1.00	6	0.000
37	N/A	0	0	1.00	10	0.000
38	N/A	0	0	1.00	8	0.000
39	N/A	0	0	1.00	6	0.000
40	N/A	0	0	1.00	10	0.000
41	N/A	0	0	1.00	10	0.000
42	N/A	0	0	1.00	8	0.000
43	N/A	0	0	1.00	12	0.000
44	N/A	0	0	1.00	10	0.000
45	N/A	0	0	1.00	8	0.000
46	N/A	0	0	1.00	12	0.000
47	N/A	0	0	1.00	10	0.000
48	N/A	0	0	1.00	8	0.000
49	N/A	0	0	1.00	12	0.000
50	N/A	0	0	1.00	10	0.000
51	N/A	0	0	1.00	8	0.000
52	N/A	0	0	1.00	12	0.000
53	A	12	9	1.00	8	1.125
54	N/A	0	0	1.00	16	0.000
55	N/A	0	0	1.00	16	0.000
56	A	2	2	1.00	14	0.143

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
57	N/A	0	0	1.00	16	0.000
58	N/A	0	0	1.00	10	0.000
59	N/A	0	0	1.00	16	0.000
60	A	5	4	1.00	14	0.286
61	A	4	3	1.00	14	0.214
62	A	4	4	1.00	14	0.286
63	A	2	2	1.00	12	0.167
64	A	4	3	1.00	14	0.214
65	A	5	5	1.00	14	0.357
66	A	4	4	1.00	14	0.286
67	A	4	3	1.00	14	0.214
68	A	11	8	1.00	14	0.571
69	A	11	8	1.00	14	0.571
70	A	11	7	1.00	10	0.700
71	A	10	7	1.00	14	0.500
72	A	11	8	1.00	14	0.571
73	A	11	8	1.00	14	0.571
74	A	12	8	1.00	16	0.500
75	A	10	9	1.00	16	0.562
76	A	7	6	1.00	16	0.375
77	A	6	6	1.00	14	0.429
78	A	7	6	1.00	16	0.375
79	A	5	5	1.00	16	0.312
80	A	9	8	1.00	16	0.500
81	A	86	27	1.00	16	1.687
82	A	69	23	1.00	12	1.917
83	A	47	23	1.00	16	1.438
84	A	64	25	1.00	16	1.562
85	A	77	25	1.00	16	1.562
86	A	9	9	1.00	16	0.562
87	A	6	7	1.00	14	0.500
88	A	9	7	1.00	16	0.438
89	A	6	7	1.00	16	0.438
90	A	8	7	1.00	16	0.438

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
91	N/A	0	0	1.00	18	0.000
92	N/A	0	0	1.00	18	0.000
93	A	2	2	1.00	16	0.125
94	N/A	0	0	1.00	18	0.000
95	N/A	0	0	1.00	18	0.000
96	A	5	4	1.00	14	0.286
97	A	4	3	1.00	14	0.214
98	A	4	4	1.00	14	0.286
99	A	2	2	1.00	14	0.143
100	A	4	3	1.00	14	0.214
101	A	5	5	1.00	14	0.357
102	A	4	4	1.00	14	0.286
103	A	4	3	1.00	14	0.214
104	A	12	8	1.00	14	0.571
105	A	9	8	1.00	10	0.800
106	A	11	7	1.00	14	0.500
107	A	9	9	1.00	14	0.643
108	A	12	8	1.00	14	0.571
109	A	9	9	1.00	14	0.643
110	A	11	7	1.00	12	0.583
111	A	8	8	1.00	14	0.571
112	A	12	8	1.00	14	0.571
113	A	12	8	1.00	16	0.500
114	A	10	9	1.00	16	0.562
115	A	7	6	1.00	16	0.375
116	A	6	6	1.00	16	0.375
117	A	7	6	1.00	16	0.375
118	A	5	5	1.00	16	0.312
119	A	9	8	1.00	16	0.500
120	A	9	8	1.00	16	0.500
121	A	13	10	1.00	16	0.625
122	A	9	9	1.00	16	0.562
123	A	6	7	1.00	16	0.438
124	A	9	7	1.00	16	0.438

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
125	A	6	7	1.00	16	0.438
126	A	8	7	1.00	16	0.438
127	N/A	0	0	1.00	18	0.000
128	N/A	0	0	1.00	18	0.000
129	A	2	2	1.00	16	0.125
130	N/A	0	0	1.00	18	0.000
131	N/A	0	0	1.00	18	0.000
132	A	5	4	1.00	14	0.286
133	A	5	4	1.00	14	0.286
134	A	4	4	1.00	12	0.333
135	A	4	3	1.00	10	0.300
136	A	4	3	1.00	14	0.214
137	A	2	2	1.00	14	0.143
138	A	4	4	1.00	14	0.286
139	A	5	4	1.00	14	0.286
140	A	14	9	1.00	16	0.562
141	A	9	8	1.00	16	0.500
142	A	9	8	1.00	14	0.571
143	A	6	6	1.00	12	0.500
144	A	7	6	1.00	16	0.375
145	A	6	6	1.00	16	0.375
146	A	7	6	1.00	16	0.375
147	A	17	9	1.00	16	0.562
148	A	15	12	1.00	16	0.750
149	A	8	7	1.00	14	0.500
150	A	6	7	1.00	12	0.583
151	A	9	7	1.00	16	0.438
152	A	6	7	1.00	16	0.438
153	A	9	9	1.00	16	0.562
154	A	6	4	1.00	10	0.400
155	A	5	4	1.00	8	0.500
156	A	4	4	1.00	6	0.667
157	A	4	3	1.00	10	0.300
158	A	4	4	1.00	10	0.400

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
159	A	5	4	1.00	10	0.400
160	A	3	2	1.00	12	0.167
161	A	3	2	1.00	12	0.167
162	A	2	2	1.00	12	0.167
163	A	4	4	1.00	12	0.333
164	A	3	2	1.00	12	0.167
165	A	4	3	1.00	10	0.300
166	A	4	3	1.00	10	0.300

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int x^5(a + b\arctan(cx))\,dx$ 70
3.2 $\int x^4(a + b\arctan(cx))\,dx$ 74
3.3 $\int x^3(a + b\arctan(cx))\,dx$ 79
3.4 $\int x^2(a + b\arctan(cx))\,dx$ 83
3.5 $\int x(a + b\arctan(cx))\,dx$ 87
3.6 $\int (a + b\arctan(cx))\,dx$ 91
3.7 $\int \frac{a+b\arctan(cx)}{x}\,dx$ 95
3.8 $\int \frac{a+b\arctan(cx)}{x^2}\,dx$ 99
3.9 $\int \frac{a+b\arctan(cx)}{x^3}\,dx$ 103
3.10 $\int \frac{a+b\arctan(cx)}{x^4}\,dx$ 107
3.11 $\int \frac{a+b\arctan(cx)}{x^5}\,dx$ 111
3.12 $\int \frac{a+b\arctan(cx)}{x^6}\,dx$ 115
3.13 $\int x^5(a + b\arctan(cx))^2\,dx$ 119
3.14 $\int x^4(a + b\arctan(cx))^2\,dx$ 125
3.15 $\int x^3(a + b\arctan(cx))^2\,dx$ 131
3.16 $\int x^2(a + b\arctan(cx))^2\,dx$ 137
3.17 $\int x(a + b\arctan(cx))^2\,dx$ 143
3.18 $\int (a + b\arctan(cx))^2\,dx$ 148
3.19 $\int \frac{(a+b\arctan(cx))^2}{x}\,dx$ 153
3.20 $\int \frac{(a+b\arctan(cx))^2}{x^2}\,dx$ 159
3.21 $\int \frac{(a+b\arctan(cx))^2}{x^3}\,dx$ 164
3.22 $\int \frac{(a+b\arctan(cx))^2}{x^4}\,dx$ 170
3.23 $\int \frac{(a+b\arctan(cx))^2}{x^5}\,dx$ 176
3.24 $\int x^5(a + b\arctan(cx))^3\,dx$ 182
3.25 $\int x^4(a + b\arctan(cx))^3\,dx$ 190
3.26 $\int x^3(a + b\arctan(cx))^3\,dx$ 198

3.27 $\int x^2(a+b\arctan(cx))^3\,dx$ 205
3.28 $\int x(a+b\arctan(cx))^3\,dx$ 212
3.29 $\int (a+b\arctan(cx))^3\,dx$ 218
3.30 $\int \frac{(a+b\arctan(cx))^3}{x}\,dx$ 224
3.31 $\int \frac{(a+b\arctan(cx))^3}{x^2}\,dx$ 232
3.32 $\int \frac{(a+b\arctan(cx))^3}{x^3}\,dx$ 238
3.33 $\int \frac{(a+b\arctan(cx))^3}{x^4}\,dx$ 244
3.34 $\int \frac{(a+b\arctan(cx))^3}{x^5}\,dx$ 252
3.35 $\int \frac{x}{\arctan(ax)}\,dx$ 259
3.36 $\int \frac{1}{\arctan(ax)}\,dx$ 262
3.37 $\int \frac{1}{x\arctan(ax)}\,dx$ 265
3.38 $\int \frac{x}{\arctan(ax)^2}\,dx$ 268
3.39 $\int \frac{1}{\arctan(ax)^2}\,dx$ 271
3.40 $\int \frac{1}{x\arctan(ax)^2}\,dx$ 274
3.41 $\int x\sqrt{\arctan(ax)}\,dx$ 277
3.42 $\int \sqrt{\arctan(ax)}\,dx$ 280
3.43 $\int \frac{\sqrt{\arctan(ax)}}{x}\,dx$ 283
3.44 $\int x\arctan(ax)^{3/2}\,dx$ 286
3.45 $\int \arctan(ax)^{3/2}\,dx$ 289
3.46 $\int \frac{\arctan(ax)^{3/2}}{x}\,dx$ 292
3.47 $\int \frac{x}{\sqrt{\arctan(ax)}}\,dx$ 295
3.48 $\int \frac{1}{\sqrt{\arctan(ax)}}\,dx$ 298
3.49 $\int \frac{1}{x\sqrt{\arctan(ax)}}\,dx$ 301
3.50 $\int \frac{x}{\arctan(ax)^{3/2}}\,dx$ 304
3.51 $\int \frac{1}{\arctan(ax)^{3/2}}\,dx$ 307
3.52 $\int \frac{1}{x\arctan(ax)^{3/2}}\,dx$ 310
3.53 $\int \sqrt{x}\arctan(x)\,dx$ 313
3.54 $\int (dx)^m(a+b\arctan(cx))^3\,dx$ 320
3.55 $\int (dx)^m(a+b\arctan(cx))^2\,dx$ 323
3.56 $\int (dx)^m(a+b\arctan(cx))\,dx$ 326
3.57 $\int \frac{(dx)^m}{a+b\arctan(cx)}\,dx$ 330
3.58 $\int (a+b\arctan(cx))^p\,dx$ 333
3.59 $\int (dx)^m(a+b\arctan(cx))^p\,dx$ 336
3.60 $\int x^7(a+b\arctan(cx^2))\,dx$ 339
3.61 $\int x^5(a+b\arctan(cx^2))\,dx$ 343
3.62 $\int x^3(a+b\arctan(cx^2))\,dx$ 347
3.63 $\int x(a+b\arctan(cx^2))\,dx$ 351
3.64 $\int \frac{a+b\arctan(cx^2)}{x}\,dx$ 355
3.65 $\int \frac{a+b\arctan(cx^2)}{x^3}\,dx$ 359

3.66 $\int \frac{a+b\arctan(cx^2)}{x^5}\,dx$ 363
3.67 $\int \frac{a+b\arctan(cx^2)}{x^7}\,dx$ 367
3.68 $\int x^4(a+b\arctan(cx^2))\,dx$ 372
3.69 $\int x^2(a+b\arctan(cx^2))\,dx$ 379
3.70 $\int (a+b\arctan(cx^2))\,dx$ 386
3.71 $\int \frac{a+b\arctan(cx^2)}{x^2}\,dx$ 392
3.72 $\int \frac{a+b\arctan(cx^2)}{x^4}\,dx$ 398
3.73 $\int \frac{a+b\arctan(cx^2)}{x^6}\,dx$ 406
3.74 $\int x^7(a+b\arctan(cx^2))^2\,dx$ 413
3.75 $\int x^5(a+b\arctan(cx^2))^2\,dx$ 419
3.76 $\int x^3(a+b\arctan(cx^2))^2\,dx$ 425
3.77 $\int x(a+b\arctan(cx^2))^2\,dx$ 430
3.78 $\int \frac{(a+b\arctan(cx^2))^2}{x}\,dx$ 435
3.79 $\int \frac{(a+b\arctan(cx^2))^2}{x^3}\,dx$ 441
3.80 $\int \frac{(a+b\arctan(cx^2))^2}{x^5}\,dx$ 446
3.81 $\int x^2(a+b\arctan(cx^2))^2\,dx$ 452
3.82 $\int (a+b\arctan(cx^2))^2\,dx$ 467
3.83 $\int \frac{(a+b\arctan(cx^2))^2}{x^2}\,dx$ 481
3.84 $\int \frac{(a+b\arctan(cx^2))^2}{x^4}\,dx$ 493
3.85 $\int \frac{(a+b\arctan(cx^2))^2}{x^6}\,dx$ 503
3.86 $\int x^3(a+b\arctan(cx^2))^3\,dx$ 513
3.87 $\int x(a+b\arctan(cx^2))^3\,dx$ 520
3.88 $\int \frac{(a+b\arctan(cx^2))^3}{x}\,dx$ 526
3.89 $\int \frac{(a+b\arctan(cx^2))^3}{x^3}\,dx$ 533
3.90 $\int \frac{(a+b\arctan(cx^2))^3}{x^5}\,dx$ 539
3.91 $\int (dx)^m\,(a+b\arctan(cx^2))^3\,dx$ 545
3.92 $\int (dx)^m\,(a+b\arctan(cx^2))^2\,dx$ 548
3.93 $\int (dx)^m\,(a+b\arctan(cx^2))\,dx$ 551
3.94 $\int \frac{(dx)^m}{a+b\arctan(cx^2)}\,dx$ 555
3.95 $\int \frac{(dx)^m}{(a+b\arctan(cx^2))^2}\,dx$ 558
3.96 $\int x^{11}(a+b\arctan(cx^3))\,dx$ 561
3.97 $\int x^8(a+b\arctan(cx^3))\,dx$ 565
3.98 $\int x^5(a+b\arctan(cx^3))\,dx$ 569
3.99 $\int x^2(a+b\arctan(cx^3))\,dx$ 573
3.100 $\int \frac{a+b\arctan(cx^3)}{x}\,dx$ 577
3.101 $\int \frac{a+b\arctan(cx^3)}{x^4}\,dx$ 581
3.102 $\int \frac{a+b\arctan(cx^3)}{x^7}\,dx$ 585

3.103 $\int \frac{a+b\arctan(cx^3)}{x^{10}}\,dx$ 589
3.104 $\int x^3(a+b\arctan(cx^3))\,dx$ 594
3.105 $\int (a+b\arctan(cx^3))\,dx$ 602
3.106 $\int \frac{a+b\arctan(cx^3)}{x^3}\,dx$ 609
3.107 $\int \frac{a+b\arctan(cx^3)}{x^6}\,dx$ 616
3.108 $\int x^7(a+b\arctan(cx^3))\,dx$ 623
3.109 $\int x^4(a+b\arctan(cx^3))\,dx$ 631
3.110 $\int x(a+b\arctan(cx^3))\,dx$ 638
3.111 $\int \frac{a+b\arctan(cx^3)}{x^2}\,dx$ 646
3.112 $\int \frac{a+b\arctan(cx^3)}{x^5}\,dx$ 653
3.113 $\int x^{11}(a+b\arctan(cx^3))^2\,dx$ 660
3.114 $\int x^8(a+b\arctan(cx^3))^2\,dx$ 666
3.115 $\int x^5(a+b\arctan(cx^3))^2\,dx$ 672
3.116 $\int x^2(a+b\arctan(cx^3))^2\,dx$ 677
3.117 $\int \frac{(a+b\arctan(cx^3))^2}{x}\,dx$ 682
3.118 $\int \frac{(a+b\arctan(cx^3))^2}{x^4}\,dx$ 688
3.119 $\int \frac{(a+b\arctan(cx^3))^2}{x^7}\,dx$ 693
3.120 $\int \frac{(a+b\arctan(cx^3))^2}{x^{10}}\,dx$ 699
3.121 $\int x^8(a+b\arctan(cx^3))^3\,dx$ 705
3.122 $\int x^5(a+b\arctan(cx^3))^3\,dx$ 712
3.123 $\int x^2(a+b\arctan(cx^3))^3\,dx$ 718
3.124 $\int \frac{(a+b\arctan(cx^3))^3}{x}\,dx$ 724
3.125 $\int \frac{(a+b\arctan(cx^3))^3}{x^4}\,dx$ 732
3.126 $\int \frac{(a+b\arctan(cx^3))^3}{x^7}\,dx$ 738
3.127 $\int (dx)^m\,(a+b\arctan(cx^3))^3\,dx$ 743
3.128 $\int (dx)^m\,(a+b\arctan(cx^3))^2\,dx$ 746
3.129 $\int (dx)^m\,(a+b\arctan(cx^3))\,dx$ 749
3.130 $\int \frac{(dx)^m}{a+b\arctan(cx^3)}\,dx$ 753
3.131 $\int \frac{(dx)^m}{(a+b\arctan(cx^3))^2}\,dx$ 756
3.132 $\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx$ 759
3.133 $\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx$ 763
3.134 $\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx$ 767
3.135 $\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx$ 771
3.136 $\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x}\,dx$ 775
3.137 $\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^2}\,dx$ 779
3.138 $\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^3}\,dx$ 783
3.139 $\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^4}\,dx$ 787
3.140 $\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx$ 791

3.141 $\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx$ 798
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3.143 $\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx$ 810
3.144 $\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x}\,dx$ 815
3.145 $\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x^2}\,dx$ 821
3.146 $\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x^3}\,dx$ 826
3.147 $\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx$ 832
3.148 $\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx$ 840
3.149 $\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx$ 849
3.150 $\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx$ 856
3.151 $\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x}\,dx$ 863
3.152 $\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x^2}\,dx$ 872
3.153 $\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x^3}\,dx$ 878
3.154 $\int x^2\arctan\left(\sqrt{x}\right)\,dx$ 885
3.155 $\int x\arctan\left(\sqrt{x}\right)\,dx$ 890
3.156 $\int \arctan\left(\sqrt{x}\right)\,dx$ 894
3.157 $\int \frac{\arctan\left(\sqrt{x}\right)}{x}\,dx$ 898
3.158 $\int \frac{\arctan\left(\sqrt{x}\right)}{x^2}\,dx$ 902
3.159 $\int \frac{\arctan\left(\sqrt{x}\right)}{x^3}\,dx$ 906
3.160 $\int x^{3/2}\arctan\left(\sqrt{x}\right)\,dx$ 911
3.161 $\int \sqrt{x}\arctan\left(\sqrt{x}\right)\,dx$ 915
3.162 $\int \frac{\arctan\left(\sqrt{x}\right)}{\sqrt{x}}\,dx$ 919
3.163 $\int \frac{\arctan\left(\sqrt{x}\right)}{x^{3/2}}\,dx$ 922
3.164 $\int \frac{\arctan\left(\sqrt{x}\right)}{x^{5/2}}\,dx$ 926
3.165 $\int \frac{\arctan(ax^5)}{x}\,dx$ 930
3.166 $\int \frac{\arctan(ax^n)}{x}\,dx$ 934

3.1 $\int x^5(a+b\arctan(cx))\,dx$

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Optimal result

Integrand size = 12, antiderivative size = 59

$$\int x^5(a+b\arctan(cx))\,dx=-\frac{bx}{6c^5}+\frac{bx^3}{18c^3}-\frac{bx^5}{30c}+\frac{b\arctan(cx)}{6c^6}+\frac{1}{6}x^6(a+b\arctan(cx))$$

```
[Out] -1/6*b*x/c^5+1/18*b*x^3/c^3-1/30*b*x^5/c+1/6*b*arctan(c*x)/c^6+1/6*x^6*(a+b
*arctan(c*x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4946, 308, 209}

$$\int x^5(a+b\arctan(cx))\,dx=\frac{1}{6}x^6(a+b\arctan(cx))+\frac{b\arctan(cx)}{6c^6}-\frac{bx}{6c^5}+\frac{bx^3}{18c^3}-\frac{bx^5}{30c}$$

```
[In] Int[x^5*(a + b*ArcTan[c*x]),x]
```

```
[Out] -1/6*(b*x)/c^5 + (b*x^3)/(18*c^3) - (b*x^5)/(30*c) + (b*ArcTan[c*x])/(6*c^6
) + (x^6*(a + b*ArcTan[c*x]))/6
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
```

```
Q[m, 2*n - 1]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{6}x^6(a+b\arctan(cx)) - \frac{1}{6}(bc)\int \frac{x^6}{1+c^2x^2}\,dx \\
&= \frac{1}{6}x^6(a+b\arctan(cx)) - \frac{1}{6}(bc)\int \left(\frac{1}{c^6} - \frac{x^2}{c^4} + \frac{x^4}{c^2} - \frac{1}{c^6\,(1+c^2x^2)}\right)dx \\
&= -\frac{bx}{6c^5} + \frac{bx^3}{18c^3} - \frac{bx^5}{30c} + \frac{1}{6}x^6(a+b\arctan(cx)) + \frac{b\int \frac{1}{1+c^2x^2}\,dx}{6c^5} \\
&= -\frac{bx}{6c^5} + \frac{bx^3}{18c^3} - \frac{bx^5}{30c} + \frac{b\arctan(cx)}{6c^6} + \frac{1}{6}x^6(a+b\arctan(cx))
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.08

$$\int x^5(a+b\arctan(cx))\,dx = -\frac{bx}{6c^5} + \frac{bx^3}{18c^3} - \frac{bx^5}{30c} + \frac{ax^6}{6} + \frac{b\arctan(cx)}{6c^6} + \frac{1}{6}bx^6\arctan(cx)$$

```
[In] Integrate[x^5*(a + b*ArcTan[c*x]),x]
[Out] -1/6*(b*x)/c^5 + (b*x^3)/(18*c^3) - (b*x^5)/(30*c) + (a*x^6)/6 + (b*ArcTan[
c*x])/(6*c^6) + (b*x^6*ArcTan[c*x])/6
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.88

method	result	size
parts	$\frac{a\,x^{6}}{6}+\frac{b\left(\frac{c^{6}x^{6}\arctan(cx)}{6}-\frac{c^{5}x^{5}}{30}+\frac{c^{3}x^{3}}{18}-\frac{cx}{6}+\frac{\arctan(cx)}{6}\right)}{c^{6}}$	52
derivativedivides	$\frac{\frac{a\,c^{6}x^{6}}{6}+b\left(\frac{c^{6}x^{6}\arctan(cx)}{6}-\frac{c^{5}x^{5}}{30}+\frac{c^{3}x^{3}}{18}-\frac{cx}{6}+\frac{\arctan(cx)}{6}\right)}{c^{6}}$	56
default	$\frac{\frac{a\,c^{6}x^{6}}{6}+b\left(\frac{c^{6}x^{6}\arctan(cx)}{6}-\frac{c^{5}x^{5}}{30}+\frac{c^{3}x^{3}}{18}-\frac{cx}{6}+\frac{\arctan(cx)}{6}\right)}{c^{6}}$	56
parallelrisch	$\frac{15b\arctan(cx)x^{6}c^{6}+15a\,c^{6}x^{6}-3b\,c^{5}x^{5}+5b\,c^{3}x^{3}-15xbc+15b\arctan(cx)}{90c^{6}}$	59
risch	$-\frac{ix^{6}b\ln(icx+1)}{12}+\frac{ix^{6}b\ln(-icx+1)}{12}+\frac{a\,x^{6}}{6}-\frac{b\,x^{5}}{30c}+\frac{b\,x^{3}}{18c^{3}}-\frac{bx}{6c^{5}}+\frac{b\arctan(cx)}{6c^{6}}$	73

```
[In] int(x^5*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
[Out] 1/6*a*x^6+b/c^6*(1/6*c^6*x^6*arctan(c*x)-1/30*c^5*x^5+1/18*c^3*x^3-1/6*c*x+
1/6*arctan(c*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

$$\int x^{5}(a+b\arctan(cx))\,dx=\frac{15\,ac^{6}x^{6}-3\,bc^{5}x^{5}+5\,bc^{3}x^{3}-15\,bcx+15\left(bc^{6}x^{6}+b\right)\arctan\left(cx\right)}{90\,c^{6}}$$

```
[In] integrate(x^5*(a+b*arctan(c*x)),x, algorithm="fricas")
[Out] 1/90*(15*a*c^6*x^6 - 3*b*c^5*x^5 + 5*b*c^3*x^3 - 15*b*c*x + 15*(b*c^6*x^6 +
 b)*arctan(c*x))/c^6
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.07

$$\int x^{5}(a+b\arctan(cx))\,dx=\begin{cases}\frac{ax^{6}}{6}+\frac{bx^{6}\operatorname{atan}\left(cx\right)}{6}-\frac{bx^{5}}{30c}+\frac{bx^{3}}{18c^{3}}-\frac{bx}{6c^{5}}+\frac{b\operatorname{atan}\left(cx\right)}{6c^{6}} & \text{for}\ c\neq 0\\ \frac{ax^{6}}{6} & \text{otherwise}\end{cases}$$

```
[In] integrate(x**5*(a+b*atan(c*x)),x)
[Out] Piecewise((a*x**6/6 + b*x**6*atan(c*x)/6 - b*x**5/(30*c) + b*x**3/(18*c**3)
 - b*x/(6*c**5) + b*atan(c*x)/(6*c**6), Ne(c, 0)), (a*x**6/6, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97

$$\int x^5(a+b\arctan(cx))\,dx$$
$$=\frac{1}{6}\,ax^6+\frac{1}{90}\left(15\,x^6\arctan\left(cx\right)-c\left(\frac{3\,c^4x^5-5\,c^2x^3+15\,x}{c^6}-\frac{15\,\arctan\left(cx\right)}{c^7}\right)\right)b$$

```
[In] integrate(x^5*(a+b*arctan(c*x)),x, algorithm="maxima")
[Out] 1/6*a*x^6 + 1/90*(15*x^6*arctan(c*x) - c*((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^
6 - 15*arctan(c*x)/c^7))*b
```

Giac [F]

$$\int x^5(a+b\arctan(cx))\,dx=\int(b\arctan\left(cx\right)+a)x^5\,dx$$

```
[In] integrate(x^5*(a+b*arctan(c*x)),x, algorithm="giac")
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.88

$$\int x^5(a+b\arctan(cx))\,dx=\frac{\frac{b\,\mathrm{atan}(c\,x)}{6}+\frac{b\,c^3\,x^3}{18}-\frac{b\,c^5\,x^5}{30}-\frac{b\,c\,x}{6}}{c^6}+\frac{a\,x^6}{6}+\frac{b\,x^6\,\mathrm{atan}(c\,x)}{6}$$

```
[In] int(x^5*(a + b*atan(c*x)),x)
[Out] ((b*atan(c*x))/6 + (b*c^3*x^3)/18 - (b*c^5*x^5)/30 - (b*c*x)/6)/c^6 + (a*x^
6)/6 + (b*x^6*atan(c*x))/6
```

3.2 $\int x^4(a+b\arctan(cx))\,dx$

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Optimal result

Integrand size = 12, antiderivative size = 56

$$\int x^4(a+b\arctan(cx))\,dx=\frac{bx^2}{10c^3}-\frac{bx^4}{20c}+\frac{1}{5}x^5(a+b\arctan(cx))-\frac{b\log(1+c^2x^2)}{10c^5}$$

```
[Out] 1/10*b*x^2/c^3-1/20*b*x^4/c+1/5*x^5*(a+b*arctan(c*x))-1/10*b*ln(c^2*x^2+1)/
c^5
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4946, 272, 45}

$$\int x^4(a+b\arctan(cx))\,dx=\frac{1}{5}x^5(a+b\arctan(cx))+\frac{bx^2}{10c^3}-\frac{b\log(c^2x^2+1)}{10c^5}-\frac{bx^4}{20c}$$

```
[In] Int[x^4*(a + b*ArcTan[c*x]),x]
```

```
[Out] (b*x^2)/(10*c^3) - (b*x^4)/(20*c) + (x^5*(a + b*ArcTan[c*x]))/5 - (b*Log[1
+ c^2*x^2])/(10*c^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{5}x^5(a+b\arctan(cx)) - \frac{1}{5}(bc)\int \frac{x^5}{1+c^2x^2}\,dx \\
&= \frac{1}{5}x^5(a+b\arctan(cx)) - \frac{1}{10}(bc)\text{Subst}\left(\int \frac{x^2}{1+c^2x}\,dx, x, x^2\right) \\
&= \frac{1}{5}x^5(a+b\arctan(cx)) - \frac{1}{10}(bc)\text{Subst}\left(\int \left(-\frac{1}{c^4}+\frac{x}{c^2}+\frac{1}{c^4\left(1+c^2x\right)}\right)\,dx, x, x^2\right) \\
&= \frac{bx^2}{10c^3} - \frac{bx^4}{20c} + \frac{1}{5}x^5(a+b\arctan(cx)) - \frac{b\log\left(1+c^2x^2\right)}{10c^5}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.09

$$\int x^4(a+b\arctan(cx))\,dx = \frac{bx^2}{10c^3} - \frac{bx^4}{20c} + \frac{ax^5}{5} + \frac{1}{5}bx^5\arctan(cx) - \frac{b\log\left(1+c^2x^2\right)}{10c^5}$$

```
[In] Integrate[x^4*(a + b*ArcTan[c*x]),x]
[Out] (b*x^2)/(10*c^3) - (b*x^4)/(20*c) + (a*x^5)/5 + (b*x^5*ArcTan[c*x])/5 - (b*
Log[1 + c^2*x^2])/(10*c^5)
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

method	result	size
parts	$\frac{a\,x^{5}}{5}+\frac{b\left(\frac{c^{5}x^{5}\arctan\left(cx\right)}{5}-\frac{c^{4}x^{4}}{20}+\frac{c^{2}x^{2}}{10}-\frac{\ln\left(c^{2}x^{2}+1\right)}{10}\right)}{c^{5}}$	54
derivativedivides	$\frac{\frac{a\,c^{5}x^{5}}{5}+b\left(\frac{c^{5}x^{5}\arctan\left(cx\right)}{5}-\frac{c^{4}x^{4}}{20}+\frac{c^{2}x^{2}}{10}-\frac{\ln\left(c^{2}x^{2}+1\right)}{10}\right)}{c^{5}}$	58
default	$\frac{\frac{a\,c^{5}x^{5}}{5}+b\left(\frac{c^{5}x^{5}\arctan\left(cx\right)}{5}-\frac{c^{4}x^{4}}{20}+\frac{c^{2}x^{2}}{10}-\frac{\ln\left(c^{2}x^{2}+1\right)}{10}\right)}{c^{5}}$	58
parallelrisch	$-\frac{-4b\arctan\left(cx\right)x^{5}c^{5}-4a\,c^{5}x^{5}+b\,c^{4}x^{4}-2b\,c^{2}x^{2}+2b\ln\left(c^{2}x^{2}+1\right)+2b}{20c^{5}}$	62
risch	$-\frac{i x^{5}b\ln\left(icx+1\right)}{10}+\frac{i x^{5}b\ln\left(-icx+1\right)}{10}+\frac{a\,x^{5}}{5}-\frac{b\,x^{4}}{20c}+\frac{b\,x^{2}}{10c^{3}}-\frac{b\ln\left(-c^{2}x^{2}-1\right)}{10c^{5}}$	73

```
[In] int(x^4*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*a*x^5+b/c^5*(1/5*c^5*x^5*arctan(c*x)-1/20*c^4*x^4+1/10*c^2*x^2-1/10*ln(
c^2*x^2+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.05

$$\int x^4(a+b\arctan(cx))\,dx = \frac{4\,bc^5x^5\arctan\left(cx\right)+4\,ac^5x^5-bc^4x^4+2\,bc^2x^2-2\,b\log\left(c^2x^2+1\right)}{20\,c^5}$$

```
[In] integrate(x^4*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] 1/20*(4*b*c^5*x^5*arctan(c*x) + 4*a*c^5*x^5 - b*c^4*x^4 + 2*b*c^2*x^2 - 2*b
*log(c^2*x^2 + 1))/c^5
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.07

$$\int x^4(a+b\arctan(cx))\,dx = \begin{cases} \frac{ax^5}{5} + \frac{bx^5\operatorname{atan}(cx)}{5} - \frac{bx^4}{20c} + \frac{bx^2}{10c^3} - \frac{b\log\left(x^2+\frac{1}{c^2}\right)}{10c^5} & \text{for } c \neq 0 \\ \frac{ax^5}{5} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**4*(a+b*atan(c*x)),x)
```

```
[Out] Piecewise((a*x**5/5 + b*x**5*atan(c*x)/5 - b*x**4/(20*c) + b*x**2/(10*c**3)
 - b*log(x**2 + c**(-2))/(10*c**5), Ne(c, 0)), (a*x**5/5, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int x^4(a+b\arctan(cx))\,dx = \frac{1}{5}\,ax^5 + \frac{1}{20}\left(4\,x^5\arctan(cx) - c\left(\frac{c^2x^4-2\,x^2}{c^4} + \frac{2\,\log(c^2x^2+1)}{c^6}\right)\right)b$$

```
[In] integrate(x^4*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*x^5 + 1/20*(4*x^5*arctan(c*x) - c*((c^2*x^4 - 2*x^2)/c^4 + 2*log(c^2*
x^2 + 1)/c^6))*b
```

Giac [F]

$$\int x^4(a+b\arctan(cx))\,dx = \int (b\arctan(cx)+a)x^4\,dx$$

```
[In] integrate(x^4*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

$$\int x^4(a+b\arctan(cx))\,dx=\frac{a\,x^5}{5}-\frac{\frac{b\,\ln\left(c^2\,x^2+1\right)}{10}-\frac{b\,c^2\,x^2}{10}+\frac{b\,c^4\,x^4}{20}}{c^5}+\frac{b\,x^5\,\mathrm{atan}(c\,x)}{5}$$

```
[In] int(x^4*(a + b*atan(c*x)),x)
[Out] (a*x^5)/5 - ((b*log(c^2*x^2 + 1))/10 - (b*c^2*x^2)/10 + (b*c^4*x^4)/20)/c^5
 + (b*x^5*atan(c*x))/5
```

3.3 $\int x^3(a+b\arctan(cx))\,dx$

Optimal result 79
Rubi [A] (verified) 79
Mathematica [A] (verified) 80
Maple [A] (verified) 80
Fricas [A] (verification not implemented) 81
Sympy [A] (verification not implemented) 81
Maxima [A] (verification not implemented) 82
Giac [F] 82
Mupad [B] (verification not implemented) 82

Optimal result

Integrand size = 12, antiderivative size = 48

$$\int x^3(a+b\arctan(cx))\,dx=\frac{bx}{4c^3}-\frac{bx^3}{12c}-\frac{b\arctan(cx)}{4c^4}+\frac{1}{4}x^4(a+b\arctan(cx))$$

```
[Out] 1/4*b*x/c^3-1/12*b*x^3/c-1/4*b*arctan(c*x)/c^4+1/4*x^4*(a+b*arctan(c*x))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4946, 308, 209}

$$\int x^3(a+b\arctan(cx))\,dx=\frac{1}{4}x^4(a+b\arctan(cx))-\frac{b\arctan(cx)}{4c^4}+\frac{bx}{4c^3}-\frac{bx^3}{12c}$$

```
[In] Int[x^3*(a + b*ArcTan[c*x]),x]
```

```
[Out] (b*x)/(4*c^3) - (b*x^3)/(12*c) - (b*ArcTan[c*x])/(4*c^4) + (x^4*(a + b*ArcT
an[c*x]))/4
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
```

```
Q[m, 2*n - 1]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}x^4(a+b\arctan(cx)) - \frac{1}{4}(bc)\int \frac{x^4}{1+c^2x^2}\,dx \\
&= \frac{1}{4}x^4(a+b\arctan(cx)) - \frac{1}{4}(bc)\int \left(-\frac{1}{c^4}+\frac{x^2}{c^2}+\frac{1}{c^4\left(1+c^2x^2\right)}\right)dx \\
&= \frac{bx}{4c^3} - \frac{bx^3}{12c} + \frac{1}{4}x^4(a+b\arctan(cx)) - \frac{b\int \frac{1}{1+c^2x^2}\,dx}{4c^3} \\
&= \frac{bx}{4c^3} - \frac{bx^3}{12c} - \frac{b\arctan(cx)}{4c^4} + \frac{1}{4}x^4(a+b\arctan(cx))
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10

$$\int x^3(a+b\arctan(cx))\,dx = \frac{bx}{4c^3} - \frac{bx^3}{12c} + \frac{ax^4}{4} - \frac{b\arctan(cx)}{4c^4} + \frac{1}{4}bx^4\arctan(cx)$$

```
[In] Integrate[x^3*(a + b*ArcTan[c*x]),x]
[Out] (b*x)/(4*c^3) - (b*x^3)/(12*c) + (a*x^4)/4 - (b*ArcTan[c*x])/(4*c^4) + (b*x
^4*ArcTan[c*x])/4
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.92

method	result	size
parts	$\frac{a\,x^{4}}{4}+\frac{b\left(\frac{c^{4} x^{4} \arctan\left(c x\right)}{4}-\frac{c^{3} x^{3}}{12}+\frac{c x}{4}-\frac{\arctan\left(c x\right)}{4}\right)}{c^{4}}$	44
derivativedivides	$\frac{\frac{a\,c^{4} x^{4}}{4}+b\left(\frac{c^{4} x^{4} \arctan\left(c x\right)}{4}-\frac{c^{3} x^{3}}{12}+\frac{c x}{4}-\frac{\arctan\left(c x\right)}{4}\right)}{c^{4}}$	48
default	$\frac{\frac{a\,c^{4} x^{4}}{4}+b\left(\frac{c^{4} x^{4} \arctan\left(c x\right)}{4}-\frac{c^{3} x^{3}}{12}+\frac{c x}{4}-\frac{\arctan\left(c x\right)}{4}\right)}{c^{4}}$	48
parallelrisch	$\frac{3 b \arctan\left(c x\right) x^{4} c^{4}+3 a\,c^{4} x^{4}-b\,c^{3} x^{3}+3 x b c-3 b \arctan\left(c x\right)}{12 c^{4}}$	50
risch	$-\frac{i x^{4} b \ln\left(i c x+1\right)}{8}+\frac{i x^{4} b \ln\left(-i c x+1\right)}{8}+\frac{a\,x^{4}}{4}-\frac{b\,x^{3}}{12 c}+\frac{b x}{4 c^{3}}-\frac{b \arctan\left(c x\right)}{4 c^{4}}$	64

[In] int(x^3*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)

[Out] 1/4*a*x^4+b/c^4*(1/4*c^4*x^4*arctan(c*x)-1/12*c^3*x^3+1/4*c*x-1/4*arctan(c*x))

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.98

$$\int x^3(a+b\arctan(cx))\,dx=\frac{3\,ac^4x^4-bc^3x^3+3\,bcx+3\,(bc^4x^4-b)\arctan{(cx)}}{12\,c^4}$$

[In] integrate(x^3*(a+b*arctan(c*x)),x, algorithm="fricas")

[Out] 1/12*(3*a*c^4*x^4 - b*c^3*x^3 + 3*b*c*x + 3*(b*c^4*x^4 - b)*arctan(c*x))/c^4

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10

$$\int x^3(a+b\arctan(cx))\,dx=\begin{cases}\frac{ax^4}{4}+\frac{bx^4\operatorname{atan}{(cx)}}{4}-\frac{bx^3}{12c}+\frac{bx}{4c^3}-\frac{b\operatorname{atan}{(cx)}}{4c^4} & \text{for}\ c\neq 0\\ \frac{ax^4}{4} & \text{otherwise}\end{cases}$$

[In] integrate(x**3*(a+b*atan(c*x)),x)

[Out] Piecewise((a*x**4/4 + b*x**4*atan(c*x)/4 - b*x**3/(12*c) + b*x/(4*c**3) - b*atan(c*x)/(4*c**4), Ne(c, 0)), (a*x**4/4, True))

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int x^3(a+b\arctan(cx))\,dx=\frac{1}{4}\,ax^4+\frac{1}{12}\left(3\,x^4\arctan\left(cx\right)-c\left(\frac{c^2x^3-3\,x}{c^4}+\frac{3\,\arctan\left(cx\right)}{c^5}\right)\right)b$$

```
[In] integrate(x^3*(a+b*arctan(c*x)),x, algorithm="maxima")
[Out] 1/4*a*x^4 + 1/12*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x
)/c^5))*b
```

Giac [F]

$$\int x^3(a+b\arctan(cx))\,dx=\int (b\arctan\left(cx\right)+a)x^3\,dx$$

```
[In] integrate(x^3*(a+b*arctan(c*x)),x, algorithm="giac")
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.92

$$\int x^3(a+b\arctan(cx))\,dx=\frac{a\,x^4}{4}-\frac{\frac{b\,\mathrm{atan}(c\,x)}{4}+\frac{b\,c^3\,x^3}{12}-\frac{b\,c\,x}{4}}{c^4}+\frac{b\,x^4\,\mathrm{atan}(c\,x)}{4}$$

```
[In] int(x^3*(a + b*atan(c*x)),x)
[Out] (a*x^4)/4 - ((b*atan(c*x))/4 + (b*c^3*x^3)/12 - (b*c*x)/4)/c^4 + (b*x^4*ata
n(c*x))/4
```

3.4 $\int x^2(a+b\arctan(cx))\,dx$

Optimal result . . . 83
Rubi [A] (verified) . . . 83
Mathematica [A] (verified) . . . 84
Maple [A] (verified) . . . 84
Fricas [A] (verification not implemented) . . . 85
Sympy [A] (verification not implemented) . . . 85
Maxima [A] (verification not implemented) . . . 86
Giac [F] . . . 86
Mupad [B] (verification not implemented) . . . 86

Optimal result

Integrand size = 12, antiderivative size = 45

$$\int x^2(a+b\arctan(cx))\,dx=-\frac{bx^2}{6c}+\frac{1}{3}x^3(a+b\arctan(cx))+\frac{b\log(1+c^2x^2)}{6c^3}$$

```
[Out] -1/6*b*x^2/c+1/3*x^3*(a+b*arctan(c*x))+1/6*b*ln(c^2*x^2+1)/c^3
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4946, 272, 45}

$$\int x^2(a+b\arctan(cx))\,dx=\frac{1}{3}x^3(a+b\arctan(cx))+\frac{b\log(c^2x^2+1)}{6c^3}-\frac{bx^2}{6c}$$

```
[In] Int[x^2*(a + b*ArcTan[c*x]),x]
```

```
[Out] -1/6*(b*x^2)/c + (x^3*(a + b*ArcTan[c*x]))/3 + (b*Log[1 + c^2*x^2])/(6*c^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{1}{3}x^3(a+b\arctan(cx)) - \frac{1}{3}(bc)\int \frac{x^3}{1+c^2x^2}\,dx \\ &= \frac{1}{3}x^3(a+b\arctan(cx)) - \frac{1}{6}(bc)\text{Subst}\left(\int \frac{x}{1+c^2x}\,dx, x, x^2\right) \\ &= \frac{1}{3}x^3(a+b\arctan(cx)) - \frac{1}{6}(bc)\text{Subst}\left(\int \left(\frac{1}{c^2} - \frac{1}{c^2(1+c^2x)}\right)\,dx, x, x^2\right) \\ &= -\frac{bx^2}{6c} + \frac{1}{3}x^3(a+b\arctan(cx)) + \frac{b\log(1+c^2x^2)}{6c^3} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.11

$$\int x^2(a+b\arctan(cx))\,dx = -\frac{bx^2}{6c} + \frac{ax^3}{3} + \frac{1}{3}bx^3\arctan(cx) + \frac{b\log(1+c^2x^2)}{6c^3}$$

```
[In] Integrate[x^2*(a + b*ArcTan[c*x]),x]
[Out] -1/6*(b*x^2)/c + (a*x^3)/3 + (b*x^3*ArcTan[c*x])/3 + (b*Log[1 + c^2*x^2])/(
6*c^3)
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

method	result	size
parts	$\frac{x^{3} a}{3}+\frac{b \left(\frac{c^{3} x^{3} \arctan \left(c x \right)}{3}-\frac{c^{2} x^{2}}{6}+\frac{\ln \left(c^{2} x^{2}+1\right)}{6}\right)}{c^{3}}$	46
derivativedivides	$\frac{\frac{a \,c^{3} x^{3}}{3}+b \left(\frac{c^{3} x^{3} \arctan \left(c x \right)}{3}-\frac{c^{2} x^{2}}{6}+\frac{\ln \left(c^{2} x^{2}+1\right)}{6}\right)}{c^{3}}$	50
default	$\frac{\frac{a \,c^{3} x^{3}}{3}+b \left(\frac{c^{3} x^{3} \arctan \left(c x \right)}{3}-\frac{c^{2} x^{2}}{6}+\frac{\ln \left(c^{2} x^{2}+1\right)}{6}\right)}{c^{3}}$	50
parallelrisch	$\frac{2 x^{3} \arctan \left(c x \right) b \,c^{3}+2 a \,c^{3} x^{3}-b \,c^{2} x^{2}+b \ln \left(c^{2} x^{2}+1\right)}{6 c^{3}}$	50
risch	$-\frac{i x^{3} b \ln \left(i c x +1\right)}{6}+\frac{i x^{3} b \ln \left(-i c x +1\right)}{6}+\frac{x^{3} a}{3}-\frac{b \,x^{2}}{6 c}+\frac{b \ln \left(-c^{2} x^{2}-1\right)}{6 c^{3}}$	64

```
[In] int(x^2*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^3*a+b/c^3*(1/3*c^3*x^3*arctan(c*x)-1/6*c^2*x^2+1/6*ln(c^2*x^2+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.09

$$\int x^2(a+b\arctan(cx))\,dx=\frac{2\,bc^{3}x^{3}\arctan\left(cx\right)+2\,ac^{3}x^{3}-bc^{2}x^{2}+b\log\left(c^{2}x^{2}+1\right)}{6\,c^{3}}$$

```
[In] integrate(x^2*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b*c^3*x^3*arctan(c*x) + 2*a*c^3*x^3 - b*c^2*x^2 + b*log(c^2*x^2 + 1)
)/c^3
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.09

$$\int x^2(a+b\arctan(cx))\,dx=\begin{cases}\frac{ax^{3}}{3}+\frac{bx^{3}\operatorname{atan}\left(cx\right)}{3}-\frac{bx^{2}}{6c}+\frac{b\log\left(x^{2}+\frac{1}{c^{2}}\right)}{6c^{3}} & \text{for}\: c\neq0\\\frac{ax^{3}}{3} & \text{otherwise}\end{cases}$$

```
[In] integrate(x**2*(a+b*atan(c*x)),x)
```

```
[Out] Piecewise((a*x**3/3 + b*x**3*atan(c*x)/3 - b*x**2/(6*c) + b*log(x**2 + c**(
-2))/(6*c**3), Ne(c, 0)), (a*x**3/3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

$$\int x^2(a + b\arctan(cx))\,dx = \frac{1}{3}\,ax^3 + \frac{1}{6}\left(2\,x^3\arctan\left(cx\right) - c\left(\frac{x^2}{c^2} - \frac{\log\left(c^2x^2 + 1\right)}{c^4}\right)\right)b$$

```
[In] integrate(x^2*(a+b*arctan(c*x)),x, algorithm="maxima")
[Out] 1/3*a*x^3 + 1/6*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*b
```

Giac [F]

$$\int x^2(a + b\arctan(cx))\,dx = \int (b\arctan\left(cx\right) + a)x^2\,dx$$

```
[In] integrate(x^2*(a+b*arctan(c*x)),x, algorithm="giac")
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

$$\int x^2(a + b\arctan(cx))\,dx = \frac{a\,x^3}{3} + \frac{b\,x^3\,\mathrm{atan}(c\,x)}{3} + \frac{b\,\ln\left(c^2\,x^2 + 1\right)}{6\,c^3} - \frac{b\,x^2}{6\,c}$$

```
[In] int(x^2*(a + b*atan(c*x)),x)
[Out] (a*x^3)/3 + (b*x^3*atan(c*x))/3 + (b*log(c^2*x^2 + 1))/(6*c^3) - (b*x^2)/(6
*c)
```

3.5 $\int x(a+b\arctan(cx))\,dx$

Optimal result 87
Rubi [A] (verified) 87
Mathematica [A] (verified) 88
Maple [A] (verified) 88
Fricas [A] (verification not implemented) 89
Sympy [A] (verification not implemented) 89
Maxima [A] (verification not implemented) 90
Giac [F] 90
Mupad [B] (verification not implemented) 90

Optimal result

Integrand size = 10, antiderivative size = 37

$$\int x(a+b\arctan(cx))\,dx=-\frac{bx}{2c}+\frac{b\arctan(cx)}{2c^2}+\frac{1}{2}x^2(a+b\arctan(cx))$$

```
[Out] -1/2*b*x/c+1/2*b*arctan(c*x)/c^2+1/2*x^2*(a+b*arctan(c*x))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4946, 327, 209}

$$\int x(a+b\arctan(cx))\,dx=\frac{1}{2}x^2(a+b\arctan(cx))+\frac{b\arctan(cx)}{2c^2}-\frac{bx}{2c}$$

```
[In] Int[x*(a + b*ArcTan[c*x]),x]
```

```
[Out] -1/2*(b*x)/c + (b*ArcTan[c*x])/(2*c^2) + (x^2*(a + b*ArcTan[c*x]))/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}x^2(a+b\arctan(cx)) - \frac{1}{2}(bc)\int \frac{x^2}{1+c^2x^2}\,dx\\ &= -\frac{bx}{2c} + \frac{1}{2}x^2(a+b\arctan(cx)) + \frac{b\int \frac{1}{1+c^2x^2}\,dx}{2c}\\ &= -\frac{bx}{2c} + \frac{b\arctan(cx)}{2c^2} + \frac{1}{2}x^2(a+b\arctan(cx))\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

$$\int x(a+b\arctan(cx))\,dx = -\frac{bx}{2c} + \frac{ax^2}{2} + \frac{b\arctan(cx)}{2c^2} + \frac{1}{2}bx^2\arctan(cx)$$

```
[In] Integrate[x*(a + b*ArcTan[c*x]),x]
[Out] -1/2*(b*x)/c + (a*x^2)/2 + (b*ArcTan[c*x])/(2*c^2) + (b*x^2*ArcTan[c*x])/2
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

method	result	size
parts	$\frac{a\,x^{2}}{2}+\frac{b\left(\frac{c^{2}x^{2}\arctan(cx)}{2}-\frac{cx}{2}+\frac{\arctan(cx)}{2}\right)}{c^{2}}$	36
parallelrisch	$\frac{\arctan(cx)b\,c^{2}x^{2}+c^{2}x^{2}a-xbc+b\arctan(cx)}{2c^{2}}$	38
derivativedivides	$\frac{\frac{c^{2}x^{2}a}{2}+b\left(\frac{c^{2}x^{2}\arctan(cx)}{2}-\frac{cx}{2}+\frac{\arctan(cx)}{2}\right)}{c^{2}}$	40
default	$\frac{\frac{c^{2}x^{2}a}{2}+b\left(\frac{c^{2}x^{2}\arctan(cx)}{2}-\frac{cx}{2}+\frac{\arctan(cx)}{2}\right)}{c^{2}}$	40
risch	$-\frac{ix^{2}b\ln(icx+1)}{4}+\frac{ix^{2}b\ln(-icx+1)}{4}+\frac{a\,x^{2}}{2}-\frac{bx}{2c}+\frac{b\arctan(cx)}{2c^{2}}$	55

```
[In] int(x*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
[Out] 1/2*a*x^2+b/c^2*(1/2*c^2*x^2*arctan(c*x)-1/2*c*x+1/2*arctan(c*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int x(a+b\arctan(cx))\,dx=\frac{ac^{2}x^{2}-bcx+(bc^{2}x^{2}+b)\arctan(cx)}{2\,c^{2}}$$

```
[In] integrate(x*(a+b*arctan(c*x)),x, algorithm="fricas")
[Out] 1/2*(a*c^2*x^2 - b*c*x + (b*c^2*x^2 + b)*arctan(c*x))/c^2
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

$$\int x(a+b\arctan(cx))\,dx=\begin{cases}\frac{ax^{2}}{2}+\frac{bx^{2}\operatorname{atan}(cx)}{2}-\frac{bx}{2c}+\frac{b\operatorname{atan}(cx)}{2c^{2}} & \text{for } c\neq 0\\ \frac{ax^{2}}{2} & \text{otherwise}\end{cases}$$

```
[In] integrate(x*(a+b*atan(c*x)),x)
[Out] Piecewise((a*x**2/2 + b*x**2*atan(c*x)/2 - b*x/(2*c) + b*atan(c*x)/(2*c**2)
, Ne(c, 0)), (a*x**2/2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int x(a+b\arctan(cx))\,dx=\frac{1}{2}\,ax^2+\frac{1}{2}\left(x^2\arctan\left(cx\right)-c\left(\frac{x}{c^2}-\frac{\arctan\left(cx\right)}{c^3}\right)\right)b$$

```
[In] integrate(x*(a+b*arctan(c*x)),x, algorithm="maxima")
[Out] 1/2*a*x^2 + 1/2*(x^2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*b
```

Giac [F]

$$\int x(a+b\arctan(cx))\,dx=\int\left(b\arctan\left(cx\right)+a\right)x\,dx$$

```
[In] integrate(x*(a+b*arctan(c*x)),x, algorithm="giac")
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int x(a+b\arctan(cx))\,dx=\frac{a\,x^2}{2}+\frac{b\,\mathrm{atan}(c\,x)}{2\,c^2}+\frac{b\,x^2\,\mathrm{atan}(c\,x)}{2}-\frac{b\,x}{2\,c}$$

```
[In] int(x*(a + b*atan(c*x)),x)
[Out] (a*x^2)/2 + (b*atan(c*x))/(2*c^2) + (b*x^2*atan(c*x))/2 - (b*x)/(2*c)
```

3.6 $\int (a + b \arctan(cx))\, dx$

Optimal result 91
Rubi [A] (verified) 91
Mathematica [A] (verified) 92
Maple [A] (verified) 92
Fricas [A] (verification not implemented) 93
Sympy [A] (verification not implemented) 93
Maxima [A] (verification not implemented) 93
Giac [A] (verification not implemented) 94
Mupad [B] (verification not implemented) 94

Optimal result

Integrand size = 8, antiderivative size = 29

$$\int (a + b \arctan(cx))\, dx = ax + bx \arctan(cx) - \frac{b \log(1 + c^2x^2)}{2c}$$

```
[Out] a*x+b*x*arctan(c*x)-1/2*b*ln(c^2*x^2+1)/c
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4930, 266}

$$\int (a + b \arctan(cx))\, dx = ax + bx \arctan(cx) - \frac{b \log(c^2x^2 + 1)}{2c}$$

```
[In] Int[a + b*ArcTan[c*x],x]
```

```
[Out] a*x + b*x*ArcTan[c*x] - (b*Log[1 + c^2*x^2])/(2*c)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= ax + b\int \arctan(cx)\,dx \\
&= ax + bx\arctan(cx) - (bc)\int \frac{x}{1+c^2x^2}\,dx \\
&= ax + bx\arctan(cx) - \frac{b\log(1+c^2x^2)}{2c}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int (a+b\arctan(cx))\,dx = ax + bx\arctan(cx) - \frac{b\log(1+c^2x^2)}{2c}$$

```
[In] Integrate[a + b*ArcTan[c*x],x]
[Out] a*x + b*x*ArcTan[c*x] - (b*Log[1 + c^2*x^2])/(2*c)
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

method	result	size
default	$ax+bx\arctan\left(cx\right)-\frac{b\ln\left(c^{2}x^{2}+1\right)}{2c}$	28
parts	$ax+bx\arctan\left(cx\right)-\frac{b\ln\left(c^{2}x^{2}+1\right)}{2c}$	28
parallelrisch	$-\frac{b\left(-2cx\arctan(cx)+\ln\left(c^{2}x^{2}+1\right)\right)}{2c}+ax$	30
derivativedivides	$\frac{cxa+b\left(cx\arctan(cx)-\frac{\ln\left(c^{2}x^{2}+1\right)}{2}\right)}{c}$	32
risch	$ax-\frac{ibx\ln(icx+1)}{2}+\frac{ibx\ln(-icx+1)}{2}-\frac{b\ln(-c^{2}x^{2}-1)}{2c}$	48

```
[In] int(a+b*arctan(c*x),x,method=_RETURNVERBOSE)
[Out] a*x+b*x*arctan(c*x)-1/2*b*ln(c^2*x^2+1)/c
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.14

$$\int (a + b\arctan(cx))\,dx = \frac{2\,bcx\arctan{(cx)} + 2\,acx - b\log{(c^2x^2 + 1)}}{2\,c}$$

```
[In] integrate(a+b*arctan(c*x),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*c*x*arctan(c*x) + 2*a*c*x - b*log(c^2*x^2 + 1))/c
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int (a + b\arctan(cx))\,dx = ax + b\left(\begin{cases} x\operatorname{atan}{(cx)} - \frac{\log{(c^2x^2+1)}}{2c} & \text{for}\ c \neq 0 \\ 0 & \text{otherwise} \end{cases}\right)$$

```
[In] integrate(a+b*atan(c*x),x)
```

```
[Out] a*x + b*Piecewise((x*atan(c*x) - log(c**2*x**2 + 1)/(2*c), Ne(c, 0)), (0, T
rue))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int (a + b\arctan(cx))\,dx = ax + \frac{(2\,cx\arctan{(cx)} - \log{(c^2x^2 + 1)})b}{2\,c}$$

```
[In] integrate(a+b*arctan(c*x),x, algorithm="maxima")
```

```
[Out] a*x + 1/2*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b/c
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int (a+b\arctan(cx))\,dx = ax + \frac{(2\,cx\arctan{(cx)} - \log{(c^2x^2+1)})b}{2\,c}$$

```
[In] integrate(a+b*arctan(c*x),x, algorithm="giac")
```

```
[Out] a*x + 1/2*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b/c
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int (a+b\arctan(cx))\,dx = a\,x - \frac{b\,\ln{(c^2\,x^2+1)}}{2\,c} + b\,x\,\mathrm{atan}(c\,x)$$

```
[In] int(a + b*atan(c*x),x)
```

```
[Out] a*x - (b*log(c^2*x^2 + 1))/(2*c) + b*x*atan(c*x)
```

3.7 $\int \frac{a+b\arctan(cx)}{x}\,dx$

Optimal result . . . 95
Rubi [A] (verified) . . . 95
Mathematica [A] (verified) . . . 96
Maple [A] (verified) . . . 96
Fricas [F] . . . 97
Sympy [F] . . . 97
Maxima [F] . . . 97
Giac [F] . . . 97
Mupad [B] (verification not implemented) . . . 98

Optimal result

Integrand size = 12, antiderivative size = 35

$$\int \frac{a+b\arctan(cx)}{x}\,dx = a\log(x)+\frac{1}{2}ib\,\mathrm{PolyLog}(2,-icx)-\frac{1}{2}ib\,\mathrm{PolyLog}(2,icx)$$

```
[Out] a*ln(x)+1/2*I*b*polylog(2,-I*c*x)-1/2*I*b*polylog(2,I*c*x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4940, 2438}

$$\int \frac{a+b\arctan(cx)}{x}\,dx = a\log(x)+\frac{1}{2}ib\,\mathrm{PolyLog}(2,-icx)-\frac{1}{2}ib\,\mathrm{PolyLog}(2,icx)$$

```
[In] Int[(a + b*ArcTan[c*x])/x,x]
```

```
[Out] a*Log[x] + (I/2)*b*PolyLog[2, (-I)*c*x] - (I/2)*b*PolyLog[2, I*c*x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= a\log(x) + \frac{1}{2}(ib)\int\frac{\log(1-icx)}{x}\,dx - \frac{1}{2}(ib)\int\frac{\log(1+icx)}{x}\,dx\\ &= a\log(x) + \frac{1}{2}ib\,\text{PolyLog}(2,-icx) - \frac{1}{2}ib\,\text{PolyLog}(2,icx)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int\frac{a+b\arctan(cx)}{x}\,dx = a\log(x) + \frac{1}{2}ib\,\text{PolyLog}(2,-icx) - \frac{1}{2}ib\,\text{PolyLog}(2,icx)$$

```
[In] Integrate[(a + b*ArcTan[c*x])/x,x]
[Out] a*Log[x] + (I/2)*b*PolyLog[2, (-I)*c*x] - (I/2)*b*PolyLog[2, I*c*x]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

method	result
risch	$-\frac{ib\,\text{dilog}(-icx+1)}{2}+a\ln(-icx)+\frac{ib\,\text{dilog}(icx+1)}{2}$
parts	$a\ln(x)+b\Big(\ln(cx)\arctan(cx)+\frac{i\ln(cx)\ln(icx+1)}{2}-\frac{i\ln(cx)\ln(-icx+1)}{2}+\frac{i\,\text{dilog}(icx+1)}{2}-\frac{i\,\text{dilog}(-$
derivativedivides	$a\ln(cx)+b\Big(\ln(cx)\arctan(cx)+\frac{i\ln(cx)\ln(icx+1)}{2}-\frac{i\ln(cx)\ln(-icx+1)}{2}+\frac{i\,\text{dilog}(icx+1)}{2}-\frac{i\,\text{dilog}(-$
default	$a\ln(cx)+b\Big(\ln(cx)\arctan(cx)+\frac{i\ln(cx)\ln(icx+1)}{2}-\frac{i\ln(cx)\ln(-icx+1)}{2}+\frac{i\,\text{dilog}(icx+1)}{2}-\frac{i\,\text{dilog}(-$

```
[In] int((a+b*arctan(c*x))/x,x,method=_RETURNVERBOSE)
[Out] -1/2*I*b*dilog(1-I*c*x)+a*ln(-I*c*x)+1/2*I*b*dilog(1+I*c*x)
```

Fricas [F]

$$\int \frac{a+b\arctan(cx)}{x}\,dx=\int \frac{b\arctan\left(cx\right)+a}{x}\,dx$$

```
[In] integrate((a+b*arctan(c*x))/x,x, algorithm="fricas")
[Out] integral((b*arctan(c*x) + a)/x, x)
```

Sympy [F]

$$\int \frac{a+b\arctan(cx)}{x}\,dx=\int \frac{a+b\operatorname{atan}\left(cx\right)}{x}\,dx$$

```
[In] integrate((a+b*atan(c*x))/x,x)
[Out] Integral((a + b*atan(c*x))/x, x)
```

Maxima [F]

$$\int \frac{a+b\arctan(cx)}{x}\,dx=\int \frac{b\arctan\left(cx\right)+a}{x}\,dx$$

```
[In] integrate((a+b*arctan(c*x))/x,x, algorithm="maxima")
[Out] b*integrate(arctan(c*x)/x, x) + a*log(x)
```

Giac [F]

$$\int \frac{a+b\arctan(cx)}{x}\,dx=\int \frac{b\arctan\left(cx\right)+a}{x}\,dx$$

```
[In] integrate((a+b*arctan(c*x))/x,x, algorithm="giac")
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

$$\int \frac{a+b\arctan(cx)}{x}\,dx = a\,\ln(x) - \frac{b\,(\mathrm{Li}_2(1-c\,x\,1\mathrm{i}) - \mathrm{Li}_2(1+c\,x\,1\mathrm{i}))\,1\mathrm{i}}{2}$$

```
[In] int((a + b*atan(c*x))/x,x)
[Out] a*log(x) - (b*(dilog(1 - c*x*1i) - dilog(c*x*1i + 1))*1i)/2
```

3.8 $\int \frac{a+b\arctan(cx)}{x^2}\,dx$

Optimal result 99
Rubi [A] (verified) 99
Mathematica [A] (verified) 100
Maple [A] (verified) 101
Fricas [A] (verification not implemented) 101
Sympy [A] (verification not implemented) 101
Maxima [A] (verification not implemented) 102
Giac [F] 102
Mupad [B] (verification not implemented) 102

Optimal result

Integrand size = 12, antiderivative size = 35

$$\int \frac{a+b\arctan(cx)}{x^2}\,dx=-\frac{a+b\arctan(cx)}{x}+bc\log(x)-\frac{1}{2}bc\log\left(1+c^2x^2\right)$$

```
[Out] (-a-b*arctan(c*x))/x+b*c*ln(x)-1/2*b*c*ln(c^2*x^2+1)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {4946, 272, 36, 29, 31}

$$\int \frac{a+b\arctan(cx)}{x^2}\,dx=-\frac{a+b\arctan(cx)}{x}-\frac{1}{2}bc\log\left(c^2x^2+1\right)+bc\log(x)$$

```
[In] Int[(a + b*ArcTan[c*x])/x^2,x]
```

```
[Out] -((a + b*ArcTan[c*x])/x) + b*c*Log[x] - (b*c*Log[1 + c^2*x^2])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a+b\arctan(cx)}{x}+(bc)\int\frac{1}{x\left(1+c^2x^2\right)}\,dx\\
&= -\frac{a+b\arctan(cx)}{x}+\frac{1}{2}(bc)\text{Subst}\left(\int\frac{1}{x\left(1+c^2x\right)}\,dx,x,x^2\right)\\
&= -\frac{a+b\arctan(cx)}{x}+\frac{1}{2}(bc)\text{Subst}\left(\int\frac{1}{x}\,dx,x,x^2\right)-\frac{1}{2}\left(bc^3\right)\text{Subst}\left(\int\frac{1}{1+c^2x}\,dx,x,x^2\right)\\
&= -\frac{a+b\arctan(cx)}{x}+bc\log(x)-\frac{1}{2}bc\log\left(1+c^2x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09

$$\int\frac{a+b\arctan(cx)}{x^2}\,dx=-\frac{a}{x}-\frac{b\arctan(cx)}{x}+bc\log(x)-\frac{1}{2}bc\log\left(1+c^2x^2\right)$$

```
[In] Integrate[(a + b*ArcTan[c*x])/x^2,x]
[Out] -(a/x) - (b*ArcTan[c*x])/x + b*c*Log[x] - (b*c*Log[1 + c^2*x^2])/2
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

method	result	size
parallelrisch	$\frac{2bc\ln(x)x-bc\ln(c^2x^2+1)x-2b\arctan(cx)-2a}{2x}$	39
parts	$-\frac{a}{x}+bc\left(-\frac{\arctan(cx)}{cx}-\frac{\ln(c^2x^2+1)}{2}+\ln(cx)\right)$	40
derivativedivides	$c\left(-\frac{a}{cx}+b\left(-\frac{\arctan(cx)}{cx}-\frac{\ln(c^2x^2+1)}{2}+\ln(cx)\right)\right)$	44
default	$c\left(-\frac{a}{cx}+b\left(-\frac{\arctan(cx)}{cx}-\frac{\ln(c^2x^2+1)}{2}+\ln(cx)\right)\right)$	44
risch	$\frac{ib\ln(icx+1)}{2x}-\frac{-2bc\ln(x)x+bc\ln(-c^2x^2-1)x+ib\ln(-icx+1)+2a}{2x}$	60

```
[In] int((a+b*arctan(c*x))/x^2,x,method=_RETURNVERBOSE)
[Out] 1/2*(2*b*c*ln(x)*x-b*c*ln(c^2*x^2+1)*x-2*b*arctan(c*x)-2*a)/x
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{a+b\arctan(cx)}{x^2}\,dx=-\frac{bcx\log\left(c^2x^2+1\right)-2\,bcx\log\left(x\right)+2\,b\arctan\left(cx\right)+2\,a}{2\,x}$$

```
[In] integrate((a+b*arctan(c*x))/x^2,x, algorithm="fricas")
[Out] -1/2*(b*c*x*log(c^2*x^2 + 1) - 2*b*c*x*log(x) + 2*b*arctan(c*x) + 2*a)/x
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{a+b\arctan(cx)}{x^2}\,dx=\begin{cases}-\frac{a}{x}+bc\log\left(x\right)-\frac{bc\log\left(x^2+\frac{1}{c^2}\right)}{2}-\frac{b\operatorname{atan}\left(cx\right)}{x} & \text{for}\ c\neq 0\\ -\frac{a}{x} & \text{otherwise}\end{cases}$$

```
[In] integrate((a+b*atan(c*x))/x**2,x)
[Out] Piecewise((-a/x + b*c*log(x) - b*c*log(x**2 + c**(-2))/2 - b*atan(c*x)/x, N
e(c, 0)), (-a/x, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

$$\int \frac{a+b\arctan(cx)}{x^2}\,dx = -\frac{1}{2}\left(c\left(\log\left(c^2x^2+1\right)-\log\left(x^2\right)\right)+\frac{2\,\arctan\left(cx\right)}{x}\right)b-\frac{a}{x}$$

```
[In] integrate((a+b*arctan(c*x))/x^2,x, algorithm="maxima")
[Out] -1/2*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b - a/x
```

Giac [F]

$$\int \frac{a+b\arctan(cx)}{x^2}\,dx = \int \frac{b\arctan\left(cx\right)+a}{x^2}\,dx$$

```
[In] integrate((a+b*arctan(c*x))/x^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

$$\int \frac{a+b\arctan(cx)}{x^2}\,dx = b\,c\,\ln\left(x\right)-\frac{a}{x}-\frac{b\,\mathrm{atan}(c\,x)}{x}-\frac{b\,c\,\ln\left(c^2\,x^2+1\right)}{2}$$

```
[In] int((a + b*atan(c*x))/x^2,x)
[Out] b*c*log(x) - a/x - (b*atan(c*x))/x - (b*c*log(c^2*x^2 + 1))/2
```

3.9 $\int \frac{a+b\arctan(cx)}{x^3}\,dx$

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Mathematica [C] (verified) . . . 104
Maple [A] (verified) . . . 104
Fricas [A] (verification not implemented) . . . 105
Sympy [A] (verification not implemented) . . . 105
Maxima [A] (verification not implemented) . . . 106
Giac [F] . . . 106
Mupad [B] (verification not implemented) . . . 106

Optimal result

Integrand size = 12, antiderivative size = 37

$$\int \frac{a+b\arctan(cx)}{x^3}\,dx=-\frac{bc}{2x}-\frac{1}{2}bc^2\arctan(cx)-\frac{a+b\arctan(cx)}{2x^2}$$

```
[Out] -1/2*b*c/x-1/2*b*c^2*arctan(c*x)+1/2*(-a-b*arctan(c*x))/x^2
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4946, 331, 209}

$$\int \frac{a+b\arctan(cx)}{x^3}\,dx=-\frac{a+b\arctan(cx)}{2x^2}-\frac{1}{2}bc^2\arctan(cx)-\frac{bc}{2x}$$

```
[In] Int[(a + b*ArcTan[c*x])/x^3,x]
```

```
[Out] -1/2*(b*c)/x - (b*c^2*ArcTan[c*x])/2 - (a + b*ArcTan[c*x])/(2*x^2)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
```

```
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{a+b\arctan(cx)}{2x^2}+\frac{1}{2}(bc)\int\frac{1}{x^2(1+c^2x^2)}\,dx\\ &= -\frac{bc}{2x}-\frac{a+b\arctan(cx)}{2x^2}-\frac{1}{2}\left(bc^3\right)\int\frac{1}{1+c^2x^2}\,dx\\ &= -\frac{bc}{2x}-\frac{1}{2}bc^2\arctan(cx)-\frac{a+b\arctan(cx)}{2x^2}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.24

$$\int\frac{a+b\arctan(cx)}{x^3}\,dx=-\frac{a}{2x^2}-\frac{b\arctan(cx)}{2x^2}-\frac{bc\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-c^2x^2\right)}{2x}$$

```
[In] Integrate[(a + b*ArcTan[c*x])/x^3,x]
[Out] -1/2*a/x^2 - (b*ArcTan[c*x])/(2*x^2) - (b*c*Hypergeometric2F1[-1/2, 1, 1/2,
 -(c^2*x^2)])/(2*x)
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.05

method	result	size
parallelrisch	$-\frac{\arctan(cx)b\,c^2x^2-c^2x^2a+xbc+b\arctan(cx)+a}{2x^2}$	39
parts	$-\frac{a}{2x^2}+b\,c^2\left(-\frac{\arctan(cx)}{2c^2x^2}-\frac{\arctan(cx)}{2}-\frac{1}{2cx}\right)$	40
derivativedivides	$c^2\left(-\frac{a}{2c^2x^2}+b\left(-\frac{\arctan(cx)}{2c^2x^2}-\frac{\arctan(cx)}{2}-\frac{1}{2cx}\right)\right)$	44
default	$c^2\left(-\frac{a}{2c^2x^2}+b\left(-\frac{\arctan(cx)}{2c^2x^2}-\frac{\arctan(cx)}{2}-\frac{1}{2cx}\right)\right)$	44
risch	$\frac{ib\ln(icx+1)}{4x^2}-\frac{-ib\,c^2\ln(-cx+i)x^2+ib\,c^2\ln(-cx-i)x^2+ib\ln(-icx+1)+2xbc+2a}{4x^2}$	79

```
[In] int((a+b*arctan(c*x))/x^3,x,method=_RETURNVERBOSE)
[Out] -1/2*(arctan(c*x)*b*c^2*x^2-c^2*x^2*a+x*b*c+b*arctan(c*x)+a)/x^2
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.70

$$\int \frac{a+b\arctan(cx)}{x^3}\,dx=-\frac{bcx+(bc^2x^2+b)\arctan{(cx)}+a}{2\,x^2}$$

```
[In] integrate((a+b*arctan(c*x))/x^3,x, algorithm="fricas")
[Out] -1/2*(b*c*x + (b*c^2*x^2 + b)*arctan(c*x) + a)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{a+b\arctan(cx)}{x^3}\,dx=-\frac{a}{2x^2}-\frac{bc^2\,\mathrm{atan}\,(cx)}{2}-\frac{bc}{2x}-\frac{b\,\mathrm{atan}\,(cx)}{2x^2}$$

```
[In] integrate((a+b*atan(c*x))/x**3,x)
[Out] -a/(2*x**2) - b*c**2*atan(c*x)/2 - b*c/(2*x) - b*atan(c*x)/(2*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int \frac{a+b\arctan(cx)}{x^3}\,dx = -\frac{1}{2}\left(\left(c\arctan\left(cx\right)+\frac{1}{x}\right)c+\frac{\arctan\left(cx\right)}{x^2}\right)b-\frac{a}{2\,x^2}$$

```
[In] integrate((a+b*arctan(c*x))/x^3,x, algorithm="maxima")
[Out] -1/2*((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b - 1/2*a/x^2
```

Giac [F]

$$\int \frac{a+b\arctan(cx)}{x^3}\,dx = \int \frac{b\arctan\left(cx\right)+a}{x^3}\,dx$$

```
[In] integrate((a+b*arctan(c*x))/x^3,x, algorithm="giac")
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

$$\int \frac{a+b\arctan(cx)}{x^3}\,dx = -\frac{\frac{a}{2}+\frac{b\,\mathrm{atan}(c\,x)}{2}+\frac{b\,c\,x}{2}}{x^2}-\frac{b\,c\,\mathrm{atan}\left(\frac{c^2\,x}{\sqrt{c^2}}\right)\,\sqrt{c^2}}{2}$$

```
[In] int((a + b*atan(c*x))/x^3,x)
[Out] - (a/2 + (b*atan(c*x))/2 + (b*c*x)/2)/x^2 - (b*c*atan((c^2*x)/(c^2)^(1/2))*
(c^2)^(1/2))/2
```

3.10 $\int \frac{a+b\arctan(cx)}{x^4}\,dx$

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Rubi [A] (verified) 107
Mathematica [A] (verified) 108
Maple [A] (verified) 109
Fricas [A] (verification not implemented) 109
Sympy [A] (verification not implemented) 109
Maxima [A] (verification not implemented) 110
Giac [F] 110
Mupad [B] (verification not implemented) 110

Optimal result

Integrand size = 12, antiderivative size = 53

$$\int \frac{a+b\arctan(cx)}{x^4}\,dx = -\frac{bc}{6x^2}-\frac{a+b\arctan(cx)}{3x^3}-\frac{1}{3}bc^3\log(x)+\frac{1}{6}bc^3\log\left(1+c^2x^2\right)$$

```
[Out] -1/6*b*c/x^2+1/3*(-a-b*arctan(c*x))/x^3-1/3*b*c^3*ln(x)+1/6*b*c^3*ln(c^2*x^
2+1)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4946, 272, 46}

$$\int \frac{a+b\arctan(cx)}{x^4}\,dx = -\frac{a+b\arctan(cx)}{3x^3}-\frac{1}{3}bc^3\log(x)+\frac{1}{6}bc^3\log\left(c^2x^2+1\right)-\frac{bc}{6x^2}$$

```
[In] Int[(a + b*ArcTan[c*x])/x^4,x]
```

```
[Out] -1/6*(b*c)/x^2 - (a + b*ArcTan[c*x])/(3*x^3) - (b*c^3*Log[x])/3 + (b*c^3*Lo
g[1 + c^2*x^2])/6
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a+b\arctan(cx)}{3x^3}+\frac{1}{3}(bc)\int\frac{1}{x^3\left(1+c^2x^2\right)}\,dx\\
&= -\frac{a+b\arctan(cx)}{3x^3}+\frac{1}{6}(bc)\text{Subst}\left(\int\frac{1}{x^2\left(1+c^2x\right)}\,dx,x,x^2\right)\\
&= -\frac{a+b\arctan(cx)}{3x^3}+\frac{1}{6}(bc)\text{Subst}\left(\int\left(\frac{1}{x^2}-\frac{c^2}{x}+\frac{c^4}{1+c^2x}\right)\,dx,x,x^2\right)\\
&= -\frac{bc}{6x^2}-\frac{a+b\arctan(cx)}{3x^3}-\frac{1}{3}bc^3\log(x)+\frac{1}{6}bc^3\log\left(1+c^2x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.02

$$\int\frac{a+b\arctan(cx)}{x^4}\,dx=-\frac{a}{3x^3}-\frac{b\arctan(cx)}{3x^3}+\frac{1}{6}bc\left(-\frac{1}{x^2}-2c^2\log(x)+c^2\log\left(1+c^2x^2\right)\right)$$

```
[In] Integrate[(a + b*ArcTan[c*x])/x^4,x]
```

```
[Out] -1/3*a/x^3 - (b*ArcTan[c*x])/(3*x^3) + (b*c*(-x^(-2) - 2*c^2*Log[x] + c^2*L
og[1 + c^2*x^2]))/6
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98

method	result	size
parts	$-\frac{a}{3x^{3}}+b\,c^{3}\left(-\frac{\arctan(cx)}{3c^{3}x^{3}}+\frac{\ln(c^{2}x^{2}+1)}{6}-\frac{1}{6c^{2}x^{2}}-\frac{\ln(cx)}{3}\right)$	52
derivativedivides	$c^{3}\left(-\frac{a}{3c^{3}x^{3}}+b\left(-\frac{\arctan(cx)}{3c^{3}x^{3}}+\frac{\ln(c^{2}x^{2}+1)}{6}-\frac{1}{6c^{2}x^{2}}-\frac{\ln(cx)}{3}\right)\right)$	56
default	$c^{3}\left(-\frac{a}{3c^{3}x^{3}}+b\left(-\frac{\arctan(cx)}{3c^{3}x^{3}}+\frac{\ln(c^{2}x^{2}+1)}{6}-\frac{1}{6c^{2}x^{2}}-\frac{\ln(cx)}{3}\right)\right)$	56
parallelrisch	$-\frac{2b\,c^{3}\ln(x)x^{3}-b\,c^{3}\ln(c^{2}x^{2}+1)x^{3}-b\,c^{3}x^{3}+xbc+2b\arctan(cx)+2a}{6x^{3}}$	60
risch	$\frac{ib\ln(icx+1)}{6x^{3}}-\frac{2b\,c^{3}\ln(x)x^{3}-b\,c^{3}\ln(c^{2}x^{2}+1)x^{3}+ib\ln(-icx+1)+xbc+2a}{6x^{3}}$	72

```
[In] int((a+b*arctan(c*x))/x^4,x,method=_RETURNVERBOSE)
[Out] -1/3*a/x^3+b*c^3*(-1/3/c^3/x^3*arctan(c*x)+1/6*ln(c^2*x^2+1)-1/6/c^2/x^2-1/
3*ln(c*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94

$$\int \frac{a+b\arctan(cx)}{x^4}\,dx=\frac{bc^3x^3\log\left(c^2x^2+1\right)-2\,bc^3x^3\log\left(x\right)-bcx-2\,b\arctan\left(cx\right)-2\,a}{6\,x^3}$$

```
[In] integrate((a+b*arctan(c*x))/x^4,x, algorithm="fricas")
[Out] 1/6*(b*c^3*x^3*log(c^2*x^2 + 1) - 2*b*c^3*x^3*log(x) - b*c*x - 2*b*arctan(c
*x) - 2*a)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.15

$$\int \frac{a+b\arctan(cx)}{x^4}\,dx=\begin{cases}-\frac{a}{3x^3}-\frac{bc^3\log\left(x\right)}{3}+\frac{bc^3\log\left(x^2+\frac{1}{c^2}\right)}{6}-\frac{bc}{6x^2}-\frac{b\,\mathrm{atan}\left(cx\right)}{3x^3} & \text{for}\; c\neq 0\\ -\frac{a}{3x^3} & \text{otherwise}\end{cases}$$

```
[In] integrate((a+b*atan(c*x))/x**4,x)
[Out] Piecewise((-a/(3*x**3) - b*c**3*log(x)/3 + b*c**3*log(x**2 + c**(-2))/6 - b
*c/(6*x**2) - b*atan(c*x)/(3*x**3), Ne(c, 0)), (-a/(3*x**3), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \frac{a+b\arctan(cx)}{x^4}\,dx = \frac{1}{6}\left(\left(c^2\log\left(c^2x^2+1\right)-c^2\log\left(x^2\right)-\frac{1}{x^2}\right)c-\frac{2\arctan\left(cx\right)}{x^3}\right)b - \frac{a}{3\,x^3}$$

```
[In] integrate((a+b*arctan(c*x))/x^4,x, algorithm="maxima")
[Out] 1/6*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*b
 - 1/3*a/x^3
```

Giac [F]

$$\int \frac{a+b\arctan(cx)}{x^4}\,dx = \int \frac{b\arctan\left(cx\right)+a}{x^4}\,dx$$

```
[In] integrate((a+b*arctan(c*x))/x^4,x, algorithm="giac")
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.87

$$\int \frac{a+b\arctan(cx)}{x^4}\,dx = \frac{b\,c^3\,\ln\left(c^2\,x^2+1\right)}{6} - \frac{\frac{a}{3}+\frac{b\,\mathrm{atan}(c\,x)}{3}+\frac{b\,c\,x}{6}}{x^3} - \frac{b\,c^3\,\ln\left(x\right)}{3}$$

```
[In] int((a + b*atan(c*x))/x^4,x)
[Out] (b*c^3*log(c^2*x^2 + 1))/6 - (a/3 + (b*atan(c*x))/3 + (b*c*x)/6)/x^3 - (b*c
^3*log(x))/3
```

3.11 $\int \frac{a+b\arctan(cx)}{x^5}\,dx$

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Mathematica [C] (verified) 112
Maple [A] (verified) 113
Fricas [A] (verification not implemented) 113
Sympy [A] (verification not implemented) 113
Maxima [A] (verification not implemented) 114
Giac [F] 114
Mupad [B] (verification not implemented) 114

Optimal result

Integrand size = 12, antiderivative size = 48

$$\int \frac{a+b\arctan(cx)}{x^5}\,dx = -\frac{bc}{12x^3}+\frac{bc^3}{4x}+\frac{1}{4}bc^4\arctan(cx)-\frac{a+b\arctan(cx)}{4x^4}$$

```
[Out] -1/12*b*c/x^3+1/4*b*c^3/x+1/4*b*c^4*arctan(c*x)+1/4*(-a-b*arctan(c*x))/x^4
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4946, 331, 209}

$$\int \frac{a+b\arctan(cx)}{x^5}\,dx = -\frac{a+b\arctan(cx)}{4x^4}+\frac{1}{4}bc^4\arctan(cx)+\frac{bc^3}{4x}-\frac{bc}{12x^3}$$

```
[In] Int[(a + b*ArcTan[c*x])/x^5,x]
```

```
[Out] -1/12*(b*c)/x^3 + (b*c^3)/(4*x) + (b*c^4*ArcTan[c*x])/4 - (a + b*ArcTan[c*x
])/(4*x^4)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
```

```
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a+b\arctan(cx)}{4x^4}+\frac{1}{4}(bc)\int\frac{1}{x^4\left(1+c^2x^2\right)}\,dx\\
&= -\frac{bc}{12x^3}-\frac{a+b\arctan(cx)}{4x^4}-\frac{1}{4}\left(bc^3\right)\int\frac{1}{x^2\left(1+c^2x^2\right)}\,dx\\
&= -\frac{bc}{12x^3}+\frac{bc^3}{4x}-\frac{a+b\arctan(cx)}{4x^4}+\frac{1}{4}\left(bc^5\right)\int\frac{1}{1+c^2x^2}\,dx\\
&= -\frac{bc}{12x^3}+\frac{bc^3}{4x}+\frac{1}{4}bc^4\arctan(cx)-\frac{a+b\arctan(cx)}{4x^4}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

$$\int\frac{a+b\arctan(cx)}{x^5}\,dx = -\frac{a}{4x^4}-\frac{b\arctan(cx)}{4x^4}-\frac{bc\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-c^2x^2\right)}{12x^3}$$

```
[In] Integrate[(a + b*ArcTan[c*x])/x^5,x]
[Out] -1/4*a/x^4 - (b*ArcTan[c*x])/(4*x^4) - (b*c*Hypergeometric2F1[-3/2, 1, -1/2
, -(c^2*x^2)])/(12*x^3)
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.92

method	result	size
parallelrisch	$\frac{3b\arctan(cx)x^4c^4+3b\,c^3x^3-xbc-3b\arctan(cx)-3a}{12x^4}$	44
parts	$-\frac{a}{4x^4}+b\,c^4\left(-\frac{\arctan(cx)}{4c^4x^4}-\frac{1}{12c^3x^3}+\frac{1}{4cx}+\frac{\arctan(cx)}{4}\right)$	48
derivativedivides	$c^4\left(-\frac{a}{4c^4x^4}+b\left(-\frac{\arctan(cx)}{4c^4x^4}-\frac{1}{12c^3x^3}+\frac{1}{4cx}+\frac{\arctan(cx)}{4}\right)\right)$	52
default	$c^4\left(-\frac{a}{4c^4x^4}+b\left(-\frac{\arctan(cx)}{4c^4x^4}-\frac{1}{12c^3x^3}+\frac{1}{4cx}+\frac{\arctan(cx)}{4}\right)\right)$	52
risch	$\frac{ib\ln(icx+1)}{8x^4}-\frac{3ib\,c^4\ln(-cx+i)x^4-3ib\,c^4\ln(-cx-i)x^4-6b\,c^3x^3+3ib\ln(-icx+1)+2xbc+6a}{24x^4}$	88

```
[In] int((a+b*arctan(c*x))/x^5,x,method=_RETURNVERBOSE)
[Out] 1/12*(3*b*arctan(c*x)*x^4*c^4+3*b*c^3*x^3-x*b*c-3*b*arctan(c*x)-3*a)/x^4
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.85

$$\int \frac{a+b\arctan(cx)}{x^5}\,dx=\frac{3\,bc^3x^3-bcx+3\left(bc^4x^4-b\right)\arctan\left(cx\right)-3\,a}{12\,x^4}$$

```
[In] integrate((a+b*arctan(c*x))/x^5,x, algorithm="fricas")
[Out] 1/12*(3*b*c^3*x^3 - b*c*x + 3*(b*c^4*x^4 - b)*arctan(c*x) - 3*a)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

$$\int \frac{a+b\arctan(cx)}{x^5}\,dx=-\frac{a}{4x^4}+\frac{bc^4\operatorname{atan}\left(cx\right)}{4}+\frac{bc^3}{4x}-\frac{bc}{12x^3}-\frac{b\operatorname{atan}\left(cx\right)}{4x^4}$$

```
[In] integrate((a+b*atan(c*x))/x**5,x)
[Out] -a/(4*x**4) + b*c**4*atan(c*x)/4 + b*c**3/(4*x) - b*c/(12*x**3) - b*atan(c*
x)/(4*x**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

$$\int \frac{a+b\arctan(cx)}{x^5}\,dx = \frac{1}{12}\left(\left(3\,c^3\arctan\left(cx\right) + \frac{3\,c^2x^2-1}{x^3}\right)c - \frac{3\,\arctan\left(cx\right)}{x^4}\right)b - \frac{a}{4\,x^4}$$

```
[In] integrate((a+b*arctan(c*x))/x^5,x, algorithm="maxima")
```

```
[Out] 1/12*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(c*x)/x^4)*b -
1/4*a/x^4
```

Giac [F]

$$\int \frac{a+b\arctan(cx)}{x^5}\,dx = \int \frac{b\arctan\left(cx\right)+a}{x^5}\,dx$$

```
[In] integrate((a+b*arctan(c*x))/x^5,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.88

$$\int \frac{a+b\arctan(cx)}{x^5}\,dx = \frac{b\,c^4\,\mathrm{atan}(c\,x)}{4} - \frac{-b\,c^3\,x^3+\frac{b\,c\,x}{3}+a}{4\,x^4} - \frac{b\,\mathrm{atan}(c\,x)}{4\,x^4}$$

```
[In] int((a + b*atan(c*x))/x^5,x)
```

```
[Out] (b*c^4*atan(c*x))/4 - (a - b*c^3*x^3 + (b*c*x)/3)/(4*x^4) - (b*atan(c*x))/(
4*x^4)
```

3.12 $\int \frac{a+b\arctan(cx)}{x^6}\,dx$

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Optimal result

Integrand size = 12, antiderivative size = 64

$$\int \frac{a+b\arctan(cx)}{x^6}\,dx=-\frac{bc}{20x^4}+\frac{bc^3}{10x^2}-\frac{a+b\arctan(cx)}{5x^5}+\frac{1}{5}bc^5\log(x)-\frac{1}{10}bc^5\log\left(1+c^2x^2\right)$$

```
[Out] -1/20*b*c/x^4+1/10*b*c^3/x^2+1/5*(-a-b*arctan(c*x))/x^5+1/5*b*c^5*ln(x)-1/1
0*b*c^5*ln(c^2*x^2+1)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4946, 272, 46}

$$\int \frac{a+b\arctan(cx)}{x^6}\,dx=-\frac{a+b\arctan(cx)}{5x^5}+\frac{1}{5}bc^5\log(x)+\frac{bc^3}{10x^2}-\frac{1}{10}bc^5\log\left(c^2x^2+1\right)-\frac{bc}{20x^4}$$

```
[In] Int[(a + b*ArcTan[c*x])/x^6,x]
```

```
[Out] -1/20*(b*c)/x^4 + (b*c^3)/(10*x^2) - (a + b*ArcTan[c*x])/(5*x^5) + (b*c^5*L
og[x])/5 - (b*c^5*Log[1 + c^2*x^2])/10
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a+b\arctan(cx)}{5x^5}+\frac{1}{5}(bc)\int\frac{1}{x^5\left(1+c^2x^2\right)}\,dx\\
&= -\frac{a+b\arctan(cx)}{5x^5}+\frac{1}{10}(bc)\text{Subst}\left(\int\frac{1}{x^3\left(1+c^2x\right)}\,dx,x,x^2\right)\\
&= -\frac{a+b\arctan(cx)}{5x^5}+\frac{1}{10}(bc)\text{Subst}\left(\int\left(\frac{1}{x^3}-\frac{c^2}{x^2}+\frac{c^4}{x}-\frac{c^6}{1+c^2x}\right)\,dx,x,x^2\right)\\
&= -\frac{bc}{20x^4}+\frac{bc^3}{10x^2}-\frac{a+b\arctan(cx)}{5x^5}+\frac{1}{5}bc^5\log(x)-\frac{1}{10}bc^5\log\left(1+c^2x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.08

$$\int\frac{a+b\arctan(cx)}{x^6}\,dx=-\frac{a}{5x^5}-\frac{bc}{20x^4}+\frac{bc^3}{10x^2}-\frac{b\arctan(cx)}{5x^5}+\frac{1}{5}bc^5\log(x)-\frac{1}{10}bc^5\log\left(1+c^2x^2\right)$$

```
[In] Integrate[(a + b*ArcTan[c*x])/x^6,x]
[Out] -1/5*a/x^5 - (b*c)/(20*x^4) + (b*c^3)/(10*x^2) - (b*ArcTan[c*x])/(5*x^5) +
(b*c^5*Log[x])/5 - (b*c^5*Log[1 + c^2*x^2])/10
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.94

method	result	size
parts	$-\frac{a}{5x^{5}}+b\,c^{5}\left(-\frac{\arctan(cx)}{5c^{5}x^{5}}-\frac{1}{20c^{4}x^{4}}+\frac{\ln(cx)}{5}+\frac{1}{10c^{2}x^{2}}-\frac{\ln(c^{2}x^{2}+1)}{10}\right)$	60
derivativedivides	$c^{5}\left(-\frac{a}{5c^{5}x^{5}}+b\left(-\frac{\arctan(cx)}{5c^{5}x^{5}}-\frac{1}{20c^{4}x^{4}}+\frac{\ln(cx)}{5}+\frac{1}{10c^{2}x^{2}}-\frac{\ln(c^{2}x^{2}+1)}{10}\right)\right)$	64
default	$c^{5}\left(-\frac{a}{5c^{5}x^{5}}+b\left(-\frac{\arctan(cx)}{5c^{5}x^{5}}-\frac{1}{20c^{4}x^{4}}+\frac{\ln(cx)}{5}+\frac{1}{10c^{2}x^{2}}-\frac{\ln(c^{2}x^{2}+1)}{10}\right)\right)$	64
parallelrisch	$\frac{4b\,c^{5}\ln(x)x^{5}-2b\,c^{5}\ln(c^{2}x^{2}+1)x^{5}-2b\,c^{5}x^{5}+2b\,c^{3}x^{3}-xbc-4b\arctan(cx)-4a}{20x^{5}}$	70
risch	$\frac{ib\ln(icx+1)}{10x^{5}}-\frac{-4b\,c^{5}\ln(x)x^{5}+2b\,c^{5}\ln(-c^{2}x^{2}-1)x^{5}-2b\,c^{3}x^{3}+2ib\ln(-icx+1)+xbc+4a}{20x^{5}}$	82

```
[In] int((a+b*arctan(c*x))/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*a/x^5+b*c^5*(-1/5/c^5/x^5*arctan(c*x)-1/20/c^4/x^4+1/5*ln(c*x)+1/10/c^
2/x^2-1/10*ln(c^2*x^2+1))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.92

$$\int\frac{a+b\arctan(cx)}{x^{6}}\,dx=-\frac{2\,bc^{5}x^{5}\log\left(c^{2}x^{2}+1\right)-4\,bc^{5}x^{5}\log\left(x\right)-2\,bc^{3}x^{3}+bcx+4\,b\arctan\left(cx\right)+4\,a}{20\,x^{5}}$$

```
[In] integrate((a+b*arctan(c*x))/x^6,x, algorithm="fricas")
```

```
[Out] -1/20*(2*b*c^5*x^5*log(c^2*x^2 + 1) - 4*b*c^5*x^5*log(x) - 2*b*c^3*x^3 + b*
c*x + 4*b*arctan(c*x) + 4*a)/x^5
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.11

$$\int\frac{a+b\arctan(cx)}{x^{6}}\,dx=\begin{cases}-\frac{a}{5x^{5}}+\frac{bc^{5}\log\left(x\right)}{5}-\frac{bc^{5}\log\left(x^{2}+\frac{1}{c^{2}}\right)}{10}+\frac{bc^{3}}{10x^{2}}-\frac{bc}{20x^{4}}-\frac{b\,\mathrm{atan}\left(cx\right)}{5x^{5}} & \text{for}\ c\neq 0\\ -\frac{a}{5x^{5}} & \text{otherwise}\end{cases}$$

```
[In] integrate((a+b*atan(c*x))/x**6,x)
```

```
[Out] Piecewise((-a/(5*x**5) + b*c**5*log(x)/5 - b*c**5*log(x**2 + c**(-2))/10 +
b*c**3/(10*x**2) - b*c/(20*x**4) - b*atan(c*x)/(5*x**5), Ne(c, 0)), (-a/(5*
x**5), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.97

$$\int \frac{a+b\arctan(cx)}{x^6}\,dx = -\frac{1}{20}\left(\left(2\,c^4\log\left(c^2x^2+1\right)-2\,c^4\log\left(x^2\right)-\frac{2\,c^2x^2-1}{x^4}\right)c+\frac{4\,\arctan\left(cx\right)}{x^5}\right)b-\frac{a}{5\,x^5}$$

```
[In] integrate((a+b*arctan(c*x))/x^6,x, algorithm="maxima")
```

```
[Out] -1/20*((2*c^4*log(c^2*x^2 + 1) - 2*c^4*log(x^2) - (2*c^2*x^2 - 1)/x^4)*c +
4*arctan(c*x)/x^5)*b - 1/5*a/x^5
```

Giac [F]

$$\int \frac{a+b\arctan(cx)}{x^6}\,dx = \int \frac{b\arctan\left(cx\right)+a}{x^6}\,dx$$

```
[In] integrate((a+b*arctan(c*x))/x^6,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88

$$\int \frac{a+b\arctan(cx)}{x^6}\,dx = \frac{b\,c^5\,\ln\left(x\right)}{5}-\frac{b\,\mathrm{atan}(c\,x)}{5\,x^5}-\frac{b\,c^5\,\ln\left(c^2\,x^2+1\right)}{10}-\frac{-\frac{b\,c^3\,x^3}{2}+\frac{b\,c\,x}{4}+a}{5\,x^5}$$

```
[In] int((a + b*atan(c*x))/x^6,x)
```

```
[Out] (b*c^5*log(x))/5 - (b*atan(c*x))/(5*x^5) - (b*c^5*log(c^2*x^2 + 1))/10 - (a
 - (b*c^3*x^3)/2 + (b*c*x)/4)/(5*x^5)
```

3.13 $\int x^5(a+b\arctan(cx))^2\,dx$

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Optimal result

Integrand size = 14, antiderivative size = 144

$$\int x^5(a+b\arctan(cx))^2\,dx=-\frac{abx}{3c^5}-\frac{4b^2x^2}{45c^4}+\frac{b^2x^4}{60c^2}-\frac{b^2x\arctan(cx)}{3c^5}+\frac{bx^3(a+b\arctan(cx))}{9c^3}-\frac{bx^5(a+b\arctan(cx))}{15c}+\frac{(a+b\arctan(cx))^2}{6c^6}+\frac{1}{6}x^6(a+b\arctan(cx))^2+\frac{23b^2\log(1+c^2x^2)}{90c^6}$$

```
[Out] -1/3*a*b*x/c^5-4/45*b^2*x^2/c^4+1/60*b^2*x^4/c^2-1/3*b^2*x*arctan(c*x)/c^5+
1/9*b*x^3*(a+b*arctan(c*x))/c^3-1/15*b*x^5*(a+b*arctan(c*x))/c+1/6*(a+b*arc
tan(c*x))^2/c^6+1/6*x^6*(a+b*arctan(c*x))^2+23/90*b^2*ln(c^2*x^2+1)/c^6
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4946, 5036, 272, 45, 4930, 266, 5004}

$$\int x^5(a+b\arctan(cx))^2\,dx=\frac{(a+b\arctan(cx))^2}{6c^6}+\frac{bx^3(a+b\arctan(cx))}{9c^3}+\frac{1}{6}x^6(a+b\arctan(cx))^2-\frac{bx^5(a+b\arctan(cx))}{15c}-\frac{abx}{3c^5}-\frac{b^2x\arctan(cx)}{3c^5}-\frac{4b^2x^2}{45c^4}+\frac{b^2x^4}{60c^2}+\frac{23b^2\log(c^2x^2+1)}{90c^6}$$

```
[In] Int[x^5*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] -1/3*(a*b*x)/c^5 - (4*b^2*x^2)/(45*c^4) + (b^2*x^4)/(60*c^2) - (b^2*x*ArcTa
n[c*x])/(3*c^5) + (b*x^3*(a + b*ArcTan[c*x]))/(9*c^3) - (b*x^5*(a + b*ArcTa
```

```
n[c*x]))/(15*c) + (a + b*ArcTan[c*x])^2/(6*c^6) + (x^6*(a + b*ArcTan[c*x])^
2)/6 + (23*b^2*Log[1 + c^2*x^2])/(90*c^6)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
```

```
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{6}x^6(a+b\arctan(cx))^2-\frac{1}{3}(bc)\int\frac{x^6(a+b\arctan(cx))}{1+c^2x^2}\,dx\\
&= \frac{1}{6}x^6(a+b\arctan(cx))^2-\frac{b\int x^4(a+b\arctan(cx))\,dx}{3c}+\frac{b\int\frac{x^4(a+b\arctan(cx))}{1+c^2x^2}\,dx}{3c}\\
&= -\frac{bx^5(a+b\arctan(cx))}{15c}+\frac{1}{6}x^6(a+b\arctan(cx))^2+\frac{1}{15}b^2\int\frac{x^5}{1+c^2x^2}\,dx\\
&\quad+\frac{b\int x^2(a+b\arctan(cx))\,dx}{3c^3}-\frac{b\int\frac{x^2(a+b\arctan(cx))}{1+c^2x^2}\,dx}{3c^3}\\
&= \frac{bx^3(a+b\arctan(cx))}{9c^3}-\frac{bx^5(a+b\arctan(cx))}{15c}\\
&\quad+\frac{1}{6}x^6(a+b\arctan(cx))^2+\frac{1}{30}b^2\text{Subst}\left(\int\frac{x^2}{1+c^2x}\,dx,x,x^2\right)\\
&\quad-\frac{b\int(a+b\arctan(cx))\,dx}{3c^5}+\frac{b\int\frac{a+b\arctan(cx)}{1+c^2x^2}\,dx}{3c^5}-\frac{b^2\int\frac{x^3}{1+c^2x^2}\,dx}{9c^2}\\
&= -\frac{abx}{3c^5}+\frac{bx^3(a+b\arctan(cx))}{9c^3}-\frac{bx^5(a+b\arctan(cx))}{15c}+\frac{(a+b\arctan(cx))^2}{6c^6}\\
&\quad+\frac{1}{6}x^6(a+b\arctan(cx))^2+\frac{1}{30}b^2\text{Subst}\left(\int\left(-\frac{1}{c^4}+\frac{x}{c^2}+\frac{1}{c^4(1+c^2x)}\right)\,dx,x,x^2\right)\\
&\quad-\frac{b^2\int\arctan(cx)\,dx}{3c^5}-\frac{b^2\text{Subst}\left(\int\frac{x}{1+c^2x}\,dx,x,x^2\right)}{18c^2}\\
&= -\frac{abx}{3c^5}-\frac{b^2x^2}{30c^4}+\frac{b^2x^4}{60c^2}-\frac{b^2x\arctan(cx)}{3c^5}+\frac{bx^3(a+b\arctan(cx))}{9c^3}\\
&\quad-\frac{bx^5(a+b\arctan(cx))}{15c}+\frac{(a+b\arctan(cx))^2}{6c^6}+\frac{1}{6}x^6(a+b\arctan(cx))^2\\
&\quad+\frac{b^2\log(1+c^2x^2)}{30c^6}+\frac{b^2\int\frac{x}{1+c^2x^2}\,dx}{3c^4}-\frac{b^2\text{Subst}\left(\int\left(\frac{1}{c^2}-\frac{1}{c^2(1+c^2x)}\right)\,dx,x,x^2\right)}{18c^2}\\
&= -\frac{abx}{3c^5}-\frac{4b^2x^2}{45c^4}+\frac{b^2x^4}{60c^2}-\frac{b^2x\arctan(cx)}{3c^5}+\frac{bx^3(a+b\arctan(cx))}{9c^3}\\
&\quad-\frac{bx^5(a+b\arctan(cx))}{15c}+\frac{(a+b\arctan(cx))^2}{6c^6}\\
&\quad+\frac{1}{6}x^6(a+b\arctan(cx))^2+\frac{23b^2\log(1+c^2x^2)}{90c^6}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.96

$$\int x^5(a+b\arctan(cx))^2\,dx$$
$$=\frac{cx(30a^2c^5x^5+b^2cx(-16+3c^2x^2)-4ab(15-5c^2x^2+3c^4x^4))+4b(bcx(-15+5c^2x^2-3c^4x^4)+15a(1+c}{180c^6}$$

[In] Integrate[x^5*(a + b*ArcTan[c*x])^2,x]

[Out] (c*x*(30*a^2*c^5*x^5 + b^2*c*x*(-16 + 3*c^2*x^2) - 4*a*b*(15 - 5*c^2*x^2 + 3*c^4*x^4)) + 4*b*(b*c*x*(-15 + 5*c^2*x^2 - 3*c^4*x^4) + 15*a*(1 + c^6*x^6))*ArcTan[c*x] + 30*b^2*(1 + c^6*x^6)*ArcTan[c*x]^2 + 46*b^2*Log[1 + c^2*x^2])/(180*c^6)

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.01

method	result
parts	$\frac{x^6a^2}{6}+\frac{b^2\left(\frac{c^6x^6\arctan(cx)^2}{6}-\frac{c^5x^5\arctan(cx)}{15}+\frac{c^3x^3\arctan(cx)}{9}-\frac{cx\arctan(cx)}{3}+\frac{\arctan(cx)^2}{6}+\frac{c^4x^4}{60}-\frac{4c^2x^2}{45}+\frac{23\ln(c^2x^2+1)}{90}\right)}{c^6}$
derivativedivides	$\frac{\frac{a^2c^6x^6}{6}+b^2\left(\frac{c^6x^6\arctan(cx)^2}{6}-\frac{c^5x^5\arctan(cx)}{15}+\frac{c^3x^3\arctan(cx)}{9}-\frac{cx\arctan(cx)}{3}+\frac{\arctan(cx)^2}{6}+\frac{c^4x^4}{60}-\frac{4c^2x^2}{45}+\frac{23\ln(c^2x^2+1)}{90}\right)}{c^6}$
default	$\frac{\frac{a^2c^6x^6}{6}+b^2\left(\frac{c^6x^6\arctan(cx)^2}{6}-\frac{c^5x^5\arctan(cx)}{15}+\frac{c^3x^3\arctan(cx)}{9}-\frac{cx\arctan(cx)}{3}+\frac{\arctan(cx)^2}{6}+\frac{c^4x^4}{60}-\frac{4c^2x^2}{45}+\frac{23\ln(c^2x^2+1)}{90}\right)}{c^6}$
parallelrisch	$\frac{30b^2\arctan(cx)^2x^6c^6+60ab\arctan(cx)x^6c^6+30a^2c^6x^6-12b^2\arctan(cx)x^5c^5-12ab\,c^5x^5+3b^2c^4x^4+20b^2\arctan(cx)x^3c^3}{180c^6}$
risch	$-\frac{b^2(c^6x^6+1)\ln(icx+1)^2}{24c^6}-\frac{ib(30a\,c^6x^6+15ib\,c^6x^6\ln(-icx+1)-6b\,c^5x^5+10b\,c^3x^3-30xbc+15ib\ln(-icx+1))\ln(icx+1}{180c^6}$

[In] int(x^5*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)

[Out] 1/6*x^6*a^2+b^2/c^6*(1/6*c^6*x^6*arctan(c*x)^2-1/15*c^5*x^5*arctan(c*x)+1/9 *c^3*x^3*arctan(c*x)-1/3*c*x*arctan(c*x)+1/6*arctan(c*x)^2+1/60*c^4*x^4-4/4 5*c^2*x^2+23/90*ln(c^2*x^2+1))+2*a*b/c^6*(1/6*c^6*x^6*arctan(c*x)-1/30*c^5* x^5+1/18*c^3*x^3-1/6*c*x+1/6*arctan(c*x))

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.06

$$\int x^5(a+b\arctan(cx))^2\,dx$$
$$=\frac{30\,a^2c^6x^6-12\,abc^5x^5+3\,b^2c^4x^4+20\,abc^3x^3-16\,b^2c^2x^2-60\,abcx+30\,(b^2c^6x^6+b^2)\arctan{(cx)}^2+46\,b^2}{180\,c^6}$$

```
[In] integrate(x^5*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/180*(30*a^2*c^6*x^6 - 12*a*b*c^5*x^5 + 3*b^2*c^4*x^4 + 20*a*b*c^3*x^3 - 1
6*b^2*c^2*x^2 - 60*a*b*c*x + 30*(b^2*c^6*x^6 + b^2)*arctan(c*x)^2 + 46*b^2*
log(c^2*x^2 + 1) + 4*(15*a*b*c^6*x^6 - 3*b^2*c^5*x^5 + 5*b^2*c^3*x^3 - 15*b
^2*c*x + 15*a*b)*arctan(c*x))/c^6
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.38

$$\int x^5(a+b\arctan(cx))^2\,dx$$
$$=\begin{cases}\frac{a^2x^6}{6}+\frac{abx^6\operatorname{atan}{(cx)}}{3}-\frac{abx^5}{15c}+\frac{abx^3}{9c^3}-\frac{abx}{3c^5}+\frac{ab\operatorname{atan}{(cx)}}{3c^6}+\frac{b^2x^6\operatorname{atan}^2{(cx)}}{6}-\frac{b^2x^5\operatorname{atan}{(cx)}}{15c}+\frac{b^2x^4}{60c^2}+\frac{b^2x^3\operatorname{atan}{(cx)}}{9c^3}-\frac{4b^2}{45} \\ \frac{a^2x^6}{6}\end{cases}$$

```
[In] integrate(x**5*(a+b*atan(c*x))**2,x)
```

```
[Out] Piecewise((a**2*x**6/6 + a*b*x**6*atan(c*x)/3 - a*b*x**5/(15*c) + a*b*x**3/
(9*c**3) - a*b*x/(3*c**5) + a*b*atan(c*x)/(3*c**6) + b**2*x**6*atan(c*x)**2
/6 - b**2*x**5*atan(c*x)/(15*c) + b**2*x**4/(60*c**2) + b**2*x**3*atan(c*x)
/(9*c**3) - 4*b**2*x**2/(45*c**4) - b**2*x*atan(c*x)/(3*c**5) + 23*b**2*log
(x**2 + c**(-2))/(90*c**6) + b**2*atan(c*x)**2/(6*c**6), Ne(c, 0)), (a**2*x
**6/6, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.13

$$\int x^5(a+b\arctan(cx))^2\,dx = \frac{1}{6}\,b^2x^6\arctan\left(cx\right)^2 + \frac{1}{6}\,a^2x^6 + \frac{1}{45}\left(15\,x^6\arctan\left(cx\right) - c\left(\frac{3\,c^4x^5 - 5\,c^2x^3 + 15\,x}{c^6} - \frac{15\,\arctan\left(cx\right)}{c^7}\right)\right)ab - \frac{1}{180}\left(4\,c\left(\frac{3\,c^4x^5 - 5\,c^2x^3 + 15\,x}{c^6} - \frac{15\,\arctan\left(cx\right)}{c^7}\right)\arctan\left(cx\right) - \frac{3\,c^4x^4 - 16\,c^2x^2 - 30\,\arctan\left(cx\right)^2 + \cdots}{c^6}\right.$$

[In] integrate(x^5*(a+b*arctan(c*x))^2,x, algorithm="maxima")

[Out] 1/6*b^2*x^6*arctan(c*x)^2 + 1/6*a^2*x^6 + 1/45*(15*x^6*arctan(c*x) - c*((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(c*x)/c^7))*a*b - 1/180*(4*c*((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(c*x)/c^7)*arctan(c*x) - (3*c^4*x^4 - 16*c^2*x^2 - 30*arctan(c*x)^2 + 46*log(c^2*x^2 + 1))/c^6)*b^2

Giac [F]

$$\int x^5(a+b\arctan(cx))^2\,dx = \int \left(b\arctan\left(cx\right) + a\right)^2 x^5\,dx$$

[In] integrate(x^5*(a+b*arctan(c*x))^2,x, algorithm="giac")

[Out] sage0*x

Mupad [B] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.19

$$\int x^5(a+b\arctan(cx))^2\,dx = \frac{30\,b^2\,\mathrm{atan}(c\,x)^2 + 46\,b^2\,\ln\left(c^2\,x^2+1\right) + 30\,a^2\,c^6\,x^6 - 16\,b^2\,c^2\,x^2 + 3\,b^2\,c^4\,x^4 + 60\,a\,b\,\mathrm{atan}(c\,x) + 20\,b^2\,c^3\,x^3\,\mathrm{at}\cdots}{\cdots}$$

[In] int(x^5*(a + b*atan(c*x))^2,x)

[Out] (30*b^2*atan(c*x)^2 + 46*b^2*log(c^2*x^2 + 1) + 30*a^2*c^6*x^6 - 16*b^2*c^2*x^2 + 3*b^2*c^4*x^4 + 60*a*b*atan(c*x) + 20*b^2*c^3*x^3*atan(c*x) - 12*b^2*c^5*x^5*atan(c*x) - 60*b^2*c*x*atan(c*x) + 30*b^2*c^6*x^6*atan(c*x)^2 + 20*a*b*c^3*x^3 - 12*a*b*c^5*x^5 - 60*a*b*c*x + 60*a*b*c^6*x^6*atan(c*x))/(180*c^6)

3.14 $\int x^4(a+b\arctan(cx))^2\,dx$

Optimal result 125
Rubi [A] (verified) 125
Mathematica [A] (verified) 128
Maple [A] (verified) 129
Fricas [F] 129
Sympy [F] 130
Maxima [F] 130
Giac [F] 130
Mupad [F(-1)] 130

Optimal result

Integrand size = 14, antiderivative size = 170

$$\begin{aligned}\int x^4(a+b\arctan(cx))^2\,dx = &-\frac{3b^2x}{10c^4}+\frac{b^2x^3}{30c^2}+\frac{3b^2\arctan(cx)}{10c^5}+\frac{bx^2(a+b\arctan(cx))}{5c^3}\\&-\frac{bx^4(a+b\arctan(cx))}{10c}+\frac{i(a+b\arctan(cx))^2}{5c^5}\\&+\frac{1}{5}x^5(a+b\arctan(cx))^2+\frac{2b(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{5c^5}\\&+\frac{ib^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{5c^5}\end{aligned}$$

```
[Out] -3/10*b^2*x/c^4+1/30*b^2*x^3/c^2+3/10*b^2*arctan(c*x)/c^5+1/5*b*x^2*(a+b*ar
ctan(c*x))/c^3-1/10*b*x^4*(a+b*arctan(c*x))/c+1/5*I*(a+b*arctan(c*x))^2/c^5
+1/5*x^5*(a+b*arctan(c*x))^2+2/5*b*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^5+1/
5*I*b^2*polylog(2,1-2/(1+I*c*x))/c^5
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {4946, 5036, 308, 209, 327, 5040, 4964, 2449, 2352}

$$\begin{aligned}\int x^4(a+b\arctan(cx))^2\,dx = &\frac{i(a+b\arctan(cx))^2}{5c^5}+\frac{2b\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{5c^5}\\&+\frac{bx^2(a+b\arctan(cx))}{5c^3}+\frac{1}{5}x^5(a+b\arctan(cx))^2\\&-\frac{bx^4(a+b\arctan(cx))}{10c}+\frac{3b^2\arctan(cx)}{10c^5}\\&+\frac{ib^2\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{5c^5}-\frac{3b^2x}{10c^4}+\frac{b^2x^3}{30c^2}\end{aligned}$$

```
[In] Int[x^4*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (-3*b^2*x)/(10*c^4) + (b^2*x^3)/(30*c^2) + (3*b^2*ArcTan[c*x])/(10*c^5) + (
b*x^2*(a + b*ArcTan[c*x]))/(5*c^3) - (b*x^4*(a + b*ArcTan[c*x]))/(10*c) + (
(I/5)*(a + b*ArcTan[c*x])^2)/c^5 + (x^5*(a + b*ArcTan[c*x])^2)/5 + (2*b*(a
+ b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(5*c^5) + ((I/5)*b^2*PolyLog[2, 1 - 2/
(1 + I*c*x)])/c^5
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{5}x^5(a+b\arctan(cx))^2-\frac{1}{5}(2bc)\int\frac{x^5(a+b\arctan(cx))}{1+c^2x^2}\,dx\\
&= \frac{1}{5}x^5(a+b\arctan(cx))^2-\frac{(2b)\int x^3(a+b\arctan(cx))\,dx}{5c}+\frac{(2b)\int\frac{x^3(a+b\arctan(cx))}{1+c^2x^2}\,dx}{5c}\\
&= -\frac{bx^4(a+b\arctan(cx))}{10c}+\frac{1}{5}x^5(a+b\arctan(cx))^2+\frac{1}{10}b^2\int\frac{x^4}{1+c^2x^2}\,dx\\
&\quad+\frac{(2b)\int x(a+b\arctan(cx))\,dx}{5c^3}-\frac{(2b)\int\frac{x(a+b\arctan(cx))}{1+c^2x^2}\,dx}{5c^3}\\
&= \frac{bx^2(a+b\arctan(cx))}{5c^3}-\frac{bx^4(a+b\arctan(cx))}{10c}+\frac{i(a+b\arctan(cx))^2}{5c^5}\\
&\quad+\frac{1}{5}x^5(a+b\arctan(cx))^2+\frac{1}{10}b^2\int\left(-\frac{1}{c^4}+\frac{x^2}{c^2}+\frac{1}{c^4\,(1+c^2x^2)}\right)dx\\
&\quad+\frac{(2b)\int\frac{a+b\arctan(cx)}{i-cx}\,dx}{5c^4}-\frac{b^2\int\frac{x^2}{1+c^2x^2}\,dx}{5c^2}\\
&= -\frac{3b^2x}{10c^4}+\frac{b^2x^3}{30c^2}+\frac{bx^2(a+b\arctan(cx))}{5c^3}-\frac{bx^4(a+b\arctan(cx))}{10c}\\
&\quad+\frac{i(a+b\arctan(cx))^2}{5c^5}+\frac{1}{5}x^5(a+b\arctan(cx))^2+\frac{2b(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{5c^5}\\
&\quad+\frac{b^2\int\frac{1}{1+c^2x^2}\,dx}{10c^4}+\frac{b^2\int\frac{1}{1+c^2x^2}\,dx}{5c^4}-\frac{(2b^2)\int\frac{\log\left(\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx}{5c^4}
\end{aligned}$$

$$= -\frac{3b^2x}{10c^4} + \frac{b^2x^3}{30c^2} + \frac{3b^2\arctan(cx)}{10c^5} + \frac{bx^2(a+b\arctan(cx))}{5c^3} - \frac{bx^4(a+b\arctan(cx))}{10c} + \frac{i(a+b\arctan(cx))^2}{5c^5} + \frac{1}{5}x^5(a+b\arctan(cx))^2 + \frac{2b(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{5c^5} + \frac{(2ib^2)\,\text{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1+icx}\right)}{5c^5}$$

$$= -\frac{3b^2x}{10c^4} + \frac{b^2x^3}{30c^2} + \frac{3b^2\arctan(cx)}{10c^5} + \frac{bx^2(a+b\arctan(cx))}{5c^3} - \frac{bx^4(a+b\arctan(cx))}{10c} + \frac{i(a+b\arctan(cx))^2}{5c^5} + \frac{1}{5}x^5(a+b\arctan(cx))^2 + \frac{2b(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{5c^5} + \frac{ib^2\,\text{PolyLog}\left(2, 1-\frac{2}{1+icx}\right)}{5c^5}$$

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.99

$$\int x^4(a+b\arctan(cx))^2\,dx$$

$$= \frac{9ab - 9b^2cx + 6abc^2x^2 + b^2c^3x^3 - 3abc^4x^4 + 6a^2c^5x^5 + 6b^2(-i+c^5x^5)\arctan(cx)^2 - 3b\arctan(cx)\left(-4ac^5\right.}{30}$$

```
[In] Integrate[x^4*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (9*a*b - 9*b^2*c*x + 6*a*b*c^2*x^2 + b^2*c^3*x^3 - 3*a*b*c^4*x^4 + 6*a^2*c^
5*x^5 + 6*b^2*(-I + c^5*x^5)*ArcTan[c*x]^2 - 3*b*ArcTan[c*x]*(-4*a*c^5*x^5
+ b*(-3 - 2*c^2*x^2 + c^4*x^4) - 4*b*Log[1 + E^((2*I)*ArcTan[c*x])]) - 6*a*
b*Log[1 + c^2*x^2] - (6*I)*b^2*PolyLog[2, -E^((2*I)*ArcTan[c*x])])/(30*c^5)
```

Maple [A] (verified)

Time = 1.99 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.56

method	result
parts	$\frac{a^2x^5}{5} + \frac{b^2\left(\frac{c^5x^5\arctan(cx)^2}{5} - \frac{c^4x^4\arctan(cx)}{10} + \frac{c^2x^2\arctan(cx)}{5} - \frac{\arctan(cx)\ln\left(c^2x^2+1\right)}{5} + \frac{c^3x^3}{30} - \frac{3cx}{10} + \frac{3\arctan(cx)}{10} - \frac{i\left(\ln(\right.}{}\right.}{}$
derivativedivides	$\frac{\frac{a^2c^5x^5}{5} + b^2\left(\frac{c^5x^5\arctan(cx)^2}{5} - \frac{c^4x^4\arctan(cx)}{10} + \frac{c^2x^2\arctan(cx)}{5} - \frac{\arctan(cx)\ln\left(c^2x^2+1\right)}{5} + \frac{c^3x^3}{30} - \frac{3cx}{10} + \frac{3\arctan(cx)}{10} - \frac{i\left(\ln(\right.}{}\right.}{}$
default	$\frac{\frac{a^2c^5x^5}{5} + b^2\left(\frac{c^5x^5\arctan(cx)^2}{5} - \frac{c^4x^4\arctan(cx)}{10} + \frac{c^2x^2\arctan(cx)}{5} - \frac{\arctan(cx)\ln\left(c^2x^2+1\right)}{5} + \frac{c^3x^3}{30} - \frac{3cx}{10} + \frac{3\arctan(cx)}{10} - \frac{i\left(\ln(\right.}{}\right.}{}$
risch	$\frac{ib^2\ln(-icx+1)x^2}{10c^3} + \frac{ib^2\ln(icx+1)x^4}{20c} + \frac{137ab}{150c^5} - \frac{ab\,x^4}{10c} + \frac{413ib^2}{2250c^5} + \frac{ia^2}{5c^5} - \frac{b^2\ln(icx+1)^2x^5}{20} - \frac{b^2\ln(-icx+1)^2}{20}$

```
[In] int(x^4*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*a^2*x^5+b^2/c^5*(1/5*c^5*x^5*arctan(c*x)^2-1/10*c^4*x^4*arctan(c*x)+1/5
*c^2*x^2*arctan(c*x)-1/5*arctan(c*x)*ln(c^2*x^2+1)+1/30*c^3*x^3-3/10*c*x+3/
10*arctan(c*x)-1/10*I*(ln(c*x-I)*ln(c^2*x^2+1)-1/2*ln(c*x-I)^2-dilog(-1/2*I
*(c*x+I))-ln(c*x-I)*ln(-1/2*I*(c*x+I)))+1/10*I*(ln(c*x+I)*ln(c^2*x^2+1)-1/2
*ln(c*x+I)^2-dilog(1/2*I*(c*x-I))-ln(c*x+I)*ln(1/2*I*(c*x-I))))+2*a*b/c^5*(
1/5*c^5*x^5*arctan(c*x)-1/20*c^4*x^4+1/10*c^2*x^2-1/10*ln(c^2*x^2+1))
```

Fricas [F]

$$\int x^4(a+b\arctan(cx))^2\,dx = \int (b\arctan(cx)+a)^2x^4\,dx$$

```
[In] integrate(x^4*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^4*arctan(c*x)^2 + 2*a*b*x^4*arctan(c*x) + a^2*x^4, x)
```

Sympy [F]

$$\int x^4(a+b\arctan(cx))^2\,dx=\int x^4(a+b\,\mathrm{atan}\,(cx))^2\,dx$$

```
[In] integrate(x**4*(a+b*atan(c*x))**2,x)
```

```
[Out] Integral(x**4*(a + b*atan(c*x))**2, x)
```

Maxima [F]

$$\int x^4(a+b\arctan(cx))^2\,dx=\int (b\arctan(cx)+a)^2x^4\,dx$$

```
[In] integrate(x^4*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/5*a^2*x^5 + 1/10*(4*x^5*arctan(c*x) - c*((c^2*x^4 - 2*x^2)/c^4 + 2*log(c^
2*x^2 + 1)/c^6))*a*b + 1/80*(4*x^5*arctan(c*x)^2 - x^5*log(c^2*x^2 + 1)^2 +
 80*integrate(1/80*(4*c^2*x^6*log(c^2*x^2 + 1) - 8*c*x^5*arctan(c*x) + 60*(
c^2*x^6 + x^4)*arctan(c*x)^2 + 5*(c^2*x^6 + x^4)*log(c^2*x^2 + 1)^2)/(c^2*x
^2 + 1), x))*b^2
```

Giac [F]

$$\int x^4(a+b\arctan(cx))^2\,dx=\int (b\arctan(cx)+a)^2x^4\,dx$$

```
[In] integrate(x^4*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int x^4(a+b\arctan(cx))^2\,dx=\int x^4\,(a+b\,\mathrm{atan}(c\,x))^2\,dx$$

```
[In] int(x^4*(a + b*atan(c*x))^2,x)
```

```
[Out] int(x^4*(a + b*atan(c*x))^2, x)
```

3.15 $\int x^3(a+b\arctan(cx))^2\,dx$

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Optimal result

Integrand size = 14, antiderivative size = 112

$$\int x^3(a+b\arctan(cx))^2\,dx=\frac{abx}{2c^3}+\frac{b^2x^2}{12c^2}+\frac{b^2x\arctan(cx)}{2c^3}-\frac{bx^3(a+b\arctan(cx))}{6c}-\frac{(a+b\arctan(cx))^2}{4c^4}+\frac{1}{4}x^4(a+b\arctan(cx))^2-\frac{b^2\log(1+c^2x^2)}{3c^4}$$

```
[Out] 1/2*a*b*x/c^3+1/12*b^2*x^2/c^2+1/2*b^2*x*arctan(c*x)/c^3-1/6*b*x^3*(a+b*arc
tan(c*x))/c-1/4*(a+b*arctan(c*x))^2/c^4+1/4*x^4*(a+b*arctan(c*x))^2-1/3*b^2
*ln(c^2*x^2+1)/c^4
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4946, 5036, 272, 45, 4930, 266, 5004}

$$\int x^3(a+b\arctan(cx))^2\,dx=-\frac{(a+b\arctan(cx))^2}{4c^4}+\frac{1}{4}x^4(a+b\arctan(cx))^2-\frac{bx^3(a+b\arctan(cx))}{6c}+\frac{abx}{2c^3}+\frac{b^2x\arctan(cx)}{2c^3}+\frac{b^2x^2}{12c^2}-\frac{b^2\log(c^2x^2+1)}{3c^4}$$

```
[In] Int[x^3*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (a*b*x)/(2*c^3) + (b^2*x^2)/(12*c^2) + (b^2*x*ArcTan[c*x])/(2*c^3) - (b*x^3
*(a + b*ArcTan[c*x]))/(6*c) - (a + b*ArcTan[c*x])^2/(4*c^4) + (x^4*(a + b*A
rcTan[c*x])^2)/4 - (b^2*Log[1 + c^2*x^2])/(3*c^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}x^4(a+b\arctan(cx))^2 - \frac{1}{2}(bc)\int \frac{x^4(a+b\arctan(cx))}{1+c^2x^2}\,dx \\
&= \frac{1}{4}x^4(a+b\arctan(cx))^2 - \frac{b\int x^2(a+b\arctan(cx))\,dx}{2c} + \frac{b\int \frac{x^2(a+b\arctan(cx))}{1+c^2x^2}\,dx}{2c} \\
&= -\frac{bx^3(a+b\arctan(cx))}{6c} + \frac{1}{4}x^4(a+b\arctan(cx))^2 \\
&\quad + \frac{1}{6}b^2\int \frac{x^3}{1+c^2x^2}\,dx + \frac{b\int(a+b\arctan(cx))\,dx}{2c^3} - \frac{b\int \frac{a+b\arctan(cx)}{1+c^2x^2}\,dx}{2c^3} \\
&= \frac{abx}{2c^3} - \frac{bx^3(a+b\arctan(cx))}{6c} - \frac{(a+b\arctan(cx))^2}{4c^4} + \frac{1}{4}x^4(a+b\arctan(cx))^2 \\
&\quad + \frac{1}{12}b^2\text{Subst}\left(\int \frac{x}{1+c^2x}\,dx, x, x^2\right) + \frac{b^2\int \arctan(cx)\,dx}{2c^3} \\
&= \frac{abx}{2c^3} + \frac{b^2x\arctan(cx)}{2c^3} - \frac{bx^3(a+b\arctan(cx))}{6c} \\
&\quad - \frac{(a+b\arctan(cx))^2}{4c^4} + \frac{1}{4}x^4(a+b\arctan(cx))^2 \\
&\quad + \frac{1}{12}b^2\text{Subst}\left(\int \left(\frac{1}{c^2} - \frac{1}{c^2(1+c^2x)}\right)\,dx, x, x^2\right) - \frac{b^2\int \frac{x}{1+c^2x^2}\,dx}{2c^2} \\
&= \frac{abx}{2c^3} + \frac{b^2x^2}{12c^2} + \frac{b^2x\arctan(cx)}{2c^3} - \frac{bx^3(a+b\arctan(cx))}{6c} \\
&\quad - \frac{(a+b\arctan(cx))^2}{4c^4} + \frac{1}{4}x^4(a+b\arctan(cx))^2 - \frac{b^2\log(1+c^2x^2)}{3c^4}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.99

$$\int x^3(a+b\arctan(cx))^2\,dx$$
$$= \frac{cx(6ab+b^2cx-2abc^2x^2+3a^2c^3x^3) - 2b(bcx(-3+c^2x^2)+a(3-3c^4x^4))\arctan(cx) + 3b^2(-1+c^4x^4)\,\mathrm{a}}{12c^4}$$

```
[In] Integrate[x^3*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (c*x*(6*a*b + b^2*c*x - 2*a*b*c^2*x^2 + 3*a^2*c^3*x^3) - 2*b*(b*c*x*(-3 + c
^2*x^2) + a*(3 - 3*c^4*x^4))*ArcTan[c*x] + 3*b^2*(-1 + c^4*x^4)*ArcTan[c*x]
^2 - 4*b^2*Log[1 + c^2*x^2])/(12*c^4)
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.05

method	result
parts	$\frac{a^2x^4}{4}+\frac{b^2\left(\frac{c^4x^4\arctan(cx)^2}{4}-\frac{c^3x^3\arctan(cx)}{6}+\frac{cx\arctan(cx)}{2}-\frac{\arctan(cx)^2}{4}+\frac{c^2x^2}{12}-\frac{\ln(c^2x^2+1)}{3}\right)}{c^4}+\frac{2ab\left(\frac{c^4x^4\arctan(cx)}{4}-\right.}{}$
derivativedivides	$\frac{\frac{a^2c^4x^4}{4}+b^2\left(\frac{c^4x^4\arctan(cx)^2}{4}-\frac{c^3x^3\arctan(cx)}{6}+\frac{cx\arctan(cx)}{2}-\frac{\arctan(cx)^2}{4}+\frac{c^2x^2}{12}-\frac{\ln(c^2x^2+1)}{3}\right)+2ab\left(\frac{c^4x^4\arctan(cx)}{4}-\frac{c^3}{1}\right.}{c^4}$
default	$\frac{\frac{a^2c^4x^4}{4}+b^2\left(\frac{c^4x^4\arctan(cx)^2}{4}-\frac{c^3x^3\arctan(cx)}{6}+\frac{cx\arctan(cx)}{2}-\frac{\arctan(cx)^2}{4}+\frac{c^2x^2}{12}-\frac{\ln(c^2x^2+1)}{3}\right)+2ab\left(\frac{c^4x^4\arctan(cx)}{4}-\frac{c^3}{1}\right.}{c^4}$
parallelrisch	$-\frac{-3x^4\arctan(cx)^2b^2c^4-6x^4\arctan(cx)ab\,c^4-3a^2c^4x^4+2b^2\arctan(cx)x^3c^3+2ab\,c^3x^3-b^2c^2x^2-6b^2\arctan(cx)xc-6abc}{12c^4}$
risch	$-\frac{b^2(c^4x^4-1)\ln(icx+1)^2}{16c^4}-\frac{ib(6a\,c^4x^4+3ib\,c^4x^4\ln(-icx+1)-2b\,c^3x^3+6xbc-3ib\ln(-icx+1))\ln(icx+1)}{24c^4}-\frac{b^2x^4\ln(-}{1}$

```
[In] int(x^3*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a^2*x^4+b^2/c^4*(1/4*c^4*x^4*arctan(c*x)^2-1/6*c^3*x^3*arctan(c*x)+1/2*
c*x*arctan(c*x)-1/4*arctan(c*x)^2+1/12*c^2*x^2-1/3*ln(c^2*x^2+1))+2*a*b/c^4
*(1/4*c^4*x^4*arctan(c*x)-1/12*c^3*x^3+1/4*c*x-1/4*arctan(c*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.08

$$\int x^3(a+b\arctan(cx))^2\,dx$$
$$=\frac{3\,a^2c^4x^4-2\,abc^3x^3+b^2c^2x^2+6\,abcx+3\,(b^2c^4x^4-b^2)\arctan{(cx)}^2-4\,b^2\log{(c^2x^2+1)}+2\,(3\,abc^4x^4-b^2c}{12\,c^4}$$

```
[In] integrate(x^3*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*a^2*c^4*x^4 - 2*a*b*c^3*x^3 + b^2*c^2*x^2 + 6*a*b*c*x + 3*(b^2*c^4*
x^4 - b^2)*arctan(c*x)^2 - 4*b^2*log(c^2*x^2 + 1) + 2*(3*a*b*c^4*x^4 - b^2*
c^3*x^3 + 3*b^2*c*x - 3*a*b)*arctan(c*x))/c^4
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.38

$$\int x^3(a+b\arctan(cx))^2\,dx$$
$$=\begin{cases}\frac{a^2x^4}{4}+\frac{abx^4\,\mathrm{atan}\,(cx)}{2}-\frac{abx^3}{6c}+\frac{abx}{2c^3}-\frac{ab\,\mathrm{atan}\,(cx)}{2c^4}+\frac{b^2x^4\,\mathrm{atan}^2\,(cx)}{4}-\frac{b^2x^3\,\mathrm{atan}\,(cx)}{6c}+\frac{b^2x^2}{12c^2}+\frac{b^2x\,\mathrm{atan}\,(cx)}{2c^3}-\frac{b^2\log\left(x^2+\frac{1}{c}\right.}{3c^4}\\ \frac{a^2x^4}{4}\end{cases}$$

```
[In] integrate(x**3*(a+b*atan(c*x))**2,x)
```

```
[Out] Piecewise((a**2*x**4/4 + a*b*x**4*atan(c*x)/2 - a*b*x**3/(6*c) + a*b*x/(2*c
**3) - a*b*atan(c*x)/(2*c**4) + b**2*x**4*atan(c*x)**2/4 - b**2*x**3*atan(c
*x)/(6*c) + b**2*x**2/(12*c**2) + b**2*x*atan(c*x)/(2*c**3) - b**2*log(x**2
 + c**(-2))/(3*c**4) - b**2*atan(c*x)**2/(4*c**4), Ne(c, 0)), (a**2*x**4/4,
 True))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.21

$$\int x^3(a+b\arctan(cx))^2\,dx$$
$$=\frac{1}{4}b^2x^4\arctan(cx)^2+\frac{1}{4}a^2x^4+\frac{1}{6}\left(3x^4\arctan(cx)-c\left(\frac{c^2x^3-3x}{c^4}+\frac{3\arctan(cx)}{c^5}\right)\right)ab$$
$$-\frac{1}{12}\left(2c\left(\frac{c^2x^3-3x}{c^4}+\frac{3\arctan(cx)}{c^5}\right)\arctan(cx)-\frac{c^2x^2+3\arctan(cx)^2-4\log(c^2x^2+1)}{c^4}\right)b^2$$

```
[In] integrate(x^3*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*x^4*arctan(c*x)^2 + 1/4*a^2*x^4 + 1/6*(3*x^4*arctan(c*x) - c*((c^2*
x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*a*b - 1/12*(2*c*((c^2*x^3 - 3*x)/c^4 +
 3*arctan(c*x)/c^5)*arctan(c*x) - (c^2*x^2 + 3*arctan(c*x)^2 - 4*log(c^2*x^
2 + 1))/c^4)*b^2
```

Giac [F]

$$\int x^3(a+b\arctan(cx))^2\,dx=\int (b\arctan{(cx)}+a)^2x^3\,dx$$

[In] integrate(x^3*(a+b*arctan(c*x))^2,x, algorithm="giac")

[Out] sage0*x

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.20

$$\int x^3(a+b\arctan(cx))^2\,dx$$
$$=\frac{3\,a^2\,c^4\,x^4-4\,b^2\,\ln{(c^2\,x^2+1)}-3\,b^2\,\mathrm{atan}(c\,x)^2+b^2\,c^2\,x^2-6\,a\,b\,\mathrm{atan}(c\,x)-2\,b^2\,c^3\,x^3\,\mathrm{atan}(c\,x)+6\,b^2\,c\,x\,\mathrm{ata}}{12\,c^4}$$

[In] int(x^3*(a + b*atan(c*x))^2,x)

[Out] (3*a^2*c^4*x^4 - 4*b^2*log(c^2*x^2 + 1) - 3*b^2*atan(c*x)^2 + b^2*c^2*x^2 - 6*a*b*atan(c*x) - 2*b^2*c^3*x^3*atan(c*x) + 6*b^2*c*x*atan(c*x) + 3*b^2*c^4*x^4*atan(c*x)^2 - 2*a*b*c^3*x^3 + 6*a*b*c*x + 6*a*b*c^4*x^4*atan(c*x))/(12*c^4)

3.16 $\int x^2(a+b\arctan(cx))^2\,dx$

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Rubi [A] (verified) 137
Mathematica [A] (verified) 140
Maple [A] (verified) 140
Fricas [F] 141
Sympy [F] 141
Maxima [F] 141
Giac [F] 141
Mupad [F(-1)] 142

Optimal result

Integrand size = 14, antiderivative size = 138

$$\int x^2(a+b\arctan(cx))^2\,dx=\frac{b^2x}{3c^2}-\frac{b^2\arctan(cx)}{3c^3}-\frac{bx^2(a+b\arctan(cx))}{3c}-\frac{i(a+b\arctan(cx))^2}{3c^3}+\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2b(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{3c^3}-\frac{ib^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{3c^3}$$

```
[Out] 1/3*b^2*x/c^2-1/3*b^2*arctan(c*x)/c^3-1/3*b*x^2*(a+b*arctan(c*x))/c-1/3*I*(
a+b*arctan(c*x))^2/c^3+1/3*x^3*(a+b*arctan(c*x))^2-2/3*b*(a+b*arctan(c*x))*
ln(2/(1+I*c*x))/c^3-1/3*I*b^2*polylog(2,1-2/(1+I*c*x))/c^3
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {4946, 5036, 327, 209, 5040, 4964, 2449, 2352}

$$\int x^2(a+b\arctan(cx))^2\,dx=-\frac{i(a+b\arctan(cx))^2}{3c^3}-\frac{2b\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{3c^3}+\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{bx^2(a+b\arctan(cx))}{3c}-\frac{b^2\arctan(cx)}{3c^3}-\frac{ib^2\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{3c^3}+\frac{b^2x}{3c^2}$$

```
[In] Int[x^2*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (b^2*x)/(3*c^2) - (b^2*ArcTan[c*x])/(3*c^3) - (b*x^2*(a + b*ArcTan[c*x]))/(
3*c) - ((I/3)*(a + b*ArcTan[c*x])^2)/c^3 + (x^3*(a + b*ArcTan[c*x])^2)/3 -
```

```
(2*b*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(3*c^3) - ((I/3)*b^2*PolyLog[2
, 1 - 2/(1 + I*c*x)])/c^3
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
```

```
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}x^3(a+b\arctan(cx))^2 - \frac{1}{3}(2bc)\int \frac{x^3(a+b\arctan(cx))}{1+c^2x^2}\,dx \\
&= \frac{1}{3}x^3(a+b\arctan(cx))^2 - \frac{(2b)\int x(a+b\arctan(cx))\,dx}{3c} + \frac{(2b)\int \frac{x(a+b\arctan(cx))}{1+c^2x^2}\,dx}{3c} \\
&= -\frac{bx^2(a+b\arctan(cx))}{3c} - \frac{i(a+b\arctan(cx))^2}{3c^3} \\
&\quad + \frac{1}{3}x^3(a+b\arctan(cx))^2 + \frac{1}{3}b^2\int \frac{x^2}{1+c^2x^2}\,dx - \frac{(2b)\int \frac{a+b\arctan(cx)}{i-cx}\,dx}{3c^2} \\
&= \frac{b^2x}{3c^2} - \frac{bx^2(a+b\arctan(cx))}{3c} - \frac{i(a+b\arctan(cx))^2}{3c^3} + \frac{1}{3}x^3(a+b\arctan(cx))^2 \\
&\quad - \frac{2b(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{3c^3} - \frac{b^2\int \frac{1}{1+c^2x^2}\,dx}{3c^2} + \frac{(2b^2)\int \frac{\log\left(\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx}{3c^2} \\
&= \frac{b^2x}{3c^2} - \frac{b^2\arctan(cx)}{3c^3} - \frac{bx^2(a+b\arctan(cx))}{3c} - \frac{i(a+b\arctan(cx))^2}{3c^3} + \frac{1}{3}x^3(a \\
&\quad + b\arctan(cx))^2 \\
&\quad - \frac{2b(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{3c^3} - \frac{(2ib^2)\,\text{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1+icx}\right)}{3c^3} \\
&= \frac{b^2x}{3c^2} - \frac{b^2\arctan(cx)}{3c^3} - \frac{bx^2(a+b\arctan(cx))}{3c} \\
&\quad - \frac{i(a+b\arctan(cx))^2}{3c^3} + \frac{1}{3}x^3(a+b\arctan(cx))^2 \\
&\quad - \frac{2b(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{3c^3} - \frac{ib^2\,\text{PolyLog}\left(2, 1-\frac{2}{1+icx}\right)}{3c^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.95

$$\int x^2(a+b\arctan(cx))^2\,dx = \frac{b^2cx-abc^2x^2+a^2c^3x^3+b^2(i+c^3x^3)\arctan(cx)^2-b\arctan(cx)\left(b+bc^2x^2-2ac^3x^3+2b\log\left(1+e^{2i\arctan\ldots}\right.\right.}{3c^3}$$

[In] Integrate[x^2*(a + b*ArcTan[c*x])^2,x]

[Out] (b^2*c*x - a*b*c^2*x^2 + a^2*c^3*x^3 + b^2*(I + c^3*x^3)*ArcTan[c*x]^2 - b*ArcTan[c*x]*(b + b*c^2*x^2 - 2*a*c^3*x^3 + 2*b*Log[1 + E^((2*I)*ArcTan[c*x])]) + a*b*Log[1 + c^2*x^2] + I*b^2*PolyLog[2, -E^((2*I)*ArcTan[c*x])])/(3*c^3)

Maple [A] (verified)

Time = 1.83 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.72

method	result
parts	$\frac{a^2x^3}{3}+\frac{b^2\left(\frac{c^3x^3\arctan(cx)^2}{3}-\frac{c^2x^2\arctan(cx)}{3}+\frac{\arctan(cx)\ln\left(c^2x^2+1\right)}{3}+\frac{cx}{3}-\frac{\arctan(cx)}{3}+\frac{i\left(\ln(cx-i)\ln\left(c^2x^2+1\right)-\frac{\ln(cx-i)^2}{2}\ldots\right.}{\ldots}\right.}{\ldots}$
derivativedivides	$\frac{\frac{a^2c^3x^3}{3}+b^2\left(\frac{c^3x^3\arctan(cx)^2}{3}-\frac{c^2x^2\arctan(cx)}{3}+\frac{\arctan(cx)\ln\left(c^2x^2+1\right)}{3}+\frac{cx}{3}-\frac{\arctan(cx)}{3}+\frac{i\left(\ln(cx-i)\ln\left(c^2x^2+1\right)-\frac{\ln(cx-i)^2}{2}\ldots\right.}{\ldots}\right.}{\ldots}$
default	$\frac{\frac{a^2c^3x^3}{3}+b^2\left(\frac{c^3x^3\arctan(cx)^2}{3}-\frac{c^2x^2\arctan(cx)}{3}+\frac{\arctan(cx)\ln\left(c^2x^2+1\right)}{3}+\frac{cx}{3}-\frac{\arctan(cx)}{3}+\frac{i\left(\ln(cx-i)\ln\left(c^2x^2+1\right)-\frac{\ln(cx-i)^2}{2}\ldots\right.}{\ldots}\right.}{\ldots}$
risch	$\frac{ib^2\ln(-icx+1)^2}{12c^3}+\frac{b^2\ln(icx+1)\ln(-icx+1)x^3}{6}-\frac{2ib^2\ln\left(c^2x^2+1\right)}{9c^3}+\frac{5ib^2\ln(-icx+1)}{36c^3}-\frac{ib^2\operatorname{dilog}\left(\frac{1}{2}-\frac{icx}{2}\right)}{3c^3}-\frac{ib^2\ldots}{}$

[In] int(x^2*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)

[Out] 1/3*a^2*x^3+b^2/c^3*(1/3*c^3*x^3*arctan(c*x)^2-1/3*c^2*x^2*arctan(c*x)+1/3*arctan(c*x)*ln(c^2*x^2+1)+1/3*c*x-1/3*arctan(c*x)+1/6*I*(ln(c*x-I)*ln(c^2*x^2+1)-1/2*ln(c*x-I)^2-dilog(-1/2*I*(c*x+I))-ln(c*x-I)*ln(-1/2*I*(c*x+I)))-1/6*I*(ln(c*x+I)*ln(c^2*x^2+1)-1/2*ln(c*x+I)^2-dilog(1/2*I*(c*x-I))-ln(c*x+I)*ln(1/2*I*(c*x-I))))+2*a*b/c^3*(1/3*c^3*x^3*arctan(c*x)-1/6*c^2*x^2+1/6*ln(c^2*x^2+1))

Fricas [F]

$$\int x^2(a+b\arctan(cx))^2\,dx=\int(b\arctan(cx)+a)^2x^2\,dx$$

```
[In] integrate(x^2*(a+b*arctan(c*x))^2,x, algorithm="fricas")
[Out] integral(b^2*x^2*arctan(c*x)^2 + 2*a*b*x^2*arctan(c*x) + a^2*x^2, x)
```

Sympy [F]

$$\int x^2(a+b\arctan(cx))^2\,dx=\int x^2(a+b\,\mathrm{atan}(cx))^2\,dx$$

```
[In] integrate(x**2*(a+b*atan(c*x))**2,x)
[Out] Integral(x**2*(a + b*atan(c*x))**2, x)
```

Maxima [F]

$$\int x^2(a+b\arctan(cx))^2\,dx=\int(b\arctan(cx)+a)^2x^2\,dx$$

```
[In] integrate(x^2*(a+b*arctan(c*x))^2,x, algorithm="maxima")
[Out] 1/3*a^2*x^3 + 1/3*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*
a*b + 1/48*(4*x^3*arctan(c*x)^2 - x^3*log(c^2*x^2 + 1)^2 + 48*integrate(1/4
8*(4*c^2*x^4*log(c^2*x^2 + 1) - 8*c*x^3*arctan(c*x) + 36*(c^2*x^4 + x^2)*ar
ctan(c*x)^2 + 3*(c^2*x^4 + x^2)*log(c^2*x^2 + 1)^2)/(c^2*x^2 + 1), x))*b^2
```

Giac [F]

$$\int x^2(a+b\arctan(cx))^2\,dx=\int(b\arctan(cx)+a)^2x^2\,dx$$

```
[In] integrate(x^2*(a+b*arctan(c*x))^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int x^2(a+b\arctan(cx))^2\,dx=\int x^2\,(a+b\,\mathrm{atan}(c\,x))^2\,dx$$

```
[In] int(x^2*(a + b*atan(c*x))^2,x)
```

```
[Out] int(x^2*(a + b*atan(c*x))^2, x)
```

3.17 $\int x(a+b\arctan(cx))^2\,dx$

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Sympy [A] (verification not implemented) 146
Maxima [A] (verification not implemented) 146
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Optimal result

Integrand size = 12, antiderivative size = 76

$$\int x(a+b\arctan(cx))^2\,dx=-\frac{abx}{c}-\frac{b^2x\arctan(cx)}{c}+\frac{(a+b\arctan(cx))^2}{2c^2}+\frac{1}{2}x^2(a+b\arctan(cx))^2+\frac{b^2\log\left(1+c^2x^2\right)}{2c^2}$$

```
[Out] -a*b*x/c-b^2*x*arctan(c*x)/c+1/2*(a+b*arctan(c*x))^2/c^2+1/2*x^2*(a+b*arcta
n(c*x))^2+1/2*b^2*ln(c^2*x^2+1)/c^2
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {4946, 5036, 4930, 266, 5004}

$$\int x(a+b\arctan(cx))^2\,dx=\frac{(a+b\arctan(cx))^2}{2c^2}+\frac{1}{2}x^2(a+b\arctan(cx))^2-\frac{abx}{c}-\frac{b^2x\arctan(cx)}{c}+\frac{b^2\log\left(c^2x^2+1\right)}{2c^2}$$

```
[In] Int[x*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] -((a*b*x)/c) - (b^2*x*ArcTan[c*x])/c + (a + b*ArcTan[c*x])^2/(2*c^2) + (x^2
*(a + b*ArcTan[c*x])^2)/2 + (b^2*Log[1 + c^2*x^2])/(2*c^2)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rubi steps

$$\text{integral} = \frac{1}{2}x^2(a+b\arctan(cx))^2 - (bc)\int \frac{x^2(a+b\arctan(cx))}{1+c^2x^2}\,dx$$

$$= \frac{1}{2}x^2(a+b\arctan(cx))^2 - \frac{b\int (a+b\arctan(cx))\,dx}{c} + \frac{b\int \frac{a+b\arctan(cx)}{1+c^2x^2}\,dx}{c}$$

$$= -\frac{abx}{c} + \frac{(a+b\arctan(cx))^2}{2c^2} + \frac{1}{2}x^2(a+b\arctan(cx))^2 - \frac{b^2\int \arctan(cx)\,dx}{c}$$

$$= -\frac{abx}{c} - \frac{b^2x\arctan(cx)}{c} + \frac{(a+b\arctan(cx))^2}{2c^2} + \frac{1}{2}x^2(a+b\arctan(cx))^2 + b^2\int \frac{x}{1+c^2x^2}\,dx$$

$$= -\frac{abx}{c} - \frac{b^2x\arctan(cx)}{c} + \frac{(a+b\arctan(cx))^2}{2c^2} + \frac{1}{2}x^2(a+b\arctan(cx))^2 + \frac{b^2\log(1+c^2x^2)}{2c^2}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.99

$$\int x(a+b\arctan(cx))^2\,dx$$
$$=\frac{acx(-2b+acx)+2b(a-bcx+ac^2x^2)\arctan(cx)+b^2(1+c^2x^2)\arctan(cx)^2+b^2\log(1+c^2x^2)}{2c^2}$$

```
[In] Integrate[x*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (a*c*x*(-2*b + a*c*x) + 2*b*(a - b*c*x + a*c^2*x^2)*ArcTan[c*x] + b^2*(1 +
c^2*x^2)*ArcTan[c*x]^2 + b^2*Log[1 + c^2*x^2])/(2*c^2)
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.16

method	result
parts	$\frac{a^2x^2}{2}+\frac{b^2\left(\frac{c^2x^2\arctan(cx)^2}{2}+\frac{\arctan(cx)^2}{2}-cx\arctan(cx)+\frac{\ln(c^2x^2+1)}{2}\right)}{c^2}+x^2ab\arctan(cx)-\frac{abx}{c}+\frac{ab\,\mathrm{arct}}{c}$
derivativedivides	$\frac{\frac{c^2x^2a^2}{2}+b^2\left(\frac{c^2x^2\arctan(cx)^2}{2}+\frac{\arctan(cx)^2}{2}-cx\arctan(cx)+\frac{\ln(c^2x^2+1)}{2}\right)+2ab\left(\frac{c^2x^2\arctan(cx)}{2}-\frac{cx}{2}+\frac{\arctan(cx)}{2}\right)}{c^2}$
default	$\frac{\frac{c^2x^2a^2}{2}+b^2\left(\frac{c^2x^2\arctan(cx)^2}{2}+\frac{\arctan(cx)^2}{2}-cx\arctan(cx)+\frac{\ln(c^2x^2+1)}{2}\right)+2ab\left(\frac{c^2x^2\arctan(cx)}{2}-\frac{cx}{2}+\frac{\arctan(cx)}{2}\right)}{c^2}$
parallelrisch	$\frac{b^2\arctan(cx)^2x^2c^2+2ab\arctan(cx)x^2c^2+c^2x^2a^2-2b^2\arctan(cx)xc-2abcx+b^2\arctan(cx)^2+b^2\ln(c^2x^2+1)+2ab\,\mathrm{arctan}}{2c^2}$
risch	$-\frac{b^2(c^2x^2+1)\ln(icx+1)^2}{8c^2}-\frac{ib(2c^2x^2a+ib\,c^2x^2\ln(-icx+1)-2xbc+ib\ln(-icx+1))\ln(icx+1)}{4c^2}+\frac{iab\,x^2\ln(-icx+1)}{2}$

```
[In] int(x*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a^2*x^2+b^2/c^2*(1/2*c^2*x^2*arctan(c*x)^2+1/2*arctan(c*x)^2-c*x*arctan
(c*x)+1/2*ln(c^2*x^2+1))+x^2*a*b*arctan(c*x)-a*b*x/c+1/c^2*a*b*arctan(c*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.09

$$\int x(a+b\arctan(cx))^2\,dx = \frac{a^2c^2x^2 - 2\,abcx + (b^2c^2x^2 + b^2)\arctan(cx)^2 + b^2\log(c^2x^2+1) + 2\,(abc^2x^2 - b^2cx + ab)\arctan(cx)}{2\,c^2}$$

```
[In] integrate(x*(a+b*arctan(c*x))^2,x, algorithm="fricas")
[Out] 1/2*(a^2*c^2*x^2 - 2*a*b*c*x + (b^2*c^2*x^2 + b^2)*arctan(c*x)^2 + b^2*log(
c^2*x^2 + 1) + 2*(a*b*c^2*x^2 - b^2*c*x + a*b)*arctan(c*x))/c^2
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.41

$$\int x(a+b\arctan(cx))^2\,dx = \begin{cases} \frac{a^2x^2}{2} + abx^2\operatorname{atan}(cx) - \frac{abx}{c} + \frac{ab\operatorname{atan}(cx)}{c^2} + \frac{b^2x^2\operatorname{atan}^2(cx)}{2} - \frac{b^2x\operatorname{atan}(cx)}{c} + \frac{b^2\log\left(x^2+\frac{1}{c^2}\right)}{2c^2} + \frac{b^2\operatorname{atan}^2(cx)}{2c^2} & \text{for } c \neq 0 \\ \frac{a^2x^2}{2} & \text{otherwise} \end{cases}$$

```
[In] integrate(x*(a+b*atan(c*x))**2,x)
[Out] Piecewise((a**2*x**2/2 + a*b*x**2*atan(c*x) - a*b*x/c + a*b*atan(c*x)/c**2
+ b**2*x**2*atan(c*x)**2/2 - b**2*x*atan(c*x)/c + b**2*log(x**2 + c**(-2))/
(2*c**2) + b**2*atan(c*x)**2/(2*c**2), Ne(c, 0)), (a**2*x**2/2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.37

$$\begin{aligned}\int x(a+b\arctan(cx))^2\,dx &= \frac{1}{2}\,b^2x^2\arctan(cx)^2 + \frac{1}{2}\,a^2x^2 + \left(x^2\arctan(cx) - c\left(\frac{x}{c^2} - \frac{\arctan(cx)}{c^3}\right)\right)ab \\ &\quad - \frac{1}{2}\left(2\,c\left(\frac{x}{c^2} - \frac{\arctan(cx)}{c^3}\right)\arctan(cx) + \frac{\arctan(cx)^2 - \log(c^2x^2+1)}{c^2}\right)b^2\end{aligned}$$

```
[In] integrate(x*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^2*arctan(c*x)^2 + 1/2*a^2*x^2 + (x^2*arctan(c*x) - c*(x/c^2 - arc
tan(c*x)/c^3))*a*b - 1/2*(2*c*(x/c^2 - arctan(c*x)/c^3)*arctan(c*x) + (arct
an(c*x)^2 - log(c^2*x^2 + 1))/c^2)*b^2
```

Giac [F]

$$\int x(a+b\arctan(cx))^2\,dx=\int (b\arctan{(cx)}+a)^2x\,dx$$

```
[In] integrate(x*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.16

$$\int x(a+b\arctan(cx))^2\,dx = \frac{\frac{b^2\,\mathrm{atan}(c\,x)^2}{2}+\frac{b^2\,\ln(c^2\,x^2+1)}{2}-c\,(x\,\mathrm{atan}(c\,x)\;b^2+a\,x\,b)+a\,b\,\mathrm{atan}(c\,x)}{c^2} + \frac{a^2\,x^2}{2}+\frac{b^2\,x^2\,\mathrm{atan}(c\,x)^2}{2}+a\,b\,x^2\,\mathrm{atan}(c\,x)$$

```
[In] int(x*(a + b*atan(c*x))^2,x)
```

```
[Out] ((b^2*atan(c*x)^2)/2 + (b^2*log(c^2*x^2 + 1))/2 - c*(b^2*x*atan(c*x) + a*b*
x) + a*b*atan(c*x))/c^2 + (a^2*x^2)/2 + (b^2*x^2*atan(c*x)^2)/2 + a*b*x^2*a
tan(c*x)
```

3.18 $\int (a+b\arctan(cx))^2\,dx$

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Optimal result

Integrand size = 10, antiderivative size = 83

$$\int (a+b\arctan(cx))^2\,dx=\frac{i(a+b\arctan(cx))^2}{c}+x(a+b\arctan(cx))^2+\frac{2b(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c}+\frac{ib^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c}$$

```
[Out] I*(a+b*arctan(c*x))^2/c+x*(a+b*arctan(c*x))^2+2*b*(a+b*arctan(c*x))*ln(2/(1
+I*c*x))/c+I*b^2*polylog(2,1-2/(1+I*c*x))/c
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4930, 5040, 4964, 2449, 2352}

$$\int (a+b\arctan(cx))^2\,dx=x(a+b\arctan(cx))^2+\frac{i(a+b\arctan(cx))^2}{c}+\frac{2b\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}+\frac{ib^2\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{c}$$

```
[In] Int[(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (I*(a + b*ArcTan[c*x])^2)/c + x*(a + b*ArcTan[c*x])^2 + (2*b*(a + b*ArcTan[
c*x])*Log[2/(1 + I*c*x)])/c + (I*b^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/c
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x(a+b\arctan(cx))^2-(2bc)\int\frac{x(a+b\arctan(cx))}{1+c^2x^2}\,dx\\
&=\frac{i(a+b\arctan(cx))^2}{c}+x(a+b\arctan(cx))^2+(2b)\int\frac{a+b\arctan(cx)}{i-cx}\,dx\\
&=\frac{i(a+b\arctan(cx))^2}{c}+x(a+b\arctan(cx))^2\\
&\quad+\frac{2b(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c}-\left(2b^2\right)\int\frac{\log\left(\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx\\
&=\frac{i(a+b\arctan(cx))^2}{c}+x(a+b\arctan(cx))^2\\
&\quad+\frac{2b(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c}+\frac{\left(2ib^2\right)\text{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx,x,\frac{1}{1+icx}\right)}{c}\\
&=\frac{i(a+b\arctan(cx))^2}{c}+x(a+b\arctan(cx))^2\\
&\quad+\frac{2b(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c}+\frac{ib^2\,\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.08

$$\int (a+b\arctan(cx))^2\,dx$$
$$= \frac{b^2(-i+cx)\arctan(cx)^2+2b\arctan(cx)\left(acx+b\log\left(1+e^{2i\arctan(cx)}\right)\right)+a(acx-b\log(1+c^2x^2))-ib^2\,\mathrm{P}}{c}$$

```
[In] Integrate[(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (b^2*(-I + c*x)*ArcTan[c*x]^2 + 2*b*ArcTan[c*x]*(a*c*x + b*Log[1 + E^((2*I)
*ArcTan[c*x])]) + a*(a*c*x - b*Log[1 + c^2*x^2]) - I*b^2*PolyLog[2, -E^((2*
I)*ArcTan[c*x])])/c
```

Maple [A] (verified)

Time = 2.19 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.48

method	result
derivativedivides	$\frac{cx\,a^2-i\arctan(cx)^2b^2+\arctan(cx)^2b^2cx+2\arctan(cx)\ln\left(1+\frac{(icx+1)^2}{c^2x^2+1}\right)b^2-i\,\mathrm{polylog}\left(2,-\frac{(icx+1)^2}{c^2x^2+1}\right)b^2+2abcx\arctan(cx}{c}$
default	$\frac{cx\,a^2-i\arctan(cx)^2b^2+\arctan(cx)^2b^2cx+2\arctan(cx)\ln\left(1+\frac{(icx+1)^2}{c^2x^2+1}\right)b^2-i\,\mathrm{polylog}\left(2,-\frac{(icx+1)^2}{c^2x^2+1}\right)b^2+2abcx\arctan(cx}{c}$
parts	$a^2x+b^2\arctan(cx)^2x-\frac{ib^2\arctan(cx)^2}{c}-\frac{ib^2\,\mathrm{polylog}\left(2,-\frac{(icx+1)^2}{c^2x^2+1}\right)}{c}+\frac{2b^2\arctan(cx)\ln\left(1+\frac{(icx+1)^2}{c^2x^2+1}\right)}{c}$
risch	$\frac{ib^2\ln\left(\frac{1}{2}+\frac{icx}{2}\right)\ln\left(\frac{1}{2}-\frac{icx}{2}\right)}{c}-\frac{ba\ln(icx+1)}{c}+\frac{b^2\ln(icx+1)\ln(-icx+1)x}{2}+\frac{ib^2\ln(icx+1)\ln(-icx+1)}{2c}+\frac{2ab}{c}-\frac{b^2\ln}{}$

```
[In] int((a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(c*x*a^2-I*arctan(c*x)^2*b^2+arctan(c*x)^2*b^2*c*x+2*arctan(c*x)*ln(1+(
1+I*c*x)^2/(c^2*x^2+1))*b^2-I*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))*b^2+2*a*b
*c*x*arctan(c*x)-a*b*ln(c^2*x^2+1))
```

Fricas [F]

$$\int (a+b\arctan(cx))^2\,dx=\int (b\arctan(cx)+a)^2\,dx$$

```
[In] integrate((a+b*arctan(c*x))^2,x, algorithm="fricas")
[Out] integral(b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2, x)
```

Sympy [F]

$$\int (a+b\arctan(cx))^2\,dx=\int (a+b\,\mathrm{atan}\,(cx))^2\ dx$$

```
[In] integrate((a+b*atan(c*x))**2,x)
[Out] Integral((a + b*atan(c*x))**2, x)
```

Maxima [F]

$$\int (a+b\arctan(cx))^2\,dx=\int (b\arctan(cx)+a)^2\,dx$$

```
[In] integrate((a+b*arctan(c*x))^2,x, algorithm="maxima")
[Out] 1/16*(4*x*arctan(c*x)^2 + 192*c^2*integrate(1/16*x^2*arctan(c*x)^2/(c^2*x^2
 + 1), x) + 16*c^2*integrate(1/16*x^2*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x)
+ 64*c^2*integrate(1/16*x^2*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) - x*log(c^2*
x^2 + 1)^2 + 4*arctan(c*x)^3/c - 128*c*integrate(1/16*x*arctan(c*x)/(c^2*x^
2 + 1), x) + 16*integrate(1/16*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x))*b^2 +
a^2*x + (2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*a*b/c
```

Giac [F]

$$\int (a+b\arctan(cx))^2\,dx=\int (b\arctan(cx)+a)^2\,dx$$

```
[In] integrate((a+b*arctan(c*x))^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\arctan(cx))^2\,dx=\int (a+b\,\mathrm{atan}(c\,x))^2\,dx$$

```
[In] int((a + b*atan(c*x))^2,x)
```

```
[Out] int((a + b*atan(c*x))^2, x)
```

3.19 $\int \frac{(a+b\arctan(cx))^2}{x}\,dx$

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Optimal result

Integrand size = 14, antiderivative size = 132

$$\begin{aligned}\int \frac{(a+b\arctan(cx))^2}{x}\,dx &= 2(a+b\arctan(cx))^2\text{arctanh}\left(1-\frac{2}{1+icx}\right)\\ &\quad - ib(a+b\arctan(cx))\,\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\\ &\quad + ib(a+b\arctan(cx))\,\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)\\ &\quad - \frac{1}{2}b^2\,\text{PolyLog}\left(3,1-\frac{2}{1+icx}\right)\\ &\quad + \frac{1}{2}b^2\,\text{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)\end{aligned}$$

```
[Out] -2*(a+b*arctan(c*x))^2*arctanh(-1+2/(1+I*c*x))-I*b*(a+b*arctan(c*x))*polylo
g(2,1-2/(1+I*c*x))+I*b*(a+b*arctan(c*x))*polylog(2,-1+2/(1+I*c*x))-1/2*b^2*
polylog(3,1-2/(1+I*c*x))+1/2*b^2*polylog(3,-1+2/(1+I*c*x))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used

$= \{4942, 5108, 5004, 5114, 6745\}$

$$\int \frac{(a+b\arctan(cx))^2}{x}\,dx = 2\text{arctanh}\left(1-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2 - ib\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx)) + ib\,\text{PolyLog}\left(2,\frac{2}{icx+1}-1\right)(a+b\arctan(cx)) - \frac{1}{2}b^2\,\text{PolyLog}\left(3,1-\frac{2}{icx+1}\right)+\frac{1}{2}b^2\,\text{PolyLog}\left(3,\frac{2}{icx+1}-1\right)$$

```
[In] Int[(a + b*ArcTan[c*x])^2/x,x]
```

```
[Out] 2*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)] - I*b*(a + b*ArcTan[c*x]
)*PolyLog[2, 1 - 2/(1 + I*c*x)] + I*b*(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2
/(1 + I*c*x)] - (b^2*PolyLog[3, 1 - 2/(1 + I*c*x)])/2 + (b^2*PolyLog[3, -1
+ 2/(1 + I*c*x)])/2
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5108

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e*
x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2(a+b\arctan(cx))^2\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right) \\
&\quad-(4bc)\int\frac{(a+b\arctan(cx))\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx \\
&= 2(a+b\arctan(cx))^2\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right) \\
&\quad+(2bc)\int\frac{(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx \\
&\quad-(2bc)\int\frac{(a+b\arctan(cx))\log\left(2-\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx \\
&= 2(a+b\arctan(cx))^2\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right) \\
&\quad-ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right) \\
&\quad+ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx}\right) \\
&\quad+\left(ib^2c\right)\int\frac{\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx-\left(ib^2c\right)\int\frac{\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx \\
&= 2(a+b\arctan(cx))^2\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right) \\
&\quad-ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right) \\
&\quad+ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx}\right) \\
&\quad-\frac{1}{2}b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)+\frac{1}{2}b^2\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.36

$$\int \frac{(a+b\arctan(cx))^2}{x}\,dx = a^2\log(cx)+iab(\text{PolyLog}(2,-icx)-\text{PolyLog}(2,icx)) + b^2\Bigg(-\frac{i\pi^3}{24}+\frac{2}{3}i\arctan(cx)^3+\arctan(cx)^2\log\left(1-e^{-2i\arctan(cx)}\right) - \arctan(cx)^2\log\left(1+e^{2i\arctan(cx)}\right) + i\arctan(cx)\,\text{PolyLog}\left(2,e^{-2i\arctan(cx)}\right) + i\arctan(cx)\,\text{PolyLog}\left(2,-e^{2i\arctan(cx)}\right) + \frac{1}{2}\text{PolyLog}\left(3,e^{-2i\arctan(cx)}\right)-\frac{1}{2}\text{PolyLog}\left(3,-e^{2i\arctan(cx)}\right)\Bigg)$$

```
[In] Integrate[(a + b*ArcTan[c*x])^2/x,x]
```

```
[Out] a^2*Log[c*x] + I*a*b*(PolyLog[2, (-I)*c*x] - PolyLog[2, I*c*x]) + b^2*((-1/
24*I)*Pi^3 + ((2*I)/3)*ArcTan[c*x]^3 + ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcT
an[c*x])] - ArcTan[c*x]^2*Log[1 + E^((2*I)*ArcTan[c*x])] + I*ArcTan[c*x]*Po
lyLog[2, E^((-2*I)*ArcTan[c*x])] + I*ArcTan[c*x]*PolyLog[2, -E^((2*I)*ArcTa
n[c*x])] + PolyLog[3, E^((-2*I)*ArcTan[c*x])]/2 - PolyLog[3, -E^((2*I)*ArcT
an[c*x])]/2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.74 (sec) , antiderivative size = 1002, normalized size of antiderivative = 7.59

Expression too large to display

```
[In] int((a+b*arctan(c*x))^2/x,x)
```

```
[Out] a^2*ln(c*x)+b^2*(ln(c*x)*arctan(c*x)^2+I*arctan(c*x)*polylog(2,-(1+I*c*x)^2
/(c^2*x^2+1))-1/2*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))-arctan(c*x)^2*ln((1+I
*c*x)^2/(c^2*x^2+1)-1)+arctan(c*x)^2*ln(1-(1+I*c*x)/(c^2*x^2+1)^(1/2))-2*I*
arctan(c*x)*polylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*polylog(3,(1+I*c*x)/(c
^2*x^2+1)^(1/2))+arctan(c*x)^2*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))-2*I*arctan
(c*x)*polylog(2,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*polylog(3,-(1+I*c*x)/(c^2*x
^2+1)^(1/2))+1/2*I*Pi*(csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c
^2*x^2+1)))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))-c
sgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2+csgn(I*((1+I
*c*x)^2/(c^2*x^2+1)-1))*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*((1+I*c*
x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))-csgn(I*((1+I*c*x)^2/(c^2*x
^2+1)-1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2
-csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+
```

```
(1+I*c*x)^2/(c^2*x^2+1)))^2+csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)
^2/(c^2*x^2+1)))^3-csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x
^2+1)))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2+csg
n(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^3+1)*arctan(c*x)
^2)+2*a*b*(ln(c*x)*arctan(c*x)+1/2*I*ln(c*x)*ln(1+I*c*x)-1/2*I*ln(c*x)*ln(1
-I*c*x)+1/2*I*dilog(1+I*c*x)-1/2*I*dilog(1-I*c*x))
```

Fricas [F]

$$\int \frac{(a+b\arctan(cx))^2}{x}\,dx = \int \frac{(b\arctan(cx)+a)^2}{x}\,dx$$

```
[In] integrate((a+b*arctan(c*x))^2/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/x, x)
```

Sympy [F]

$$\int \frac{(a+b\arctan(cx))^2}{x}\,dx = \int \frac{(a+b\,\mathrm{atan}(cx))^2}{x}\,dx$$

```
[In] integrate((a+b*atan(c*x))**2/x,x)
```

```
[Out] Integral((a + b*atan(c*x))**2/x, x)
```

Maxima [F]

$$\int \frac{(a+b\arctan(cx))^2}{x}\,dx = \int \frac{(b\arctan(cx)+a)^2}{x}\,dx$$

```
[In] integrate((a+b*arctan(c*x))^2/x,x, algorithm="maxima")
```

```
[Out] a^2*log(x) + 1/16*integrate((12*b^2*arctan(c*x)^2 + b^2*log(c^2*x^2 + 1)^2
+ 32*a*b*arctan(c*x))/x, x)
```

Giac [F]

$$\int \frac{(a+b\arctan(cx))^2}{x}\,dx = \int \frac{(b\arctan(cx)+a)^2}{x}\,dx$$

```
[In] integrate((a+b*arctan(c*x))^2/x,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{x}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^2}{x}\,dx$$

```
[In] int((a + b*atan(c*x))^2/x,x)
```

```
[Out] int((a + b*atan(c*x))^2/x, x)
```

3.20 $\int \frac{(a+b\arctan(cx))^2}{x^2}\,dx$

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Optimal result

Integrand size = 14, antiderivative size = 82

$$\int \frac{(a+b\arctan(cx))^2}{x^2}\,dx = -ic(a+b\arctan(cx))^2 - \frac{(a+b\arctan(cx))^2}{x} + 2bc(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right) - ib^2c\,\text{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)$$

```
[Out] -I*c*(a+b*arctan(c*x))^2-(a+b*arctan(c*x))^2/x+2*b*c*(a+b*arctan(c*x))*ln(2
-2/(1-I*c*x))-I*b^2*c*polylog(2,-1+2/(1-I*c*x))
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4946, 5044, 4988, 2497}

$$\int \frac{(a+b\arctan(cx))^2}{x^2}\,dx = -ic(a+b\arctan(cx))^2 - \frac{(a+b\arctan(cx))^2}{x} + 2bc\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx)) - ib^2c\,\text{PolyLog}\left(2,\frac{2}{1-icx}-1\right)$$

```
[In] Int[(a + b*ArcTan[c*x])^2/x^2,x]
```

```
[Out] (-I)*c*(a + b*ArcTan[c*x])^2 - (a + b*ArcTan[c*x])^2/x + 2*b*c*(a + b*ArcTa
n[c*x])*Log[2 - 2/(1 - I*c*x)] - I*b^2*c*PolyLog[2, -1 + 2/(1 - I*c*x)]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(a+b\arctan(cx))^2}{x} + (2bc)\int \frac{a+b\arctan(cx)}{x\,(1+c^2x^2)}\,dx \\
&= -ic(a+b\arctan(cx))^2 - \frac{(a+b\arctan(cx))^2}{x} + (2ibc)\int \frac{a+b\arctan(cx)}{x(i+cx)}\,dx \\
&= -ic(a+b\arctan(cx))^2 - \frac{(a+b\arctan(cx))^2}{x} \\
&\quad + 2bc(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right) - (2b^2c^2)\int \frac{\log\left(2-\frac{2}{1-icx}\right)}{1+c^2x^2}\,dx \\
&= -ic(a+b\arctan(cx))^2 - \frac{(a+b\arctan(cx))^2}{x} \\
&\quad + 2bc(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right) - ib^2c\,\text{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.24

$$\int \frac{(a+b\arctan(cx))^2}{x^2}\,dx$$
$$= \frac{b^2(-1-icx)\arctan(cx)^2+2b\arctan(cx)\left(-a+bcx\log\left(1-e^{2i\arctan(cx)}\right)\right)-a(a-2bcx\log(cx)+bcx\log}{x}$$

```
[In] Integrate[(a + b*ArcTan[c*x])^2/x^2,x]
```

```
[Out] (b^2*(-1 - I*c*x)*ArcTan[c*x]^2 + 2*b*ArcTan[c*x]*(-a + b*c*x*Log[1 - E^((2
*I)*ArcTan[c*x])]) - a*(a - 2*b*c*x*Log[c*x] + b*c*x*Log[1 + c^2*x^2]) - I*
b^2*c*x*PolyLog[2, E^((2*I)*ArcTan[c*x])])/x
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 269 vs. 2(78) = 156.

Time = 2.35 (sec) , antiderivative size = 270, normalized size of antiderivative = 3.29

method	result
parts	$-\frac{a^2}{x}+b^2c\left(-\frac{\arctan(cx)^2}{cx}+2\ln(cx)\arctan(cx)-\arctan(cx)\ln(c^2x^2+1)+i\ln(cx)\ln(\right.$
derivativedivides	$c\left(-\frac{a^2}{cx}+b^2\left(-\frac{\arctan(cx)^2}{cx}+2\ln(cx)\arctan(cx)-\arctan(cx)\ln(c^2x^2+1)+i\ln(cx)\ln\right.\right.$
default	$c\left(-\frac{a^2}{cx}+b^2\left(-\frac{\arctan(cx)^2}{cx}+2\ln(cx)\arctan(cx)-\arctan(cx)\ln(c^2x^2+1)+i\ln(cx)\ln\right.\right.$

```
[In] int((a+b*arctan(c*x))^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a^2/x+b^2*c*(-1/c/x*arctan(c*x)^2+2*ln(c*x)*arctan(c*x)-arctan(c*x)*ln(c^2
*x^2+1)+I*ln(c*x)*ln(1+I*c*x)-I*ln(c*x)*ln(1-I*c*x)+I*dilog(1+I*c*x)-I*dilo
g(1-I*c*x)-1/2*I*(ln(c*x-I)*ln(c^2*x^2+1)-1/2*ln(c*x-I)^2-dilog(-1/2*I*(c*x
+I))-ln(c*x-I)*ln(-1/2*I*(c*x+I)))+1/2*I*(ln(c*x+I)*ln(c^2*x^2+1)-1/2*ln(c*
x+I)^2-dilog(1/2*I*(c*x-I))-ln(c*x+I)*ln(1/2*I*(c*x-I))))+2*a*b*c*(-1/c/x*a
rctan(c*x)+ln(c*x)-1/2*ln(c^2*x^2+1))
```

Fricas [F]

$$\int \frac{(a + b\arctan(cx))^2}{x^2} \, dx = \int \frac{(b\arctan(cx) + a)^2}{x^2} \, dx$$

```
[In] integrate((a+b*arctan(c*x))^2/x^2,x, algorithm="fricas")
[Out] integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/x^2, x)
```

Sympy [F]

$$\int \frac{(a + b\arctan(cx))^2}{x^2} \, dx = \int \frac{(a + b\operatorname{atan}(cx))^2}{x^2} \, dx$$

```
[In] integrate((a+b*atan(c*x))**2/x**2,x)
[Out] Integral((a + b*atan(c*x))**2/x**2, x)
```

Maxima [F]

$$\int \frac{(a + b\arctan(cx))^2}{x^2} \, dx = \int \frac{(b\arctan(cx) + a)^2}{x^2} \, dx$$

```
[In] integrate((a+b*arctan(c*x))^2/x^2,x, algorithm="maxima")
[Out] -(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*a*b + 1/16*(4*(c*arcta
n(c*x)^3 + 4*c^2*integrate(1/16*x^2*log(c^2*x^2 + 1)^2/(c^2*x^4 + x^2), x)
- 16*c^2*integrate(1/16*x^2*log(c^2*x^2 + 1)/(c^2*x^4 + x^2), x) + 32*c*int
egrate(1/16*x*arctan(c*x)/(c^2*x^4 + x^2), x) + 48*integrate(1/16*arctan(c*
x)^2/(c^2*x^4 + x^2), x) + 4*integrate(1/16*log(c^2*x^2 + 1)^2/(c^2*x^4 + x
^2), x))*x - 4*arctan(c*x)^2 + log(c^2*x^2 + 1)^2)*b^2/x - a^2/x
```

Giac [F]

$$\int \frac{(a + b\arctan(cx))^2}{x^2} \, dx = \int \frac{(b\arctan(cx) + a)^2}{x^2} \, dx$$

```
[In] integrate((a+b*arctan(c*x))^2/x^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{x^2}\,dx = \int \frac{\left(a+b\,\mathrm{atan}(c\,x)\right)^2}{x^2}\,dx$$

```
[In] int((a + b*atan(c*x))^2/x^2,x)
[Out] int((a + b*atan(c*x))^2/x^2, x)
```

3.21 $\int \frac{(a+b\arctan(cx))^2}{x^3}\,dx$

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Mupad [B] (verification not implemented) 168

Optimal result

Integrand size = 14, antiderivative size = 79

$$\int \frac{(a+b\arctan(cx))^2}{x^3}\,dx = -\frac{bc(a+b\arctan(cx))}{x} - \frac{1}{2}c^2(a+b\arctan(cx))^2 - \frac{(a+b\arctan(cx))^2}{2x^2} + b^2c^2\log(x) - \frac{1}{2}b^2c^2\log\left(1+c^2x^2\right)$$

```
[Out] -b*c*(a+b*arctan(c*x))/x-1/2*c^2*(a+b*arctan(c*x))^2-1/2*(a+b*arctan(c*x))^
2/x^2+b^2*c^2*ln(x)-1/2*b^2*c^2*ln(c^2*x^2+1)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4946, 5038, 272, 36, 29, 31, 5004}

$$\int \frac{(a+b\arctan(cx))^2}{x^3}\,dx = -\frac{1}{2}c^2(a+b\arctan(cx))^2 - \frac{(a+b\arctan(cx))^2}{2x^2} - \frac{bc(a+b\arctan(cx))}{x} - \frac{1}{2}b^2c^2\log\left(c^2x^2+1\right) + b^2c^2\log(x)$$

```
[In] Int[(a + b*ArcTan[c*x])^2/x^3,x]
```

```
[Out] -((b*c*(a + b*ArcTan[c*x]))/x) - (c^2*(a + b*ArcTan[c*x])^2)/2 - (a + b*Arc
Tan[c*x])^2/(2*x^2) + b^2*c^2*Log[x] - (b^2*c^2*Log[1 + c^2*x^2])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(a+b\arctan(cx))^2}{2x^2}+(bc)\int\frac{a+b\arctan(cx)}{x^2\left(1+c^2x^2\right)}\,dx\\
&= -\frac{(a+b\arctan(cx))^2}{2x^2}+(bc)\int\frac{a+b\arctan(cx)}{x^2}\,dx-\left(bc^3\right)\int\frac{a+b\arctan(cx)}{1+c^2x^2}\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{bc(a+b\arctan(cx))}{x} - \frac{1}{2}c^2(a+b\arctan(cx))^2 \\
&\quad - \frac{(a+b\arctan(cx))^2}{2x^2} + \left(b^2c^2\right)\int \frac{1}{x\left(1+c^2x^2\right)}\,dx \\
&= -\frac{bc(a+b\arctan(cx))}{x} - \frac{1}{2}c^2(a+b\arctan(cx))^2 \\
&\quad - \frac{(a+b\arctan(cx))^2}{2x^2} + \frac{1}{2}\left(b^2c^2\right)\operatorname{Subst}\left(\int \frac{1}{x\left(1+c^2x\right)}\,dx, x, x^2\right) \\
&= -\frac{bc(a+b\arctan(cx))}{x} - \frac{1}{2}c^2(a+b\arctan(cx))^2 - \frac{(a+b\arctan(cx))^2}{2x^2} \\
&\quad + \frac{1}{2}\left(b^2c^2\right)\operatorname{Subst}\left(\int \frac{1}{x}\,dx, x, x^2\right) - \frac{1}{2}\left(b^2c^4\right)\operatorname{Subst}\left(\int \frac{1}{1+c^2x}\,dx, x, x^2\right) \\
&= -\frac{bc(a+b\arctan(cx))}{x} - \frac{1}{2}c^2(a+b\arctan(cx))^2 \\
&\quad - \frac{(a+b\arctan(cx))^2}{2x^2} + b^2c^2\log(x) - \frac{1}{2}b^2c^2\log\left(1+c^2x^2\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.14

$$
\int \frac{(a+b\arctan(cx))^2}{x^3}\,dx = -\frac{a^2+2abcx+2b(a+bcx+ac^2x^2)\arctan(cx)+b^2(1+c^2x^2)\arctan(cx)^2-2b^2c^2x^2\log(x)+b^2c^2x^2\log(1}{2x^2}
$$

```
[In] Integrate[(a + b*ArcTan[c*x])^2/x^3,x]
```

```
[Out] -1/2*(a^2 + 2*a*b*c*x + 2*b*(a + b*c*x + a*c^2*x^2)*ArcTan[c*x] + b^2*(1 +
c^2*x^2)*ArcTan[c*x]^2 - 2*b^2*c^2*x^2*Log[x] + b^2*c^2*x^2*Log[1 + c^2*x^2
])/x^2
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.24

method	result
parts	$-\frac{a^2}{2x^2}+b^2c^2\left(-\frac{\arctan(cx)^2}{2c^2x^2}-\frac{\arctan(cx)}{cx}-\frac{\arctan(cx)^2}{2}-\frac{\ln(c^2x^2+1)}{2}+\ln(cx)\right)-\frac{ab\arctan(cx)}{x^2}-$
derivativedivides	$c^2\left(-\frac{a^2}{2c^2x^2}+b^2\left(-\frac{\arctan(cx)^2}{2c^2x^2}-\frac{\arctan(cx)}{cx}-\frac{\arctan(cx)^2}{2}-\frac{\ln(c^2x^2+1)}{2}+\ln(cx)\right)-\frac{ab\arctan(cx)}{c^2x^2}\right.$
default	$c^2\left(-\frac{a^2}{2c^2x^2}+b^2\left(-\frac{\arctan(cx)^2}{2c^2x^2}-\frac{\arctan(cx)}{cx}-\frac{\arctan(cx)^2}{2}-\frac{\ln(c^2x^2+1)}{2}+\ln(cx)\right)-\frac{ab\arctan(cx)}{c^2x^2}\right.$
parallelrisch	$\frac{-b^2\arctan(cx)^2x^2c^2+2b^2c^2\ln(x)x^2-b^2c^2\ln(c^2x^2+1)x^2-2ab\arctan(cx)x^2c^2+c^2x^2a^2-2b^2\arctan(cx)xc-2abcx-b^2 a}{2x^2}$
risch	$\frac{b^2(c^2x^2+1)\ln(icx+1)^2}{8x^2}+\frac{ib(ibc^2x^2\ln(-icx+1)+2xbc+2a+ib\ln(-icx+1))\ln(icx+1)}{4x^2}-\frac{-4i\ln((-3ibc-ac)x-3b+}{}$

[In] int((a+b*arctan(c*x))^2/x^3,x,method=_RETURNVERBOSE)

[Out] -1/2*a^2/x^2+b^2*c^2*(-1/2/c^2/x^2*arctan(c*x)^2-1/c/x*arctan(c*x)-1/2*arctan(c*x)^2-1/2*ln(c^2*x^2+1)+ln(c*x))-a*b/x^2*arctan(c*x)-a*b*c^2*arctan(c*x)-a*b*c/x

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.19

$$\int\frac{(a+b\arctan(cx))^2}{x^3}\,dx=$$
$$-\frac{b^2c^2x^2\log(c^2x^2+1)-2\,b^2c^2x^2\log(x)+2\,abcx+(b^2c^2x^2+b^2)\arctan(cx)^2+a^2+2\,(abc^2x^2+b^2cx+}{2\,x^2}$$

[In] integrate((a+b*arctan(c*x))^2/x^3,x, algorithm="fricas")

[Out] -1/2*(b^2*c^2*x^2*log(c^2*x^2 + 1) - 2*b^2*c^2*x^2*log(x) + 2*a*b*c*x + (b^2*c^2*x^2 + b^2)*arctan(c*x)^2 + a^2 + 2*(a*b*c^2*x^2 + b^2*c*x + a*b)*arctan(c*x))/x^2

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.51

$$\int\frac{(a+b\arctan(cx))^2}{x^3}\,dx$$
$$=\begin{cases}-\frac{a^2}{2x^2}-abc^2\,\mathrm{atan}\,(cx)-\frac{abc}{x}-\frac{ab\,\mathrm{atan}\,(cx)}{x^2}+b^2c^2\log(x)-\frac{b^2c^2\log\left(x^2+\frac{1}{c^2}\right)}{2}-\frac{b^2c^2\,\mathrm{atan}^2\,(cx)}{2}-\frac{b^2c\,\mathrm{atan}\,(cx)}{x}-\frac{b^2\,\mathrm{at}}{} \\ -\frac{a^2}{2x^2}\end{cases}$$

[In] integrate((a+b*atan(c*x))**2/x**3,x)

```
[Out] Piecewise((-a**2/(2*x**2) - a*b*c**2*atan(c*x) - a*b*c/x - a*b*atan(c*x)/x**2 + b**2*c**2*log(x) - b**2*c**2*log(x**2 + c**(-2))/2 - b**2*c**2*atan(c*x)**2/2 - b**2*c*atan(c*x)/x - b**2*atan(c*x)**2/(2*x**2), Ne(c, 0)), (-a**2/(2*x**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.24

$$\int \frac{(a+b\arctan(cx))^2}{x^3}\,dx = -\left(\left(c\arctan(cx)+\frac{1}{x}\right)c+\frac{\arctan(cx)}{x^2}\right)ab + \frac{1}{2}\left(\left(\arctan(cx)^2-\log\left(c^2x^2+1\right)+2\log(x)\right)c^2-2\left(c\arctan(cx)+\frac{1}{x}\right)c\arctan(cx)\right)b^2 - \frac{b^2\arctan(cx)^2}{2x^2}-\frac{a^2}{2x^2}$$

```
[In] integrate((a+b*arctan(c*x))^2/x^3,x, algorithm="maxima")
```

```
[Out] -((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*a*b + 1/2*((arctan(c*x)^2 - log(c^2*x^2 + 1) + 2*log(x))*c^2 - 2*(c*arctan(c*x) + 1/x)*c*arctan(c*x))*b^2 - 1/2*b^2*arctan(c*x)^2/x^2 - 1/2*a^2/x^2
```

Giac [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^3}\,dx = \int \frac{(b\arctan(cx)+a)^2}{x^3}\,dx$$

```
[In] integrate((a+b*arctan(c*x))^2/x^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 2.58 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.77

$$\int \frac{(a+b\arctan(cx))^2}{x^3}\,dx = b^2\,c^2\,\ln(x)-\frac{a^2}{2\,x^2}-\frac{b^2\,c^2\,\mathrm{atan}(c\,x)^2}{2}-\frac{b^2\,c^2\,\ln(c\,x+1\mathrm{i})}{2} - \frac{b^2\,c^2\,\ln(1+c\,x\,1\mathrm{i})}{2}-\frac{b^2\,\mathrm{atan}(c\,x)^2}{2\,x^2}-\frac{a\,b\,c}{x}-\frac{a\,b\,\mathrm{atan}(c\,x)}{x^2} - \frac{b^2\,c\,\mathrm{atan}(c\,x)}{x}-\frac{a\,b\,c^2\,\ln(c\,x+1\mathrm{i})\,1\mathrm{i}}{2}+\frac{a\,b\,c^2\,\ln(1+c\,x\,1\mathrm{i})\,1\mathrm{i}}{2}$$

```
[In] int((a + b*atan(c*x))^2/x^3,x)
```

```
[Out] b^2*c^2*log(x) - a^2/(2*x^2) - (b^2*c^2*atan(c*x)^2)/2 - (b^2*c^2*log(c*x +
 1i))/2 - (b^2*c^2*log(c*x*1i + 1))/2 - (b^2*atan(c*x)^2)/(2*x^2) - (a*b*c)
/x - (a*b*atan(c*x))/x^2 - (a*b*c^2*log(c*x + 1i)*1i)/2 + (a*b*c^2*log(c*x*
1i + 1)*1i)/2 - (b^2*c*atan(c*x))/x
```

3.22 $\int \frac{(a+b\arctan(cx))^2}{x^4}\,dx$

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Optimal result

Integrand size = 14, antiderivative size = 140

$$\int \frac{(a+b\arctan(cx))^2}{x^4}\,dx=-\frac{b^2c^2}{3x}-\frac{1}{3}b^2c^3\arctan(cx)-\frac{bc(a+b\arctan(cx))}{3x^2}+\frac{1}{3}ic^3(a+b\arctan(cx))^2-\frac{(a+b\arctan(cx))^2}{3x^3}-\frac{2}{3}bc^3(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right)+\frac{1}{3}ib^2c^3\operatorname{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)$$

```
[Out] -1/3*b^2*c^2/x-1/3*b^2*c^3*arctan(c*x)-1/3*b*c*(a+b*arctan(c*x))/x^2+1/3*I*
c^3*(a+b*arctan(c*x))^2-1/3*(a+b*arctan(c*x))^2/x^3-2/3*b*c^3*(a+b*arctan(c
*x))*ln(2-2/(1-I*c*x))+1/3*I*b^2*c^3*polylog(2,-1+2/(1-I*c*x))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4946, 5038, 331, 209, 5044, 4988, 2497}

$$\int \frac{(a+b\arctan(cx))^2}{x^4}\,dx=\frac{1}{3}ic^3(a+b\arctan(cx))^2-\frac{2}{3}bc^3\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))-\frac{(a+b\arctan(cx))^2}{3x^3}-\frac{bc(a+b\arctan(cx))}{3x^2}-\frac{1}{3}b^2c^3\arctan(cx)+\frac{1}{3}ib^2c^3\operatorname{PolyLog}\left(2,\frac{2}{1-icx}-1\right)-\frac{b^2c^2}{3x}$$

```
[In] Int[(a + b*ArcTan[c*x])^2/x^4,x]
```

```
[Out] -1/3*(b^2*c^2)/x - (b^2*c^3*ArcTan[c*x])/3 - (b*c*(a + b*ArcTan[c*x]))/(3*x
^2) + (I/3)*c^3*(a + b*ArcTan[c*x])^2 - (a + b*ArcTan[c*x])^2/(3*x^3) - (2*
b*c^3*(a + b*ArcTan[c*x])*Log[2 - 2/(1 - I*c*x)])/3 + (I/3)*b^2*c^3*PolyLog
[2, -1 + 2/(1 - I*c*x)]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
```

```
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(a+b\arctan(cx))^2}{3x^3}+\frac{1}{3}(2bc)\int\frac{a+b\arctan(cx)}{x^3\left(1+c^2x^2\right)}\,dx\\
&= -\frac{(a+b\arctan(cx))^2}{3x^3}+\frac{1}{3}(2bc)\int\frac{a+b\arctan(cx)}{x^3}\,dx-\frac{1}{3}\left(2bc^3\right)\int\frac{a+b\arctan(cx)}{x\left(1+c^2x^2\right)}\,dx\\
&= -\frac{bc(a+b\arctan(cx))}{3x^2}+\frac{1}{3}ic^3(a+b\arctan(cx))^2-\frac{(a+b\arctan(cx))^2}{3x^3}\\
&\quad+\frac{1}{3}\left(b^2c^2\right)\int\frac{1}{x^2\left(1+c^2x^2\right)}\,dx-\frac{1}{3}\left(2ibc^3\right)\int\frac{a+b\arctan(cx)}{x(i+cx)}\,dx\\
&= -\frac{b^2c^2}{3x}-\frac{bc(a+b\arctan(cx))}{3x^2}+\frac{1}{3}ic^3(a+b\arctan(cx))^2\\
&\quad-\frac{(a+b\arctan(cx))^2}{3x^3}-\frac{2}{3}bc^3(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right)\\
&\quad-\frac{1}{3}\left(b^2c^4\right)\int\frac{1}{1+c^2x^2}\,dx+\frac{1}{3}\left(2b^2c^4\right)\int\frac{\log\left(2-\frac{2}{1-icx}\right)}{1+c^2x^2}\,dx\\
&= -\frac{b^2c^2}{3x}-\frac{1}{3}b^2c^3\arctan(cx)-\frac{bc(a+b\arctan(cx))}{3x^2}\\
&\quad+\frac{1}{3}ic^3(a+b\arctan(cx))^2-\frac{(a+b\arctan(cx))^2}{3x^3}\\
&\quad-\frac{2}{3}bc^3(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right)+\frac{1}{3}ib^2c^3\operatorname{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\arctan(cx))^2}{x^4}\,dx =$$
$$-\frac{a^2+abcx+b^2c^2x^2+b^2(1-ic^3x^3)\arctan(cx)^2+b\arctan(cx)\left(2a+bcx+bc^3x^3+2bc^3x^3\log\left(1-e^{2i\arctan(cx)}\right)\right)\ldots}{3x^3}$$

[In] Integrate[(a + b*ArcTan[c*x])^2/x^4,x]

[Out] -1/3*(a^2 + a*b*c*x + b^2*c^2*x^2 + b^2*(1 - I*c^3*x^3)*ArcTan[c*x]^2 + b*ArcTan[c*x]*(2*a + b*c*x + b*c^3*x^3 + 2*b*c^3*x^3*Log[1 - E^((2*I)*ArcTan[c*x])]) + 2*a*b*c^3*x^3*Log[c*x] - a*b*c^3*x^3*Log[1 + c^2*x^2] - I*b^2*c^3*x^3*PolyLog[2, E^((2*I)*ArcTan[c*x])])/x^3

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 309 vs. 2(122) = 244.

Time = 2.91 (sec) , antiderivative size = 310, normalized size of antiderivative = 2.21

method	result
parts	$-\frac{a^2}{3x^3}+b^2c^3\left(-\frac{\arctan(cx)^2}{3c^3x^3}+\frac{\arctan(cx)\ln(c^2x^2+1)}{3}-\frac{\arctan(cx)}{3c^2x^2}-\frac{2\ln(cx)\arctan(cx)}{3}+\frac{i\left(\ln(cx-i)\ln(\ldots\right.}{\ldots}\right.$
derivativedivides	$c^3\left(-\frac{a^2}{3c^3x^3}+b^2\left(-\frac{\arctan(cx)^2}{3c^3x^3}+\frac{\arctan(cx)\ln(c^2x^2+1)}{3}-\frac{\arctan(cx)}{3c^2x^2}-\frac{2\ln(cx)\arctan(cx)}{3}+\frac{i\left(\ln(cx-\ldots\right.}{\ldots}\right.\right.$
default	$c^3\left(-\frac{a^2}{3c^3x^3}+b^2\left(-\frac{\arctan(cx)^2}{3c^3x^3}+\frac{\arctan(cx)\ln(c^2x^2+1)}{3}-\frac{\arctan(cx)}{3c^2x^2}-\frac{2\ln(cx)\arctan(cx)}{3}+\frac{i\left(\ln(cx-\ldots\right.}{\ldots}\right.\right.$

[In] int((a+b*arctan(c*x))^2/x^4,x,method=_RETURNVERBOSE)

[Out] -1/3*a^2/x^3+b^2*c^3*(-1/3/c^3/x^3*arctan(c*x)^2+1/3*arctan(c*x)*ln(c^2*x^2+1)-1/3/c^2/x^2*arctan(c*x)-2/3*ln(c*x)*arctan(c*x)+1/6*I*(ln(c*x-I)*ln(c^2*x^2+1)-1/2*ln(c*x-I)^2-dilog(-1/2*I*(c*x+I))-ln(c*x-I)*ln(-1/2*I*(c*x+I)))-1/6*I*(ln(c*x+I)*ln(c^2*x^2+1)-1/2*ln(c*x+I)^2-dilog(1/2*I*(c*x-I))-ln(c*x+I)*ln(1/2*I*(c*x-I)))-1/3/c/x-1/3*arctan(c*x)-1/3*I*ln(c*x)*ln(1+I*c*x)+1/3*I*ln(c*x)*ln(1-I*c*x)-1/3*I*dilog(1+I*c*x)+1/3*I*dilog(1-I*c*x))+2*a*b*c^3*(-1/3/c^3/x^3*arctan(c*x)+1/6*ln(c^2*x^2+1)-1/6/c^2/x^2-1/3*ln(c*x))

Fricas [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^4}\,dx = \int \frac{(b\arctan(cx)+a)^2}{x^4}\,dx$$

```
[In] integrate((a+b*arctan(c*x))^2/x^4,x, algorithm="fricas")
[Out] integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/x^4, x)
```

Sympy [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^4}\,dx = \int \frac{(a+b\,\mathrm{atan}(cx))^2}{x^4}\,dx$$

```
[In] integrate((a+b*atan(c*x))**2/x**4,x)
[Out] Integral((a + b*atan(c*x))**2/x**4, x)
```

Maxima [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^4}\,dx = \int \frac{(b\arctan(cx)+a)^2}{x^4}\,dx$$

```
[In] integrate((a+b*arctan(c*x))^2/x^4,x, algorithm="maxima")
[Out] 1/3*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*a
*b + 1/48*(48*x^3*integrate(-1/48*(4*c^2*x^2*log(c^2*x^2 + 1) - 8*c*x*arcta
n(c*x) - 36*(c^2*x^2 + 1)*arctan(c*x)^2 - 3*(c^2*x^2 + 1)*log(c^2*x^2 + 1)^
2)/(c^2*x^6 + x^4), x) - 4*arctan(c*x)^2 + log(c^2*x^2 + 1)^2)*b^2/x^3 - 1/
3*a^2/x^3
```

Giac [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^4}\,dx = \int \frac{(b\arctan(cx)+a)^2}{x^4}\,dx$$

```
[In] integrate((a+b*arctan(c*x))^2/x^4,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{x^4}\,dx = \int \frac{\left(a+b\,\mathrm{atan}(c\,x)\right)^2}{x^4}\,dx$$

```
[In] int((a + b*atan(c*x))^2/x^4,x)
[Out] int((a + b*atan(c*x))^2/x^4, x)
```

3.23 $\int \frac{(a+b\arctan(cx))^2}{x^5}\,dx$

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Optimal result

Integrand size = 14, antiderivative size = 116

$$\int \frac{(a+b\arctan(cx))^2}{x^5}\,dx = -\frac{b^2c^2}{12x^2}-\frac{bc(a+b\arctan(cx))}{6x^3}+\frac{bc^3(a+b\arctan(cx))}{2x}+\frac{1}{4}c^4(a+b\arctan(cx))^2-\frac{(a+b\arctan(cx))^2}{4x^4}-\frac{2}{3}b^2c^4\log(x)+\frac{1}{3}b^2c^4\log\left(1+c^2x^2\right)$$

```
[Out] -1/12*b^2*c^2/x^2-1/6*b*c*(a+b*arctan(c*x))/x^3+1/2*b*c^3*(a+b*arctan(c*x))
/x+1/4*c^4*(a+b*arctan(c*x))^2-1/4*(a+b*arctan(c*x))^2/x^4-2/3*b^2*c^4*ln(x
)+1/3*b^2*c^4*ln(c^2*x^2+1)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {4946, 5038, 272, 46, 36, 29, 31, 5004}

$$\int \frac{(a+b\arctan(cx))^2}{x^5}\,dx = \frac{1}{4}c^4(a+b\arctan(cx))^2+\frac{bc^3(a+b\arctan(cx))}{2x}-\frac{(a+b\arctan(cx))^2}{4x^4}-\frac{bc(a+b\arctan(cx))}{6x^3}-\frac{2}{3}b^2c^4\log(x)-\frac{b^2c^2}{12x^2}+\frac{1}{3}b^2c^4\log\left(c^2x^2+1\right)$$

```
[In] Int[(a + b*ArcTan[c*x])^2/x^5,x]
```

```
[Out] -1/12*(b^2*c^2)/x^2 - (b*c*(a + b*ArcTan[c*x]))/(6*x^3) + (b*c^3*(a + b*Arc
Tan[c*x]))/(2*x) + (c^4*(a + b*ArcTan[c*x])^2)/4 - (a + b*ArcTan[c*x])^2/(4
*x^4) - (2*b^2*c^4*Log[x])/3 + (b^2*c^4*Log[1 + c^2*x^2])/3
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(a+b\arctan(cx))^2}{4x^4}+\frac{1}{2}(bc)\int\frac{a+b\arctan(cx)}{x^4\left(1+c^2x^2\right)}\,dx \\
&= -\frac{(a+b\arctan(cx))^2}{4x^4}+\frac{1}{2}(bc)\int\frac{a+b\arctan(cx)}{x^4}\,dx-\frac{1}{2}\left(bc^3\right)\int\frac{a+b\arctan(cx)}{x^2\left(1+c^2x^2\right)}\,dx \\
&= -\frac{bc(a+b\arctan(cx))}{6x^3}-\frac{(a+b\arctan(cx))^2}{4x^4}+\frac{1}{6}\left(b^2c^2\right)\int\frac{1}{x^3\left(1+c^2x^2\right)}\,dx \\
&\quad -\frac{1}{2}\left(bc^3\right)\int\frac{a+b\arctan(cx)}{x^2}\,dx+\frac{1}{2}\left(bc^5\right)\int\frac{a+b\arctan(cx)}{1+c^2x^2}\,dx \\
&= -\frac{bc(a+b\arctan(cx))}{6x^3}+\frac{bc^3(a+b\arctan(cx))}{2x} \\
&\quad +\frac{1}{4}c^4(a+b\arctan(cx))^2-\frac{(a+b\arctan(cx))^2}{4x^4} \\
&\quad +\frac{1}{12}\left(b^2c^2\right)\operatorname{Subst}\left(\int\frac{1}{x^2\left(1+c^2x\right)}\,dx,x,x^2\right)-\frac{1}{2}\left(b^2c^4\right)\int\frac{1}{x\left(1+c^2x^2\right)}\,dx \\
&= -\frac{bc(a+b\arctan(cx))}{6x^3}+\frac{bc^3(a+b\arctan(cx))}{2x}+\frac{1}{4}c^4(a+b\arctan(cx))^2 \\
&\quad -\frac{(a+b\arctan(cx))^2}{4x^4}+\frac{1}{12}\left(b^2c^2\right)\operatorname{Subst}\left(\int\left(\frac{1}{x^2}-\frac{c^2}{x}+\frac{c^4}{1+c^2x}\right)dx,x,x^2\right) \\
&\quad -\frac{1}{4}\left(b^2c^4\right)\operatorname{Subst}\left(\int\frac{1}{x\left(1+c^2x\right)}\,dx,x,x^2\right) \\
&= -\frac{b^2c^2}{12x^2}-\frac{bc(a+b\arctan(cx))}{6x^3}+\frac{bc^3(a+b\arctan(cx))}{2x}+\frac{1}{4}c^4(a+b\arctan(cx))^2 \\
&\quad -\frac{(a+b\arctan(cx))^2}{4x^4}-\frac{1}{6}b^2c^4\log(x)+\frac{1}{12}b^2c^4\log\left(1+c^2x^2\right) \\
&\quad -\frac{1}{4}\left(b^2c^4\right)\operatorname{Subst}\left(\int\frac{1}{x}\,dx,x,x^2\right)+\frac{1}{4}\left(b^2c^6\right)\operatorname{Subst}\left(\int\frac{1}{1+c^2x}\,dx,x,x^2\right) \\
&= -\frac{b^2c^2}{12x^2}-\frac{bc(a+b\arctan(cx))}{6x^3}+\frac{bc^3(a+b\arctan(cx))}{2x}+\frac{1}{4}c^4(a+b\arctan(cx))^2 \\
&\quad -\frac{(a+b\arctan(cx))^2}{4x^4}-\frac{2}{3}b^2c^4\log(x)+\frac{1}{3}b^2c^4\log\left(1+c^2x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.10

$$\int \frac{(a+b\arctan(cx))^2}{x^5}\,dx$$
$$= \frac{-3a^2-2abcx-b^2c^2x^2+6abc^3x^3+2b(bcx(-1+3c^2x^2)+3a(-1+c^4x^4))\arctan(cx)+3b^2(-1+c^4x^4)\,a}{12x^4}$$

```
[In] Integrate[(a + b*ArcTan[c*x])^2/x^5,x]
```

```
[Out] (-3*a^2 - 2*a*b*c*x - b^2*c^2*x^2 + 6*a*b*c^3*x^3 + 2*b*(b*c*x*(-1 + 3*c^2*
x^2) + 3*a*(-1 + c^4*x^4))*ArcTan[c*x] + 3*b^2*(-1 + c^4*x^4)*ArcTan[c*x]^2
 - 8*b^2*c^4*x^4*Log[x] + 4*b^2*c^4*x^4*Log[1 + c^2*x^2])/(12*x^4)
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.14

method	result
parts	$-\frac{a^2}{4x^4}+b^2c^4\left(-\frac{\arctan(cx)^2}{4c^4x^4}-\frac{\arctan(cx)}{6c^3x^3}+\frac{\arctan(cx)}{2cx}+\frac{\arctan(cx)^2}{4}-\frac{1}{12c^2x^2}-\frac{2\ln(cx)}{3}+\frac{\ln(c^2x^2+1)}{3}\right.$
derivativedivides	$c^4\left(-\frac{a^2}{4c^4x^4}+b^2\left(-\frac{\arctan(cx)^2}{4c^4x^4}-\frac{\arctan(cx)}{6c^3x^3}+\frac{\arctan(cx)}{2cx}+\frac{\arctan(cx)^2}{4}-\frac{1}{12c^2x^2}-\frac{2\ln(cx)}{3}+\frac{\ln(c^2x}{3}\right.\right.$
default	$c^4\left(-\frac{a^2}{4c^4x^4}+b^2\left(-\frac{\arctan(cx)^2}{4c^4x^4}-\frac{\arctan(cx)}{6c^3x^3}+\frac{\arctan(cx)}{2cx}+\frac{\arctan(cx)^2}{4}-\frac{1}{12c^2x^2}-\frac{2\ln(cx)}{3}+\frac{\ln(c^2x}{3}\right.\right.$
parallelrisch	$-\frac{-3x^4\arctan(cx)^2b^2c^4+8b^2c^4\ln(x)x^4-4b^2c^4\ln(c^2x^2+1)x^4-6x^4\arctan(cx)ab\,c^4-b^2c^4x^4-6b^2\arctan(cx)x^3c^3-6ab\,c}{12x^4}$
risch	$-\frac{b^2(c^4x^4-1)\ln(icx+1)^2}{16x^4}+\frac{ib(-3ib\,c^4x^4\ln(-icx+1)-6b\,c^3x^3+2xbc+6a+3ib\ln(-icx+1))\ln(icx+1)}{24x^4}-\frac{12i\ln((-4}{}$

```
[In] int((a+b*arctan(c*x))^2/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/x^4*a^2+b^2*c^4*(-1/4/c^4/x^4*arctan(c*x)^2-1/6/c^3/x^3*arctan(c*x)+1/
2/c/x*arctan(c*x)+1/4*arctan(c*x)^2-1/12/c^2/x^2-2/3*ln(c*x)+1/3*ln(c^2*x^2
+1))+2*a*b*c^4*(-1/4/c^4/x^4*arctan(c*x)-1/12/c^3/x^3+1/4/c/x+1/4*arctan(c*
x))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.16

$$\int \frac{(a+b\arctan(cx))^2}{x^5}\,dx = \frac{4\,b^2c^4x^4\log\left(c^2x^2+1\right)-8\,b^2c^4x^4\log\left(x\right)+6\,abc^3x^3-b^2c^2x^2-2\,abcx+3\left(b^2c^4x^4-b^2\right)\arctan\left(cx\right)^2-3\,a^2+\ldots}{12\,x^4}$$

```
[In] integrate((a+b*arctan(c*x))^2/x^5,x, algorithm="fricas")
[Out] 1/12*(4*b^2*c^4*x^4*log(c^2*x^2 + 1) - 8*b^2*c^4*x^4*log(x) + 6*a*b*c^3*x^3
 - b^2*c^2*x^2 - 2*a*b*c*x + 3*(b^2*c^4*x^4 - b^2)*arctan(c*x)^2 - 3*a^2 +
2*(3*a*b*c^4*x^4 + 3*b^2*c^3*x^3 - b^2*c*x - 3*a*b)*arctan(c*x))/x^4
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.47

$$\int \frac{(a+b\arctan(cx))^2}{x^5}\,dx = \begin{cases} -\frac{a^2}{4x^4}+\frac{abc^4\operatorname{atan}(cx)}{2}+\frac{abc^3}{2x}-\frac{abc}{6x^3}-\frac{ab\operatorname{atan}(cx)}{2x^4}-\frac{2b^2c^4\log(x)}{3}+\frac{b^2c^4\log\left(x^2+\frac{1}{c^2}\right)}{3}+\frac{b^2c^4\operatorname{atan}^2(cx)}{4}+\frac{b^2c^3\operatorname{atan}(cx)}{2x}-\frac{b^2\ldots}{12\ldots} \\ -\frac{a^2}{4x^4} \end{cases}$$

```
[In] integrate((a+b*atan(c*x))**2/x**5,x)
[Out] Piecewise((-a**2/(4*x**4) + a*b*c**4*atan(c*x)/2 + a*b*c**3/(2*x) - a*b*c/(
6*x**3) - a*b*atan(c*x)/(2*x**4) - 2*b**2*c**4*log(x)/3 + b**2*c**4*log(x**
2 + c**(-2))/3 + b**2*c**4*atan(c*x)**2/4 + b**2*c**3*atan(c*x)/(2*x) - b**
2*c**2/(12*x**2) - b**2*c*atan(c*x)/(6*x**3) - b**2*atan(c*x)**2/(4*x**4),
Ne(c, 0)), (-a**2/(4*x**4), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.31

$$\int \frac{(a+b\arctan(cx))^2}{x^5}\,dx = \frac{1}{6}\left(\left(3\,c^3\arctan\left(cx\right)+\frac{3\,c^2x^2-1}{x^3}\right)c-\frac{3\,\arctan\left(cx\right)}{x^4}\right)ab$$
$$+\frac{1}{12}\left(2\left(3\,c^3\arctan\left(cx\right)+\frac{3\,c^2x^2-1}{x^3}\right)c\arctan\left(cx\right)-\frac{\left(3\,c^2x^2\arctan\left(cx\right)^2-4\,c^2x^2\log\left(c^2x^2+1\right)+8\,c^2x\ldots\right.}{x^2}\right.$$
$$-\frac{b^2\arctan\left(cx\right)^2}{4\,x^4}-\frac{a^2}{4\,x^4}$$

```
[In] integrate((a+b*arctan(c*x))^2/x^5,x, algorithm="maxima")
```

```
[Out] 1/6*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(c*x)/x^4)*a*b +
 1/12*(2*(3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c*arctan(c*x) - (3*c^2*x
^2*arctan(c*x)^2 - 4*c^2*x^2*log(c^2*x^2 + 1) + 8*c^2*x^2*log(x) + 1)*c^2/x
^2)*b^2 - 1/4*b^2*arctan(c*x)^2/x^4 - 1/4*a^2/x^4
```

Giac [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^5}\,dx = \int \frac{(b\arctan{(cx)}+a)^2}{x^5}\,dx$$

```
[In] integrate((a+b*arctan(c*x))^2/x^5,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B] (verification not implemented)

Time = 2.55 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.47

$$\int \frac{(a+b\arctan(cx))^2}{x^5}\,dx$$
$$= \frac{b^2\,c^4\,\mathrm{atan}(c\,x)^2}{4} - \frac{2\,b^2\,c^4\,\ln{(x)}}{3} - \frac{\frac{b^2\,\mathrm{atan}(c\,x)^2}{4} + \frac{a^2}{4} + x\left(\frac{c\,\mathrm{atan}(c\,x)\,b^2}{6} + \frac{a\,c\,b}{6}\right) - x^3\left(\frac{b^2\,c^3\,\mathrm{atan}(c\,x)}{2} + \frac{a\,b\,c^3}{2}\right) + \frac{b^2\,c^2\,x^2}{12} + \frac{a\,b\,\mathrm{atan}(c\,x)}{2}}{x^4} + \frac{b^2\,c^4\,\ln{(c\,x+1\mathrm{i})}}{3} + \frac{b^2\,c^4\,\ln{(1+c\,x\,1\mathrm{i})}}{3} + \frac{a\,b\,c^4\,\ln{(c\,x+1\mathrm{i})}\,1\mathrm{i}}{4} - \frac{a\,b\,c^4\,\ln{(1+c\,x\,1\mathrm{i})}\,1\mathrm{i}}{4}$$

```
[In] int((a + b*atan(c*x))^2/x^5,x)
```

```
[Out] (b^2*c^4*atan(c*x)^2)/4 - (2*b^2*c^4*log(x))/3 - ((b^2*atan(c*x)^2)/4 + a^2
/4 + x*((b^2*c*atan(c*x))/6 + (a*b*c)/6) - x^3*((b^2*c^3*atan(c*x))/2 + (a*
b*c^3)/2) + (b^2*c^2*x^2)/12 + (a*b*atan(c*x))/2)/x^4 + (b^2*c^4*log(c*x +
1i))/3 + (b^2*c^4*log(c*x*1i + 1))/3 + (a*b*c^4*log(c*x + 1i)*1i)/4 - (a*b*
c^4*log(c*x*1i + 1)*1i)/4
```

3.24 $\int x^5(a+b\arctan(cx))^3\,dx$

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Optimal result

Integrand size = 14, antiderivative size = 255

$$\begin{aligned}\int x^5(a+b\arctan(cx))^3\,dx &= \frac{19b^3x}{60c^5}-\frac{b^3x^3}{60c^3}-\frac{19b^3\arctan(cx)}{60c^6}-\frac{4b^2x^2(a+b\arctan(cx))}{15c^4}\\&+\frac{b^2x^4(a+b\arctan(cx))}{20c^2}-\frac{23ib(a+b\arctan(cx))^2}{30c^6}\\&-\frac{bx(a+b\arctan(cx))^2}{2c^5}+\frac{bx^3(a+b\arctan(cx))^2}{6c^3}\\&-\frac{bx^5(a+b\arctan(cx))^2}{10c}+\frac{(a+b\arctan(cx))^3}{6c^6}\\&+\frac{1}{6}x^6(a+b\arctan(cx))^3-\frac{23b^2(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{15c^6}\\&-\frac{23ib^3\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{30c^6}\end{aligned}$$

```
[Out] 19/60*b^3*x/c^5-1/60*b^3*x^3/c^3-19/60*b^3*arctan(c*x)/c^6-4/15*b^2*x^2*(a+
b*arctan(c*x))/c^4+1/20*b^2*x^4*(a+b*arctan(c*x))/c^2-23/30*I*b*(a+b*arctan
(c*x))^2/c^6-1/2*b*x*(a+b*arctan(c*x))^2/c^5+1/6*b*x^3*(a+b*arctan(c*x))^2/
c^3-1/10*b*x^5*(a+b*arctan(c*x))^2/c+1/6*(a+b*arctan(c*x))^3/c^6+1/6*x^6*(a
+b*arctan(c*x))^3-23/15*b^2*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^6-23/30*I*b
^3*polylog(2,1-2/(1+I*c*x))/c^6
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 33, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {4946, 5036, 308, 209, 327, 5040, 4964, 2449, 2352, 4930, 5004}

$$\int x^5(a+b\arctan(cx))^3\,dx=-\frac{23b^2\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{15c^6}-\frac{4b^2x^2(a+b\arctan(cx))}{15c^4}+\frac{b^2x^4(a+b\arctan(cx))}{20c^2}+\frac{(a+b\arctan(cx))^3}{6c^6}-\frac{23ib(a+b\arctan(cx))^2}{30c^6}-\frac{bx(a+b\arctan(cx))^2}{2c^5}+\frac{bx^3(a+b\arctan(cx))^2}{6c^3}+\frac{1}{6}x^6(a+b\arctan(cx))^3-\frac{bx^5(a+b\arctan(cx))^2}{10c}-\frac{19b^3\arctan(cx)}{60c^6}-\frac{23ib^3\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{30c^6}+\frac{19b^3x}{60c^5}-\frac{b^3x^3}{60c^3}$$

```
[In] Int[x^5*(a + b*ArcTan[c*x])^3,x]
```

```
[Out] (19*b^3*x)/(60*c^5) - (b^3*x^3)/(60*c^3) - (19*b^3*ArcTan[c*x])/(60*c^6) -
(4*b^2*x^2*(a + b*ArcTan[c*x]))/(15*c^4) + (b^2*x^4*(a + b*ArcTan[c*x]))/(2
0*c^2) - (((23*I)/30)*b*(a + b*ArcTan[c*x])^2)/c^6 - (b*x*(a + b*ArcTan[c*x
])^2)/(2*c^5) + (b*x^3*(a + b*ArcTan[c*x])^2)/(6*c^3) - (b*x^5*(a + b*ArcTa
n[c*x])^2)/(10*c) + (a + b*ArcTan[c*x])^3/(6*c^6) + (x^6*(a + b*ArcTan[c*x]
)^3)/6 - (23*b^2*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(15*c^6) - (((23*I
)/30)*b^3*PolyLog[2, 1 - 2/(1 + I*c*x)])/c^6
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
```

```
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{6}x^6(a+b\arctan(cx))^3 - \frac{1}{2}(bc)\int \frac{x^6(a+b\arctan(cx))^2}{1+c^2x^2}\,dx$$

$$= \frac{1}{6}x^6(a+b\arctan(cx))^3 - \frac{b\int x^4(a+b\arctan(cx))^2\,dx}{2c} + \frac{b\int \frac{x^4(a+b\arctan(cx))^2}{1+c^2x^2}\,dx}{2c}$$

$$= -\frac{bx^5(a+b\arctan(cx))^2}{10c} + \frac{1}{6}x^6(a+b\arctan(cx))^3 + \frac{1}{5}b^2\int \frac{x^5(a+b\arctan(cx))}{1+c^2x^2}\,dx + \frac{b\int x^2(a+b\arctan(cx))^2\,dx}{2c^3} - \frac{b\int \frac{x^2(a+b\arctan(cx))^2}{1+c^2x^2}\,dx}{2c^3}$$

$$= \frac{bx^3(a+b\arctan(cx))^2}{6c^3} - \frac{bx^5(a+b\arctan(cx))^2}{10c} + \frac{1}{6}x^6(a+b\arctan(cx))^3 - \frac{b\int (a+b\arctan(cx))^2\,dx}{2c^5} + \frac{b\int \frac{(a+b\arctan(cx))^2}{1+c^2x^2}\,dx}{2c^5} + \frac{b^2\int x^3(a+b\arctan(cx))\,dx}{5c^2} - \frac{b^2\int \frac{x^3(a+b\arctan(cx))}{1+c^2x^2}\,dx}{5c^2} - \frac{b^2\int \frac{x^3(a+b\arctan(cx))}{1+c^2x^2}\,dx}{3c^2}$$

$$= \frac{b^2x^4(a+b\arctan(cx))}{20c^2} - \frac{bx(a+b\arctan(cx))^2}{2c^5} + \frac{bx^3(a+b\arctan(cx))^2}{6c^3} - \frac{bx^5(a+b\arctan(cx))^2}{10c} + \frac{(a+b\arctan(cx))^3}{6c^6} + \frac{1}{6}x^6(a+b\arctan(cx))^3 - \frac{b^2\int x(a+b\arctan(cx))\,dx}{5c^4} + \frac{b^2\int \frac{x(a+b\arctan(cx))}{1+c^2x^2}\,dx}{5c^4} - \frac{b^2\int x(a+b\arctan(cx))\,dx}{3c^4} + \frac{b^2\int \frac{x(a+b\arctan(cx))}{1+c^2x^2}\,dx}{3c^4} + \frac{b^2\int \frac{x(a+b\arctan(cx))}{1+c^2x^2}\,dx}{c^4} - \frac{b^3\int \frac{x^4}{1+c^2x^2}\,dx}{20c}$$

$$= -\frac{4b^2x^2(a+b\arctan(cx))}{15c^4} + \frac{b^2x^4(a+b\arctan(cx))}{20c^2} - \frac{23ib(a+b\arctan(cx))^2}{30c^6} - \frac{bx(a+b\arctan(cx))^2}{2c^5} + \frac{bx^3(a+b\arctan(cx))^2}{6c^3} - \frac{bx^5(a+b\arctan(cx))^2}{10c} + \frac{(a+b\arctan(cx))^3}{6c^6} + \frac{1}{6}x^6(a+b\arctan(cx))^3 - \frac{b^2\int\frac{a+b\arctan(cx)}{i-cx}\,dx}{5c^5} - \frac{b^2\int\frac{a+b\arctan(cx)}{i-cx}\,dx}{3c^5} - \frac{b^2\int\frac{a+b\arctan(cx)}{i-cx}\,dx}{c^5} + \frac{b^3\int\frac{x^2}{1+c^2x^2}\,dx}{10c^3} + \frac{b^3\int\frac{x^2}{1+c^2x^2}\,dx}{6c^3} - \frac{b^3\int\left(-\frac{1}{c^4}+\frac{x^2}{c^2}+\frac{1}{c^4(1+c^2x^2)}\right)dx}{20c}$$

$$= \frac{19b^3x}{60c^5} - \frac{b^3x^3}{60c^3} - \frac{4b^2x^2(a+b\arctan(cx))}{15c^4} + \frac{b^2x^4(a+b\arctan(cx))}{20c^2} - \frac{23ib(a+b\arctan(cx))^2}{30c^6} - \frac{bx(a+b\arctan(cx))^2}{2c^5} + \frac{bx^3(a+b\arctan(cx))^2}{6c^3} - \frac{bx^5(a+b\arctan(cx))^2}{10c} + \frac{(a+b\arctan(cx))^3}{6c^6} + \frac{1}{6}x^6(a+b\arctan(cx))^3 - \frac{23b^2(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{15c^6} - \frac{b^3\int\frac{1}{1+c^2x^2}\,dx}{20c^5} - \frac{b^3\int\frac{1}{1+c^2x^2}\,dx}{10c^5} - \frac{b^3\int\frac{1}{1+c^2x^2}\,dx}{6c^5} + \frac{b^3\int\frac{\log\left(\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx}{5c^5} + \frac{b^3\int\frac{\log\left(\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx}{3c^5} + \frac{b^3\int\frac{\log\left(\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx}{c^5}$$

$$= \frac{19b^3x}{60c^5} - \frac{b^3x^3}{60c^3} - \frac{19b^3\arctan(cx)}{60c^6} - \frac{4b^2x^2(a+b\arctan(cx))}{15c^4} + \frac{b^2x^4(a+b\arctan(cx))}{20c^2} - \frac{23ib(a+b\arctan(cx))^2}{30c^6} - \frac{bx(a+b\arctan(cx))^2}{2c^5} + \frac{bx^3(a+b\arctan(cx))^2}{6c^3} - \frac{bx^5(a+b\arctan(cx))^2}{10c} + \frac{(a+b\arctan(cx))^3}{6c^6} + \frac{1}{6}x^6(a+b\arctan(cx))^3 - \frac{23b^2(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{15c^6} - \frac{(ib^3)\,\text{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1+icx}\right)}{5c^6} - \frac{(ib^3)\,\text{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1+icx}\right)}{3c^6} - \frac{(ib^3)\,\text{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1+icx}\right)}{c^6}$$

$$= \frac{19b^3x}{60c^5} - \frac{b^3x^3}{60c^3} - \frac{19b^3\arctan(cx)}{60c^6} - \frac{4b^2x^2(a+b\arctan(cx))}{15c^4} + \frac{b^2x^4(a+b\arctan(cx))}{20c^2} - \frac{23ib(a+b\arctan(cx))^2}{30c^6} - \frac{bx(a+b\arctan(cx))^2}{2c^5} + \frac{bx^3(a+b\arctan(cx))^2}{6c^3} - \frac{bx^5(a+b\arctan(cx))^2}{10c} + \frac{(a+b\arctan(cx))^3}{6c^6} + \frac{1}{6}x^6(a+b\arctan(cx))^3 - \frac{23b^2(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{15c^6} - \frac{23ib^3\,\text{PolyLog}\left(2, 1-\frac{2}{1+icx}\right)}{30c^6}$$

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.14

$$\int x^5(a+b\arctan(cx))^3\,dx$$
$$=\frac{-19ab^2-30a^2bcx+19b^3cx-16ab^2c^2x^2+10a^2bc^3x^3-b^3c^3x^3+3ab^2c^4x^4-6a^2bc^5x^5+10a^3c^6x^6+2b^2(b}{}$$

```
[In] Integrate[x^5*(a + b*ArcTan[c*x])^3,x]
```

```
[Out] (-19*a*b^2 - 30*a^2*b*c*x + 19*b^3*c*x - 16*a*b^2*c^2*x^2 + 10*a^2*b*c^3*x^
3 - b^3*c^3*x^3 + 3*a*b^2*c^4*x^4 - 6*a^2*b*c^5*x^5 + 10*a^3*c^6*x^6 + 2*b^
2*(b*(23*I - 15*c*x + 5*c^3*x^3 - 3*c^5*x^5) + 15*a*(1 + c^6*x^6))*ArcTan[c
*x]^2 + 10*b^3*(1 + c^6*x^6)*ArcTan[c*x]^3 + b*ArcTan[c*x]*(b^2*(-19 - 16*c
^2*x^2 + 3*c^4*x^4) - 4*a*b*c*x*(15 - 5*c^2*x^2 + 3*c^4*x^4) + 30*a^2*(1 +
c^6*x^6) - 92*b^2*Log[1 + E^((2*I)*ArcTan[c*x])]) + 46*a*b^2*Log[1 + c^2*x^
2] + (46*I)*b^3*PolyLog[2, -E^((2*I)*ArcTan[c*x])])/(60*c^6)
```

Maple [A] (verified)

Time = 2.52 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.58

method	result
derivativedivides	$\frac{\frac{a^3c^6x^6}{6}+b^3\left(\frac{c^6x^6\arctan(cx)^3}{6}-\frac{c^5x^5\arctan(cx)^2}{10}+\frac{c^3x^3\arctan(cx)^2}{6}-\frac{\arctan(cx)^2cx}{2}+\frac{\arctan(cx)^3}{6}+\frac{c^4x^4\arctan(cx)}{20}-\frac{4c^2x^2\arctan}{1}\right.}{}$
default	$\frac{\frac{a^3c^6x^6}{6}+b^3\left(\frac{c^6x^6\arctan(cx)^3}{6}-\frac{c^5x^5\arctan(cx)^2}{10}+\frac{c^3x^3\arctan(cx)^2}{6}-\frac{\arctan(cx)^2cx}{2}+\frac{\arctan(cx)^3}{6}+\frac{c^4x^4\arctan(cx)}{20}-\frac{4c^2x^2\arctan}{1}\right.}{}$
parts	$\frac{a^3x^6}{6}+\frac{b^3\left(\frac{c^6x^6\arctan(cx)^3}{6}-\frac{c^5x^5\arctan(cx)^2}{10}+\frac{c^3x^3\arctan(cx)^2}{6}-\frac{\arctan(cx)^2cx}{2}+\frac{\arctan(cx)^3}{6}+\frac{c^4x^4\arctan(cx)}{20}-\frac{4c^2x^2\arctan}{1}\right.}{}$
risch	Expression too large to display

```
[In] int(x^5*(a+b*arctan(c*x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^6*(1/6*a^3*c^6*x^6+b^3*(1/6*c^6*x^6*arctan(c*x)^3-1/10*c^5*x^5*arctan(c
*x)^2+1/6*c^3*x^3*arctan(c*x)^2-1/2*arctan(c*x)^2*c*x+1/6*arctan(c*x)^3+1/2
0*c^4*x^4*arctan(c*x)-4/15*c^2*x^2*arctan(c*x)+23/30*arctan(c*x)*ln(c^2*x^2
+1)-1/60*c^3*x^3+19/60*c*x-19/60*arctan(c*x)+23/60*I*(ln(c*x-I)*ln(c^2*x^2+
1)-1/2*ln(c*x-I)^2-dilog(-1/2*I*(c*x+I))-ln(c*x-I)*ln(-1/2*I*(c*x+I)))-23/6
0*I*(ln(c*x+I)*ln(c^2*x^2+1)-1/2*ln(c*x+I)^2-dilog(1/2*I*(c*x-I))-ln(c*x+I)
*ln(1/2*I*(c*x-I))))+3*a*b^2*(1/6*c^6*x^6*arctan(c*x)^2-1/15*c^5*x^5*arctan
```

```
(c*x)+1/9*c^3*x^3*arctan(c*x)-1/3*c*x*arctan(c*x)+1/6*arctan(c*x)^2+1/60*c^
4*x^4-4/45*c^2*x^2+23/90*ln(c^2*x^2+1))+3*a^2*b*(1/6*c^6*x^6*arctan(c*x)-1/
30*c^5*x^5+1/18*c^3*x^3-1/6*c*x+1/6*arctan(c*x)))
```

Fricas [F]

$$\int x^5(a+b\arctan(cx))^3\,dx=\int (b\arctan(cx)+a)^3x^5\,dx$$

```
[In] integrate(x^5*(a+b*arctan(c*x))^3,x, algorithm="fricas")
[Out] integral(b^3*x^5*arctan(c*x)^3 + 3*a*b^2*x^5*arctan(c*x)^2 + 3*a^2*b*x^5*ar
ctan(c*x) + a^3*x^5, x)
```

Sympy [F]

$$\int x^5(a+b\arctan(cx))^3\,dx=\int x^5(a+b\,\mathrm{atan}\,(cx))^3\;dx$$

```
[In] integrate(x**5*(a+b*atan(c*x))**3,x)
[Out] Integral(x**5*(a + b*atan(c*x))**3, x)
```

Maxima [F]

$$\int x^5(a+b\arctan(cx))^3\,dx=\int (b\arctan(cx)+a)^3x^5\,dx$$

```
[In] integrate(x^5*(a+b*arctan(c*x))^3,x, algorithm="maxima")
[Out] 1/2*a*b^2*x^6*arctan(c*x)^2 + 1/6*a^3*x^6 + 1/30*(15*x^6*arctan(c*x) - c*((
3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(c*x)/c^7))*a^2*b - 1/60*(4*c*
((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(c*x)/c^7)*arctan(c*x) - (3*
c^4*x^4 - 16*c^2*x^2 - 30*arctan(c*x)^2 + 46*log(c^2*x^2 + 1))/c^6)*a*b^2 +
 1/480*(20*(5760*c^7*integrate(1/480*x^7*arctan(c*x)^3/(c^7*x^2 + c^5), x)
- 1440*c^6*integrate(1/480*x^6*arctan(c*x)^2/(c^7*x^2 + c^5), x) - 360*c^6*
integrate(1/480*x^6*log(c^2*x^2 + 1)^2/(c^7*x^2 + c^5), x) - 288*c^6*integr
ate(1/480*x^6*log(c^2*x^2 + 1)/(c^7*x^2 + c^5), x) + 5760*c^5*integrate(1/4
80*x^5*arctan(c*x)^3/(c^7*x^2 + c^5), x) + 576*c^5*integrate(1/480*x^5*arct
an(c*x)/(c^7*x^2 + c^5), x) + 480*c^4*integrate(1/480*x^4*log(c^2*x^2 + 1)/
(c^7*x^2 + c^5), x) - 960*c^3*integrate(1/480*x^3*arctan(c*x)/(c^7*x^2 + c^
5), x) - 1440*c^2*integrate(1/480*x^2*log(c^2*x^2 + 1)/(c^7*x^2 + c^5), x)
```

```
+ 2880*c*integrate(1/480*x*arctan(c*x)/(c^7*x^2 + c^5), x) - arctan(c*x)^3/
c^6 - 360*integrate(1/480*log(c^2*x^2 + 1)^2/(c^7*x^2 + c^5), x))*c^6 + 40*
(c^6*x^6 + 1)*arctan(c*x)^3 - 4*(3*c^5*x^5 - 5*c^3*x^3 + 15*c*x)*arctan(c*x
)^2 + (3*c^5*x^5 - 5*c^3*x^3 + 15*c*x)*log(c^2*x^2 + 1)^2)*b^3/c^6
```

Giac [F]

$$\int x^5(a+b\arctan(cx))^3\,dx=\int(b\arctan(cx)+a)^3x^5\,dx$$

```
[In] integrate(x^5*(a+b*arctan(c*x))^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int x^5(a+b\arctan(cx))^3\,dx=\int x^5\,(a+b\,\mathrm{atan}(c\,x))^3\,dx$$

```
[In] int(x^5*(a + b*atan(c*x))^3,x)
```

```
[Out] int(x^5*(a + b*atan(c*x))^3, x)
```

3.25 $\int x^4(a+b\arctan(cx))^3\,dx$

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Optimal result

Integrand size = 14, antiderivative size = 271

$$\begin{aligned}\int x^4(a+b\arctan(cx))^3\,dx = & -\frac{9ab^2x}{10c^4}-\frac{b^3x^2}{20c^3}-\frac{9b^3x\arctan(cx)}{10c^4} \\ & +\frac{b^2x^3(a+b\arctan(cx))}{10c^2}+\frac{9b(a+b\arctan(cx))^2}{20c^5} \\ & +\frac{3bx^2(a+b\arctan(cx))^2}{10c^3}-\frac{3bx^4(a+b\arctan(cx))^2}{20c} \\ & +\frac{i(a+b\arctan(cx))^3}{5c^5}+\frac{1}{5}x^5(a+b\arctan(cx))^3 \\ & +\frac{3b(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{5c^5}+\frac{b^3\log\left(1+c^2x^2\right)}{2c^5} \\ & +\frac{3ib^2(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{5c^5} \\ & +\frac{3b^3\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{10c^5}\end{aligned}$$

```
[Out] -9/10*a*b^2*x/c^4-1/20*b^3*x^2/c^3-9/10*b^3*x*arctan(c*x)/c^4+1/10*b^2*x^3*
(a+b*arctan(c*x))/c^2+9/20*b*(a+b*arctan(c*x))^2/c^5+3/10*b*x^2*(a+b*arctan
(c*x))^2/c^3-3/20*b*x^4*(a+b*arctan(c*x))^2/c+1/5*I*(a+b*arctan(c*x))^3/c^5
+1/5*x^5*(a+b*arctan(c*x))^3+3/5*b*(a+b*arctan(c*x))^2*ln(2/(1+I*c*x))/c^5+
1/2*b^3*ln(c^2*x^2+1)/c^5+3/5*I*b^2*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*
x))/c^5+3/10*b^3*polylog(3,1-2/(1+I*c*x))/c^5
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {4946, 5036, 272, 45, 4930, 266, 5004, 5040, 4964, 5114, 6745}

$$\int x^4(a+b\arctan(cx))^3\,dx = \frac{3ib^2\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{5c^5} + \frac{b^2x^3(a+b\arctan(cx))}{10c^2} + \frac{i(a+b\arctan(cx))^3}{5c^5} + \frac{9b(a+b\arctan(cx))^2}{20c^5} + \frac{3b\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{5c^5} + \frac{3bx^2(a+b\arctan(cx))^2}{10c^3} + \frac{1}{5}x^5(a+b\arctan(cx))^3 - \frac{3bx^4(a+b\arctan(cx))^2}{20c} - \frac{9ab^2x}{10c^4} - \frac{9b^3x\arctan(cx)}{10c^4} + \frac{3b^3\operatorname{PolyLog}\left(3,1-\frac{2}{icx+1}\right)}{10c^5} - \frac{b^3x^2}{20c^3} + \frac{b^3\log(c^2x^2+1)}{2c^5}$$

```
[In] Int[x^4*(a + b*ArcTan[c*x])^3,x]
```

```
[Out] (-9*a*b^2*x)/(10*c^4) - (b^3*x^2)/(20*c^3) - (9*b^3*x*ArcTan[c*x])/(10*c^4)
 + (b^2*x^3*(a + b*ArcTan[c*x]))/(10*c^2) + (9*b*(a + b*ArcTan[c*x])^2)/(20
*c^5) + (3*b*x^2*(a + b*ArcTan[c*x])^2)/(10*c^3) - (3*b*x^4*(a + b*ArcTan[c
*x])^2)/(20*c) + ((I/5)*(a + b*ArcTan[c*x])^3)/c^5 + (x^5*(a + b*ArcTan[c*x
])^3)/5 + (3*b*(a + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/(5*c^5) + (b^3*Log
[1 + c^2*x^2])/(2*c^5) + (((3*I)/5)*b^2*(a + b*ArcTan[c*x])*PolyLog[2, 1 -
2/(1 + I*c*x)])/c^5 + (3*b^3*PolyLog[3, 1 - 2/(1 + I*c*x)])/(10*c^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
```

```
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{5}x^5(a+b\arctan(cx))^3-\frac{1}{5}(3bc)\int\frac{x^5(a+b\arctan(cx))^2}{1+c^2x^2}\,dx\\
&= \frac{1}{5}x^5(a+b\arctan(cx))^3-\frac{(3b)\int x^3(a+b\arctan(cx))^2\,dx}{5c}+\frac{(3b)\int\frac{x^3(a+b\arctan(cx))^2}{1+c^2x^2}\,dx}{5c}\\
&= -\frac{3bx^4(a+b\arctan(cx))^2}{20c}+\frac{1}{5}x^5(a+b\arctan(cx))^3\\
&\quad+\frac{1}{10}(3b^2)\int\frac{x^4(a+b\arctan(cx))}{1+c^2x^2}\,dx\\
&\quad+\frac{(3b)\int x(a+b\arctan(cx))^2\,dx}{5c^3}-\frac{(3b)\int\frac{x(a+b\arctan(cx))^2}{1+c^2x^2}\,dx}{5c^3}\\
&= \frac{3bx^2(a+b\arctan(cx))^2}{10c^3}-\frac{3bx^4(a+b\arctan(cx))^2}{20c}\\
&\quad+\frac{i(a+b\arctan(cx))^3}{5c^5}+\frac{1}{5}x^5(a+b\arctan(cx))^3\\
&\quad+\frac{(3b)\int\frac{(a+b\arctan(cx))^2}{i-cx}\,dx}{5c^4}+\frac{(3b^2)\int x^2(a+b\arctan(cx))\,dx}{10c^2}\\
&\quad-\frac{(3b^2)\int\frac{x^2(a+b\arctan(cx))}{1+c^2x^2}\,dx}{10c^2}-\frac{(3b^2)\int\frac{x^2(a+b\arctan(cx))}{1+c^2x^2}\,dx}{5c^2}\\
&= \frac{b^2x^3(a+b\arctan(cx))}{10c^2}+\frac{3bx^2(a+b\arctan(cx))^2}{10c^3}-\frac{3bx^4(a+b\arctan(cx))^2}{20c}\\
&\quad+\frac{i(a+b\arctan(cx))^3}{5c^5}+\frac{1}{5}x^5(a+b\arctan(cx))^3+\frac{3b(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{5c^5}\\
&\quad-\frac{(3b^2)\int(a+b\arctan(cx))\,dx}{10c^4}+\frac{(3b^2)\int\frac{a+b\arctan(cx)}{1+c^2x^2}\,dx}{10c^4}-\frac{(3b^2)\int(a+b\arctan(cx))\,dx}{5c^4}\\
&\quad+\frac{(3b^2)\int\frac{a+b\arctan(cx)}{1+c^2x^2}\,dx}{5c^4}-\frac{(6b^2)\int\frac{(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx}{5c^4}-\frac{b^3\int\frac{x^3}{1+c^2x^2}\,dx}{10c}
\end{aligned}$$

$$= -\frac{9ab^2x}{10c^4} + \frac{b^2x^3(a+b\arctan(cx))}{10c^2} + \frac{9b(a+b\arctan(cx))^2}{20c^5} + \frac{3bx^2(a+b\arctan(cx))^2}{10c^3} - \frac{3bx^4(a+b\arctan(cx))^2}{20c} + \frac{i(a+b\arctan(cx))^3}{5c^5} + \frac{1}{5}x^5(a+b\arctan(cx))^3 + \frac{3b(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{5c^5} + \frac{3ib^2(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{5c^5} - \frac{(3ib^3)\int\frac{\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{1+c^2x^2}dx}{5c^4} - \frac{(3b^3)\int\arctan(cx)\,dx}{10c^4} - \frac{(3b^3)\int\arctan(cx)\,dx}{5c^4} - \frac{b^3\operatorname{Subst}\left(\int\frac{x}{1+c^2x}\,dx,x,x^2\right)}{20c}$$

$$= -\frac{9ab^2x}{10c^4} - \frac{9b^3x\arctan(cx)}{10c^4} + \frac{b^2x^3(a+b\arctan(cx))}{10c^2} + \frac{9b(a+b\arctan(cx))^2}{20c^5} + \frac{3bx^2(a+b\arctan(cx))^2}{10c^3} - \frac{3bx^4(a+b\arctan(cx))^2}{20c} + \frac{i(a+b\arctan(cx))^3}{5c^5} + \frac{1}{5}x^5(a+b\arctan(cx))^3 + \frac{3b(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{5c^5} + \frac{3ib^2(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{5c^5} + \frac{3b^3\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{10c^5} + \frac{(3b^3)\int\frac{x}{1+c^2x^2}dx}{10c^3} + \frac{(3b^3)\int\frac{x}{1+c^2x^2}dx}{5c^3} - \frac{b^3\operatorname{Subst}\left(\int\left(\frac{1}{c^2}-\frac{1}{c^2(1+c^2x)}\right)dx,x,x^2\right)}{20c}$$

$$= -\frac{9ab^2x}{10c^4} - \frac{b^3x^2}{20c^3} - \frac{9b^3x\arctan(cx)}{10c^4} + \frac{b^2x^3(a+b\arctan(cx))}{10c^2} + \frac{9b(a+b\arctan(cx))^2}{20c^5} + \frac{3bx^2(a+b\arctan(cx))^2}{10c^3} - \frac{3bx^4(a+b\arctan(cx))^2}{20c} + \frac{i(a+b\arctan(cx))^3}{5c^5} + \frac{1}{5}x^5(a+b\arctan(cx))^3 + \frac{3b(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{5c^5} + \frac{b^3\log(1+c^2x^2)}{2c^5} + \frac{3ib^2(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{5c^5} + \frac{3b^3\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{10c^5}$$

Mathematica [A] (verified)

Time = 1.12 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.46

$$\int x^4(a+b\arctan(cx))^3\,dx = \frac{-b^3 - 18ab^2cx + 6a^2bc^2x^2 - b^3c^2x^2 + 2ab^2c^3x^3 - 3a^2bc^4x^4 + 4a^3c^5x^5 + 18ab^2\arctan(cx) - 18b^3cx\arctan(}{}$$

```
[In] Integrate[x^4*(a + b*ArcTan[c*x])^3,x]
```

```
[Out] (-b^3 - 18*a*b^2*c*x + 6*a^2*b*c^2*x^2 - b^3*c^2*x^2 + 2*a*b^2*c^3*x^3 - 3*
a^2*b*c^4*x^4 + 4*a^3*c^5*x^5 + 18*a*b^2*ArcTan[c*x] - 18*b^3*c*x*ArcTan[c*
```

```
x] + 12*a*b^2*c^2*x^2*ArcTan[c*x] + 2*b^3*c^3*x^3*ArcTan[c*x] - 6*a*b^2*c^4
*x^4*ArcTan[c*x] + 12*a^2*b*c^5*x^5*ArcTan[c*x] - (12*I)*a*b^2*ArcTan[c*x]^
2 + 9*b^3*ArcTan[c*x]^2 + 6*b^3*c^2*x^2*ArcTan[c*x]^2 - 3*b^3*c^4*x^4*ArcTa
n[c*x]^2 + 12*a*b^2*c^5*x^5*ArcTan[c*x]^2 - (4*I)*b^3*ArcTan[c*x]^3 + 4*b^3
*c^5*x^5*ArcTan[c*x]^3 + 24*a*b^2*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])
] + 12*b^3*ArcTan[c*x]^2*Log[1 + E^((2*I)*ArcTan[c*x])] - 6*a^2*b*Log[1 + c
^2*x^2] + 10*b^3*Log[1 + c^2*x^2] - (12*I)*b^2*(a + b*ArcTan[c*x])*PolyLog[
2, -E^((2*I)*ArcTan[c*x])] + 6*b^3*PolyLog[3, -E^((2*I)*ArcTan[c*x])])/(20*
c^5)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.28 (sec) , antiderivative size = 1185, normalized size of antiderivative = 4.37

Expression too large to display

```
[In] int(x^4*(a+b*arctan(c*x))^3,x)
```

```
[Out] 1/c^5*(1/5*a^3*c^5*x^5+b^3*(1/5*c^5*x^5*arctan(c*x)^3-3/20*c^4*x^4*arctan(c
*x)^2+3/10*c^2*x^2*arctan(c*x)^2-3/10*arctan(c*x)^2*ln(c^2*x^2+1)+3/5*arcta
n(c*x)^2*ln((1+I*c*x)/(c^2*x^2+1)^(1/2))-1/20*I*(3*Pi*csgn(I*(1+I*c*x)^2/(c
^2*x^2+1))^3*arctan(c*x)^2-6*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^2*csgn(I*(1
+I*c*x)/(c^2*x^2+1)^(1/2))*arctan(c*x)^2+3*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1
))*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/(1+
(1+I*c*x)^2/(c^2*x^2+1))^2)*arctan(c*x)^2-3*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+
1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)^2*arctan(
c*x)^2+3*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1
/2))^2*arctan(c*x)^2-3*Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)*csgn(I*(1+I
*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)^2*arctan(c*x)^2+3*Pi*csg
n(I*(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)^3*arctan(c*x)^2-
3*Pi*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1
))^2)*arctan(c*x)^2+6*Pi*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*(1+(1+I
*c*x)^2/(c^2*x^2+1))^2)^2*arctan(c*x)^2-3*Pi*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2
+1))^2)^3*arctan(c*x)^2+4*arctan(c*x)^3+2*I*arctan(c*x)*c^3*x^3-I*c^2*x^2-2
0*arctan(c*x)-18*I*arctan(c*x)*c*x+12*I*ln(2)*arctan(c*x)^2+9*I*arctan(c*x)
^2-I)-ln(1+(1+I*c*x)^2/(c^2*x^2+1))-3/5*I*arctan(c*x)*polylog(2,-(1+I*c*x)^
2/(c^2*x^2+1))+3/10*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1)))+3*a*b^2*(1/5*c^5*x
^5*arctan(c*x)^2-1/10*c^4*x^4*arctan(c*x)+1/5*c^2*x^2*arctan(c*x)-1/5*arcta
n(c*x)*ln(c^2*x^2+1)+1/30*c^3*x^3-3/10*c*x+3/10*arctan(c*x)-1/10*I*(ln(c*x-
I)*ln(c^2*x^2+1)-1/2*ln(c*x-I)^2-dilog(-1/2*I*(c*x+I))-ln(c*x-I)*ln(-1/2*I*
(c*x+I)))+1/10*I*(ln(c*x+I)*ln(c^2*x^2+1)-1/2*ln(c*x+I)^2-dilog(1/2*I*(c*x-
I))-ln(c*x+I)*ln(1/2*I*(c*x-I))))+3*a^2*b*(1/5*c^5*x^5*arctan(c*x)-1/20*c^4
*x^4+1/10*c^2*x^2-1/10*ln(c^2*x^2+1)))
```

Fricas [F]

$$\int x^4(a+b\arctan(cx))^3\,dx=\int (b\arctan(cx)+a)^3x^4\,dx$$

```
[In] integrate(x^4*(a+b*arctan(c*x))^3,x, algorithm="fricas")
[Out] integral(b^3*x^4*arctan(c*x)^3 + 3*a*b^2*x^4*arctan(c*x)^2 + 3*a^2*b*x^4*ar
ctan(c*x) + a^3*x^4, x)
```

Sympy [F]

$$\int x^4(a+b\arctan(cx))^3\,dx=\int x^4(a+b\,\mathrm{atan}(cx))^3\,dx$$

```
[In] integrate(x**4*(a+b*atan(c*x))**3,x)
[Out] Integral(x**4*(a + b*atan(c*x))**3, x)
```

Maxima [F]

$$\int x^4(a+b\arctan(cx))^3\,dx=\int (b\arctan(cx)+a)^3x^4\,dx$$

```
[In] integrate(x^4*(a+b*arctan(c*x))^3,x, algorithm="maxima")
[Out] 1/40*b^3*x^5*arctan(c*x)^3 - 3/160*b^3*x^5*arctan(c*x)*log(c^2*x^2 + 1)^2 +
 1/5*a^3*x^5 + 3/20*(4*x^5*arctan(c*x) - c*((c^2*x^4 - 2*x^2)/c^4 + 2*log(c
^2*x^2 + 1)/c^6))*a^2*b + integrate(1/160*(12*b^3*c^2*x^6*arctan(c*x)*log(c
^2*x^2 + 1) + 140*(b^3*c^2*x^6 + b^3*x^4)*arctan(c*x)^3 + 12*(40*a*b^2*c^2*
x^6 - b^3*c*x^5 + 40*a*b^2*x^4)*arctan(c*x)^2 + 3*(b^3*c*x^5 + 5*(b^3*c^2*x
^6 + b^3*x^4)*arctan(c*x))*log(c^2*x^2 + 1)^2)/(c^2*x^2 + 1), x)
```

Giac [F]

$$\int x^4(a+b\arctan(cx))^3\,dx=\int (b\arctan(cx)+a)^3x^4\,dx$$

```
[In] integrate(x^4*(a+b*arctan(c*x))^3,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int x^4(a+b\arctan(cx))^3\,dx=\int x^4\,(a+b\,\mathrm{atan}(c\,x))^3\,dx$$

```
[In] int(x^4*(a + b*atan(c*x))^3,x)
[Out] int(x^4*(a + b*atan(c*x))^3, x)
```

3.26 $\int x^3(a+b\arctan(cx))^3\,dx$

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Optimal result

Integrand size = 14, antiderivative size = 194

$$\begin{aligned}\int x^3(a+b\arctan(cx))^3\,dx = &-\frac{b^3x}{4c^3}+\frac{b^3\arctan(cx)}{4c^4}+\frac{b^2x^2(a+b\arctan(cx))}{4c^2}\\&+\frac{ib(a+b\arctan(cx))^2}{c^4}+\frac{3bx(a+b\arctan(cx))^2}{4c^3}\\&-\frac{bx^3(a+b\arctan(cx))^2}{4c}-\frac{(a+b\arctan(cx))^3}{4c^4}\\&+\frac{1}{4}x^4(a+b\arctan(cx))^3+\frac{2b^2(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c^4}\\&+\frac{ib^3\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^4}\end{aligned}$$

```
[Out] -1/4*b^3*x/c^3+1/4*b^3*arctan(c*x)/c^4+1/4*b^2*x^2*(a+b*arctan(c*x))/c^2+I*
b*(a+b*arctan(c*x))^2/c^4+3/4*b*x*(a+b*arctan(c*x))^2/c^3-1/4*b*x^3*(a+b*ar
ctan(c*x))^2/c-1/4*(a+b*arctan(c*x))^3/c^4+1/4*x^4*(a+b*arctan(c*x))^3+2*b^
2*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^4+I*b^3*polylog(2,1-2/(1+I*c*x))/c^4
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules

used = {4946, 5036, 327, 209, 5040, 4964, 2449, 2352, 4930, 5004}

$$\int x^3(a+b\arctan(cx))^3\,dx = \frac{2b^2\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c^4} + \frac{b^2x^2(a+b\arctan(cx))}{4c^2} - \frac{(a+b\arctan(cx))^3}{4c^4} + \frac{ib(a+b\arctan(cx))^2}{c^4} + \frac{3bx(a+b\arctan(cx))^2}{4c^3} + \frac{1}{4}x^4(a+b\arctan(cx))^3 - \frac{bx^3(a+b\arctan(cx))^2}{4c} + \frac{b^3\arctan(cx)}{4c^4} + \frac{ib^3\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{c^4} - \frac{b^3x}{4c^3}$$

```
[In] Int[x^3*(a + b*ArcTan[c*x])^3,x]
```

```
[Out] -1/4*(b^3*x)/c^3 + (b^3*ArcTan[c*x])/(4*c^4) + (b^2*x^2*(a + b*ArcTan[c*x])
)/(4*c^2) + (I*b*(a + b*ArcTan[c*x])^2)/c^4 + (3*b*x*(a + b*ArcTan[c*x])^2)
/(4*c^3) - (b*x^3*(a + b*ArcTan[c*x])^2)/(4*c) - (a + b*ArcTan[c*x])^3/(4*c
^4) + (x^4*(a + b*ArcTan[c*x])^3)/4 + (2*b^2*(a + b*ArcTan[c*x])*Log[2/(1 +
 I*c*x)])/c^4 + (I*b^3*PolyLog[2, 1 - 2/(1 + I*c*x)])/c^4
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule steps

$$\text{integral} = \frac{1}{4}x^4(a+b\arctan(cx))^3 - \frac{1}{4}(3bc)\int \frac{x^4(a+b\arctan(cx))^2}{1+c^2x^2}\,dx$$

$$= \frac{1}{4}x^4(a+b\arctan(cx))^3 - \frac{(3b)\int x^2(a+b\arctan(cx))^2\,dx}{4c} + \frac{(3b)\int \frac{x^2(a+b\arctan(cx))^2}{1+c^2x^2}\,dx}{4c}$$

$$= -\frac{bx^3(a+b\arctan(cx))^2}{4c} + \frac{1}{4}x^4(a+b\arctan(cx))^3 + \frac{1}{2}b^2\int \frac{x^3(a+b\arctan(cx))}{1+c^2x^2}\,dx + \frac{(3b)\int (a+b\arctan(cx))^2\,dx}{4c^3} - \frac{(3b)\int \frac{(a+b\arctan(cx))^2}{1+c^2x^2}\,dx}{4c^3}$$

$$= \frac{3bx(a+b\arctan(cx))^2}{4c^3} - \frac{bx^3(a+b\arctan(cx))^2}{4c} - \frac{(a+b\arctan(cx))^3}{4c^4} + \frac{1}{4}x^4(a+b\arctan(cx))^3 + \frac{b^2\int x(a+b\arctan(cx))\,dx}{2c^2} - \frac{b^2\int \frac{x(a+b\arctan(cx))}{1+c^2x^2}\,dx}{2c^2} - \frac{(3b^2)\int \frac{x(a+b\arctan(cx))}{1+c^2x^2}\,dx}{2c^2}$$

$$= \frac{b^2x^2(a+b\arctan(cx))}{4c^2} + \frac{ib(a+b\arctan(cx))^2}{c^4} + \frac{3bx(a+b\arctan(cx))^2}{4c^3} - \frac{bx^3(a+b\arctan(cx))^2}{4c} - \frac{(a+b\arctan(cx))^3}{4c^4} + \frac{1}{4}x^4(a+b\arctan(cx))^3 + \frac{b^2\int \frac{a+b\arctan(cx)}{i-cx}\,dx}{2c^3} + \frac{(3b^2)\int \frac{a+b\arctan(cx)}{i-cx}\,dx}{2c^3} - \frac{b^3\int \frac{x^2}{1+c^2x^2}\,dx}{4c}$$

$$= -\frac{b^3x}{4c^3} + \frac{b^2x^2(a+b\arctan(cx))}{4c^2} + \frac{ib(a+b\arctan(cx))^2}{c^4} + \frac{3bx(a+b\arctan(cx))^2}{4c^3} - \frac{bx^3(a+b\arctan(cx))^2}{4c} - \frac{(a+b\arctan(cx))^3}{4c^4} + \frac{1}{4}x^4(a+b\arctan(cx))^3 + \frac{2b^2(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c^4} + \frac{b^3\int \frac{1}{1+c^2x^2}\,dx}{4c^3} - \frac{b^3\int \frac{\log\left(\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx}{2c^3} - \frac{(3b^3)\int \frac{\log\left(\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx}{2c^3}$$

$$= -\frac{b^3x}{4c^3} + \frac{b^3\arctan(cx)}{4c^4} + \frac{b^2x^2(a+b\arctan(cx))}{4c^2} + \frac{ib(a+b\arctan(cx))^2}{c^4} + \frac{3bx(a+b\arctan(cx))^2}{4c^3} - \frac{bx^3(a+b\arctan(cx))^2}{4c} - \frac{(a+b\arctan(cx))^3}{4c^4} + \frac{1}{4}x^4(a+b\arctan(cx))^3 + \frac{2b^2(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c^4} + \frac{(ib^3)\,\text{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1+icx}\right)}{2c^4} + \frac{(3ib^3)\,\text{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1+icx}\right)}{2c^4}$$

$$= -\frac{b^3x}{4c^3} + \frac{b^3\arctan(cx)}{4c^4} + \frac{b^2x^2(a+b\arctan(cx))}{4c^2} + \frac{ib(a+b\arctan(cx))^2}{c^4} + \frac{3bx(a+b\arctan(cx))^2}{4c^3} - \frac{bx^3(a+b\arctan(cx))^2}{4c} - \frac{(a+b\arctan(cx))^3}{4c^4} + \frac{1}{4}x^4(a+b\arctan(cx))^3 + \frac{2b^2(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c^4} + \frac{ib^3\,\text{PolyLog}\left(2, 1-\frac{2}{1+icx}\right)}{c^4}$$

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.16

$$\int x^3(a+b\arctan(cx))^3\,dx$$
$$= \frac{ab^2+3a^2bcx-b^3cx+ab^2c^2x^2-a^2bc^3x^3+a^3c^4x^4-b^2(b(4i-3cx+c^3x^3)+a(3-3c^4x^4))\arctan(cx)^2+b}{}$$

```
[In] Integrate[x^3*(a + b*ArcTan[c*x])^3,x]
```

```
[Out] (a*b^2 + 3*a^2*b*c*x - b^3*c*x + a*b^2*c^2*x^2 - a^2*b*c^3*x^3 + a^3*c^4*x^
4 - b^2*(b*(4*I - 3*c*x + c^3*x^3) + a*(3 - 3*c^4*x^4))*ArcTan[c*x]^2 + b^3
*(-1 + c^4*x^4)*ArcTan[c*x]^3 + b*ArcTan[c*x]*(-2*a*b*c*x*(-3 + c^2*x^2) +
b^2*(1 + c^2*x^2) + 3*a^2*(-1 + c^4*x^4) + 8*b^2*Log[1 + E^((2*I)*ArcTan[c*
x])]) - 4*a*b^2*Log[1 + c^2*x^2] - (4*I)*b^3*PolyLog[2, -E^((2*I)*ArcTan[c*
x])])/(4*c^4)
```

Maple [A] (verified)

Time = 2.56 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.75

method	result
derivativedivides	$\frac{\frac{a^3c^4x^4}{4}+b^3\left(\frac{c^4x^4\arctan(cx)^3}{4}-\frac{c^3x^3\arctan(cx)^2}{4}+\frac{3\arctan(cx)^2cx}{4}-\frac{\arctan(cx)^3}{4}+\frac{c^2x^2\arctan(cx)}{4}-\arctan(cx)\ln(c^2x^2+1)-\right.}{}$
default	$\frac{\frac{a^3c^4x^4}{4}+b^3\left(\frac{c^4x^4\arctan(cx)^3}{4}-\frac{c^3x^3\arctan(cx)^2}{4}+\frac{3\arctan(cx)^2cx}{4}-\frac{\arctan(cx)^3}{4}+\frac{c^2x^2\arctan(cx)}{4}-\arctan(cx)\ln(c^2x^2+1)-\right.}{}$
parts	$\frac{a^3x^4}{4}+\frac{b^3\left(\frac{c^4x^4\arctan(cx)^3}{4}-\frac{c^3x^3\arctan(cx)^2}{4}+\frac{3\arctan(cx)^2cx}{4}-\frac{\arctan(cx)^3}{4}+\frac{c^2x^2\arctan(cx)}{4}-\arctan(cx)\ln(c^2x^2+1)-\right.}{}$
risch	Expression too large to display

```
[In] int(x^3*(a+b*arctan(c*x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(1/4*a^3*c^4*x^4+b^3*(1/4*c^4*x^4*arctan(c*x)^3-1/4*c^3*x^3*arctan(c*
x)^2+3/4*arctan(c*x)^2*c*x-1/4*arctan(c*x)^3+1/4*c^2*x^2*arctan(c*x)-arctan
(c*x)*ln(c^2*x^2+1)-1/4*c*x+1/4*arctan(c*x)-1/2*I*(ln(c*x-I)*ln(c^2*x^2+1)-
1/2*ln(c*x-I)^2-dilog(-1/2*I*(c*x+I))-ln(c*x-I)*ln(-1/2*I*(c*x+I)))+1/2*I*(
ln(c*x+I)*ln(c^2*x^2+1)-1/2*ln(c*x+I)^2-dilog(1/2*I*(c*x-I))-ln(c*x+I)*ln(1
/2*I*(c*x-I))))+3*a*b^2*(1/4*c^4*x^4*arctan(c*x)^2-1/6*c^3*x^3*arctan(c*x)+
1/2*c*x*arctan(c*x)-1/4*arctan(c*x)^2+1/12*c^2*x^2-1/3*ln(c^2*x^2+1))+3*a^2
*b*(1/4*c^4*x^4*arctan(c*x)-1/12*c^3*x^3+1/4*c*x-1/4*arctan(c*x)))
```

Fricas [F]

$$\int x^3(a+b\arctan(cx))^3\,dx=\int\left(b\arctan\left(cx\right)+a\right)^3x^3\,dx$$

```
[In] integrate(x^3*(a+b*arctan(c*x))^3,x, algorithm="fricas")
[Out] integral(b^3*x^3*arctan(c*x)^3 + 3*a*b^2*x^3*arctan(c*x)^2 + 3*a^2*b*x^3*ar
ctan(c*x) + a^3*x^3, x)
```

Sympy [F]

$$\int x^3(a+b\arctan(cx))^3\,dx=\int x^3(a+b\,\mathrm{atan}\,(cx))^3\;dx$$

```
[In] integrate(x**3*(a+b*atan(c*x))**3,x)
[Out] Integral(x**3*(a + b*atan(c*x))**3, x)
```

Maxima [F]

$$\int x^3(a+b\arctan(cx))^3\,dx=\int\left(b\arctan\left(cx\right)+a\right)^3x^3\,dx$$

```
[In] integrate(x^3*(a+b*arctan(c*x))^3,x, algorithm="maxima")
[Out] 3/4*a*b^2*x^4*arctan(c*x)^2 + 1/4*a^3*x^4 + 1/4*(3*x^4*arctan(c*x) - c*((c^
2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*a^2*b - 1/4*(2*c*((c^2*x^3 - 3*x)/c^
4 + 3*arctan(c*x)/c^5)*arctan(c*x) - (c^2*x^2 + 3*arctan(c*x)^2 - 4*log(c^2
*x^2 + 1))/c^4)*a*b^2 + 1/64*(4*(512*c^5*integrate(1/64*x^5*arctan(c*x)^3/(
c^5*x^2 + c^3), x) - 192*c^4*integrate(1/64*x^4*arctan(c*x)^2/(c^5*x^2 + c^
3), x) - 48*c^4*integrate(1/64*x^4*log(c^2*x^2 + 1)^2/(c^5*x^2 + c^3), x) -
 64*c^4*integrate(1/64*x^4*log(c^2*x^2 + 1)/(c^5*x^2 + c^3), x) + 512*c^3*i
ntegrate(1/64*x^3*arctan(c*x)^3/(c^5*x^2 + c^3), x) + 128*c^3*integrate(1/6
4*x^3*arctan(c*x)/(c^5*x^2 + c^3), x) + 192*c^2*integrate(1/64*x^2*log(c^2*
x^2 + 1)/(c^5*x^2 + c^3), x) - 384*c*integrate(1/64*x*arctan(c*x)/(c^5*x^2
+ c^3), x) + arctan(c*x)^3/c^4 + 48*integrate(1/64*log(c^2*x^2 + 1)^2/(c^5*
x^2 + c^3), x))*c^4 + 8*(c^4*x^4 - 1)*arctan(c*x)^3 - 4*(c^3*x^3 - 3*c*x)*a
rctan(c*x)^2 + (c^3*x^3 - 3*c*x)*log(c^2*x^2 + 1)^2)*b^3/c^4
```

Giac [F]

$$\int x^3(a+b\arctan(cx))^3\,dx = \int (b\arctan(cx)+a)^3x^3\,dx$$

```
[In] integrate(x^3*(a+b*arctan(c*x))^3,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int x^3(a+b\arctan(cx))^3\,dx = \int x^3\,(a+b\,\mathrm{atan}(c\,x))^3\,dx$$

```
[In] int(x^3*(a + b*atan(c*x))^3,x)
[Out] int(x^3*(a + b*atan(c*x))^3, x)
```

3.27 $\int x^2(a+b\arctan(cx))^3\,dx$

Optimal result . . . 205
Rubi [A] (verified) . . . 206
Mathematica [A] (verified) . . . 209
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Fricas [F] . . . 210
Sympy [F] . . . 210
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Mupad [F(-1)] . . . 211

Optimal result

Integrand size = 14, antiderivative size = 206

$$\int x^2(a+b\arctan(cx))^3\,dx = \frac{ab^2x}{c^2}+\frac{b^3x\arctan(cx)}{c^2}-\frac{b(a+b\arctan(cx))^2}{2c^3} -\frac{bx^2(a+b\arctan(cx))^2}{2c}-\frac{i(a+b\arctan(cx))^3}{3c^3} +\frac{1}{3}x^3(a+b\arctan(cx))^3 -\frac{b(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{c^3}-\frac{b^3\log(1+c^2x^2)}{2c^3} -\frac{ib^2(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^3} -\frac{b^3\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2c^3}$$

```
[Out] a*b^2*x/c^2+b^3*x*arctan(c*x)/c^2-1/2*b*(a+b*arctan(c*x))^2/c^3-1/2*b*x^2*(
a+b*arctan(c*x))^2/c-1/3*I*(a+b*arctan(c*x))^3/c^3+1/3*x^3*(a+b*arctan(c*x)
)^3-b*(a+b*arctan(c*x))^2*ln(2/(1+I*c*x))/c^3-1/2*b^3*ln(c^2*x^2+1)/c^3-I*b
^2*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*x))/c^3-1/2*b^3*polylog(3,1-2/(1+
I*c*x))/c^3
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {4946, 5036, 4930, 266, 5004, 5040, 4964, 5114, 6745}

$$\int x^2(a+b\arctan(cx))^3\,dx = -\frac{ib^2\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{c^3} - \frac{b(a+b\arctan(cx))^2}{2c^3} - \frac{i(a+b\arctan(cx))^3}{3c^3} - \frac{b\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c^3} + \frac{1}{3}x^3(a+b\arctan(cx))^3 - \frac{bx^2(a+b\arctan(cx))^2}{2c} + \frac{ab^2x}{c^2} + \frac{b^3x\arctan(cx)}{c^2} - \frac{b^3\operatorname{PolyLog}\left(3,1-\frac{2}{icx+1}\right)}{2c^3} - \frac{b^3\log\left(c^2x^2+1\right)}{2c^3}$$

```
[In] Int[x^2*(a + b*ArcTan[c*x])^3,x]
```

```
[Out] (a*b^2*x)/c^2 + (b^3*x*ArcTan[c*x])/c^2 - (b*(a + b*ArcTan[c*x])^2)/(2*c^3)
 - (b*x^2*(a + b*ArcTan[c*x])^2)/(2*c) - ((I/3)*(a + b*ArcTan[c*x])^3)/c^3
+ (x^3*(a + b*ArcTan[c*x])^3)/3 - (b*(a + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x
)])/c^3 - (b^3*Log[1 + c^2*x^2])/(2*c^3) - (I*b^2*(a + b*ArcTan[c*x])*PolyL
og[2, 1 - 2/(1 + I*c*x)])/c^3 - (b^3*PolyLog[3, 1 - 2/(1 + I*c*x)])/(2*c^3)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}x^3(a+b\arctan(cx))^3-(bc)\int\frac{x^3(a+b\arctan(cx))^2}{1+c^2x^2}\,dx\\ &= \frac{1}{3}x^3(a+b\arctan(cx))^3-\frac{b\int x(a+b\arctan(cx))^2\,dx}{c}+\frac{b\int\frac{x(a+b\arctan(cx))^2}{1+c^2x^2}\,dx}{c}\end{aligned}$$

$$= -\frac{bx^2(a+b\arctan(cx))^2}{2c} - \frac{i(a+b\arctan(cx))^3}{3c^3} + \frac{1}{3}x^3(a+b\arctan(cx))^3 + b^2\int \frac{x^2(a+b\arctan(cx))}{1+c^2x^2}\,dx - \frac{b\int \frac{(a+b\arctan(cx))^2}{i-cx}\,dx}{c^2}$$

$$= -\frac{bx^2(a+b\arctan(cx))^2}{2c} - \frac{i(a+b\arctan(cx))^3}{3c^3} + \frac{1}{3}x^3(a+b\arctan(cx))^3 - \frac{b(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{c^3} + \frac{b^2\int(a+b\arctan(cx))\,dx}{c^2} - \frac{b^2\int \frac{a+b\arctan(cx)}{1+c^2x^2}\,dx}{c^2} + \frac{(2b^2)\int \frac{(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx}{c^2}$$

$$= \frac{ab^2x}{c^2} - \frac{b(a+b\arctan(cx))^2}{2c^3} - \frac{bx^2(a+b\arctan(cx))^2}{2c} - \frac{i(a+b\arctan(cx))^3}{3c^3} + \frac{1}{3}x^3(a+b\arctan(cx))^3 - \frac{b(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{c^3} - \frac{ib^2(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^3} + \frac{(ib^3)\int \frac{\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx}{c^2} + \frac{b^3\int \arctan(cx)\,dx}{c^2}$$

$$= \frac{ab^2x}{c^2} + \frac{b^3x\arctan(cx)}{c^2} - \frac{b(a+b\arctan(cx))^2}{2c^3} - \frac{bx^2(a+b\arctan(cx))^2}{2c} - \frac{i(a+b\arctan(cx))^3}{3c^3} + \frac{1}{3}x^3(a+b\arctan(cx))^3 - \frac{b(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{c^3} - \frac{ib^2(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^3} - \frac{b^3\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2c^3} - \frac{b^3\int \frac{x}{1+c^2x^2}\,dx}{c}$$

$$= \frac{ab^2x}{c^2} + \frac{b^3x\arctan(cx)}{c^2} - \frac{b(a+b\arctan(cx))^2}{2c^3} - \frac{bx^2(a+b\arctan(cx))^2}{2c} - \frac{i(a+b\arctan(cx))^3}{3c^3} + \frac{1}{3}x^3(a+b\arctan(cx))^3 - \frac{b(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{c^3} - \frac{b^3\log(1+c^2x^2)}{2c^3} - \frac{ib^2(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^3} - \frac{b^3\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2c^3}$$

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.31

$$\int x^2(a+b\arctan(cx))^3\,dx$$
$$=\frac{-3a^2bc^2x^2+2a^3c^3x^3+6a^2bc^3x^3\arctan(cx)+3a^2b\log(1+c^2x^2)+6ab^2(cx+(i+c^3x^3)\arctan(cx)^2-a}{}$$

```
[In] Integrate[x^2*(a + b*ArcTan[c*x])^3,x]
```

```
[Out] (-3*a^2*b*c^2*x^2 + 2*a^3*c^3*x^3 + 6*a^2*b*c^3*x^3*ArcTan[c*x] + 3*a^2*b*L
og[1 + c^2*x^2] + 6*a*b^2*(c*x + (I + c^3*x^3)*ArcTan[c*x]^2 - ArcTan[c*x]*
(1 + c^2*x^2 + 2*Log[1 + E^((2*I)*ArcTan[c*x])]) + I*PolyLog[2, -E^((2*I)*A
rcTan[c*x])]) + b^3*(6*c*x*ArcTan[c*x] - 3*ArcTan[c*x]^2 - 3*c^2*x^2*ArcTan
[c*x]^2 + (2*I)*ArcTan[c*x]^3 + 2*c^3*x^3*ArcTan[c*x]^3 - 6*ArcTan[c*x]^2*L
og[1 + E^((2*I)*ArcTan[c*x])] - 3*Log[1 + c^2*x^2] + (6*I)*ArcTan[c*x]*Poly
Log[2, -E^((2*I)*ArcTan[c*x])] - 3*PolyLog[3, -E^((2*I)*ArcTan[c*x])]))/(6*
c^3)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.19 (sec) , antiderivative size = 1088, normalized size of antiderivative = 5.28

Expression too large to display

```
[In] int(x^2*(a+b*arctan(c*x))^3,x)
```

```
[Out] 1/c^3*(1/3*a^3*c^3*x^3+b^3*(1/3*c^3*x^3*arctan(c*x)^3-1/2*c^2*x^2*arctan(c*
x)^2+1/2*arctan(c*x)^2*ln(c^2*x^2+1)-arctan(c*x)^2*ln((1+I*c*x)/(c^2*x^2+1)
^(1/2))+I*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))-1/2*polylog(3,-(1
+I*c*x)^2/(c^2*x^2+1))+1/12*I*arctan(c*x)*(-3*Pi*arctan(c*x)*csgn(I/(1+(1+I
*c*x)^2/(c^2*x^2+1))^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*
x^2+1))^2)^2+3*Pi*arctan(c*x)*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)*csgn(I*
(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)*csgn(I*(1+I*c*x)^2/(
c^2*x^2+1))+3*Pi*arctan(c*x)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/
(c^2*x^2+1))^2)^3-3*Pi*arctan(c*x)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c
*x)^2/(c^2*x^2+1))^2)^2*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))-3*Pi*arctan(c*x)*cs
gn(I*(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1))^2)+6
*Pi*arctan(c*x)*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*(1+(1+I*c*x)^2/(
c^2*x^2+1))^2)^2-3*Pi*arctan(c*x)*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1))^2)^3+3
*Pi*arctan(c*x)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^3-6*Pi*arctan(c*x)*csgn(I*(
1+I*c*x)^2/(c^2*x^2+1))^2*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+3*Pi*arctan(c
*x)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))^2+4
*arctan(c*x)^2+12*I*ln(2)*arctan(c*x)-12+6*I*arctan(c*x)-12*I*c*x)+ln(1+(1+
```

```
I*c*x)^2/(c^2*x^2+1)))+3*a*b^2*(1/3*c^3*x^3*arctan(c*x)^2-1/3*c^2*x^2*arcta
n(c*x)+1/3*arctan(c*x)*ln(c^2*x^2+1)+1/3*c*x-1/3*arctan(c*x)+1/6*I*(ln(c*x-
I)*ln(c^2*x^2+1)-1/2*ln(c*x-I)^2-dilog(-1/2*I*(c*x+I))-ln(c*x-I)*ln(-1/2*I*
(c*x+I)))-1/6*I*(ln(c*x+I)*ln(c^2*x^2+1)-1/2*ln(c*x+I)^2-dilog(1/2*I*(c*x-I
))-ln(c*x+I)*ln(1/2*I*(c*x-I))))+3*a^2*b*(1/3*c^3*x^3*arctan(c*x)-1/6*c^2*x
^2+1/6*ln(c^2*x^2+1)))
```

Fricas [F]

$$\int x^2(a+b\arctan(cx))^3\,dx = \int (b\arctan(cx)+a)^3x^2\,dx$$

```
[In] integrate(x^2*(a+b*arctan(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^2*arctan(c*x)^3 + 3*a*b^2*x^2*arctan(c*x)^2 + 3*a^2*b*x^2*ar
ctan(c*x) + a^3*x^2, x)
```

Sympy [F]

$$\int x^2(a+b\arctan(cx))^3\,dx = \int x^2(a+b\operatorname{atan}(cx))^3\,dx$$

```
[In] integrate(x**2*(a+b*atan(c*x))**3,x)
```

```
[Out] Integral(x**2*(a + b*atan(c*x))**3, x)
```

Maxima [F]

$$\int x^2(a+b\arctan(cx))^3\,dx = \int (b\arctan(cx)+a)^3x^2\,dx$$

```
[In] integrate(x^2*(a+b*arctan(c*x))^3,x, algorithm="maxima")
```

```
[Out] 1/24*b^3*x^3*arctan(c*x)^3 - 1/32*b^3*x^3*arctan(c*x)*log(c^2*x^2 + 1)^2 +
1/3*a^3*x^3 + 1/2*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*
a^2*b + integrate(1/32*(4*b^3*c^2*x^4*arctan(c*x)*log(c^2*x^2 + 1) + 28*(b^
3*c^2*x^4 + b^3*x^2)*arctan(c*x)^3 + 4*(24*a*b^2*c^2*x^4 - b^3*c*x^3 + 24*a
*b^2*x^2)*arctan(c*x)^2 + (b^3*c*x^3 + 3*(b^3*c^2*x^4 + b^3*x^2)*arctan(c*x
))*log(c^2*x^2 + 1)^2)/(c^2*x^2 + 1), x)
```

Giac [F]

$$\int x^2(a+b\arctan(cx))^3\,dx=\int (b\arctan(cx)+a)^3x^2\,dx$$

```
[In] integrate(x^2*(a+b*arctan(c*x))^3,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int x^2(a+b\arctan(cx))^3\,dx=\int x^2\,(a+b\,\mathrm{atan}(c\,x))^3\,dx$$

```
[In] int(x^2*(a + b*atan(c*x))^3,x)
[Out] int(x^2*(a + b*atan(c*x))^3, x)
```

3.28 $\int x(a+b\arctan(cx))^3\,dx$

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Optimal result

Integrand size = 12, antiderivative size = 131

$$\int x(a+b\arctan(cx))^3\,dx=-\frac{3ib(a+b\arctan(cx))^2}{2c^2}-\frac{3bx(a+b\arctan(cx))^2}{2c}+\frac{(a+b\arctan(cx))^3}{2c^2}+\frac{1}{2}x^2(a+b\arctan(cx))^3-\frac{3b^2(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c^2}-\frac{3ib^3\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2c^2}$$

```
[Out] -3/2*I*b*(a+b*arctan(c*x))^2/c^2-3/2*b*x*(a+b*arctan(c*x))^2/c+1/2*(a+b*arc
tan(c*x))^3/c^2+1/2*x^2*(a+b*arctan(c*x))^3-3*b^2*(a+b*arctan(c*x))*ln(2/(1
+I*c*x))/c^2-3/2*I*b^3*polylog(2,1-2/(1+I*c*x))/c^2
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {4946, 5036, 4930, 5040, 4964, 2449, 2352, 5004}

$$\int x(a+b\arctan(cx))^3\,dx=-\frac{3b^2\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c^2}-\frac{3ib(a+b\arctan(cx))^2}{2c^2}+\frac{(a+b\arctan(cx))^3}{2c^2}+\frac{1}{2}x^2(a+b\arctan(cx))^3-\frac{3bx(a+b\arctan(cx))^2}{2c}-\frac{3ib^3\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{2c^2}$$

```
[In] Int[x*(a + b*ArcTan[c*x])^3,x]
```

```
[Out] (((-3*I)/2)*b*(a + b*ArcTan[c*x])^2)/c^2 - (3*b*x*(a + b*ArcTan[c*x])^2)/(2
*c) + (a + b*ArcTan[c*x])^3/(2*c^2) + (x^2*(a + b*ArcTan[c*x])^3)/2 - (3*b^
2*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/c^2 - (((3*I)/2)*b^3*PolyLog[2, 1
 - 2/(1 + I*c*x)])/c^2
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
```

```
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}x^2(a+b\arctan(cx))^3 - \frac{1}{2}(3bc)\int \frac{x^2(a+b\arctan(cx))^2}{1+c^2x^2}\,dx \\
&= \frac{1}{2}x^2(a+b\arctan(cx))^3 - \frac{(3b)\int (a+b\arctan(cx))^2\,dx}{2c} + \frac{(3b)\int \frac{(a+b\arctan(cx))^2}{1+c^2x^2}\,dx}{2c} \\
&= -\frac{3bx(a+b\arctan(cx))^2}{2c} + \frac{(a+b\arctan(cx))^3}{2c^2} \\
&\quad + \frac{1}{2}x^2(a+b\arctan(cx))^3 + \left(3b^2\right)\int \frac{x(a+b\arctan(cx))}{1+c^2x^2}\,dx \\
&= -\frac{3ib(a+b\arctan(cx))^2}{2c^2} - \frac{3bx(a+b\arctan(cx))^2}{2c} + \frac{(a+b\arctan(cx))^3}{2c^2} \\
&\quad + \frac{1}{2}x^2(a+b\arctan(cx))^3 - \frac{\left(3b^2\right)\int \frac{a+b\arctan(cx)}{i-cx}\,dx}{c} \\
&= -\frac{3ib(a+b\arctan(cx))^2}{2c^2} - \frac{3bx(a+b\arctan(cx))^2}{2c} + \frac{(a+b\arctan(cx))^3}{2c^2} \\
&\quad + \frac{1}{2}x^2(a+b\arctan(cx))^3 - \frac{3b^2(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c^2} + \frac{\left(3b^3\right)\int \frac{\log\left(\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx}{c} \\
&= -\frac{3ib(a+b\arctan(cx))^2}{2c^2} - \frac{3bx(a+b\arctan(cx))^2}{2c} \\
&\quad + \frac{(a+b\arctan(cx))^3}{2c^2} + \frac{1}{2}x^2(a+b\arctan(cx))^3 \\
&\quad - \frac{3b^2(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c^2} - \frac{\left(3ib^3\right)\text{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1+icx}\right)}{c^2} \\
&= -\frac{3ib(a+b\arctan(cx))^2}{2c^2} - \frac{3bx(a+b\arctan(cx))^2}{2c} + \frac{(a+b\arctan(cx))^3}{2c^2} + \frac{1}{2}x^2(a \\
&\quad + b\arctan(cx))^3 - \frac{3b^2(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c^2} - \frac{3ib^3\,\text{PolyLog}\left(2, 1-\frac{2}{1+icx}\right)}{2c^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.16

$$\int x(a+b\arctan(cx))^3\,dx$$
$$=\frac{3b^2(a+ac^2x^2+b(i-cx))\arctan(cx)^2+b^3(1+c^2x^2)\arctan(cx)^3+3b\arctan(cx)\left(a(a-2bcx+ac^2x^2)\right.}{2c^2}$$

```
[In] Integrate[x*(a + b*ArcTan[c*x])^3,x]
```

```
[Out] (3*b^2*(a + a*c^2*x^2 + b*(I - c*x))*ArcTan[c*x]^2 + b^3*(1 + c^2*x^2)*ArcT
an[c*x]^3 + 3*b*ArcTan[c*x]*(a*(a - 2*b*c*x + a*c^2*x^2) - 2*b^2*Log[1 + E^
((2*I)*ArcTan[c*x])]) + a*(a*c*x*(-3*b + a*c*x) + 3*b^2*Log[1 + c^2*x^2]) +
 (3*I)*b^3*PolyLog[2, -E^((2*I)*ArcTan[c*x])])/(2*c^2)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 275 vs. 2(117) = 234.

Time = 2.89 (sec) , antiderivative size = 276, normalized size of antiderivative = 2.11

method	result
derivativedivides	$\frac{\frac{c^2x^2a^3}{2}+b^3\left(\frac{c^2x^2\arctan(cx)^3}{2}+\frac{\arctan(cx)^3}{2}-\frac{3\arctan(cx)^2cx}{2}+\frac{3\arctan(cx)\ln\left(c^2x^2+1\right)}{2}+\frac{3i\left(\ln(cx-i)\ln\left(c^2x^2+1\right)-\frac{\ln(cx-i)^2}{2}\right.}{}\right.}{}$
default	$\frac{\frac{c^2x^2a^3}{2}+b^3\left(\frac{c^2x^2\arctan(cx)^3}{2}+\frac{\arctan(cx)^3}{2}-\frac{3\arctan(cx)^2cx}{2}+\frac{3\arctan(cx)\ln\left(c^2x^2+1\right)}{2}+\frac{3i\left(\ln(cx-i)\ln\left(c^2x^2+1\right)-\frac{\ln(cx-i)^2}{2}\right.}{}\right.}{}$
parts	$\frac{a^3x^2}{2}+\frac{b^3\left(\frac{c^2x^2\arctan(cx)^3}{2}+\frac{\arctan(cx)^3}{2}-\frac{3\arctan(cx)^2cx}{2}+\frac{3\arctan(cx)\ln\left(c^2x^2+1\right)}{2}+\frac{3i\left(\ln(cx-i)\ln\left(c^2x^2+1\right)-\frac{\ln(cx-i)^2}{2}\right.}{}\right.}{}$
risch	$-\frac{ib^3\ln(-icx+1)^3}{16c^2}+\frac{9ib^3\ln(-icx+1)^2}{32c^2}-\frac{3ib^3\ln(-icx+1)x^2}{32}-\frac{ib^3\ln(-icx+1)^3x^2}{16}+\frac{3ib^3\ln(-icx+1)^2x^2}{32}-$

```
[In] int(x*(a+b*arctan(c*x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(1/2*c^2*x^2*a^3+b^3*(1/2*c^2*x^2*arctan(c*x)^3+1/2*arctan(c*x)^3-3/2
*arctan(c*x)^2*c*x+3/2*arctan(c*x)*ln(c^2*x^2+1)+3/4*I*(ln(c*x-I)*ln(c^2*x^
2+1)-1/2*ln(c*x-I)^2-dilog(-1/2*I*(c*x+I))-ln(c*x-I)*ln(-1/2*I*(c*x+I)))-3/
4*I*(ln(c*x+I)*ln(c^2*x^2+1)-1/2*ln(c*x+I)^2-dilog(1/2*I*(c*x-I))-ln(c*x+I)
*ln(1/2*I*(c*x-I))))+3*a*b^2*(1/2*c^2*x^2*arctan(c*x)^2+1/2*arctan(c*x)^2-c
*x*arctan(c*x)+1/2*ln(c^2*x^2+1))+3*a^2*b*(1/2*c^2*x^2*arctan(c*x)-1/2*c*x+
1/2*arctan(c*x)))
```

Fricas [F]

$$\int x(a+b\arctan(cx))^3\,dx=\int(b\arctan(cx)+a)^3x\,dx$$

```
[In] integrate(x*(a+b*arctan(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x*arctan(c*x)^3 + 3*a*b^2*x*arctan(c*x)^2 + 3*a^2*b*x*arctan(c
*x) + a^3*x, x)
```

Sympy [F]

$$\int x(a+b\arctan(cx))^3\,dx=\int x(a+b\,\mathrm{atan}(cx))^3\,dx$$

```
[In] integrate(x*(a+b*atan(c*x))**3,x)
```

```
[Out] Integral(x*(a + b*atan(c*x))**3, x)
```

Maxima [F]

$$\int x(a+b\arctan(cx))^3\,dx=\int(b\arctan(cx)+a)^3x\,dx$$

```
[In] integrate(x*(a+b*arctan(c*x))^3,x, algorithm="maxima")
```

```
[Out] 3/2*a*b^2*x^2*arctan(c*x)^2 + 1/2*a^3*x^2 + 3/2*(x^2*arctan(c*x) - c*(x/c^2
 - arctan(c*x)/c^3))*a^2*b - 3/2*(2*c*(x/c^2 - arctan(c*x)/c^3)*arctan(c*x)
 + (arctan(c*x)^2 - log(c^2*x^2 + 1))/c^2)*a*b^2 - 1/32*(12*c*x*arctan(c*x)
^2 - 8*(c^2*x^2 + 1)*arctan(c*x)^3 - 3*c*x*log(c^2*x^2 + 1)^2 - 4*(128*c^3*
integrate(1/32*x^3*arctan(c*x)^3/(c^3*x^2 + c), x) - 96*c^2*integrate(1/32*
x^2*arctan(c*x)^2/(c^3*x^2 + c), x) - 24*c^2*integrate(1/32*x^2*log(c^2*x^2
 + 1)^2/(c^3*x^2 + c), x) - 96*c^2*integrate(1/32*x^2*log(c^2*x^2 + 1)/(c^3
*x^2 + c), x) + 128*c*integrate(1/32*x*arctan(c*x)^3/(c^3*x^2 + c), x) + 19
2*c*integrate(1/32*x*arctan(c*x)/(c^3*x^2 + c), x) - arctan(c*x)^3/c^2 - 24
*integrate(1/32*log(c^2*x^2 + 1)^2/(c^3*x^2 + c), x))*c^2)*b^3/c^2
```

Giac [F]

$$\int x(a+b\arctan(cx))^3\,dx = \int (b\arctan(cx)+a)^3x\,dx$$

```
[In] integrate(x*(a+b*arctan(c*x))^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int x(a+b\arctan(cx))^3\,dx = \int x\,(a+b\,\mathrm{atan}(c\,x))^3\,dx$$

```
[In] int(x*(a + b*atan(c*x))^3,x)
```

```
[Out] int(x*(a + b*atan(c*x))^3, x)
```

3.29 $\int (a+b\arctan(cx))^3\,dx$

Optimal result 218
Rubi [A] (verified) 218
Mathematica [A] (verified) 220
Maple [B] (verified) 221
Fricas [F] 221
Sympy [F] 222
Maxima [F] 222
Giac [F] 222
Mupad [F(-1)] 223

Optimal result

Integrand size = 10, antiderivative size = 119

$$\int (a+b\arctan(cx))^3\,dx=\frac{i(a+b\arctan(cx))^3}{c}+x(a+b\arctan(cx))^3+\frac{3b(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{c}+\frac{3ib^2(a+b\arctan(cx))\,\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c}+\frac{3b^3\,\text{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2c}$$

```
[Out] I*(a+b*arctan(c*x))^3/c+x*(a+b*arctan(c*x))^3+3*b*(a+b*arctan(c*x))^2*ln(2/
(1+I*c*x))/c+3*I*b^2*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*x))/c+3/2*b^3*p
olylog(3,1-2/(1+I*c*x))/c
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {4930, 5040, 4964, 5004, 5114, 6745}

$$\int (a+b\arctan(cx))^3\,dx=\frac{3ib^2\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{c}+x(a+b\arctan(cx))^3+\frac{i(a+b\arctan(cx))^3}{c}+\frac{3b\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}+\frac{3b^3\,\text{PolyLog}\left(3,1-\frac{2}{icx+1}\right)}{2c}$$

```
[In] Int[(a + b*ArcTan[c*x])^3,x]
```

```
[Out] (I*(a + b*ArcTan[c*x])^3)/c + x*(a + b*ArcTan[c*x])^3 + (3*b*(a + b*ArcTan[
c*x])^2*Log[2/(1 + I*c*x)])/c + ((3*I)*b^2*(a + b*ArcTan[c*x])*PolyLog[2, 1
 - 2/(1 + I*c*x)])/c + (3*b^3*PolyLog[3, 1 - 2/(1 + I*c*x)])/(2*c)
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= x(a+b\arctan(cx))^3-(3bc)\int\frac{x(a+b\arctan(cx))^2}{1+c^2x^2}\,dx\\
&=\frac{i(a+b\arctan(cx))^3}{c}+x(a+b\arctan(cx))^3+(3b)\int\frac{(a+b\arctan(cx))^2}{i-cx}\,dx\\
&=\frac{i(a+b\arctan(cx))^3}{c}+x(a+b\arctan(cx))^3+\frac{3b(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{c}\\
&\quad-\left(6b^2\right)\int\frac{(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx\\
&=\frac{i(a+b\arctan(cx))^3}{c}+x(a+b\arctan(cx))^3+\frac{3b(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{c}\\
&\quad+\frac{3ib^2(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c}-\left(3ib^3\right)\int\frac{\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx\\
&=\frac{i(a+b\arctan(cx))^3}{c}+x(a+b\arctan(cx))^3+\frac{3b(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{c}\\
&\quad+\frac{3ib^2(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c}+\frac{3b^3\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2c}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.61

$$
\begin{aligned}
\int(a+b\arctan(cx))^3\,dx &= a^3x+3a^2bx\arctan(cx)-\frac{3a^2b\log\left(1+c^2x^2\right)}{2c}\\
&+\frac{3ab^2\left(-i\arctan(cx)^2+cx\arctan(cx)^2+2\arctan(cx)\log\left(1+e^{2i\arctan(cx)}\right)-i\operatorname{PolyLog}\left(2,-e^{2i\arctan(cx)}\right)\right.}{c}\\
&+\frac{b^3\left(-i\arctan(cx)^3+cx\arctan(cx)^3+3\arctan(cx)^2\log\left(1+e^{2i\arctan(cx)}\right)-3i\arctan(cx)\operatorname{PolyLog}\left(2,-e\right.\right.}{c}
\end{aligned}
$$

```
[In] Integrate[(a + b*ArcTan[c*x])^3,x]
```

```
[Out] a^3*x + 3*a^2*b*x*ArcTan[c*x] - (3*a^2*b*Log[1 + c^2*x^2])/(2*c) + (3*a*b^2
*((-I)*ArcTan[c*x]^2 + c*x*ArcTan[c*x]^2 + 2*ArcTan[c*x]*Log[1 + E^((2*I)*A
rcTan[c*x])] - I*PolyLog[2, -E^((2*I)*ArcTan[c*x])]))/c + (b^3*((-I)*ArcTan
[c*x]^3 + c*x*ArcTan[c*x]^3 + 3*ArcTan[c*x]^2*Log[1 + E^((2*I)*ArcTan[c*x])
] - (3*I)*ArcTan[c*x]*PolyLog[2, -E^((2*I)*ArcTan[c*x])] + (3*PolyLog[3, -E
^((2*I)*ArcTan[c*x])])/2))/c
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 239 vs. 2(112) = 224.

Time = 5.71 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.02

method	result
derivativedivides	$\frac{c\,a^3x+b^3\left(\arctan(cx)^3(cx+i)-2i\arctan(cx)^3+3\arctan(cx)^2\ln\left(1+\frac{(icx+1)^2}{c^2x^2+1}\right)-3i\arctan(cx)\,\mathrm{polylog}\left(2,-\frac{(icx+1)^2}{c^2x^2+1}\right)+\right.}{}$
default	$\frac{c\,a^3x+b^3\left(\arctan(cx)^3(cx+i)-2i\arctan(cx)^3+3\arctan(cx)^2\ln\left(1+\frac{(icx+1)^2}{c^2x^2+1}\right)-3i\arctan(cx)\,\mathrm{polylog}\left(2,-\frac{(icx+1)^2}{c^2x^2+1}\right)+\right.}{}$
parts	$x\,a^3+\frac{b^3\left(\arctan(cx)^3(cx+i)-2i\arctan(cx)^3+3\arctan(cx)^2\ln\left(1+\frac{(icx+1)^2}{c^2x^2+1}\right)-3i\arctan(cx)\,\mathrm{polylog}\left(2,-\frac{(icx+1)^2}{c^2x^2+1}\right.\right.}{c}$

```
[In] int((a+b*arctan(c*x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(c*a^3*x+b^3*(arctan(c*x)^3*(c*x+I)-2*I*arctan(c*x)^3+3*arctan(c*x)^2*l
n(1+(1+I*c*x)^2/(c^2*x^2+1))-3*I*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^
2+1))+3/2*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1)))+3*a^2*b*(c*x*arctan(c*x)-1/2
*ln(c^2*x^2+1))+3*b^2*a*(arctan(c*x)^2*(c*x+I)+2*arctan(c*x)*ln(1+(1+I*c*x)
^2/(c^2*x^2+1))-2*I*arctan(c*x)^2-I*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))))
```

Fricas [F]

$$\int (a+b\arctan(cx))^3\,dx=\int (b\arctan{(cx)}+a)^3\,dx$$

```
[In] integrate((a+b*arctan(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*arctan(c*x)^3 + 3*a*b^2*arctan(c*x)^2 + 3*a^2*b*arctan(c*x) +
a^3, x)
```

Sympy [F]

$$\int (a+b\arctan(cx))^3\,dx=\int \left(a+b\,\mathrm{atan}\,(cx)\right)^3\,dx$$

[In] integrate((a+b*atan(c*x))**3,x)

[Out] Integral((a + b*atan(c*x))**3, x)

Maxima [F]

$$\int (a+b\arctan(cx))^3\,dx=\int (b\arctan(cx)+a)^3\,dx$$

[In] integrate((a+b*arctan(c*x))^3,x, algorithm="maxima")

[Out] 1/8*b^3*x*arctan(c*x)^3 - 3/32*b^3*x*arctan(c*x)*log(c^2*x^2 + 1)^2 + 7/32*b^3*arctan(c*x)^4/c + 28*b^3*c^2*integrate(1/32*x^2*arctan(c*x)^3/(c^2*x^2 + 1), x) + 3*b^3*c^2*integrate(1/32*x^2*arctan(c*x)*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 96*a*b^2*c^2*integrate(1/32*x^2*arctan(c*x)^2/(c^2*x^2 + 1), x) + 12*b^3*c^2*integrate(1/32*x^2*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) + a*b^2*arctan(c*x)^3/c - 12*b^3*c*integrate(1/32*x*arctan(c*x)^2/(c^2*x^2 + 1), x) + 3*b^3*c*integrate(1/32*x*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + a^3*x + 3*b^3*integrate(1/32*arctan(c*x)*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 3/2*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*a^2*b/c

Giac [F]

$$\int (a+b\arctan(cx))^3\,dx=\int (b\arctan(cx)+a)^3\,dx$$

[In] integrate((a+b*arctan(c*x))^3,x, algorithm="giac")

[Out] sage0*x

Mupad [F(-1)]

Timed out.

$$\int (a + b \arctan(cx))^3 \, dx = \int (a + b \operatorname{atan}(c\,x))^3 \, dx$$

```
[In] int((a + b*atan(c*x))^3,x)
```

```
[Out] int((a + b*atan(c*x))^3, x)
```

3.30 $\int \frac{(a+b\arctan(cx))^3}{x}\,dx$

Optimal result . . . 224
Rubi [A] (verified) . . . 225
Mathematica [A] (verified) . . . 228
Maple [C] (warning: unable to verify) . . . 229
Fricas [F] . . . 230
Sympy [F] . . . 230
Maxima [F] . . . 230
Giac [F(-1)] . . . 230
Mupad [F(-1)] . . . 231

Optimal result

Integrand size = 14, antiderivative size = 206

$$\begin{aligned}\int \frac{(a+b\arctan(cx))^3}{x}\,dx &= 2(a+b\arctan(cx))^3\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right)\\ &-\frac{3}{2}ib(a+b\arctan(cx))^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\\ &+\frac{3}{2}ib(a+b\arctan(cx))^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)\\ &-\frac{3}{2}b^2(a+b\arctan(cx))\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)\\ &+\frac{3}{2}b^2(a+b\arctan(cx))\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)\\ &+\frac{3}{4}ib^3\operatorname{PolyLog}\left(4,1-\frac{2}{1+icx}\right)\\ &-\frac{3}{4}ib^3\operatorname{PolyLog}\left(4,-1+\frac{2}{1+icx}\right)\end{aligned}$$

```
[Out] -2*(a+b*arctan(c*x))^3*arctanh(-1+2/(1+I*c*x))-3/2*I*b*(a+b*arctan(c*x))^2*
polylog(2,1-2/(1+I*c*x))+3/2*I*b*(a+b*arctan(c*x))^2*polylog(2,-1+2/(1+I*c*
x))-3/2*b^2*(a+b*arctan(c*x))*polylog(3,1-2/(1+I*c*x))+3/2*b^2*(a+b*arctan(
c*x))*polylog(3,-1+2/(1+I*c*x))+3/4*I*b^3*polylog(4,1-2/(1+I*c*x))-3/4*I*b^
3*polylog(4,-1+2/(1+I*c*x))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4942, 5108, 5004, 5114, 5118, 6745}

$$\int \frac{(a+b\arctan(cx))^3}{x}\,dx = 2\text{arctanh}\left(1-\frac{2}{1+icx}\right)(a+b\arctan(cx))^3 - \frac{3}{2}b^2\,\text{PolyLog}\left(3,1-\frac{2}{icx+1}\right)(a+b\arctan(cx)) + \frac{3}{2}b^2\,\text{PolyLog}\left(3,\frac{2}{icx+1}-1\right)(a+b\arctan(cx)) - \frac{3}{2}ib\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))^2 + \frac{3}{2}ib\,\text{PolyLog}\left(2,\frac{2}{icx+1}-1\right)(a+b\arctan(cx))^2 + \frac{3}{4}ib^3\,\text{PolyLog}\left(4,1-\frac{2}{icx+1}\right) - \frac{3}{4}ib^3\,\text{PolyLog}\left(4,\frac{2}{icx+1}-1\right)$$

```
[In] Int[(a + b*ArcTan[c*x])^3/x,x]
```

```
[Out] 2*(a + b*ArcTan[c*x])^3*ArcTanh[1 - 2/(1 + I*c*x)] - ((3*I)/2)*b*(a + b*Arc
Tan[c*x])^2*PolyLog[2, 1 - 2/(1 + I*c*x)] + ((3*I)/2)*b*(a + b*ArcTan[c*x])
^2*PolyLog[2, -1 + 2/(1 + I*c*x)] - (3*b^2*(a + b*ArcTan[c*x])*PolyLog[3, 1
 - 2/(1 + I*c*x)])/2 + (3*b^2*(a + b*ArcTan[c*x])*PolyLog[3, -1 + 2/(1 + I*
c*x)])/2 + ((3*I)/4)*b^3*PolyLog[4, 1 - 2/(1 + I*c*x)] - ((3*I)/4)*b^3*Poly
Log[4, -1 + 2/(1 + I*c*x)]
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5108

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e*
x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5118

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k + 1,
 u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2(a+b\arctan(cx))^3\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right) \\
&\quad -(6bc)\int\frac{(a+b\arctan(cx))^2\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx \\
&= 2(a+b\arctan(cx))^3\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right) \\
&\quad +(3bc)\int\frac{(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx \\
&\quad -(3bc)\int\frac{(a+b\arctan(cx))^2\log\left(2-\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx
\end{aligned}$$

$$
\begin{aligned}
&= 2(a+b\arctan(cx))^3\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right) \\
&\quad -\frac{3}{2}ib(a+b\arctan(cx))^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right) \\
&\quad +\frac{3}{2}ib(a+b\arctan(cx))^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx}\right) \\
&\quad +\left(3ib^2c\right)\int\frac{(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx \\
&\quad -\left(3ib^2c\right)\int\frac{(a+b\arctan(cx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= 2(a+b\arctan(cx))^3\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right) \\
&\quad -\frac{3}{2}ib(a+b\arctan(cx))^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right) \\
&\quad +\frac{3}{2}ib(a+b\arctan(cx))^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx}\right) \\
&\quad -\frac{3}{2}b^2(a+b\arctan(cx))\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right) \\
&\quad +\frac{3}{2}b^2(a+b\arctan(cx))\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx}\right) \\
&\quad +\frac{1}{2}\left(3b^3c\right)\int\frac{\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx-\frac{1}{2}\left(3b^3c\right)\int\frac{\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx
\end{aligned}
$$

$$
\begin{aligned}
&= 2(a+b\arctan(cx))^3\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right) \\
&\quad -\frac{3}{2}ib(a+b\arctan(cx))^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right) \\
&\quad +\frac{3}{2}ib(a+b\arctan(cx))^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx}\right) \\
&\quad -\frac{3}{2}b^2(a+b\arctan(cx))\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right) \\
&\quad +\frac{3}{2}b^2(a+b\arctan(cx))\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx}\right) \\
&\quad +\frac{3}{4}ib^3\operatorname{PolyLog}\left(4,1-\frac{2}{1+icx}\right)-\frac{3}{4}ib^3\operatorname{PolyLog}\left(4,-1+\frac{2}{1+icx}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.79

$$\int \frac{(a+b\arctan(cx))^3}{x}\,dx = a^3\log(cx)+\frac{3}{2}ia^2b(\text{PolyLog}(2,-icx)-\text{PolyLog}(2,icx)) + 3ab^2\left(-\frac{i\pi^3}{24}+\frac{2}{3}i\arctan(cx)^3 + \arctan(cx)^2\log\left(1-e^{-2i\arctan(cx)}\right) - \arctan(cx)^2\log\left(1+e^{2i\arctan(cx)}\right) + i\arctan(cx)\,\text{PolyLog}\left(2,e^{-2i\arctan(cx)}\right) + i\arctan(cx)\,\text{PolyLog}\left(2,-e^{2i\arctan(cx)}\right) + \frac{1}{2}\text{PolyLog}\left(3,e^{-2i\arctan(cx)}\right) - \frac{1}{2}\text{PolyLog}\left(3,-e^{2i\arctan(cx)}\right)\right) - \frac{1}{64}ib^3\left(\pi^4-32\arctan(cx)^4 + 64i\arctan(cx)^3\log\left(1-e^{-2i\arctan(cx)}\right) - 64i\arctan(cx)^3\log\left(1+e^{2i\arctan(cx)}\right) - 96\arctan(cx)^2\,\text{PolyLog}\left(2,e^{-2i\arctan(cx)}\right) - 96\arctan(cx)^2\,\text{PolyLog}\left(2,-e^{2i\arctan(cx)}\right) + 96i\arctan(cx)\,\text{PolyLog}\left(3,e^{-2i\arctan(cx)}\right) - 96i\arctan(cx)\,\text{PolyLog}\left(3,-e^{2i\arctan(cx)}\right) + 48\,\text{PolyLog}\left(4,e^{-2i\arctan(cx)}\right) + 48\,\text{PolyLog}\left(4,-e^{2i\arctan(cx)}\right)\right)$$

```
[In] Integrate[(a + b*ArcTan[c*x])^3/x,x]
```

```
[Out] a^3*Log[c*x] + ((3*I)/2)*a^2*b*(PolyLog[2, (-I)*c*x] - PolyLog[2, I*c*x]) +
 3*a*b^2*((-1/24*I)*Pi^3 + ((2*I)/3)*ArcTan[c*x]^3 + ArcTan[c*x]^2*Log[1 -
E^((-2*I)*ArcTan[c*x])] - ArcTan[c*x]^2*Log[1 + E^((2*I)*ArcTan[c*x])] + I*
ArcTan[c*x]*PolyLog[2, E^((-2*I)*ArcTan[c*x])] + I*ArcTan[c*x]*PolyLog[2, -
E^((2*I)*ArcTan[c*x])] + PolyLog[3, E^((-2*I)*ArcTan[c*x])]/2 - PolyLog[3,
-E^((2*I)*ArcTan[c*x])]/2) - (I/64)*b^3*(Pi^4 - 32*ArcTan[c*x]^4 + (64*I)*A
rcTan[c*x]^3*Log[1 - E^((-2*I)*ArcTan[c*x])] - (64*I)*ArcTan[c*x]^3*Log[1 +
 E^((2*I)*ArcTan[c*x])] - 96*ArcTan[c*x]^2*PolyLog[2, E^((-2*I)*ArcTan[c*x]
)] - 96*ArcTan[c*x]^2*PolyLog[2, -E^((2*I)*ArcTan[c*x])] + (96*I)*ArcTan[c*
x]*PolyLog[3, E^((-2*I)*ArcTan[c*x])] - (96*I)*ArcTan[c*x]*PolyLog[3, -E^((
2*I)*ArcTan[c*x])] + 48*PolyLog[4, E^((-2*I)*ArcTan[c*x])] + 48*PolyLog[4,
-E^((2*I)*ArcTan[c*x])])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.58 (sec) , antiderivative size = 2026, normalized size of antiderivative = 9.83

Expression too large to display

```
[In] int((a+b*arctan(c*x))^3/x,x)
```

```
[Out] a^3*ln(c*x)+b^3*(ln(c*x)*arctan(c*x)^3-arctan(c*x)^3*ln((1+I*c*x)^2/(c^2*x^
2+1)-1)+arctan(c*x)^3*ln(1-(1+I*c*x)/(c^2*x^2+1)^(1/2))-3*I*arctan(c*x)^2*p
olylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))+6*arctan(c*x)*polylog(3,(1+I*c*x)/(c^
2*x^2+1)^(1/2))+6*I*polylog(4,(1+I*c*x)/(c^2*x^2+1)^(1/2))+arctan(c*x)^3*ln
(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))-3*I*arctan(c*x)^2*polylog(2,-(1+I*c*x)/(c^2
*x^2+1)^(1/2))+6*arctan(c*x)*polylog(3,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+6*I*po
lylog(4,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/2*I*Pi*(csgn(I*((1+I*c*x)^2/(c^2*x^
2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1
+I*c*x)^2/(c^2*x^2+1)))-csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^
2*x^2+1)))^2+csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I/(1+(1+I*c*x)^2/(c^2
*x^2+1)))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))-c
sgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1
+I*c*x)^2/(c^2*x^2+1)))^2-csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*((1+I*
c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2+csgn(I*((1+I*c*x)^2/(c
^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^3-csgn(I*((1+I*c*x)^2/(c^2*x^2+1)
-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c
*x)^2/(c^2*x^2+1)))^2+csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*
x^2+1)))^3+1)*arctan(c*x)^3+3/2*I*arctan(c*x)^2*polylog(2,-(1+I*c*x)^2/(c^2
*x^2+1))-3/2*arctan(c*x)*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))-3/4*I*polylog(
4,-(1+I*c*x)^2/(c^2*x^2+1)))+3*a*b^2*(ln(c*x)*arctan(c*x)^2+I*arctan(c*x)*p
olylog(2,-(1+I*c*x)^2/(c^2*x^2+1))-1/2*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))-
arctan(c*x)^2*ln((1+I*c*x)^2/(c^2*x^2+1)-1)+arctan(c*x)^2*ln(1-(1+I*c*x)/(c
^2*x^2+1)^(1/2))-2*I*arctan(c*x)*polylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*p
olylog(3,(1+I*c*x)/(c^2*x^2+1)^(1/2))+arctan(c*x)^2*ln(1+(1+I*c*x)/(c^2*x^2
+1)^(1/2))-2*I*arctan(c*x)*polylog(2,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*polylo
g(3,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/2*I*Pi*(csgn(I*((1+I*c*x)^2/(c^2*x^2+1)
-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c
*x)^2/(c^2*x^2+1)))-csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^
2+1)))^2+csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2
+1)))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))-csgn(
I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c
*x)^2/(c^2*x^2+1)))^2-csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*((1+I*c*x)
^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2+csgn(I*((1+I*c*x)^2/(c^2*x
^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^3-csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/
(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^
2/(c^2*x^2+1)))^2+csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+
1)))^3+1)*arctan(c*x)^2)+3*a^2*b*(ln(c*x)*arctan(c*x)+1/2*I*ln(c*x)*ln(1+I*
```

```
c*x)-1/2*I*ln(c*x)*ln(1-I*c*x)+1/2*I*dilog(1+I*c*x)-1/2*I*dilog(1-I*c*x))
```

Fricas [F]

$$\int \frac{(a+b\arctan(cx))^3}{x}\,dx = \int \frac{(b\arctan(cx)+a)^3}{x}\,dx$$

```
[In] integrate((a+b*arctan(c*x))^3/x,x, algorithm="fricas")
[Out] integral((b^3*arctan(c*x)^3 + 3*a*b^2*arctan(c*x)^2 + 3*a^2*b*arctan(c*x) +
 a^3)/x, x)
```

Sympy [F]

$$\int \frac{(a+b\arctan(cx))^3}{x}\,dx = \int \frac{(a+b\operatorname{atan}(cx))^3}{x}\,dx$$

```
[In] integrate((a+b*atan(c*x))**3/x,x)
[Out] Integral((a + b*atan(c*x))**3/x, x)
```

Maxima [F]

$$\int \frac{(a+b\arctan(cx))^3}{x}\,dx = \int \frac{(b\arctan(cx)+a)^3}{x}\,dx$$

```
[In] integrate((a+b*arctan(c*x))^3/x,x, algorithm="maxima")
[Out] a^3*log(x) + 1/32*integrate((28*b^3*arctan(c*x)^3 + 3*b^3*arctan(c*x)*log(c
^2*x^2 + 1)^2 + 96*a*b^2*arctan(c*x)^2 + 96*a^2*b*arctan(c*x))/x, x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^3}{x}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*arctan(c*x))^3/x,x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^3}{x}\,dx=\int \frac{{\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}^3}{x} \,d x$$

```
[In] int((a + b*atan(c*x))^3/x,x)
[Out] int((a + b*atan(c*x))^3/x, x)
```

3.31 $\int \frac{(a+b\arctan(cx))^3}{x^2}\,dx$

Optimal result 232
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Fricas [F] 236
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Optimal result

Integrand size = 14, antiderivative size = 116

$$\int \frac{(a+b\arctan(cx))^3}{x^2}\,dx = -ic(a+b\arctan(cx))^3-\frac{(a+b\arctan(cx))^3}{x} +3bc(a+b\arctan(cx))^2\log\left(2-\frac{2}{1-icx}\right) -3ib^2c(a+b\arctan(cx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1-icx}\right) +\frac{3}{2}b^3c\operatorname{PolyLog}\left(3,-1+\frac{2}{1-icx}\right)$$

```
[Out] -I*c*(a+b*arctan(c*x))^3-(a+b*arctan(c*x))^3/x+3*b*c*(a+b*arctan(c*x))^2*ln
(2-2/(1-I*c*x))-3*I*b^2*c*(a+b*arctan(c*x))*polylog(2,-1+2/(1-I*c*x))+3/2*b
^3*c*polylog(3,-1+2/(1-I*c*x))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4946, 5044, 4988, 5004, 5112, 6745}

$$\int \frac{(a+b\arctan(cx))^3}{x^2}\,dx = -3ib^2c\operatorname{PolyLog}\left(2,\frac{2}{1-icx}-1\right)(a+b\arctan(cx)) -ic(a+b\arctan(cx))^3-\frac{(a+b\arctan(cx))^3}{x} +3bc\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))^2 +\frac{3}{2}b^3c\operatorname{PolyLog}\left(3,\frac{2}{1-icx}-1\right)$$

```
[In] Int[(a + b*ArcTan[c*x])^3/x^2,x]
```

```
[Out] (-I)*c*(a + b*ArcTan[c*x])^3 - (a + b*ArcTan[c*x])^3/x + 3*b*c*(a + b*ArcTa
n[c*x])^2*Log[2 - 2/(1 - I*c*x)] - (3*I)*b^2*c*(a + b*ArcTan[c*x])*PolyLog[
2, -1 + 2/(1 - I*c*x)] + (3*b^3*c*PolyLog[3, -1 + 2/(1 - I*c*x)])/2
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5112

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\text{integral} = -\frac{(a+b\arctan(cx))^3}{x} + (3bc)\int \frac{(a+b\arctan(cx))^2}{x(1+c^2x^2)}\,dx$$
$$= -ic(a+b\arctan(cx))^3 - \frac{(a+b\arctan(cx))^3}{x} + (3ibc)\int \frac{(a+b\arctan(cx))^2}{x(i+cx)}\,dx$$
$$= -ic(a+b\arctan(cx))^3 - \frac{(a+b\arctan(cx))^3}{x} + 3bc(a+b\arctan(cx))^2\log\left(2-\frac{2}{1-icx}\right) - (6b^2c^2)\int \frac{(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right)}{1+c^2x^2}\,dx$$
$$= -ic(a+b\arctan(cx))^3 - \frac{(a+b\arctan(cx))^3}{x} + 3bc(a+b\arctan(cx))^2\log\left(2-\frac{2}{1-icx}\right) - 3ib^2c(a+b\arctan(cx))\,\text{PolyLog}\left(2,-1+\frac{2}{1-icx}\right) + (3ib^3c^2)\int \frac{\text{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)}{1+c^2x^2}\,dx$$
$$= -ic(a+b\arctan(cx))^3 - \frac{(a+b\arctan(cx))^3}{x} + 3bc(a+b\arctan(cx))^2\log\left(2-\frac{2}{1-icx}\right) - 3ib^2c(a+b\arctan(cx))\,\text{PolyLog}\left(2,-1+\frac{2}{1-icx}\right) + \frac{3}{2}b^3c\,\text{PolyLog}\left(3,-1+\frac{2}{1-icx}\right)$$

Mathematica [A] (verified)

Time = 1.30 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.84

$$\int \frac{(a+b\arctan(cx))^3}{x^2}\,dx = -\frac{a^3}{x} - \frac{3a^2b\arctan(cx)}{x} + 3a^2bc\log(x) - \frac{3}{2}a^2bc\log\left(1+c^2x^2\right) + 3ab^2c\left(-\frac{\arctan(cx)^2}{cx} + 2\arctan(cx)\log\left(1-e^{2i\arctan(cx)}\right) - i\left(\arctan(cx)^2 + \text{PolyLog}\left(2,e^{2i\arctan(cx)}\right)\right)\right) + b^3c\left(-\frac{i\pi^3}{8} + i\arctan(cx)^3 - \frac{\arctan(cx)^3}{cx} + 3\arctan(cx)^2\log\left(1-e^{-2i\arctan(cx)}\right) + 3i\arctan(cx)\,\text{PolyLog}\left(2,e^{-2i\arctan(cx)}\right) + \frac{3}{2}\text{PolyLog}\left(3,e^{-2i\arctan(cx)}\right)\right)$$

```
[In] Integrate[(a + b*ArcTan[c*x])^3/x^2,x]
```

```
[Out] -(a^3/x) - (3*a^2*b*ArcTan[c*x])/x + 3*a^2*b*c*Log[x] - (3*a^2*b*c*Log[1 +
c^2*x^2])/2 + 3*a*b^2*c*(-(ArcTan[c*x]^2/(c*x)) + 2*ArcTan[c*x]*Log[1 - E^(
(2*I)*ArcTan[c*x])] - I*(ArcTan[c*x]^2 + PolyLog[2, E^((2*I)*ArcTan[c*x])])
) + b^3*c*((-1/8*I)*Pi^3 + I*ArcTan[c*x]^3 - ArcTan[c*x]^3/(c*x) + 3*ArcTan
[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] + (3*I)*ArcTan[c*x]*PolyLog[2, E^((
-2*I)*ArcTan[c*x])] + (3*PolyLog[3, E^((-2*I)*ArcTan[c*x])])/2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.91 (sec) , antiderivative size = 1862, normalized size of antiderivative = 16.05

Expression too large to display

```
[In] int((a+b*arctan(c*x))^3/x^2,x)
```

```
[Out] c*(-a^3/c/x+b^3*(-1/c/x*arctan(c*x)^3+3*ln(c*x)*arctan(c*x)^2-3/2*arctan(c*
x)^2*ln(c^2*x^2+1)+3*arctan(c*x)^2*ln((1+I*c*x)/(c^2*x^2+1)^(1/2))-3*arctan
(c*x)^2*ln((1+I*c*x)^2/(c^2*x^2+1)-1)-I*arctan(c*x)^3+3/4*(I*Pi*csgn(I*(1+I
*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2
+1))^2)^2-2*I*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1
)))^2-I*Pi*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))^2*csgn(I*(1+I*c*x)^2/(c^2*x^
2+1))-2*I*Pi*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*(1+(1+I*c*x)^2/(c^2
*x^2+1))^2)^2+I*Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)*csgn(I*(1+I*c*x)^2
/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)^2+2*I*Pi*csgn(I*((1+I*c*x)^2/(c
^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^3-2*I*Pi*csgn(I*((1+I*c*x)^2/(c^2
*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1
+(1+I*c*x)^2/(c^2*x^2+1)))^2+I*Pi*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*csg
n(I*(1+(1+I*c*x)^2/(c^2*x^2+1))^2)+2*I*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/
(1+(1+I*c*x)^2/(c^2*x^2+1)))^3+2*I*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1
+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/
(c^2*x^2+1)))-I*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^3-I*Pi*csgn(I*(1+I*c*x)^
2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)^3-2*I*Pi*csgn(I*((1+I*c*x)^2/(
c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*csgn(I*((1+I*c*x)^2/(c^2*x^2+1
)-1))+2*I*Pi*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))*csgn(I*(1+I*c*x)^2/(c^2*x^
2+1))^2+2*I*Pi-I*Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)*csgn(I*(1+I*c*x)^
2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1))^2
)+2*I*Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1
)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))+I*Pi*
csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1))^2)^3-2*I*Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x
^2+1)))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2+4
*ln(2))*arctan(c*x)^2+3*arctan(c*x)^2*ln(1-(1+I*c*x)/(c^2*x^2+1)^(1/2))-6*I
*arctan(c*x)*polylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))+6*polylog(3,(1+I*c*x)/(
c^2*x^2+1)^(1/2))+3*arctan(c*x)^2*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))-6*I*arc
```

```
tan(c*x)*polylog(2,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+6*polylog(3,-(1+I*c*x)/(c^
2*x^2+1)^(1/2)))+3*a*b^2*(-arctan(c*x)^2/c/x+2*ln(c*x)*arctan(c*x)-arctan(c
*x)*ln(c^2*x^2+1)+I*ln(c*x)*ln(1+I*c*x)-I*ln(c*x)*ln(1-I*c*x)+I*dilog(1+I*c
*x)-I*dilog(1-I*c*x)-1/2*I*(ln(c*x-I)*ln(c^2*x^2+1)-1/2*ln(c*x-I)^2-dilog(-
1/2*I*(c*x+I))-ln(c*x-I)*ln(-1/2*I*(c*x+I)))+1/2*I*(ln(c*x+I)*ln(c^2*x^2+1)
-1/2*ln(c*x+I)^2-dilog(1/2*I*(c*x-I))-ln(c*x+I)*ln(1/2*I*(c*x-I))))+3*a^2*b
*(-1/c/x*arctan(c*x)+ln(c*x)-1/2*ln(c^2*x^2+1)))
```

Fricas [F]

$$\int \frac{(a + b \arctan(cx))^3}{x^2} \, dx = \int \frac{(b \arctan(cx) + a)^3}{x^2} \, dx$$

```
[In] integrate((a+b*arctan(c*x))^3/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(c*x)^3 + 3*a*b^2*arctan(c*x)^2 + 3*a^2*b*arctan(c*x) +
 a^3)/x^2, x)
```

Sympy [F]

$$\int \frac{(a + b \arctan(cx))^3}{x^2} \, dx = \int \frac{(a + b \operatorname{atan}(cx))^3}{x^2} \, dx$$

```
[In] integrate((a+b*atan(c*x))**3/x**2,x)
```

```
[Out] Integral((a + b*atan(c*x))**3/x**2, x)
```

Maxima [F]

$$\int \frac{(a + b \arctan(cx))^3}{x^2} \, dx = \int \frac{(b \arctan(cx) + a)^3}{x^2} \, dx$$

```
[In] integrate((a+b*arctan(c*x))^3/x^2,x, algorithm="maxima")
```

```
[Out] -3/2*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*a^2*b - a^3/x - 1/
32*(4*b^3*arctan(c*x)^3 - 3*b^3*arctan(c*x)*log(c^2*x^2 + 1)^2 - (7*b^3*c*a
rctan(c*x)^4 + 32*a*b^2*c*arctan(c*x)^3 + 96*b^3*c^2*integrate(1/32*x^2*arc
tan(c*x)*log(c^2*x^2 + 1)^2/(c^2*x^4 + x^2), x) - 384*b^3*c^2*integrate(1/3
2*x^2*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^4 + x^2), x) + 384*b^3*c*integrat
e(1/32*x*arctan(c*x)^2/(c^2*x^4 + x^2), x) - 96*b^3*c*integrate(1/32*x*log(
c^2*x^2 + 1)^2/(c^2*x^4 + x^2), x) + 896*b^3*integrate(1/32*arctan(c*x)^3/(
c^2*x^4 + x^2), x) + 96*b^3*integrate(1/32*arctan(c*x)*log(c^2*x^2 + 1)^2/(
c^2*x^4 + x^2), x) + 3072*a*b^2*integrate(1/32*arctan(c*x)^2/(c^2*x^4 + x^2
), x))*x)/x
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^3}{x^2}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*arctan(c*x))^3/x^2,x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^3}{x^2}\,dx = \int \frac{\left(a+b\,\mathrm{atan}(c\,x)\right)^3}{x^2}\,dx$$

```
[In] int((a + b*atan(c*x))^3/x^2,x)
[Out] int((a + b*atan(c*x))^3/x^2, x)
```

3.32 $\int \frac{(a+b\arctan(cx))^3}{x^3}\,dx$

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Rubi [A] (verified) . . . 238
Mathematica [A] (verified) . . . 240
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Fricas [F] . . . 241
Sympy [F] . . . 242
Maxima [F] . . . 242
Giac [F(-1)] . . . 242
Mupad [F(-1)] . . . 243

Optimal result

Integrand size = 14, antiderivative size = 133

$$\begin{aligned}\int \frac{(a+b\arctan(cx))^3}{x^3}\,dx &= -\frac{3}{2}ibc^2(a+b\arctan(cx))^2-\frac{3bc(a+b\arctan(cx))^2}{2x}\\&\quad-\frac{1}{2}c^2(a+b\arctan(cx))^3-\frac{(a+b\arctan(cx))^3}{2x^2}\\&\quad+3b^2c^2(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right)\\&\quad-\frac{3}{2}ib^3c^2\,\mathrm{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)\end{aligned}$$

```
[Out] -3/2*I*b*c^2*(a+b*arctan(c*x))^2-3/2*b*c*(a+b*arctan(c*x))^2/x-1/2*c^2*(a+b
*arctan(c*x))^3-1/2*(a+b*arctan(c*x))^3/x^2+3*b^2*c^2*(a+b*arctan(c*x))*ln(
2-2/(1-I*c*x))-3/2*I*b^3*c^2*polylog(2,-1+2/(1-I*c*x))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4946, 5038, 5044, 4988, 2497, 5004}

$$\begin{aligned}\int \frac{(a+b\arctan(cx))^3}{x^3}\,dx &= 3b^2c^2\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))\\&\quad-\frac{3}{2}ibc^2(a+b\arctan(cx))^2-\frac{1}{2}c^2(a+b\arctan(cx))^3\\&\quad-\frac{(a+b\arctan(cx))^3}{2x^2}-\frac{3bc(a+b\arctan(cx))^2}{2x}\\&\quad-\frac{3}{2}ib^3c^2\,\mathrm{PolyLog}\left(2,\frac{2}{1-icx}-1\right)\end{aligned}$$

```
[In] Int[(a + b*ArcTan[c*x])^3/x^3,x]
```

```
[Out] ((-3*I)/2)*b*c^2*(a + b*ArcTan[c*x])^2 - (3*b*c*(a + b*ArcTan[c*x])^2)/(2*x
) - (c^2*(a + b*ArcTan[c*x])^3)/2 - (a + b*ArcTan[c*x])^3/(2*x^2) + 3*b^2*c
^2*(a + b*ArcTan[c*x])*Log[2 - 2/(1 - I*c*x)] - ((3*I)/2)*b^3*c^2*PolyLog[2
, -1 + 2/(1 - I*c*x)]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(a+b\arctan(cx))^3}{2x^2}+\frac{1}{2}(3bc)\int\frac{(a+b\arctan(cx))^2}{x^2(1+c^2x^2)}\,dx\\
&= -\frac{(a+b\arctan(cx))^3}{2x^2}+\frac{1}{2}(3bc)\int\frac{(a+b\arctan(cx))^2}{x^2}\,dx-\frac{1}{2}\left(3bc^3\right)\int\frac{(a+b\arctan(cx))^2}{1+c^2x^2}\,dx\\
&= -\frac{3bc(a+b\arctan(cx))^2}{2x}-\frac{1}{2}c^2(a+b\arctan(cx))^3\\
&\quad-\frac{(a+b\arctan(cx))^3}{2x^2}+\left(3b^2c^2\right)\int\frac{a+b\arctan(cx)}{x(1+c^2x^2)}\,dx\\
&= -\frac{3}{2}ibc^2(a+b\arctan(cx))^2-\frac{3bc(a+b\arctan(cx))^2}{2x}-\frac{1}{2}c^2(a+b\arctan(cx))^3\\
&\quad-\frac{(a+b\arctan(cx))^3}{2x^2}+\left(3ib^2c^2\right)\int\frac{a+b\arctan(cx)}{x(i+cx)}\,dx\\
&= -\frac{3}{2}ibc^2(a+b\arctan(cx))^2-\frac{3bc(a+b\arctan(cx))^2}{2x}\\
&\quad-\frac{1}{2}c^2(a+b\arctan(cx))^3-\frac{(a+b\arctan(cx))^3}{2x^2}\\
&\quad+3b^2c^2(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right)-\left(3b^3c^3\right)\int\frac{\log\left(2-\frac{2}{1-icx}\right)}{1+c^2x^2}\,dx\\
&= -\frac{3}{2}ibc^2(a+b\arctan(cx))^2-\frac{3bc(a+b\arctan(cx))^2}{2x}\\
&\quad-\frac{1}{2}c^2(a+b\arctan(cx))^3-\frac{(a+b\arctan(cx))^3}{2x^2}\\
&\quad+3b^2c^2(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right)-\frac{3}{2}ib^3c^2\,\mathrm{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.32

$$\int\frac{(a+b\arctan(cx))^3}{x^3}\,dx=$$

$$-\frac{3b^2(a+ac^2x^2+bcx(1+icx))\arctan(cx)^2+b^3(1+c^2x^2)\arctan(cx)^3+3b\arctan(cx)\left(a(a+2bcx+ac^2\right.}{}$$

```
[In] Integrate[(a + b*ArcTan[c*x])^3/x^3,x]
```

```
[Out] -1/2*(3*b^2*(a + a*c^2*x^2 + b*c*x*(1 + I*c*x))*ArcTan[c*x]^2 + b^3*(1 + c^
2*x^2)*ArcTan[c*x]^3 + 3*b*ArcTan[c*x]*(a*(a + 2*b*c*x + a*c^2*x^2) - 2*b^2
*c^2*x^2*Log[1 - E^((2*I)*ArcTan[c*x])]) + a*(a*(a + 3*b*c*x) - 6*b^2*c^2*x
^2*Log[(c*x)/Sqrt[1 + c^2*x^2]]) + (3*I)*b^3*c^2*x^2*PolyLog[2, E^((2*I)*Ar
cTan[c*x])])/x^2
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 353 vs. 2(119) = 238.

Time = 4.62 (sec) , antiderivative size = 354, normalized size of antiderivative = 2.66

method	result
derivativedivides	$c^2\left(-\frac{a^3}{2c^2x^2}+b^3\left(-\frac{\arctan(cx)^3}{2c^2x^2}-\frac{3\arctan(cx)^2}{2cx}-\frac{\arctan(cx)^3}{2}-\frac{3\arctan(cx)\ln(c^2x^2+1)}{2}+3\ln(cx)\,a\right.\right.$
default	$c^2\left(-\frac{a^3}{2c^2x^2}+b^3\left(-\frac{\arctan(cx)^3}{2c^2x^2}-\frac{3\arctan(cx)^2}{2cx}-\frac{\arctan(cx)^3}{2}-\frac{3\arctan(cx)\ln(c^2x^2+1)}{2}+3\ln(cx)\,a\right.\right.$
parts	$-\frac{a^3}{2x^2}+b^3c^2\left(-\frac{\arctan(cx)^3}{2c^2x^2}-\frac{3\arctan(cx)^2}{2cx}-\frac{\arctan(cx)^3}{2}-\frac{3\arctan(cx)\ln(c^2x^2+1)}{2}+3\ln(cx)\,\mathrm{arct}\right.$

```
[In] int((a+b*arctan(c*x))^3/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(-1/2*a^3/c^2/x^2+b^3*(-1/2/c^2/x^2*arctan(c*x)^3-3/2*arctan(c*x)^2/c/x
-1/2*arctan(c*x)^3-3/2*arctan(c*x)*ln(c^2*x^2+1)+3*ln(c*x)*arctan(c*x)+3/2*
I*ln(c*x)*ln(1+I*c*x)-3/2*I*ln(c*x)*ln(1-I*c*x)+3/2*I*dilog(1+I*c*x)-3/2*I*
dilog(1-I*c*x)-3/4*I*(ln(c*x-I)*ln(c^2*x^2+1)-1/2*ln(c*x-I)^2-dilog(-1/2*I*
(c*x+I))-ln(c*x-I)*ln(-1/2*I*(c*x+I)))+3/4*I*(ln(c*x+I)*ln(c^2*x^2+1)-1/2*l
n(c*x+I)^2-dilog(1/2*I*(c*x-I))-ln(c*x+I)*ln(1/2*I*(c*x-I))))+3*a*b^2*(-1/2
/c^2/x^2*arctan(c*x)^2-1/c/x*arctan(c*x)-1/2*arctan(c*x)^2-1/2*ln(c^2*x^2+1
)+ln(c*x))+3*a^2*b*(-1/2/c^2/x^2*arctan(c*x)-1/2*arctan(c*x)-1/2/c/x))
```

Fricas [F]

$$\int \frac{(a+b\arctan(cx))^3}{x^3}\,dx=\int \frac{(b\arctan(cx)+a)^3}{x^3}\,dx$$

```
[In] integrate((a+b*arctan(c*x))^3/x^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(c*x)^3 + 3*a*b^2*arctan(c*x)^2 + 3*a^2*b*arctan(c*x) +
 a^3)/x^3, x)
```

Sympy [F]

$$\int \frac{(a+b\arctan(cx))^3}{x^3}\,dx = \int \frac{(a+b\operatorname{atan}(cx))^3}{x^3}\,dx$$

```
[In] integrate((a+b*atan(c*x))**3/x**3,x)
```

```
[Out] Integral((a + b*atan(c*x))**3/x**3, x)
```

Maxima [F]

$$\int \frac{(a+b\arctan(cx))^3}{x^3}\,dx = \int \frac{(b\arctan(cx)+a)^3}{x^3}\,dx$$

```
[In] integrate((a+b*arctan(c*x))^3/x^3,x, algorithm="maxima")
```

```
[Out] -3/2*((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*a^2*b + 3/2*((arctan(c*x)^
2 - log(c^2*x^2 + 1) + 2*log(x))*c^2 - 2*(c*arctan(c*x) + 1/x)*c*arctan(c*x
))*a*b^2 - 3/2*a*b^2*arctan(c*x)^2/x^2 - 1/32*(12*c*x*arctan(c*x)^2 + 8*(c^
2*x^2 + 1)*arctan(c*x)^3 - 3*c*x*log(c^2*x^2 + 1)^2 - 4*(c^2*arctan(c*x)^3
+ 24*c^3*integrate(1/32*x^3*log(c^2*x^2 + 1)^2/(c^2*x^5 + x^3), x) - 96*c^3
*integrate(1/32*x^3*log(c^2*x^2 + 1)/(c^2*x^5 + x^3), x) + 128*c^2*integrat
e(1/32*x^2*arctan(c*x)^3/(c^2*x^5 + x^3), x) + 192*c^2*integrate(1/32*x^2*a
rctan(c*x)/(c^2*x^5 + x^3), x) + 96*c*integrate(1/32*x*arctan(c*x)^2/(c^2*x
^5 + x^3), x) + 24*c*integrate(1/32*x*log(c^2*x^2 + 1)^2/(c^2*x^5 + x^3), x
) + 128*integrate(1/32*arctan(c*x)^3/(c^2*x^5 + x^3), x))*x^2)*b^3/x^2 - 1/
2*a^3/x^2
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^3}{x^3}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*arctan(c*x))^3/x^3,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^3}{x^3}\,dx = \int \frac{\left(a+b\,\mathrm{atan}(c\,x)\right)^3}{x^3}\,dx$$

```
[In] int((a + b*atan(c*x))^3/x^3,x)
```

```
[Out] int((a + b*atan(c*x))^3/x^3, x)
```

3.33 $\int \frac{(a+b\arctan(cx))^3}{x^4}\,dx$

Optimal result 244
Rubi [A] (verified) 245
Mathematica [A] (verified) 248
Maple [C] (warning: unable to verify) 249
Fricas [F] 250
Sympy [F] 250
Maxima [F] 250
Giac [F(-1)] 251
Mupad [F(-1)] 251

Optimal result

Integrand size = 14, antiderivative size = 213

$$\begin{aligned}\int \frac{(a+b\arctan(cx))^3}{x^4}\,dx = &-\frac{b^2c^2(a+b\arctan(cx))}{x}-\frac{1}{2}bc^3(a+b\arctan(cx))^2\\&-\frac{bc(a+b\arctan(cx))^2}{2x^2}+\frac{1}{3}ic^3(a+b\arctan(cx))^3\\&-\frac{(a+b\arctan(cx))^3}{3x^3}+b^3c^3\log(x)-\frac{1}{2}b^3c^3\log\left(1+c^2x^2\right)\\&-bc^3(a+b\arctan(cx))^2\log\left(2-\frac{2}{1-icx}\right)\\&+ib^2c^3(a+b\arctan(cx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)\\&-\frac{1}{2}b^3c^3\operatorname{PolyLog}\left(3,-1+\frac{2}{1-icx}\right)\end{aligned}$$

```
[Out] -b^2*c^2*(a+b*arctan(c*x))/x-1/2*b*c^3*(a+b*arctan(c*x))^2-1/2*b*c*(a+b*arc
tan(c*x))^2/x^2+1/3*I*c^3*(a+b*arctan(c*x))^3-1/3*(a+b*arctan(c*x))^3/x^3+b
^3*c^3*ln(x)-1/2*b^3*c^3*ln(c^2*x^2+1)-b*c^3*(a+b*arctan(c*x))^2*ln(2-2/(1-
I*c*x))+I*b^2*c^3*(a+b*arctan(c*x))*polylog(2,-1+2/(1-I*c*x))-1/2*b^3*c^3*p
olylog(3,-1+2/(1-I*c*x))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.786$, Rules used = {4946, 5038, 272, 36, 29, 31, 5004, 5044, 4988, 5112, 6745}

$$\int \frac{(a+b\arctan(cx))^3}{x^4}\,dx = ib^2c^3\operatorname{PolyLog}\left(2,\frac{2}{1-icx}-1\right)(a+b\arctan(cx)) - \frac{b^2c^2(a+b\arctan(cx))}{x} + \frac{1}{3}ic^3(a+b\arctan(cx))^3 - \frac{1}{2}bc^3(a+b\arctan(cx))^2 - bc^3\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))^2 - \frac{(a+b\arctan(cx))^3}{3x^3} - \frac{bc(a+b\arctan(cx))^2}{2x^2} - \frac{1}{2}b^3c^3\operatorname{PolyLog}\left(3,\frac{2}{1-icx}-1\right) + b^3c^3\log(x) - \frac{1}{2}b^3c^3\log\left(c^2x^2+1\right)$$

```
[In] Int[(a + b*ArcTan[c*x])^3/x^4,x]
```

```
[Out] -((b^2*c^2*(a + b*ArcTan[c*x]))/x) - (b*c^3*(a + b*ArcTan[c*x])^2)/2 - (b*c
*(a + b*ArcTan[c*x])^2)/(2*x^2) + (I/3)*c^3*(a + b*ArcTan[c*x])^3 - (a + b*
ArcTan[c*x])^3/(3*x^3) + b^3*c^3*Log[x] - (b^3*c^3*Log[1 + c^2*x^2])/2 - b*
c^3*(a + b*ArcTan[c*x])^2*Log[2 - 2/(1 - I*c*x)] + I*b^2*c^3*(a + b*ArcTan[
c*x])*PolyLog[2, -1 + 2/(1 - I*c*x)] - (b^3*c^3*PolyLog[3, -1 + 2/(1 - I*c*
x)])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5112

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(a+b\arctan(cx))^3}{3x^3}+(bc)\int\frac{(a+b\arctan(cx))^2}{x^3(1+c^2x^2)}\,dx\\
&= -\frac{(a+b\arctan(cx))^3}{3x^3}+(bc)\int\frac{(a+b\arctan(cx))^2}{x^3}\,dx-\left(bc^3\right)\int\frac{(a+b\arctan(cx))^2}{x(1+c^2x^2)}\,dx\\
&= -\frac{bc(a+b\arctan(cx))^2}{2x^2}+\frac{1}{3}ic^3(a+b\arctan(cx))^3-\frac{(a+b\arctan(cx))^3}{3x^3}\\
&\quad+\left(b^2c^2\right)\int\frac{a+b\arctan(cx)}{x^2(1+c^2x^2)}\,dx-\left(ibc^3\right)\int\frac{(a+b\arctan(cx))^2}{x(i+cx)}\,dx\\
&= -\frac{bc(a+b\arctan(cx))^2}{2x^2}+\frac{1}{3}ic^3(a+b\arctan(cx))^3-\frac{(a+b\arctan(cx))^3}{3x^3}\\
&\quad-bc^3(a+b\arctan(cx))^2\log\left(2-\frac{2}{1-icx}\right)+\left(b^2c^2\right)\int\frac{a+b\arctan(cx)}{x^2}\,dx\\
&\quad-\left(b^2c^4\right)\int\frac{a+b\arctan(cx)}{1+c^2x^2}\,dx+\left(2b^2c^4\right)\int\frac{(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right)}{1+c^2x^2}\,dx\\
&= -\frac{b^2c^2(a+b\arctan(cx))}{x}-\frac{1}{2}bc^3(a+b\arctan(cx))^2\\
&\quad-\frac{bc(a+b\arctan(cx))^2}{2x^2}+\frac{1}{3}ic^3(a+b\arctan(cx))^3\\
&\quad-\frac{(a+b\arctan(cx))^3}{3x^3}-bc^3(a+b\arctan(cx))^2\log\left(2-\frac{2}{1-icx}\right)\\
&\quad+ib^2c^3(a+b\arctan(cx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)\\
&\quad+\left(b^3c^3\right)\int\frac{1}{x(1+c^2x^2)}\,dx-\left(ib^3c^4\right)\int\frac{\operatorname{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)}{1+c^2x^2}\,dx\\
&= -\frac{b^2c^2(a+b\arctan(cx))}{x}-\frac{1}{2}bc^3(a+b\arctan(cx))^2\\
&\quad-\frac{bc(a+b\arctan(cx))^2}{2x^2}+\frac{1}{3}ic^3(a+b\arctan(cx))^3\\
&\quad-\frac{(a+b\arctan(cx))^3}{3x^3}-bc^3(a+b\arctan(cx))^2\log\left(2-\frac{2}{1-icx}\right)\\
&\quad+ib^2c^3(a+b\arctan(cx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)\\
&\quad-\frac{1}{2}b^3c^3\operatorname{PolyLog}\left(3,-1+\frac{2}{1-icx}\right)+\frac{1}{2}\left(b^3c^3\right)\operatorname{Subst}\left(\int\frac{1}{x(1+c^2x)}\,dx,x,x^2\right)
\end{aligned}$$

$$= -\frac{b^2c^2(a+b\arctan(cx))}{x} - \frac{1}{2}bc^3(a+b\arctan(cx))^2 - \frac{bc(a+b\arctan(cx))^2}{2x^2} + \frac{1}{3}ic^3(a+b\arctan(cx))^3 - \frac{(a+b\arctan(cx))^3}{3x^3} - bc^3(a+b\arctan(cx))^2\log\left(2-\frac{2}{1-icx}\right) + ib^2c^3(a+b\arctan(cx))\operatorname{PolyLog}\left(2, -1+\frac{2}{1-icx}\right) - \frac{1}{2}b^3c^3\operatorname{PolyLog}\left(3,-1+\frac{2}{1-icx}\right) + \frac{1}{2}(b^3c^3)\operatorname{Subst}\left(\int\frac{1}{x}\,dx,x,x^2\right) - \frac{1}{2}(b^3c^5)\operatorname{Subst}\left(\int\frac{1}{1+c^2x}\,dx,x,x^2\right)$$

$$= -\frac{b^2c^2(a+b\arctan(cx))}{x} - \frac{1}{2}bc^3(a+b\arctan(cx))^2 - \frac{bc(a+b\arctan(cx))^2}{2x^2} + \frac{1}{3}ic^3(a+b\arctan(cx))^3 - \frac{(a+b\arctan(cx))^3}{3x^3} + b^3c^3\log(x) - \frac{1}{2}b^3c^3\log\left(1+c^2x^2\right) - bc^3(a+b\arctan(cx))^2\log\left(2-\frac{2}{1-icx}\right) + ib^2c^3(a+b\arctan(cx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1-icx}\right) - \frac{1}{2}b^3c^3\operatorname{PolyLog}\left(3,-1+\frac{2}{1-icx}\right)$$

Mathematica [A] (verified)

Time = 1.38 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.43

$$\int\frac{(a+b\arctan(cx))^3}{x^4}\,dx$$

$$= \frac{1}{6}\left(-\frac{2a^3}{x^3} - \frac{3a^2bc}{x^2} - \frac{6a^2b\arctan(cx)}{x^3} - 6a^2bc^3\log(x) + 3a^2bc^3\log\left(1+c^2x^2\right) + \frac{6iab^2\left(ic^2x^2+(i+c^3x^3)\arctan(cx)^2+icx\arctan(cx)\left(1+c^2x^2+2c^2x^2\log\left(1-e^{2i\arctan(cx)}\right)\right)+c^3x^3\,\mathrm{Po}\right.}{x^3} + 6b^3c^3\left(-i\arctan(cx)\operatorname{PolyLog}\left(2,e^{-2i\arctan(cx)}\right)\right) + \frac{1}{24}\left(i\pi^3-\frac{24\arctan(cx)}{cx}+\left(-8i-\frac{8}{c^3x^3}\right)\arctan(cx)^3+\arctan(cx)^2\left(-12-\frac{12}{c^2x^2}-24\log\left(1-e^{-2i\arctan(cx)}\right.\right.\right.$$

```
[In] Integrate[(a + b*ArcTan[c*x])^3/x^4,x]
```

```
[Out] ((-2*a^3)/x^3 - (3*a^2*b*c)/x^2 - (6*a^2*b*ArcTan[c*x])/x^3 - 6*a^2*b*c^3*L
og[x] + 3*a^2*b*c^3*Log[1 + c^2*x^2] + ((6*I)*a*b^2*(I*c^2*x^2 + (I + c^3*x
^3)*ArcTan[c*x]^2 + I*c*x*ArcTan[c*x]*(1 + c^2*x^2 + 2*c^2*x^2*Log[1 - E^((
2*I)*ArcTan[c*x])]) + c^3*x^3*PolyLog[2, E^((2*I)*ArcTan[c*x])]))/x^3 + 6*b
```

```
^3*c^3*((-I)*ArcTan[c*x]*PolyLog[2, E^((-2*I)*ArcTan[c*x])] + (I*Pi^3 - (24
*ArcTan[c*x])/(c*x) + (-8*I - 8/(c^3*x^3))*ArcTan[c*x]^3 + ArcTan[c*x]^2*(-
12 - 12/(c^2*x^2) - 24*Log[1 - E^((-2*I)*ArcTan[c*x])]) + 24*Log[c*x] - 12*
Log[1 + c^2*x^2] - 12*PolyLog[3, E^((-2*I)*ArcTan[c*x])])/24))/6
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.80 (sec) , antiderivative size = 2097, normalized size of antiderivative = 9.85

Expression too large to display

```
[In] int((a+b*arctan(c*x))^3/x^4,x)
```

```
[Out] c^3*(-1/3/c^3/x^3*a^3+b^3*(-1/3/c^3/x^3*arctan(c*x)^3-1/2/c^2/x^2*arctan(c*
x)^2-ln(c*x)*arctan(c*x)^2+1/2*arctan(c*x)^2*ln(c^2*x^2+1)-arctan(c*x)^2*ln
((1+I*c*x)/(c^2*x^2+1)^(1/2))+arctan(c*x)^2*ln((1+I*c*x)^2/(c^2*x^2+1)-1)+1
/12*arctan(c*x)*(3*I*arctan(c*x)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x
)^2/(c^2*x^2+1))^2)*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)*csgn(I*(1+I*c*x)^
2/(c^2*x^2+1))*Pi*c*x-6*I*arctan(c*x)*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1
+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/
(c^2*x^2+1)))*Pi*c*x-3*I*arctan(c*x)*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1))^2)^
3*Pi*c*x-3*I*arctan(c*x)*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1))^2)*csgn(I*(1+(1
+I*c*x)^2/(c^2*x^2+1)))^2*Pi*c*x-3*I*arctan(c*x)*csgn(I*(1+I*c*x)^2/(c^2*x^
2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)^2*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)
*Pi*c*x+6*I*arctan(c*x)*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(
c^2*x^2+1)))^2*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*Pi*c*x+3*I*arctan(c*x)*c
sgn(I*(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)^3*Pi*c*x-6*I*a
rctan(c*x)*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^3*
Pi*c*x-6*I*arctan(c*x)*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c
^2*x^2+1)))^3*Pi*c*x-12*I*c*x-6*I*arctan(c*x)*Pi*c*x+6*I*arctan(c*x)*csgn(I
*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*csgn(I*((1+I*c*
x)^2/(c^2*x^2+1)-1))*Pi*c*x+3*I*arctan(c*x)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))
^3*Pi*c*x-6*I*arctan(c*x)*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2
/(c^2*x^2+1)))*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*((1+I*c*x)^2/(c^2
*x^2+1)-1))*Pi*c*x-3*I*arctan(c*x)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c
*x)^2/(c^2*x^2+1))^2)^2*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*Pi*c*x+6*I*arctan(c
*x)*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(((
1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*Pi*c*x+3*I*arctan(
c*x)*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))^2*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*
Pi*c*x+6*I*arctan(c*x)*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2
*x^2+1)))^2*Pi*c*x+6*I*arctan(c*x)*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1))^2)^2*
csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1)))*Pi*c*x+4*I*arctan(c*x)^2*c*x-12*arctan(
c*x)*ln(2)*c*x-6*c*x*arctan(c*x)-12-6*I*arctan(c*x)*csgn(I*(1+I*c*x)/(c^2*x
^2+1)^(1/2))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^2*Pi*c*x)/c/x+ln((1+I*c*x)/(c^
2*x^2+1)^(1/2)-1)+ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))-arctan(c*x)^2*ln(1-(1+I
```

```
*c*x)/(c^2*x^2+1)^(1/2))+2*I*arctan(c*x)*polylog(2,(1+I*c*x)/(c^2*x^2+1)^(1
/2))-2*polylog(3,(1+I*c*x)/(c^2*x^2+1)^(1/2))-arctan(c*x)^2*ln(1+(1+I*c*x)/
(c^2*x^2+1)^(1/2))+2*I*arctan(c*x)*polylog(2,-(1+I*c*x)/(c^2*x^2+1)^(1/2))-
2*polylog(3,-(1+I*c*x)/(c^2*x^2+1)^(1/2)))+3*a*b^2*(-1/3/c^3/x^3*arctan(c*x
)^2+1/3*arctan(c*x)*ln(c^2*x^2+1)-1/3/c^2/x^2*arctan(c*x)-2/3*ln(c*x)*arcta
n(c*x)+1/6*I*(ln(c*x-I)*ln(c^2*x^2+1)-1/2*ln(c*x-I)^2-dilog(-1/2*I*(c*x+I))
-ln(c*x-I)*ln(-1/2*I*(c*x+I)))-1/6*I*(ln(c*x+I)*ln(c^2*x^2+1)-1/2*ln(c*x+I)
^2-dilog(1/2*I*(c*x-I))-ln(c*x+I)*ln(1/2*I*(c*x-I)))-1/3/c/x-1/3*arctan(c*x
)-1/3*I*ln(c*x)*ln(1+I*c*x)+1/3*I*ln(c*x)*ln(1-I*c*x)-1/3*I*dilog(1+I*c*x)+
1/3*I*dilog(1-I*c*x))+3*a^2*b*(-1/3/c^3/x^3*arctan(c*x)+1/6*ln(c^2*x^2+1)-1
/6/c^2/x^2-1/3*ln(c*x)))
```

Fricas [F]

$$\int \frac{(a+b\arctan(cx))^3}{x^4}\,dx=\int \frac{\left(b\arctan\left(cx\right)+a\right)^3}{x^4}\,dx$$

```
[In] integrate((a+b*arctan(c*x))^3/x^4,x, algorithm="fricas")
[Out] integral((b^3*arctan(c*x)^3 + 3*a*b^2*arctan(c*x)^2 + 3*a^2*b*arctan(c*x) +
 a^3)/x^4, x)
```

Sympy [F]

$$\int \frac{(a+b\arctan(cx))^3}{x^4}\,dx=\int \frac{\left(a+b\,\mathrm{atan}\left(cx\right)\right)^3}{x^4}\,dx$$

```
[In] integrate((a+b*atan(c*x))**3/x**4,x)
[Out] Integral((a + b*atan(c*x))**3/x**4, x)
```

Maxima [F]

$$\int \frac{(a+b\arctan(cx))^3}{x^4}\,dx=\int \frac{\left(b\arctan\left(cx\right)+a\right)^3}{x^4}\,dx$$

```
[In] integrate((a+b*arctan(c*x))^3/x^4,x, algorithm="maxima")
[Out] 1/2*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*a
^2*b - 1/3*a^3/x^3 - 1/96*(4*b^3*arctan(c*x)^3 - 3*b^3*arctan(c*x)*log(c^2*
x^2 + 1)^2 - 96*x^3*integrate(-1/32*(4*b^3*c^2*x^2*arctan(c*x)*log(c^2*x^2
+ 1) - 28*(b^3*c^2*x^2 + b^3)*arctan(c*x)^3 - 4*(24*a*b^2*c^2*x^2 + b^3*c*x
 + 24*a*b^2)*arctan(c*x)^2 + (b^3*c*x - 3*(b^3*c^2*x^2 + b^3)*arctan(c*x))*
log(c^2*x^2 + 1)^2)/(c^2*x^6 + x^4), x))/x^3
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^3}{x^4}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*arctan(c*x))^3/x^4,x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^3}{x^4}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^3}{x^4}\,dx$$

```
[In] int((a + b*atan(c*x))^3/x^4,x)
[Out] int((a + b*atan(c*x))^3/x^4, x)
```

3.34 $\int \frac{(a+b\arctan(cx))^3}{x^5}\,dx$

Optimal result 252
Rubi [A] (verified) 253
Mathematica [A] (verified) 255
Maple [B] (verified) 256
Fricas [F] 257
Sympy [F] 257
Maxima [F(-1)] 257
Giac [F(-1)] 257
Mupad [F(-1)] 258

Optimal result

Integrand size = 14, antiderivative size = 198

$$\begin{aligned}\int \frac{(a+b\arctan(cx))^3}{x^5}\,dx = & -\frac{b^3c^3}{4x}-\frac{1}{4}b^3c^4\arctan(cx)-\frac{b^2c^2(a+b\arctan(cx))}{4x^2}\\ & +ibc^4(a+b\arctan(cx))^2-\frac{bc(a+b\arctan(cx))^2}{4x^3}\\ & +\frac{3bc^3(a+b\arctan(cx))^2}{4x}\\ & +\frac{1}{4}c^4(a+b\arctan(cx))^3-\frac{(a+b\arctan(cx))^3}{4x^4}\\ & -2b^2c^4(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right)\\ & +ib^3c^4\operatorname{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)\end{aligned}$$

```
[Out] -1/4*b^3*c^3/x-1/4*b^3*c^4*arctan(c*x)-1/4*b^2*c^2*(a+b*arctan(c*x))/x^2+I*
b*c^4*(a+b*arctan(c*x))^2-1/4*b*c*(a+b*arctan(c*x))^2/x^3+3/4*b*c^3*(a+b*ar
ctan(c*x))^2/x+1/4*c^4*(a+b*arctan(c*x))^3-1/4*(a+b*arctan(c*x))^3/x^4-2*b^
2*c^4*(a+b*arctan(c*x))*ln(2-2/(1-I*c*x))+I*b^3*c^4*polylog(2,-1+2/(1-I*c*x
))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {4946, 5038, 331, 209, 5044, 4988, 2497, 5004}

$$\int \frac{(a+b\arctan(cx))^3}{x^5}\,dx = -2b^2c^4\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx)) - \frac{b^2c^2(a+b\arctan(cx))}{4x^2} + \frac{1}{4}c^4(a+b\arctan(cx))^3 + ibc^4(a+b\arctan(cx))^2 + \frac{3bc^3(a+b\arctan(cx))^2}{4x} - \frac{(a+b\arctan(cx))^3}{4x^4} - \frac{bc(a+b\arctan(cx))^2}{4x^3} - \frac{1}{4}b^3c^4\arctan(cx) + ib^3c^4\,\text{PolyLog}\left(2,\frac{2}{1-icx}-1\right) - \frac{b^3c^3}{4x}$$

```
[In] Int[(a + b*ArcTan[c*x])^3/x^5,x]
```

```
[Out] -1/4*(b^3*c^3)/x - (b^3*c^4*ArcTan[c*x])/4 - (b^2*c^2*(a + b*ArcTan[c*x]))/
(4*x^2) + I*b*c^4*(a + b*ArcTan[c*x])^2 - (b*c*(a + b*ArcTan[c*x])^2)/(4*x^
3) + (3*b*c^3*(a + b*ArcTan[c*x])^2)/(4*x) + (c^4*(a + b*ArcTan[c*x])^3)/4
- (a + b*ArcTan[c*x])^3/(4*x^4) - 2*b^2*c^4*(a + b*ArcTan[c*x])*Log[2 - 2/(
1 - I*c*x)] + I*b^3*c^4*PolyLog[2, -1 + 2/(1 - I*c*x)]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(a+b\arctan(cx))^3}{4x^4}+\frac{1}{4}(3bc)\int\frac{(a+b\arctan(cx))^2}{x^4\left(1+c^2x^2\right)}\,dx\\
&= -\frac{(a+b\arctan(cx))^3}{4x^4}+\frac{1}{4}(3bc)\int\frac{(a+b\arctan(cx))^2}{x^4}\,dx-\frac{1}{4}\left(3bc^3\right)\int\frac{(a+b\arctan(cx))^2}{x^2\left(1+c^2x^2\right)}\,dx\\
&= -\frac{bc(a+b\arctan(cx))^2}{4x^3}-\frac{(a+b\arctan(cx))^3}{4x^4}+\frac{1}{2}\left(b^2c^2\right)\int\frac{a+b\arctan(cx)}{x^3\left(1+c^2x^2\right)}\,dx\\
&\quad -\frac{1}{4}\left(3bc^3\right)\int\frac{(a+b\arctan(cx))^2}{x^2}\,dx+\frac{1}{4}\left(3bc^5\right)\int\frac{(a+b\arctan(cx))^2}{1+c^2x^2}\,dx
\end{aligned}
$$

$$= -\frac{bc(a+b\arctan(cx))^2}{4x^3} + \frac{3bc^3(a+b\arctan(cx))^2}{4x} + \frac{1}{4}c^4(a+b\arctan(cx))^3 - \frac{(a+b\arctan(cx))^3}{4x^4} + \frac{1}{2}(b^2c^2)\int \frac{a+b\arctan(cx)}{x^3}\,dx - \frac{1}{2}(b^2c^4)\int \frac{a+b\arctan(cx)}{x(1+c^2x^2)}\,dx - \frac{1}{2}(3b^2c^4)\int \frac{a+b\arctan(cx)}{x(1+c^2x^2)}\,dx$$

$$= -\frac{b^2c^2(a+b\arctan(cx))}{4x^2} + ibc^4(a+b\arctan(cx))^2 - \frac{bc(a+b\arctan(cx))^2}{4x^3} + \frac{3bc^3(a+b\arctan(cx))^2}{4x} + \frac{1}{4}c^4(a+b\arctan(cx))^3 - \frac{(a+b\arctan(cx))^3}{4x^4} + \frac{1}{4}(b^3c^3)\int \frac{1}{x^2(1+c^2x^2)}\,dx - \frac{1}{2}(ib^2c^4)\int \frac{a+b\arctan(cx)}{x(i+cx)}\,dx - \frac{1}{2}(3ib^2c^4)\int \frac{a+b\arctan(cx)}{x(i+cx)}\,dx$$

$$= -\frac{b^3c^3}{4x} - \frac{b^2c^2(a+b\arctan(cx))}{4x^2} + ibc^4(a+b\arctan(cx))^2 - \frac{bc(a+b\arctan(cx))^2}{4x^3} + \frac{3bc^3(a+b\arctan(cx))^2}{4x} + \frac{1}{4}c^4(a+b\arctan(cx))^3 - \frac{(a+b\arctan(cx))^3}{4x^4} - 2b^2c^4(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right) - \frac{1}{4}(b^3c^5)\int \frac{1}{1+c^2x^2}\,dx + \frac{1}{2}(b^3c^5)\int \frac{\log\left(2-\frac{2}{1-icx}\right)}{1+c^2x^2}\,dx + \frac{1}{2}(3b^3c^5)\int \frac{\log\left(2-\frac{2}{1-icx}\right)}{1+c^2x^2}\,dx$$

$$= -\frac{b^3c^3}{4x} - \frac{1}{4}b^3c^4\arctan(cx) - \frac{b^2c^2(a+b\arctan(cx))}{4x^2} + ibc^4(a+b\arctan(cx))^2 - \frac{bc(a+b\arctan(cx))^2}{4x^3} + \frac{3bc^3(a+b\arctan(cx))^2}{4x} + \frac{1}{4}c^4(a+b\arctan(cx))^3 - \frac{(a+b\arctan(cx))^3}{4x^4} - 2b^2c^4(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right) + ib^3c^4\,\text{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)$$

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.34

$$\int \frac{(a+b\arctan(cx))^3}{x^5}\,dx =$$

$$-\frac{a^3 + a^2bcx + ab^2c^2x^2 - 3a^2bc^3x^3 + b^3c^3x^3 + ab^2c^4x^4 + b^2(bcx(1-3c^2x^2-4ic^3x^3) + a(3-3c^4x^4))\,\text{arct}}{}$$

```
[In] Integrate[(a + b*ArcTan[c*x])^3/x^5,x]
```

```
[Out] -1/4*(a^3 + a^2*b*c*x + a*b^2*c^2*x^2 - 3*a^2*b*c^3*x^3 + b^3*c^3*x^3 + a*b
^2*c^4*x^4 + b^2*(b*c*x*(1 - 3*c^2*x^2 - (4*I)*c^3*x^3) + a*(3 - 3*c^4*x^4)
)*ArcTan[c*x]^2 - b^3*(-1 + c^4*x^4)*ArcTan[c*x]^3 + b*ArcTan[c*x]*(b^2*c^2
*x^2*(1 + c^2*x^2) + a*b*(2*c*x - 6*c^3*x^3) + a^2*(3 - 3*c^4*x^4) + 8*b^2*
c^4*x^4*Log[1 - E^((2*I)*ArcTan[c*x])]) + 8*a*b^2*c^4*x^4*Log[(c*x)/Sqrt[1
+ c^2*x^2]] - (4*I)*b^3*c^4*x^4*PolyLog[2, E^((2*I)*ArcTan[c*x])])/x^4
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 422 vs. 2(180) = 360.

Time = 4.50 (sec) , antiderivative size = 423, normalized size of antiderivative = 2.14

method	result
derivativedivides	$c^4\left(-\frac{a^3}{4c^4x^4}+b^3\left(-\frac{\arctan(cx)^3}{4c^4x^4}+\frac{\arctan(cx)^3}{4}-\frac{\arctan(cx)^2}{4c^3x^3}+\frac{3\arctan(cx)^2}{4cx}+\arctan(cx)\ln(c^2x^2\right.\right.$
default	$c^4\left(-\frac{a^3}{4c^4x^4}+b^3\left(-\frac{\arctan(cx)^3}{4c^4x^4}+\frac{\arctan(cx)^3}{4}-\frac{\arctan(cx)^2}{4c^3x^3}+\frac{3\arctan(cx)^2}{4cx}+\arctan(cx)\ln(c^2x^2\right.\right.$
parts	$-\frac{a^3}{4x^4}+b^3c^4\left(-\frac{\arctan(cx)^3}{4c^4x^4}+\frac{\arctan(cx)^3}{4}-\frac{\arctan(cx)^2}{4c^3x^3}+\frac{3\arctan(cx)^2}{4cx}+\arctan(cx)\ln(c^2x^2+1\right.$

```
[In] int((a+b*arctan(c*x))^3/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] c^4*(-1/4*a^3/c^4/x^4+b^3*(-1/4/c^4/x^4*arctan(c*x)^3+1/4*arctan(c*x)^3-1/4
/c^3/x^3*arctan(c*x)^2+3/4*arctan(c*x)^2/c/x+arctan(c*x)*ln(c^2*x^2+1)-1/4/
c^2/x^2*arctan(c*x)-2*ln(c*x)*arctan(c*x)-1/4*arctan(c*x)-1/4/c/x-I*ln(c*x)
*ln(1+I*c*x)+I*ln(c*x)*ln(1-I*c*x)-I*dilog(1+I*c*x)+I*dilog(1-I*c*x)+1/2*I*
(ln(c*x-I)*ln(c^2*x^2+1)-1/2*ln(c*x-I)^2-dilog(-1/2*I*(c*x+I))-ln(c*x-I)*ln
(-1/2*I*(c*x+I)))-1/2*I*(ln(c*x+I)*ln(c^2*x^2+1)-1/2*ln(c*x+I)^2-dilog(1/2*
I*(c*x-I))-ln(c*x+I)*ln(1/2*I*(c*x-I)))))+3*a*b^2*(-1/4/c^4/x^4*arctan(c*x)^
2-1/6/c^3/x^3*arctan(c*x)+1/2/c/x*arctan(c*x)+1/4*arctan(c*x)^2-1/12/c^2/x^
2-2/3*ln(c*x)+1/3*ln(c^2*x^2+1))+3*a^2*b*(-1/4/c^4/x^4*arctan(c*x)-1/12/c^3
/x^3+1/4/c/x+1/4*arctan(c*x)))
```

Fricas [F]

$$\int \frac{(a+b\arctan(cx))^3}{x^5}\,dx = \int \frac{\left(b\arctan\left(cx\right)+a\right)^3}{x^5}\,dx$$

```
[In] integrate((a+b*arctan(c*x))^3/x^5,x, algorithm="fricas")
[Out] integral((b^3*arctan(c*x)^3 + 3*a*b^2*arctan(c*x)^2 + 3*a^2*b*arctan(c*x) +
 a^3)/x^5, x)
```

Sympy [F]

$$\int \frac{(a+b\arctan(cx))^3}{x^5}\,dx = \int \frac{\left(a+b\operatorname{atan}\left(cx\right)\right)^3}{x^5}\,dx$$

```
[In] integrate((a+b*atan(c*x))**3/x**5,x)
[Out] Integral((a + b*atan(c*x))**3/x**5, x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^3}{x^5}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*arctan(c*x))^3/x^5,x, algorithm="maxima")
[Out] Timed out
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^3}{x^5}\,dx = \text{Timed out}$$

```
[In] integrate((a+b*arctan(c*x))^3/x^5,x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^3}{x^5}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^3}{x^5}\,dx$$

```
[In] int((a + b*atan(c*x))^3/x^5,x)
```

```
[Out] int((a + b*atan(c*x))^3/x^5, x)
```

3.35 $\int \frac{x}{\arctan(ax)}\,dx$

Optimal result . . . 259
Rubi [N/A] . . . 259
Mathematica [N/A] . . . 260
Maple [N/A] (verified) . . . 260
Fricas [N/A] . . . 260
Sympy [N/A] . . . 260
Maxima [N/A] . . . 261
Giac [N/A] . . . 261
Mupad [N/A] . . . 261

Optimal result

Integrand size = 8, antiderivative size = 8

$$\int \frac{x}{\arctan(ax)}\,dx = \text{Int}\left(\frac{x}{\arctan(ax)},x\right)$$

```
[Out] Unintegrable(x/arctan(a*x),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x}{\arctan(ax)}\,dx = \int \frac{x}{\arctan(ax)}\,dx$$

```
[In] Int[x/ArcTan[a*x],x]
[Out] Defer[Int][x/ArcTan[a*x], x]
```

Rubi steps

$$\text{integral} = \int \frac{x}{\arctan(ax)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \frac{x}{\arctan(ax)}\,dx = \int \frac{x}{\arctan(ax)}\,dx$$

```
[In] Integrate[x/ArcTan[a*x],x]
[Out] Integrate[x/ArcTan[a*x], x]
```

Maple [N/A] (verified)

Not integrable

Time = 10.58 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{x}{\arctan\left(ax\right)}dx$$

```
[In] int(x/arctan(a*x),x)
[Out] int(x/arctan(a*x),x)
```

Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \frac{x}{\arctan(ax)}\,dx = \int \frac{x}{\arctan\left(ax\right)}\,dx$$

```
[In] integrate(x/arctan(a*x),x, algorithm="fricas")
[Out] integral(x/arctan(a*x), x)
```

Sympy [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \frac{x}{\arctan(ax)}\,dx = \int \frac{x}{\operatorname{atan}\left(ax\right)}\,dx$$

```
[In] integrate(x/atan(a*x),x)
[Out] Integral(x/atan(a*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \frac{x}{\arctan(ax)}\,dx = \int \frac{x}{\arctan{(ax)}}\,dx$$

```
[In] integrate(x/arctan(a*x),x, algorithm="maxima")
[Out] integrate(x/arctan(a*x), x)
```

Giac [N/A]

Not integrable

Time = 23.92 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.38

$$\int \frac{x}{\arctan(ax)}\,dx = \int \frac{x}{\arctan{(ax)}}\,dx$$

```
[In] integrate(x/arctan(a*x),x, algorithm="giac")
[Out] sage0*x
```

Mupad [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \frac{x}{\arctan(ax)}\,dx = \int \frac{x}{\mathrm{atan}\left(a\,x\right)}\,dx$$

```
[In] int(x/atan(a*x),x)
[Out] int(x/atan(a*x), x)
```

3.36 $\int \frac{1}{\arctan(ax)}\,dx$

Optimal result . . . 262
Rubi [N/A] . . . 262
Mathematica [N/A] . . . 263
Maple [N/A] (verified) . . . 263
Fricas [N/A] . . . 263
Sympy [N/A] . . . 263
Maxima [N/A] . . . 264
Giac [N/A] . . . 264
Mupad [N/A] . . . 264

Optimal result

Integrand size = 6, antiderivative size = 6

$$\int \frac{1}{\arctan(ax)}\,dx = \operatorname{Int}\left(\frac{1}{\arctan(ax)},x\right)$$

```
[Out] Unintegrable(1/arctan(a*x),x)
```

Rubi [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\arctan(ax)}\,dx = \int \frac{1}{\arctan(ax)}\,dx$$

```
[In] Int[ArcTan[a*x]^(-1),x]
[Out] Defer[Int][ArcTan[a*x]^(-1), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{\arctan(ax)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.33

$$\int \frac{1}{\arctan(ax)}\,dx = \int \frac{1}{\arctan(ax)}\,dx$$

```
[In] Integrate[ArcTan[a*x]^(-1),x]
[Out] Integrate[ArcTan[a*x]^(-1), x]
```

Maple [N/A] (verified)

Not integrable

Time = 4.12 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{1}{\arctan\left(ax\right)}dx$$

```
[In] int(1/arctan(a*x),x)
[Out] int(1/arctan(a*x),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.33

$$\int \frac{1}{\arctan(ax)}\,dx = \int \frac{1}{\arctan\left(ax\right)}\,dx$$

```
[In] integrate(1/arctan(a*x),x, algorithm="fricas")
[Out] integral(1/arctan(a*x), x)
```

Sympy [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

$$\int \frac{1}{\arctan(ax)}\,dx = \int \frac{1}{\operatorname{atan}\left(ax\right)}\,dx$$

```
[In] integrate(1/atan(a*x),x)
[Out] Integral(1/atan(a*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.33

$$\int \frac{1}{\arctan(ax)} \, dx = \int \frac{1}{\arctan(ax)} \, dx$$

```
[In] integrate(1/arctan(a*x),x, algorithm="maxima")
[Out] integrate(1/arctan(a*x), x)
```

Giac [N/A]

Not integrable

Time = 21.74 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.50

$$\int \frac{1}{\arctan(ax)} \, dx = \int \frac{1}{\arctan(ax)} \, dx$$

```
[In] integrate(1/arctan(a*x),x, algorithm="giac")
[Out] sage0*x
```

Mupad [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.33

$$\int \frac{1}{\arctan(ax)} \, dx = \int \frac{1}{\mathrm{atan}(a\,x)} \, dx$$

```
[In] int(1/atan(a*x),x)
[Out] int(1/atan(a*x), x)
```

3.37 $\int \frac{1}{x \arctan(ax)} \, dx$

Optimal result 265
Rubi [N/A] 265
Mathematica [N/A] 266
Maple [N/A] (verified) 266
Fricas [N/A] 266
Sympy [N/A] 266
Maxima [N/A] 267
Giac [N/A] 267
Mupad [N/A] 267

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{1}{x \arctan(ax)} \, dx = \text{Int}\left(\frac{1}{x \arctan(ax)}, x\right)$$

```
[Out] Unintegrable(1/x/arctan(a*x),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x \arctan(ax)} \, dx = \int \frac{1}{x \arctan(ax)} \, dx$$

```
[In] Int[1/(x*ArcTan[a*x]),x]
```

```
[Out] Defer[Int][1/(x*ArcTan[a*x]), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x \arctan(ax)} \, dx$$

Mathematica [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x \arctan(ax)} dx = \int \frac{1}{x \arctan(ax)} dx$$

```
[In] Integrate[1/(x*ArcTan[a*x]),x]
[Out] Integrate[1/(x*ArcTan[a*x]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 4.59 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \arctan(ax)} dx$$

```
[In] int(1/x/arctan(a*x),x)
[Out] int(1/x/arctan(a*x),x)
```

Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x \arctan(ax)} dx = \int \frac{1}{x \arctan(ax)} dx$$

```
[In] integrate(1/x/arctan(a*x),x, algorithm="fricas")
[Out] integral(1/(x*arctan(a*x)), x)
```

Sympy [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1}{x \arctan(ax)} dx = \int \frac{1}{x \operatorname{atan}(ax)} dx$$

```
[In] integrate(1/x/atan(a*x),x)
[Out] Integral(1/(x*atan(a*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x \arctan(ax)}\,dx = \int \frac{1}{x \arctan{(ax)}}\,dx$$

```
[In] integrate(1/x/arctan(a*x),x, algorithm="maxima")
[Out] integrate(1/(x*arctan(a*x)), x)
```

Giac [N/A]

Not integrable

Time = 24.44 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.30

$$\int \frac{1}{x \arctan(ax)}\,dx = \int \frac{1}{x \arctan{(ax)}}\,dx$$

```
[In] integrate(1/x/arctan(a*x),x, algorithm="giac")
[Out] sage0*x
```

Mupad [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x \arctan(ax)}\,dx = \int \frac{1}{x \,\mathrm{atan}\left(a\,x\right)}\,dx$$

```
[In] int(1/(x*atan(a*x)),x)
[Out] int(1/(x*atan(a*x)), x)
```

3.38 $\int \frac{x}{\arctan(ax)^2} \, dx$

Optimal result 268
Rubi [N/A] 268
Mathematica [N/A] 269
Maple [N/A] (verified) 269
Fricas [N/A] 269
Sympy [N/A] 269
Maxima [N/A] 270
Giac [N/A] 270
Mupad [N/A] 270

Optimal result

Integrand size = 8, antiderivative size = 8

$$\int \frac{x}{\arctan(ax)^2} \, dx = \text{Int}\left(\frac{x}{\arctan(ax)^2}, x\right)$$

```
[Out] Unintegrable(x/arctan(a*x)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x}{\arctan(ax)^2} \, dx = \int \frac{x}{\arctan(ax)^2} \, dx$$

```
[In] Int[x/ArcTan[a*x]^2,x]
```

```
[Out] Defer[Int][x/ArcTan[a*x]^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{x}{\arctan(ax)^2} \, dx$$

Mathematica [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \frac{x}{\arctan(ax)^2} \, dx = \int \frac{x}{\arctan(ax)^2} \, dx$$

```
[In] Integrate[x/ArcTan[a*x]^2,x]
[Out] Integrate[x/ArcTan[a*x]^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 10.51 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{x}{\arctan(ax)^2} dx$$

```
[In] int(x/arctan(a*x)^2,x)
[Out] int(x/arctan(a*x)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \frac{x}{\arctan(ax)^2} \, dx = \int \frac{x}{\arctan(ax)^2} \, dx$$

```
[In] integrate(x/arctan(a*x)^2,x, algorithm="fricas")
[Out] integral(x/arctan(a*x)^2, x)
```

Sympy [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{x}{\arctan(ax)^2} \, dx = \int \frac{x}{\operatorname{atan}^2(ax)} \, dx$$

```
[In] integrate(x/atan(a*x)**2,x)
[Out] Integral(x/atan(a*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 5.62

$$\int \frac{x}{\arctan(ax)^2} \, dx = \int \frac{x}{\arctan(ax)^2} \, dx$$

```
[In] integrate(x/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^2*x^3 - arctan(a*x)*integrate((3*a^2*x^2 + 1)/arctan(a*x), x) + x)/(a*a
rctan(a*x))
```

Giac [N/A]

Not integrable

Time = 49.11 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.38

$$\int \frac{x}{\arctan(ax)^2} \, dx = \int \frac{x}{\arctan(ax)^2} \, dx$$

```
[In] integrate(x/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \frac{x}{\arctan(ax)^2} \, dx = \int \frac{x}{\mathrm{atan}\left(a\,x\right)^2} \, dx$$

```
[In] int(x/atan(a*x)^2,x)
```

```
[Out] int(x/atan(a*x)^2, x)
```

3.39 $\int \frac{1}{\arctan(ax)^2} dx$

Optimal result 271
Rubi [N/A] 271
Mathematica [N/A] 272
Maple [N/A] (verified) 272
Fricas [N/A] 272
Sympy [N/A] 272
Maxima [N/A] 273
Giac [N/A] 273
Mupad [N/A] 273

Optimal result

Integrand size = 6, antiderivative size = 6

$$\int \frac{1}{\arctan(ax)^2} dx = \operatorname{Int}\left(\frac{1}{\arctan(ax)^2}, x\right)$$

```
[Out] Unintegrable(1/arctan(a*x)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\arctan(ax)^2} dx = \int \frac{1}{\arctan(ax)^2} dx$$

```
[In] Int[ArcTan[a*x]^(-2),x]
```

```
[Out] Defer[Int][ArcTan[a*x]^(-2), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{\arctan(ax)^2} dx$$

Mathematica [N/A]

Not integrable

Time = 1.10 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.33

$$\int \frac{1}{\arctan(ax)^2}\,dx = \int \frac{1}{\arctan(ax)^2}\,dx$$

```
[In] Integrate[ArcTan[a*x]^(-2),x]
[Out] Integrate[ArcTan[a*x]^(-2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 5.54 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{1}{\arctan\left(ax\right)^{2}}dx$$

```
[In] int(1/arctan(a*x)^2,x)
[Out] int(1/arctan(a*x)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.33

$$\int \frac{1}{\arctan(ax)^2}\,dx = \int \frac{1}{\arctan\left(ax\right)^{2}}\,dx$$

```
[In] integrate(1/arctan(a*x)^2,x, algorithm="fricas")
[Out] integral(arctan(a*x)^(-2), x)
```

Sympy [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.33

$$\int \frac{1}{\arctan(ax)^2}\,dx = \int \frac{1}{\operatorname{atan}^{2}\left(ax\right)}\,dx$$

```
[In] integrate(1/atan(a*x)**2,x)
[Out] Integral(atan(a*x)**(-2), x)
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 39, normalized size of antiderivative = 6.50

$$\int \frac{1}{\arctan(ax)^2}\,dx = \int \frac{1}{\arctan(ax)^2}\,dx$$

```
[In] integrate(1/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^2*x^2 - 2*a^2*arctan(a*x)*integrate(x/arctan(a*x), x) + 1)/(a*arctan(a*
x))
```

Giac [N/A]

Not integrable

Time = 47.80 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.50

$$\int \frac{1}{\arctan(ax)^2}\,dx = \int \frac{1}{\arctan(ax)^2}\,dx$$

```
[In] integrate(1/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.33

$$\int \frac{1}{\arctan(ax)^2}\,dx = \int \frac{1}{\mathrm{atan}\left(a\,x\right)^2}\,dx$$

```
[In] int(1/atan(a*x)^2,x)
```

```
[Out] int(1/atan(a*x)^2, x)
```

3.40 $\int \frac{1}{x \arctan(ax)^2} \, dx$

Optimal result . . . 274
Rubi [N/A] . . . 274
Mathematica [N/A] . . . 275
Maple [N/A] (verified) . . . 275
Fricas [N/A] . . . 275
Sympy [N/A] . . . 275
Maxima [N/A] . . . 276
Giac [N/A] . . . 276
Mupad [N/A] . . . 276

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{1}{x \arctan(ax)^2} \, dx = \text{Int}\left(\frac{1}{x \arctan(ax)^2}, x\right)$$

```
[Out] Unintegrable(1/x/arctan(a*x)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x \arctan(ax)^2} \, dx = \int \frac{1}{x \arctan(ax)^2} \, dx$$

```
[In] Int[1/(x*ArcTan[a*x]^2),x]
```

```
[Out] Defer[Int][1/(x*ArcTan[a*x]^2), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x \arctan(ax)^2} \, dx$$

Mathematica [N/A]

Not integrable

Time = 1.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x\arctan(ax)^2}\,dx = \int \frac{1}{x\arctan(ax)^2}\,dx$$

```
[In] Integrate[1/(x*ArcTan[a*x]^2),x]
[Out] Integrate[1/(x*ArcTan[a*x]^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 4.23 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\arctan\left(ax\right)^2}dx$$

```
[In] int(1/x/arctan(a*x)^2,x)
[Out] int(1/x/arctan(a*x)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x\arctan(ax)^2}\,dx = \int \frac{1}{x\arctan\left(ax\right)^2}\,dx$$

```
[In] integrate(1/x/arctan(a*x)^2,x, algorithm="fricas")
[Out] integral(1/(x*arctan(a*x)^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\arctan(ax)^2}\,dx = \int \frac{1}{x\operatorname{atan}^2\left(ax\right)}\,dx$$

```
[In] integrate(1/x/atan(a*x)**2,x)
[Out] Integral(1/(x*atan(a*x)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 5.10

$$\int \frac{1}{x \arctan(ax)^2} \, dx = \int \frac{1}{x \arctan{(ax)}^2} \, dx$$

```
[In] integrate(1/x/arctan(a*x)^2,x, algorithm="maxima")
[Out] -(a^2*x^2 - x*arctan(a*x)*integrate((a^2*x^2 - 1)/(x^2*arctan(a*x)), x) + 1
)/(a*x*arctan(a*x))
```

Giac [N/A]

Not integrable

Time = 49.41 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.30

$$\int \frac{1}{x \arctan(ax)^2} \, dx = \int \frac{1}{x \arctan{(ax)}^2} \, dx$$

```
[In] integrate(1/x/arctan(a*x)^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x \arctan(ax)^2} \, dx = \int \frac{1}{x \,\mathrm{atan}{(a\,x)}^2} \, dx$$

```
[In] int(1/(x*atan(a*x)^2),x)
[Out] int(1/(x*atan(a*x)^2), x)
```

3.41 $\int x\sqrt{\arctan(ax)}\,dx$

Optimal result 277
Rubi [N/A] 277
Mathematica [N/A] 278
Maple [N/A] (verified) 278
Fricas [F(-2)] 278
Sympy [N/A] 278
Maxima [F(-2)] 279
Giac [N/A] 279
Mupad [N/A] 279

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int x\sqrt{\arctan(ax)}\,dx = \text{Int}\left(x\sqrt{\arctan(ax)},x\right)$$

```
[Out] Unintegrable(x*arctan(a*x)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x\sqrt{\arctan(ax)}\,dx = \int x\sqrt{\arctan(ax)}\,dx$$

```
[In] Int[x*Sqrt[ArcTan[a*x]],x]
[Out] Defer[Int][x*Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\text{integral} = \int x\sqrt{\arctan(ax)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.48 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x\sqrt{\arctan(ax)}\,dx = \int x\sqrt{\arctan(ax)}\,dx$$

```
[In] Integrate[x*Sqrt[ArcTan[a*x]],x]
[Out] Integrate[x*Sqrt[ArcTan[a*x]], x]
```

Maple [N/A] (verified)

Not integrable

Time = 2.09 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int x\sqrt{\arctan(ax)}dx$$

```
[In] int(x*arctan(a*x)^(1/2),x)
[Out] int(x*arctan(a*x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x\sqrt{\arctan(ax)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*arctan(a*x)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int x\sqrt{\arctan(ax)}\,dx = \int x\sqrt{\operatorname{atan}(ax)}\,dx$$

```
[In] integrate(x*atan(a*x)**(1/2),x)
[Out] Integral(x*sqrt(atan(a*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x\sqrt{\arctan(ax)}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x*arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [N/A]

Not integrable

Time = 53.54 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.30

$$\int x\sqrt{\arctan(ax)}\,dx = \int x\sqrt{\arctan(ax)}\,dx$$

```
[In] integrate(x*arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int x\sqrt{\arctan(ax)}\,dx = \int x\,\sqrt{\mathrm{atan}\left(a\,x\right)}\,dx$$

```
[In] int(x*atan(a*x)^(1/2),x)
```

```
[Out] int(x*atan(a*x)^(1/2), x)
```

3.42 $\int \sqrt{\arctan(ax)}\,dx$

Optimal result 280
Rubi [N/A] 280
Mathematica [N/A] 281
Maple [N/A] (verified) 281
Fricas [F(-2)] 281
Sympy [N/A] 281
Maxima [F(-2)] 282
Giac [N/A] 282
Mupad [N/A] 282

Optimal result

Integrand size = 8, antiderivative size = 8

$$\int \sqrt{\arctan(ax)}\,dx = \text{Int}\left(\sqrt{\arctan(ax)},x\right)$$

```
[Out] Unintegrable(arctan(a*x)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sqrt{\arctan(ax)}\,dx = \int \sqrt{\arctan(ax)}\,dx$$

```
[In] Int[Sqrt[ArcTan[a*x]],x]
```

```
[Out] Defer[Int][Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\text{integral} = \int \sqrt{\arctan(ax)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.81 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \sqrt{\arctan(ax)}\,dx = \int \sqrt{\arctan(ax)}\,dx$$

```
[In] Integrate[Sqrt[ArcTan[a*x]],x]
[Out] Integrate[Sqrt[ArcTan[a*x]], x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.64 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \sqrt{\arctan\left(ax\right)}dx$$

```
[In] int(arctan(a*x)^(1/2),x)
[Out] int(arctan(a*x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{\arctan(ax)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(arctan(a*x)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \sqrt{\arctan(ax)}\,dx = \int \sqrt{\operatorname{atan}\left(ax\right)}\,dx$$

```
[In] integrate(atan(a*x)**(1/2),x)
[Out] Integral(sqrt(atan(a*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{\arctan(ax)}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(arctan(a*x)^(1/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [N/A]

Not integrable

Time = 53.95 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.38

$$\int \sqrt{\arctan(ax)}\,dx = \int \sqrt{\arctan(ax)}\,dx$$

```
[In] integrate(arctan(a*x)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \sqrt{\arctan(ax)}\,dx = \int \sqrt{\mathrm{atan}(a\,x)}\,dx$$

```
[In] int(atan(a*x)^(1/2),x)
[Out] int(atan(a*x)^(1/2), x)
```

3.43 $\int \frac{\sqrt{\arctan(ax)}}{x} dx$

Optimal result . . . 283
Rubi [N/A] . . . 283
Mathematica [N/A] . . . 284
Maple [N/A] (verified) . . . 284
Fricas [F(-2)] . . . 284
Sympy [N/A] . . . 284
Maxima [F(-2)] . . . 285
Giac [N/A] . . . 285
Mupad [N/A] . . . 285

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\sqrt{\arctan(ax)}}{x} dx = \operatorname{Int}\left(\frac{\sqrt{\arctan(ax)}}{x}, x\right)$$

```
[Out] Unintegrable(arctan(a*x)^(1/2)/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{\arctan(ax)}}{x} dx = \int \frac{\sqrt{\arctan(ax)}}{x} dx$$

```
[In] Int[Sqrt[ArcTan[a*x]]/x,x]
[Out] Defer[Int][Sqrt[ArcTan[a*x]]/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\sqrt{\arctan(ax)}}{x} dx$$

Mathematica [N/A]

Not integrable

Time = 1.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{\arctan(ax)}}{x}\,dx = \int \frac{\sqrt{\arctan(ax)}}{x}\,dx$$

```
[In] Integrate[Sqrt[ArcTan[a*x]]/x,x]
[Out] Integrate[Sqrt[ArcTan[a*x]]/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.95 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{\arctan\left(ax\right)}}{x}dx$$

```
[In] int(arctan(a*x)^(1/2)/x,x)
[Out] int(arctan(a*x)^(1/2)/x,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{x}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(arctan(a*x)^(1/2)/x,x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{\arctan(ax)}}{x}\,dx = \int \frac{\sqrt{\operatorname{atan}\left(ax\right)}}{x}\,dx$$

```
[In] integrate(atan(a*x)**(1/2)/x,x)
[Out] Integral(sqrt(atan(a*x))/x, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{x}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(arctan(a*x)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [N/A]

Not integrable

Time = 148.20 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.25

$$\int \frac{\sqrt{\arctan(ax)}}{x}\,dx = \int \frac{\sqrt{\arctan(ax)}}{x}\,dx$$

```
[In] integrate(arctan(a*x)^(1/2)/x,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\arctan(ax)}}{x}\,dx = \int \frac{\sqrt{\mathrm{atan}(a\,x)}}{x}\,dx$$

```
[In] int(atan(a*x)^(1/2)/x,x)
```

```
[Out] int(atan(a*x)^(1/2)/x, x)
```

3.44 $\int x \arctan(ax)^{3/2} \, dx$

Optimal result 286
Rubi [N/A] 286
Mathematica [N/A] 287
Maple [N/A] (verified) 287
Fricas [F(-2)] 287
Sympy [N/A] 287
Maxima [F(-2)] 288
Giac [N/A] 288
Mupad [N/A] 288

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int x \arctan(ax)^{3/2} \, dx = \text{Int}\left(x \arctan(ax)^{3/2}, x\right)$$

```
[Out] Unintegrable(x*arctan(a*x)^(3/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x \arctan(ax)^{3/2} \, dx = \int x \arctan(ax)^{3/2} \, dx$$

```
[In] Int[x*ArcTan[a*x]^(3/2),x]
```

```
[Out] Defer[Int][x*ArcTan[a*x]^(3/2), x]
```

Rubi steps

$$\text{integral} = \int x \arctan(ax)^{3/2} \, dx$$

Mathematica [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x \arctan(ax)^{3/2} \, dx = \int x \arctan(ax)^{3/2} \, dx$$

```
[In] Integrate[x*ArcTan[a*x]^(3/2),x]
[Out] Integrate[x*ArcTan[a*x]^(3/2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.79 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int x \arctan(ax)^{\frac{3}{2}} \, dx$$

```
[In] int(x*arctan(a*x)^(3/2),x)
[Out] int(x*arctan(a*x)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x \arctan(ax)^{3/2} \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*arctan(a*x)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 1.41 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int x \arctan(ax)^{3/2} \, dx = \int x \operatorname{atan}^{\frac{3}{2}}(ax) \, dx$$

```
[In] integrate(x*atan(a*x)**(3/2),x)
[Out] Integral(x*atan(a*x)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x\arctan(ax)^{3/2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x*arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [N/A]

Not integrable

Time = 81.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.30

$$\int x\arctan(ax)^{3/2}\,dx = \int x\arctan(ax)^{\frac{3}{2}}\,dx$$

```
[In] integrate(x*arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int x\arctan(ax)^{3/2}\,dx = \int x\,\mathrm{atan}(a\,x)^{3/2}\,dx$$

```
[In] int(x*atan(a*x)^(3/2),x)
```

```
[Out] int(x*atan(a*x)^(3/2), x)
```

3.45 $\int \arctan(ax)^{3/2}\,dx$

Optimal result 289
Rubi [N/A] 289
Mathematica [N/A] 290
Maple [N/A] (verified) 290
Fricas [F(-2)] 290
Sympy [N/A] 290
Maxima [F(-2)] 291
Giac [N/A] 291
Mupad [N/A] 291

Optimal result

Integrand size = 8, antiderivative size = 8

$$\int \arctan(ax)^{3/2}\,dx = \operatorname{Int}\left(\arctan(ax)^{3/2},x\right)$$

```
[Out] Unintegrable(arctan(a*x)^(3/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \arctan(ax)^{3/2}\,dx = \int \arctan(ax)^{3/2}\,dx$$

```
[In] Int[ArcTan[a*x]^(3/2),x]
[Out] Defer[Int][ArcTan[a*x]^(3/2), x]
```

Rubi steps

$$\text{integral} = \int \arctan(ax)^{3/2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.71 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \arctan(ax)^{3/2}\,dx = \int \arctan(ax)^{3/2}\,dx$$

```
[In] Integrate[ArcTan[a*x]^(3/2),x]
[Out] Integrate[ArcTan[a*x]^(3/2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.53 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \arctan\left(ax\right)^{\frac{3}{2}}dx$$

```
[In] int(arctan(a*x)^(3/2),x)
[Out] int(arctan(a*x)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \arctan(ax)^{3/2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(arctan(a*x)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \arctan(ax)^{3/2}\,dx = \int \operatorname{atan}^{\frac{3}{2}}(ax)\,dx$$

```
[In] integrate(atan(a*x)**(3/2),x)
[Out] Integral(atan(a*x)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \arctan(ax)^{3/2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [N/A]

Not integrable

Time = 81.25 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.38

$$\int \arctan(ax)^{3/2}\,dx = \int \arctan\left(ax\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate(arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \arctan(ax)^{3/2}\,dx = \int \mathrm{atan}(a\,x)^{3/2}\,dx$$

```
[In] int(atan(a*x)^(3/2),x)
```

```
[Out] int(atan(a*x)^(3/2), x)
```

3.46 $\int \frac{\arctan(ax)^{3/2}}{x} dx$

Optimal result 292
Rubi [N/A] 292
Mathematica [N/A] 293
Maple [N/A] (verified) 293
Fricas [F(-2)] 293
Sympy [N/A] 293
Maxima [F(-2)] 294
Giac [N/A] 294
Mupad [N/A] 294

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\arctan(ax)^{3/2}}{x} dx = \text{Int}\left(\frac{\arctan(ax)^{3/2}}{x}, x\right)$$

```
[Out] Unintegrable(arctan(a*x)^(3/2)/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\arctan(ax)^{3/2}}{x} dx = \int \frac{\arctan(ax)^{3/2}}{x} dx$$

```
[In] Int[ArcTan[a*x]^(3/2)/x,x]
[Out] Defer[Int][ArcTan[a*x]^(3/2)/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\arctan(ax)^{3/2}}{x} dx$$

Mathematica [N/A]

Not integrable

Time = 0.96 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\arctan(ax)^{3/2}}{x} \, dx = \int \frac{\arctan(ax)^{3/2}}{x} \, dx$$

```
[In] Integrate[ArcTan[a*x]^(3/2)/x,x]
[Out] Integrate[ArcTan[a*x]^(3/2)/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 2.73 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\arctan(ax)^{\frac{3}{2}}}{x} dx$$

```
[In] int(arctan(a*x)^(3/2)/x,x)
[Out] int(arctan(a*x)^(3/2)/x,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{x} \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(arctan(a*x)^(3/2)/x,x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\arctan(ax)^{3/2}}{x} \, dx = \int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{x} \, dx$$

```
[In] integrate(atan(a*x)**(3/2)/x,x)
[Out] Integral(atan(a*x)**(3/2)/x, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{x}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(arctan(a*x)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [N/A]

Not integrable

Time = 142.20 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.25

$$\int \frac{\arctan(ax)^{3/2}}{x}\,dx = \int \frac{\arctan\left(ax\right)^{\frac{3}{2}}}{x}\,dx$$

```
[In] integrate(arctan(a*x)^(3/2)/x,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)^{3/2}}{x}\,dx = \int \frac{\operatorname{atan}(a\,x)^{3/2}}{x}\,dx$$

```
[In] int(atan(a*x)^(3/2)/x,x)
```

```
[Out] int(atan(a*x)^(3/2)/x, x)
```

3.47 $\int \frac{x}{\sqrt{\arctan(ax)}} dx$

Optimal result 295
Rubi [N/A] 295
Mathematica [N/A] 296
Maple [N/A] (verified) 296
Fricas [F(-2)] 296
Sympy [N/A] 296
Maxima [F(-2)] 297
Giac [N/A] 297
Mupad [N/A] 297

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{x}{\sqrt{\arctan(ax)}} dx = \text{Int}\left(\frac{x}{\sqrt{\arctan(ax)}}, x\right)$$

```
[Out] Unintegrable(x/arctan(a*x)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x}{\sqrt{\arctan(ax)}} dx = \int \frac{x}{\sqrt{\arctan(ax)}} dx$$

```
[In] Int[x/Sqrt[ArcTan[a*x]],x]
[Out] Defer[Int][x/Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\text{integral} = \int \frac{x}{\sqrt{\arctan(ax)}} dx$$

Mathematica [N/A]

Not integrable

Time = 0.93 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{x}{\sqrt{\arctan(ax)}}\,dx = \int \frac{x}{\sqrt{\arctan(ax)}}\,dx$$

```
[In] Integrate[x/Sqrt[ArcTan[a*x]],x]
[Out] Integrate[x/Sqrt[ArcTan[a*x]], x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.75 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{x}{\sqrt{\arctan(ax)}}dx$$

```
[In] int(x/arctan(a*x)^(1/2),x)
[Out] int(x/arctan(a*x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x/arctan(a*x)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{\arctan(ax)}}\,dx = \int \frac{x}{\sqrt{\operatorname{atan}(ax)}}\,dx$$

```
[In] integrate(x/atan(a*x)**(1/2),x)
[Out] Integral(x/sqrt(atan(a*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [N/A]

Not integrable

Time = 70.97 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.30

$$\int \frac{x}{\sqrt{\arctan(ax)}}\,dx = \int \frac{x}{\sqrt{\arctan(ax)}}\,dx$$

```
[In] integrate(x/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{\arctan(ax)}}\,dx = \int \frac{x}{\sqrt{\mathrm{atan}\left(a\,x\right)}}\,dx$$

```
[In] int(x/atan(a*x)^(1/2),x)
```

```
[Out] int(x/atan(a*x)^(1/2), x)
```

3.48 $\int \frac{1}{\sqrt{\arctan(ax)}}\,dx$

Optimal result 298
Rubi [N/A] 298
Mathematica [N/A] 299
Maple [N/A] (verified) 299
Fricas [F(-2)] 299
Sympy [N/A] 299
Maxima [F(-2)] 300
Giac [N/A] 300
Mupad [N/A] 300

Optimal result

Integrand size = 8, antiderivative size = 8

$$\int \frac{1}{\sqrt{\arctan(ax)}}\,dx = \text{Int}\left(\frac{1}{\sqrt{\arctan(ax)}},x\right)$$

```
[Out] Unintegrable(1/arctan(a*x)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\sqrt{\arctan(ax)}}\,dx = \int \frac{1}{\sqrt{\arctan(ax)}}\,dx$$

```
[In] Int[1/Sqrt[ArcTan[a*x]],x]
[Out] Defer[Int][1/Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{\sqrt{\arctan(ax)}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \frac{1}{\sqrt{\arctan(ax)}} \, dx = \int \frac{1}{\sqrt{\arctan(ax)}} \, dx$$

```
[In] Integrate[1/Sqrt[ArcTan[a*x]],x]
[Out] Integrate[1/Sqrt[ArcTan[a*x]], x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.59 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{\arctan (ax)}} dx$$

```
[In] int(1/arctan(a*x)^(1/2),x)
[Out] int(1/arctan(a*x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{\arctan(ax)}} \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/arctan(a*x)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \frac{1}{\sqrt{\arctan(ax)}} \, dx = \int \frac{1}{\sqrt{\operatorname{atan}(ax)}} \, dx$$

```
[In] integrate(1/atan(a*x)**(1/2),x)
[Out] Integral(1/sqrt(atan(a*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(1/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [N/A]

Not integrable

Time = 64.90 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.38

$$\int \frac{1}{\sqrt{\arctan(ax)}}\,dx = \int \frac{1}{\sqrt{\arctan{(ax)}}}\,dx$$

```
[In] integrate(1/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{\arctan(ax)}}\,dx = \int \frac{1}{\sqrt{\mathrm{atan}\left(a\,x\right)}}\,dx$$

```
[In] int(1/atan(a*x)^(1/2),x)
```

```
[Out] int(1/atan(a*x)^(1/2), x)
```

3.49 $\int \frac{1}{x\sqrt{\arctan(ax)}}\,dx$

Optimal result . . . 301
Rubi [N/A] . . . 301
Mathematica [N/A] . . . 302
Maple [N/A] (verified) . . . 302
Fricas [F(-2)] . . . 302
Sympy [N/A] . . . 302
Maxima [F(-2)] . . . 303
Giac [N/A] . . . 303
Mupad [N/A] . . . 303

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{1}{x\sqrt{\arctan(ax)}}\,dx = \mathrm{Int}\left(\frac{1}{x\sqrt{\arctan(ax)}},x\right)$$

```
[Out] Unintegrable(1/x/arctan(a*x)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\sqrt{\arctan(ax)}}\,dx = \int \frac{1}{x\sqrt{\arctan(ax)}}\,dx$$

```
[In] Int[1/(x*Sqrt[ArcTan[a*x]]),x]
[Out] Defer[Int][1/(x*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x\sqrt{\arctan(ax)}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.30 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{x\sqrt{\arctan(ax)}}\,dx = \int \frac{1}{x\sqrt{\arctan(ax)}}\,dx$$

```
[In] Integrate[1/(x*Sqrt[ArcTan[a*x]]),x]
[Out] Integrate[1/(x*Sqrt[ArcTan[a*x]]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 2.35 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{x\sqrt{\arctan\left(ax\right)}}dx$$

```
[In] int(1/x/arctan(a*x)^(1/2),x)
[Out] int(1/x/arctan(a*x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/x/arctan(a*x)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{\arctan(ax)}}\,dx = \int \frac{1}{x\sqrt{\operatorname{atan}\left(ax\right)}}\,dx$$

```
[In] integrate(1/x/atan(a*x)**(1/2),x)
[Out] Integral(1/(x*sqrt(atan(a*x))), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(1/x/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [N/A]

Not integrable

Time = 68.15 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.25

$$\int \frac{1}{x\sqrt{\arctan(ax)}}\,dx = \int \frac{1}{x\sqrt{\arctan(ax)}}\,dx$$

```
[In] integrate(1/x/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{\arctan(ax)}}\,dx = \int \frac{1}{x\,\sqrt{\mathrm{atan}\left(a\,x\right)}}\,dx$$

```
[In] int(1/(x*atan(a*x)^(1/2)),x)
```

```
[Out] int(1/(x*atan(a*x)^(1/2)), x)
```

3.50 $\int \frac{x}{\arctan(ax)^{3/2}}\,dx$

Optimal result 304
Rubi [N/A] 304
Mathematica [N/A] 305
Maple [N/A] (verified) 305
Fricas [F(-2)] 305
Sympy [N/A] 305
Maxima [F(-2)] 306
Giac [N/A] 306
Mupad [N/A] 306

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{x}{\arctan(ax)^{3/2}}\,dx = \text{Int}\left(\frac{x}{\arctan(ax)^{3/2}}, x\right)$$

```
[Out] Unintegrable(x/arctan(a*x)^(3/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x}{\arctan(ax)^{3/2}}\,dx = \int \frac{x}{\arctan(ax)^{3/2}}\,dx$$

```
[In] Int[x/ArcTan[a*x]^(3/2),x]
[Out] Defer[Int][x/ArcTan[a*x]^(3/2), x]
```

Rubi steps

$$\text{integral} = \int \frac{x}{\arctan(ax)^{3/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{x}{\arctan(ax)^{3/2}} \, dx = \int \frac{x}{\arctan(ax)^{3/2}} \, dx$$

```
[In] Integrate[x/ArcTan[a*x]^(3/2),x]
[Out] Integrate[x/ArcTan[a*x]^(3/2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.94 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{x}{\arctan (ax)^{\frac{3}{2}}}dx$$

```
[In] int(x/arctan(a*x)^(3/2),x)
[Out] int(x/arctan(a*x)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{\arctan(ax)^{3/2}} \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x/arctan(a*x)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{x}{\arctan(ax)^{3/2}} \, dx = \int \frac{x}{\operatorname{atan}^{\frac{3}{2}}(ax)} \, dx$$

```
[In] integrate(x/atan(a*x)**(3/2),x)
[Out] Integral(x/atan(a*x)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x/arctan(a*x)^(3/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [N/A]

Not integrable

Time = 249.37 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.30

$$\int \frac{x}{\arctan(ax)^{3/2}}\,dx = \int \frac{x}{\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x/arctan(a*x)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

Mupad [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{x}{\arctan(ax)^{3/2}}\,dx = \int \frac{x}{{\mathrm{atan}\left(a\,x\right)}^{3/2}}\,dx$$

```
[In] int(x/atan(a*x)^(3/2),x)
[Out] int(x/atan(a*x)^(3/2), x)
```

3.51 $\int \frac{1}{\arctan(ax)^{3/2}}\,dx$

Optimal result . . . 307
Rubi [N/A] . . . 307
Mathematica [N/A] . . . 308
Maple [N/A] (verified) . . . 308
Fricas [F(-2)] . . . 308
Sympy [N/A] . . . 308
Maxima [F(-2)] . . . 309
Giac [N/A] . . . 309
Mupad [N/A] . . . 309

Optimal result

Integrand size = 8, antiderivative size = 8

$$\int \frac{1}{\arctan(ax)^{3/2}}\,dx = \text{Int}\left(\frac{1}{\arctan(ax)^{3/2}},x\right)$$

```
[Out] Unintegrable(1/arctan(a*x)^(3/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\arctan(ax)^{3/2}}\,dx = \int \frac{1}{\arctan(ax)^{3/2}}\,dx$$

```
[In] Int[ArcTan[a*x]^(-3/2),x]
```

```
[Out] Defer[Int][ArcTan[a*x]^(-3/2), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{\arctan(ax)^{3/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.74 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \frac{1}{\arctan(ax)^{3/2}}\,dx = \int \frac{1}{\arctan(ax)^{3/2}}\,dx$$

```
[In] Integrate[ArcTan[a*x]^(-3/2),x]
[Out] Integrate[ArcTan[a*x]^(-3/2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.52 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{\arctan\left(ax\right)^{\frac{3}{2}}}dx$$

```
[In] int(1/arctan(a*x)^(3/2),x)
[Out] int(1/arctan(a*x)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/arctan(a*x)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \frac{1}{\arctan(ax)^{3/2}}\,dx = \int \frac{1}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx$$

```
[In] integrate(1/atan(a*x)**(3/2),x)
[Out] Integral(atan(a*x)**(-3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(1/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [N/A]

Not integrable

Time = 239.10 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.38

$$\int \frac{1}{\arctan(ax)^{3/2}}\,dx = \int \frac{1}{\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1}{\arctan(ax)^{3/2}}\,dx = \int \frac{1}{{\mathrm{atan}\left(a\,x\right)}^{3/2}}\,dx$$

```
[In] int(1/atan(a*x)^(3/2),x)
```

```
[Out] int(1/atan(a*x)^(3/2), x)
```

3.52 $\int \frac{1}{x \arctan(ax)^{3/2}} dx$

Optimal result . . . 310
Rubi [N/A] . . . 310
Mathematica [N/A] . . . 311
Maple [N/A] (verified) . . . 311
Fricas [F(-2)] . . . 311
Sympy [N/A] . . . 311
Maxima [F(-2)] . . . 312
Giac [N/A] . . . 312
Mupad [N/A] . . . 312

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{1}{x \arctan(ax)^{3/2}} dx = \text{Int}\left(\frac{1}{x \arctan(ax)^{3/2}}, x\right)$$

```
[Out] Unintegrable(1/x/arctan(a*x)^(3/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x \arctan(ax)^{3/2}} dx = \int \frac{1}{x \arctan(ax)^{3/2}} dx$$

```
[In] Int[1/(x*ArcTan[a*x]^(3/2)),x]
[Out] Defer[Int][1/(x*ArcTan[a*x]^(3/2)), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x \arctan(ax)^{3/2}} dx$$

Mathematica [N/A]

Not integrable

Time = 2.34 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{x \arctan(ax)^{3/2}}\,dx = \int \frac{1}{x \arctan(ax)^{3/2}}\,dx$$

```
[In] Integrate[1/(x*ArcTan[a*x]^(3/2)),x]
[Out] Integrate[1/(x*ArcTan[a*x]^(3/2)), x]
```

Maple [N/A] (verified)

Not integrable

Time = 2.64 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{x \arctan\left(ax\right)^{\frac{3}{2}}}dx$$

```
[In] int(1/x/arctan(a*x)^(3/2),x)
[Out] int(1/x/arctan(a*x)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x \arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/x/arctan(a*x)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \arctan(ax)^{3/2}}\,dx = \int \frac{1}{x \operatorname{atan}^{\frac{3}{2}}(ax)}\,dx$$

```
[In] integrate(1/x/atan(a*x)**(3/2),x)
[Out] Integral(1/(x*atan(a*x)**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(1/x/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [N/A]

Not integrable

Time = 184.34 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.25

$$\int \frac{1}{x\arctan(ax)^{3/2}}\,dx = \int \frac{1}{x\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\arctan(ax)^{3/2}}\,dx = \int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)^{3/2}}\,dx$$

```
[In] int(1/(x*atan(a*x)^(3/2)),x)
```

```
[Out] int(1/(x*atan(a*x)^(3/2)), x)
```

3.53 $\int \sqrt{x} \arctan(x) \, dx$

Optimal result 313
Rubi [A] (verified) 313
Mathematica [A] (verified) 316
Maple [A] (verified) 316
Fricas [C] (verification not implemented) 317
Sympy [A] (verification not implemented) 317
Maxima [A] (verification not implemented) 318
Giac [A] (verification not implemented) 318
Mupad [B] (verification not implemented) 319

Optimal result

Integrand size = 8, antiderivative size = 117

$$\int \sqrt{x} \arctan(x) \, dx = -\frac{4\sqrt{x}}{3} - \frac{1}{3}\sqrt{2} \arctan\left(1 - \sqrt{2}\sqrt{x}\right) + \frac{1}{3}\sqrt{2} \arctan\left(1 + \sqrt{2}\sqrt{x}\right) + \frac{2}{3}x^{3/2} \arctan(x) - \frac{\log\left(1 - \sqrt{2}\sqrt{x} + x\right)}{3\sqrt{2}} + \frac{\log\left(1 + \sqrt{2}\sqrt{x} + x\right)}{3\sqrt{2}}$$

```
[Out] 2/3*x^(3/2)*arctan(x)-1/6*ln(1+x-2^(1/2)*x^(1/2))*2^(1/2)+1/6*ln(1+x+2^(1/2
)*x^(1/2))*2^(1/2)+1/3*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)+1/3*arctan(1+2^(1
/2)*x^(1/2))*2^(1/2)-4/3*x^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.125, Rules used = {4946, 327, 335, 217, 1179, 642, 1176, 631, 210}

$$\int \sqrt{x} \arctan(x) \, dx = \frac{2}{3}x^{3/2} \arctan(x) - \frac{1}{3}\sqrt{2} \arctan\left(1 - \sqrt{2}\sqrt{x}\right) + \frac{1}{3}\sqrt{2} \arctan\left(\sqrt{2}\sqrt{x} + 1\right) - \frac{4\sqrt{x}}{3} - \frac{\log\left(x - \sqrt{2}\sqrt{x} + 1\right)}{3\sqrt{2}} + \frac{\log\left(x + \sqrt{2}\sqrt{x} + 1\right)}{3\sqrt{2}}$$

```
[In] Int[Sqrt[x]*ArcTan[x],x]
```

```
[Out] (-4*Sqrt[x])/3 - (Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[x]])/3 + (Sqrt[2]*ArcTan[
1 + Sqrt[2]*Sqrt[x]])/3 + (2*x^(3/2)*ArcTan[x])/3 - Log[1 - Sqrt[2]*Sqrt[x]
 + x]/(3*Sqrt[2]) + Log[1 + Sqrt[2]*Sqrt[x] + x]/(3*Sqrt[2])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2}{3}x^{3/2}\arctan(x) - \frac{2}{3}\int \frac{x^{3/2}}{1+x^2}\,dx \\
&= -\frac{4\sqrt{x}}{3} + \frac{2}{3}x^{3/2}\arctan(x) + \frac{2}{3}\int \frac{1}{\sqrt{x}\,(1+x^2)}\,dx \\
&= -\frac{4\sqrt{x}}{3} + \frac{2}{3}x^{3/2}\arctan(x) + \frac{4}{3}\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \sqrt{x}\right) \\
&= -\frac{4\sqrt{x}}{3} + \frac{2}{3}x^{3/2}\arctan(x) + \frac{2}{3}\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{x}\right) + \frac{2}{3}\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{x}\right) \\
&= -\frac{4\sqrt{x}}{3} + \frac{2}{3}x^{3/2}\arctan(x) + \frac{1}{3}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \sqrt{x}\right) \\
&\quad + \frac{1}{3}\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \sqrt{x}\right) - \frac{\text{Subst}\left(\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx, x, \sqrt{x}\right)}{3\sqrt{2}} \\
&\quad - \frac{\text{Subst}\left(\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx, x, \sqrt{x}\right)}{3\sqrt{2}} \\
&= -\frac{4\sqrt{x}}{3} + \frac{2}{3}x^{3/2}\arctan(x) - \frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{3\sqrt{2}} + \frac{\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{3\sqrt{2}} \\
&\quad + \frac{1}{3}\sqrt{2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}\sqrt{x}\right) \\
&\quad - \frac{1}{3}\sqrt{2}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\sqrt{2}\sqrt{x}\right) \\
&= -\frac{4\sqrt{x}}{3} - \frac{1}{3}\sqrt{2}\arctan\left(1-\sqrt{2}\sqrt{x}\right) + \frac{1}{3}\sqrt{2}\arctan\left(1+\sqrt{2}\sqrt{x}\right) \\
&\quad + \frac{2}{3}x^{3/2}\arctan(x) - \frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{3\sqrt{2}} + \frac{\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{3\sqrt{2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.92

$$\int \sqrt{x}\arctan(x)\,dx = \frac{1}{6}\Big(-8\sqrt{x}-2\sqrt{2}\arctan\left(1-\sqrt{2}\sqrt{x}\right)+2\sqrt{2}\arctan\left(1+\sqrt{2}\sqrt{x}\right)+4x^{3/2}\arctan(x)-\sqrt{2}\log\left(1-\sqrt{2}\sqrt{x}+x\right)+\sqrt{2}\log\left(1+\sqrt{2}\sqrt{x}+x\right)\Big)$$

```
[In] Integrate[Sqrt[x]*ArcTan[x],x]
```

```
[Out] (-8*Sqrt[x] - 2*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[x]] + 2*Sqrt[2]*ArcTan[1 +
Sqrt[2]*Sqrt[x]] + 4*x^(3/2)*ArcTan[x] - Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[x] +
x] + Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[x] + x])/6
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.59

method	result
derivativedivides	$\frac{2x^{\frac{3}{2}}\arctan(x)}{3}-\frac{4\sqrt{x}}{3}+\frac{\sqrt{2}\left(\ln\left(\frac{1+x+\sqrt{2}\sqrt{x}}{1+x-\sqrt{2}\sqrt{x}}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{6}$
default	$\frac{2x^{\frac{3}{2}}\arctan(x)}{3}-\frac{4\sqrt{x}}{3}+\frac{\sqrt{2}\left(\ln\left(\frac{1+x+\sqrt{2}\sqrt{x}}{1+x-\sqrt{2}\sqrt{x}}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{6}$
meijerg	$-\frac{4\sqrt{x}}{3}+\frac{\sqrt{x}\left(-\frac{\sqrt{2}\ln\left(1-\sqrt{2}\left(x^{2}\right)^{\frac{1}{4}}+\sqrt{x^{2}}\right)}{2\left(x^{2}\right)^{\frac{1}{4}}}+\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(x^{2}\right)^{\frac{1}{4}}}{2-\sqrt{2}\left(x^{2}\right)^{\frac{1}{4}}}\right)}{\left(x^{2}\right)^{\frac{1}{4}}}+\frac{\sqrt{2}\ln\left(1+\sqrt{2}\left(x^{2}\right)^{\frac{1}{4}}+\sqrt{x^{2}}\right)}{2\left(x^{2}\right)^{\frac{1}{4}}}+\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\right.}{2+\sqrt{2}}\right.}{\left(x^{2}\right)^{\frac{1}{4}}}\right.}{3}$

```
[In] int(arctan(x)*x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*x^(3/2)*arctan(x)-4/3*x^(1/2)+1/6*2^(1/2)*(ln((1+x+2^(1/2)*x^(1/2))/(1+
x-2^(1/2)*x^(1/2)))+2*arctan(1+2^(1/2)*x^(1/2))+2*arctan(-1+2^(1/2)*x^(1/2)
))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.68

$$\int \sqrt{x}\arctan(x)\,dx = \frac{2}{3}\left(x\arctan\left(x\right)-2\right)\sqrt{x}+\left(\frac{1}{6}i+\frac{1}{6}\right)\sqrt{2}\log\left(\left(i+1\right)\sqrt{2}+2\sqrt{x}\right) - \left(\frac{1}{6}i-\frac{1}{6}\right)\sqrt{2}\log\left(-\left(i-1\right)\sqrt{2}+2\sqrt{x}\right) + \left(\frac{1}{6}i-\frac{1}{6}\right)\sqrt{2}\log\left(\left(i-1\right)\sqrt{2}+2\sqrt{x}\right) - \left(\frac{1}{6}i+\frac{1}{6}\right)\sqrt{2}\log\left(-\left(i+1\right)\sqrt{2}+2\sqrt{x}\right)$$

```
[In] integrate(arctan(x)*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(x*arctan(x) - 2)*sqrt(x) + (1/6*I + 1/6)*sqrt(2)*log((I + 1)*sqrt(2) +
 2*sqrt(x)) - (1/6*I - 1/6)*sqrt(2)*log(-(I - 1)*sqrt(2) + 2*sqrt(x)) + (1/
6*I - 1/6)*sqrt(2)*log((I - 1)*sqrt(2) + 2*sqrt(x)) - (1/6*I + 1/6)*sqrt(2)
*log(-(I + 1)*sqrt(2) + 2*sqrt(x))
```

Sympy [A] (verification not implemented)

Time = 1.95 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.94

$$\int \sqrt{x}\arctan(x)\,dx = \frac{2x^{\frac{3}{2}}\operatorname{atan}\left(x\right)}{3}-\frac{4\sqrt{x}}{3}-\frac{\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{6} + \frac{\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{6} + \frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{3}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{3}$$

```
[In] integrate(atan(x)*x**(1/2),x)
```

```
[Out] 2*x**(3/2)*atan(x)/3 - 4*sqrt(x)/3 - sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x +
 4)/6 + sqrt(2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/6 + sqrt(2)*atan(sqrt(2)*s
qrt(x) - 1)/3 + sqrt(2)*atan(sqrt(2)*sqrt(x) + 1)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.74

$$\int \sqrt{x}\arctan(x)\,dx = \frac{2}{3}\,x^{\frac{3}{2}}\arctan\left(x\right) + \frac{1}{3}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2} + 2\,\sqrt{x}\right)\right) + \frac{1}{3}\,\sqrt{2}\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2} - 2\,\sqrt{x}\right)\right) + \frac{1}{6}\,\sqrt{2}\log\left(\sqrt{2}\sqrt{x} + x + 1\right) - \frac{1}{6}\,\sqrt{2}\log\left(-\sqrt{2}\sqrt{x} + x + 1\right) - \frac{4}{3}\,\sqrt{x}$$

```
[In] integrate(arctan(x)*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*x^(3/2)*arctan(x) + 1/3*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x)
)) + 1/3*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 1/6*sqrt(2)*l
og(sqrt(2)*sqrt(x) + x + 1) - 1/6*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 4
/3*sqrt(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.74

$$\int \sqrt{x}\arctan(x)\,dx = \frac{2}{3}\,x^{\frac{3}{2}}\arctan\left(x\right) + \frac{1}{3}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2} + 2\,\sqrt{x}\right)\right) + \frac{1}{3}\,\sqrt{2}\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2} - 2\,\sqrt{x}\right)\right) + \frac{1}{6}\,\sqrt{2}\log\left(\sqrt{2}\sqrt{x} + x + 1\right) - \frac{1}{6}\,\sqrt{2}\log\left(-\sqrt{2}\sqrt{x} + x + 1\right) - \frac{4}{3}\,\sqrt{x}$$

```
[In] integrate(arctan(x)*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*x^(3/2)*arctan(x) + 1/3*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x)
)) + 1/3*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 1/6*sqrt(2)*l
og(sqrt(2)*sqrt(x) + x + 1) - 1/6*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 4
/3*sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.42

$$\int \sqrt{x}\arctan(x)\,dx = \frac{2\,x^{3/2}\,\mathrm{atan}(x)}{3} - \frac{4\,\sqrt{x}}{3} + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{1}{3}+\frac{1}{3}\mathrm{i}\right) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{1}{3}-\frac{1}{3}\mathrm{i}\right)$$

```
[In] int(x^(1/2)*atan(x),x)
```

```
[Out] (2*x^(3/2)*atan(x))/3 + 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(1/3 + 1
i/3) + 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 + 1i/2))*(1/3 - 1i/3) - (4*x^(1/2)
)/3
```

3.54 $\int (dx)^m (a + b \arctan(cx))^3 \, dx$

Optimal result 320
Rubi [N/A] 320
Mathematica [N/A] 321
Maple [N/A] (verified) 321
Fricas [N/A] 321
Sympy [N/A] 321
Maxima [N/A] 322
Giac [N/A] 322
Mupad [N/A] 322

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int (dx)^m (a + b \arctan(cx))^3 \, dx = \operatorname{Int}\left((dx)^m (a + b \arctan(cx))^3, x\right)$$

```
[Out] Unintegrable((d*x)^m*(a+b*arctan(c*x))^3,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (dx)^m (a + b \arctan(cx))^3 \, dx = \int (dx)^m (a + b \arctan(cx))^3 \, dx$$

```
[In] Int[(d*x)^m*(a + b*ArcTan[c*x])^3,x]
```

```
[Out] Defer[Int][(d*x)^m*(a + b*ArcTan[c*x])^3, x]
```

Rubi steps

$$\text{integral} = \int (dx)^m (a + b \arctan(cx))^3 \, dx$$

Mathematica [N/A]

Not integrable

Time = 4.83 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m(a+b\arctan(cx))^3\,dx=\int (dx)^m(a+b\arctan(cx))^3\,dx$$

```
[In] Integrate[(d*x)^m*(a + b*ArcTan[c*x])^3,x]
[Out] Integrate[(d*x)^m*(a + b*ArcTan[c*x])^3, x]
```

Maple [N/A] (verified)

Not integrable

Time = 2.56 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (dx)^m\left(a+b\arctan\left(cx\right)\right)^3dx$$

```
[In] int((d*x)^m*(a+b*arctan(c*x))^3,x)
[Out] int((d*x)^m*(a+b*arctan(c*x))^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.75

$$\int (dx)^m(a+b\arctan(cx))^3\,dx=\int (b\arctan\left(cx\right)+a)^3(dx)^m\,\,dx$$

```
[In] integrate((d*x)^m*(a+b*arctan(c*x))^3,x, algorithm="fricas")
[Out] integral((b^3*arctan(c*x)^3 + 3*a*b^2*arctan(c*x)^2 + 3*a^2*b*arctan(c*x) +
 a^3)*(d*x)^m, x)
```

Sympy [N/A]

Not integrable

Time = 7.45 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int (dx)^m(a+b\arctan(cx))^3\,dx=\int (dx)^m\left(a+b\,\mathrm{atan}\left(cx\right)\right)^3\,dx$$

```
[In] integrate((d*x)**m*(a+b*atan(c*x))**3,x)
[Out] Integral((d*x)**m*(a + b*atan(c*x))**3, x)
```

Maxima [N/A]

Not integrable

Time = 3.74 (sec) , antiderivative size = 387, normalized size of antiderivative = 24.19

$$\int (dx)^m(a + b\arctan(cx))^3\,dx = \int (b\arctan(cx) + a)^3(dx)^m\,dx$$

```
[In] integrate((d*x)^m*(a+b*arctan(c*x))^3,x, algorithm="maxima")
```

```
[Out] (d*x)^(m + 1)*a^3/(d*(m + 1)) + 1/32*(4*b^3*d^m*x*x^m*arctan(c*x)^3 - 3*b^3
*d^m*x*x^m*arctan(c*x)*log(c^2*x^2 + 1)^2 + 32*(m + 1)*integrate(1/32*(12*b
^3*c^2*d^m*x^2*x^m*arctan(c*x)*log(c^2*x^2 + 1) + 28*(b^3*d^m*m + b^3*d^m +
 (b^3*c^2*d^m*m + b^3*c^2*d^m)*x^2)*x^m*arctan(c*x)^3 - 12*(b^3*c*d^m*x - 8
*a*b^2*d^m*m - 8*a*b^2*d^m - 8*(a*b^2*c^2*d^m*m + a*b^2*c^2*d^m)*x^2)*x^m*a
rctan(c*x)^2 + 96*(a^2*b*d^m*m + a^2*b*d^m + (a^2*b*c^2*d^m*m + a^2*b*c^2*d
^m)*x^2)*x^m*arctan(c*x) + 3*(b^3*c*d^m*x*x^m + (b^3*d^m*m + b^3*d^m + (b^3
*c^2*d^m*m + b^3*c^2*d^m)*x^2)*x^m*arctan(c*x))*log(c^2*x^2 + 1)^2)/((c^2*m
 + c^2)*x^2 + m + 1), x))/(m + 1)
```

Giac [N/A]

Not integrable

Time = 72.75 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.19

$$\int (dx)^m(a + b\arctan(cx))^3\,dx = \int (b\arctan(cx) + a)^3(dx)^m\,dx$$

```
[In] integrate((d*x)^m*(a+b*arctan(c*x))^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m(a + b\arctan(cx))^3\,dx = \int {\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}^3\,{\left(d\,x\right)}^m \,dx$$

```
[In] int((a + b*atan(c*x))^3*(d*x)^m,x)
```

```
[Out] int((a + b*atan(c*x))^3*(d*x)^m, x)
```

3.55 $\int (dx)^m (a+b\arctan(cx))^2\,dx$

Optimal result . . . 323
Rubi [N/A] . . . 323
Mathematica [N/A] . . . 324
Maple [N/A] (verified) . . . 324
Fricas [N/A] . . . 324
Sympy [N/A] . . . 324
Maxima [N/A] . . . 325
Giac [N/A] . . . 325
Mupad [N/A] . . . 325

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int (dx)^m (a+b\arctan(cx))^2\,dx = \mathrm{Int}\big((dx)^m (a+b\arctan(cx))^2,x\big)$$

```
[Out] Unintegrable((d*x)^m*(a+b*arctan(c*x))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (dx)^m (a+b\arctan(cx))^2\,dx = \int (dx)^m (a+b\arctan(cx))^2\,dx$$

```
[In] Int[(d*x)^m*(a + b*ArcTan[c*x])^2,x]
[Out] Defer[Int][(d*x)^m*(a + b*ArcTan[c*x])^2, x]
```

Rubi steps

$$\text{integral} = \int (dx)^m (a+b\arctan(cx))^2\,dx$$

Mathematica [N/A]

Not integrable

Time = 3.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m(a+b\arctan(cx))^2\,dx=\int (dx)^m(a+b\arctan(cx))^2\,dx$$

```
[In] Integrate[(d*x)^m*(a + b*ArcTan[c*x])^2,x]
[Out] Integrate[(d*x)^m*(a + b*ArcTan[c*x])^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 3.82 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (dx)^m\left(a+b\arctan\left(cx\right)\right)^2dx$$

```
[In] int((d*x)^m*(a+b*arctan(c*x))^2,x)
[Out] int((d*x)^m*(a+b*arctan(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.88

$$\int (dx)^m(a+b\arctan(cx))^2\,dx=\int \left(b\arctan\left(cx\right)+a\right)^2(dx)^m\,dx$$

```
[In] integrate((d*x)^m*(a+b*arctan(c*x))^2,x, algorithm="fricas")
[Out] integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)*(d*x)^m, x)
```

Sympy [N/A]

Not integrable

Time = 4.35 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int (dx)^m(a+b\arctan(cx))^2\,dx=\int (dx)^m\left(a+b\,\mathrm{atan}\left(cx\right)\right)^2\,dx$$

```
[In] integrate((d*x)**m*(a+b*atan(c*x))**2,x)
[Out] Integral((d*x)**m*(a + b*atan(c*x))**2, x)
```

Maxima [N/A]

Not integrable

Time = 2.20 (sec) , antiderivative size = 295, normalized size of antiderivative = 18.44

$$\int (dx)^m(a+b\arctan(cx))^2\,dx = \int (b\arctan{(cx)}+a)^2(dx)^m\;dx$$

```
[In] integrate((d*x)^m*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

```
[Out] (d*x)^(m + 1)*a^2/(d*(m + 1)) + 1/16*(4*b^2*d^m*x*x^m*arctan(c*x)^2 - b^2*d
^m*x*x^m*log(c^2*x^2 + 1)^2 + 16*(m + 1)*integrate(1/16*(4*b^2*c^2*d^m*x^2*
x^m*log(c^2*x^2 + 1) + 12*(b^2*d^m*m + b^2*d^m + (b^2*c^2*d^m*m + b^2*c^2*d
^m)*x^2)*x^m*arctan(c*x)^2 + (b^2*d^m*m + b^2*d^m + (b^2*c^2*d^m*m + b^2*c^
2*d^m)*x^2)*x^m*log(c^2*x^2 + 1)^2 - 8*(b^2*c*d^m*x - 4*a*b*d^m*m - 4*a*b*d
^m - 4*(a*b*c^2*d^m*m + a*b*c^2*d^m)*x^2)*x^m*arctan(c*x))/((c^2*m + c^2)*x
^2 + m + 1), x))/(m + 1)
```

Giac [N/A]

Not integrable

Time = 72.16 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.19

$$\int (dx)^m(a+b\arctan(cx))^2\,dx = \int (b\arctan{(cx)}+a)^2(dx)^m\;dx$$

```
[In] integrate((d*x)^m*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m(a+b\arctan(cx))^2\,dx = \int {\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}^2\,{\left(d\,x\right)}^m \,d x$$

```
[In] int((a + b*atan(c*x))^2*(d*x)^m,x)
```

```
[Out] int((a + b*atan(c*x))^2*(d*x)^m, x)
```

3.56 $\int (dx)^m (a + b\arctan(cx))\, dx$

Optimal result 326
Rubi [A] (verified) 326
Mathematica [A] (verified) 327
Maple [F] 327
Fricas [F] 328
Sympy [F] 328
Maxima [F] 328
Giac [F] 328
Mupad [F(-1)] 329

Optimal result

Integrand size = 14, antiderivative size = 73

$$\int (dx)^m (a + b\arctan(cx))\, dx = \frac{(dx)^{1+m}(a + b\arctan(cx))}{d(1+m)} - \frac{bc(dx)^{2+m}\,\text{Hypergeometric2F1}\left(1, \frac{2+m}{2}, \frac{4+m}{2}, -c^2x^2\right)}{d^2(1+m)(2+m)}$$

```
[Out] (d*x)^(1+m)*(a+b*arctan(c*x))/d/(1+m)-b*c*(d*x)^(2+m)*hypergeom([1, 1+1/2*m
],[2+1/2*m],-c^2*x^2)/d^2/(1+m)/(2+m)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4958, 371}

$$\int (dx)^m (a + b\arctan(cx))\, dx = \frac{(dx)^{m+1}(a + b\arctan(cx))}{d(m+1)} - \frac{bc(dx)^{m+2}\,\text{Hypergeometric2F1}\left(1, \frac{m+2}{2}, \frac{m+4}{2}, -c^2x^2\right)}{d^2(m+1)(m+2)}$$

```
[In] Int[(d*x)^m*(a + b*ArcTan[c*x]),x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*ArcTan[c*x]))/(d*(1 + m)) - (b*c*(d*x)^(2 + m)*Hyperg
eometric2F1[1, (2 + m)/2, (4 + m)/2, -(c^2*x^2)])/(d^2*(1 + m)*(2 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4958

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))*((d_)*(x_))^(m_), x_Symbol] :>
 Simp[(d*x)^(m + 1)*((a + b*ArcTan[c*x^n])/(d*(m + 1))), x] - Dist[b*c*(n/(
d^n*(m + 1))), Int[(d*x)^(m + n)/(1 + c^2*x^(2*n)), x], x] /; FreeQ[{a, b,
c, d, m, n}, x] && IntegerQ[n] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(dx)^{1+m}(a+b\arctan(cx))}{d(1+m)} - \frac{(bc)\int \frac{(dx)^{1+m}}{1+c^2x^2}\,dx}{d(1+m)}\\ &= \frac{(dx)^{1+m}(a+b\arctan(cx))}{d(1+m)} - \frac{bc(dx)^{2+m}\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},-c^2x^2\right)}{d^2(1+m)(2+m)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.82

$$\int (dx)^m(a+b\arctan(cx))\,dx = -\frac{x(dx)^m\left(-((2+m)(a+b\arctan(cx)))+bcx\,\text{Hypergeometric2F1}\left(1,1+\frac{m}{2},2+\frac{m}{2},-c^2x^2\right)\right)}{(1+m)(2+m)}$$

```
[In] Integrate[(d*x)^m*(a + b*ArcTan[c*x]),x]
[Out] -((x*(d*x)^m*(-((2 + m)*(a + b*ArcTan[c*x])) + b*c*x*Hypergeometric2F1[1, 1
 + m/2, 2 + m/2, -(c^2*x^2)]))/((1 + m)*(2 + m)))
```

Maple [F]

$$\int (dx)^m\,(a+b\arctan(cx))\,dx$$

```
[In] int((d*x)^m*(a+b*arctan(c*x)),x)
[Out] int((d*x)^m*(a+b*arctan(c*x)),x)
```

Fricas [F]

$$\int (dx)^m(a+b\arctan(cx))\,dx = \int (b\arctan(cx)+a)(dx)^m\,dx$$

```
[In] integrate((d*x)^m*(a+b*arctan(c*x)),x, algorithm="fricas")
[Out] integral((b*arctan(c*x) + a)*(d*x)^m, x)
```

Sympy [F]

$$\int (dx)^m(a+b\arctan(cx))\,dx = \int (dx)^m\,(a+b\,\mathrm{atan}\,(cx))\,dx$$

```
[In] integrate((d*x)**m*(a+b*atan(c*x)),x)
[Out] Integral((d*x)**m*(a + b*atan(c*x)), x)
```

Maxima [F]

$$\int (dx)^m(a+b\arctan(cx))\,dx = \int (b\arctan(cx)+a)(dx)^m\,dx$$

```
[In] integrate((d*x)^m*(a+b*arctan(c*x)),x, algorithm="maxima")
[Out] (d^m*x*x^m*arctan(c*x) - (c*d^m*m + c*d^m)*integrate(x*x^m/((c^2*m + c^2)*x
^2 + m + 1), x))*b/(m + 1) + (d*x)^(m + 1)*a/(d*(m + 1))
```

Giac [F]

$$\int (dx)^m(a+b\arctan(cx))\,dx = \int (b\arctan(cx)+a)(dx)^m\,dx$$

```
[In] integrate((d*x)^m*(a+b*arctan(c*x)),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m(a+b\arctan(cx))\,dx=\int \left(a+b\,\mathrm{atan}(c\,x)\right)\,(d\,x)^m\,dx$$

```
[In] int((a + b*atan(c*x))*(d*x)^m,x)
[Out] int((a + b*atan(c*x))*(d*x)^m, x)
```

3.57 $\int \frac{(dx)^m}{a+b\arctan(cx)}\,dx$

Optimal result . . . 330
Rubi [N/A] . . . 330
Mathematica [N/A] . . . 331
Maple [N/A] (verified) . . . 331
Fricas [N/A] . . . 331
Sympy [N/A] . . . 331
Maxima [N/A] . . . 332
Giac [N/A] . . . 332
Mupad [N/A] . . . 332

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{(dx)^m}{a+b\arctan(cx)}\,dx = \text{Int}\left(\frac{(dx)^m}{a+b\arctan(cx)},x\right)$$

```
[Out] Unintegrable((d*x)^m/(a+b*arctan(c*x)),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(dx)^m}{a+b\arctan(cx)}\,dx = \int \frac{(dx)^m}{a+b\arctan(cx)}\,dx$$

```
[In] Int[(d*x)^m/(a + b*ArcTan[c*x]),x]
[Out] Defer[Int][(d*x)^m/(a + b*ArcTan[c*x]), x]
```

Rubi steps

$$\text{integral} = \int \frac{(dx)^m}{a+b\arctan(cx)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{a+b\arctan(cx)}\,dx=\int \frac{(dx)^m}{a+b\arctan(cx)}\,dx$$

```
[In] Integrate[(d*x)^m/(a + b*ArcTan[c*x]),x]
[Out] Integrate[(d*x)^m/(a + b*ArcTan[c*x]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 3.53 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^m}{a+b\arctan(cx)}dx$$

```
[In] int((d*x)^m/(a+b*arctan(c*x)),x)
[Out] int((d*x)^m/(a+b*arctan(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{a+b\arctan(cx)}\,dx=\int \frac{(dx)^m}{b\arctan(cx)+a}\,dx$$

```
[In] integrate((d*x)^m/(a+b*arctan(c*x)),x, algorithm="fricas")
[Out] integral((d*x)^m/(b*arctan(c*x) + a), x)
```

Sympy [N/A]

Not integrable

Time = 1.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{(dx)^m}{a+b\arctan(cx)}\,dx=\int \frac{(dx)^m}{a+b\operatorname{atan}(cx)}\,dx$$

```
[In] integrate((d*x)**m/(a+b*atan(c*x)),x)
[Out] Integral((d*x)**m/(a + b*atan(c*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{a + b\arctan(cx)}\,dx = \int \frac{(dx)^m}{b\arctan(cx) + a}\,dx$$

```
[In] integrate((d*x)^m/(a+b*arctan(c*x)),x, algorithm="maxima")
[Out] integrate((d*x)^m/(b*arctan(c*x) + a), x)
```

Giac [N/A]

Not integrable

Time = 87.38 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.19

$$\int \frac{(dx)^m}{a + b\arctan(cx)}\,dx = \int \frac{(dx)^m}{b\arctan(cx) + a}\,dx$$

```
[In] integrate((d*x)^m/(a+b*arctan(c*x)),x, algorithm="giac")
[Out] sage0*x
```

Mupad [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{a + b\arctan(cx)}\,dx = \int \frac{(d\,x)^m}{a + b\,\mathrm{atan}(c\,x)}\,dx$$

```
[In] int((d*x)^m/(a + b*atan(c*x)),x)
[Out] int((d*x)^m/(a + b*atan(c*x)), x)
```

3.58 $\int (a+b\arctan(cx))^p\,dx$

Optimal result 333
Rubi [N/A] 333
Mathematica [N/A] 334
Maple [N/A] (verified) 334
Fricas [N/A] 334
Sympy [N/A] 334
Maxima [N/A] 335
Giac [N/A] 335
Mupad [N/A] 335

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int (a+b\arctan(cx))^p\,dx=\mathrm{Int}((a+b\arctan(cx))^p,x)$$

```
[Out] Unintegrable((a+b*arctan(c*x))^p,x)
```

Rubi [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (a+b\arctan(cx))^p\,dx=\int (a+b\arctan(cx))^p\,dx$$

```
[In] Int[(a + b*ArcTan[c*x])^p,x]
```

```
[Out] Defer[Int][(a + b*ArcTan[c*x])^p, x]
```

Rubi steps

$$\text{integral}=\int (a+b\arctan(cx))^p\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int (a+b\arctan(cx))^p\,dx=\int (a+b\arctan(cx))^p\,dx$$

```
[In] Integrate[(a + b*ArcTan[c*x])^p,x]
[Out] Integrate[(a + b*ArcTan[c*x])^p, x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.69 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \left(a+b\arctan\left(cx\right)\right)^{p}dx$$

```
[In] int((a+b*arctan(c*x))^p,x)
[Out] int((a+b*arctan(c*x))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int (a+b\arctan(cx))^p\,dx=\int \left(b\arctan\left(cx\right)+a\right)^{p}dx$$

```
[In] integrate((a+b*arctan(c*x))^p,x, algorithm="fricas")
[Out] integral((b*arctan(c*x) + a)^p, x)
```

Sympy [N/A]

Not integrable

Time = 1.81 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int (a+b\arctan(cx))^p\,dx=\int \left(a+b\operatorname{atan}\left(cx\right)\right)^{p}\,dx$$

```
[In] integrate((a+b*atan(c*x))**p,x)
[Out] Integral((a + b*atan(c*x))**p, x)
```

Maxima [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int (a+b\arctan(cx))^p\,dx=\int (b\arctan(cx)+a)^p\,dx$$

```
[In] integrate((a+b*arctan(c*x))^p,x, algorithm="maxima")
[Out] integrate((b*arctan(c*x) + a)^p, x)
```

Giac [N/A]

Not integrable

Time = 80.21 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.30

$$\int (a+b\arctan(cx))^p\,dx=\int (b\arctan(cx)+a)^p\,dx$$

```
[In] integrate((a+b*arctan(c*x))^p,x, algorithm="giac")
[Out] sage0*x
```

Mupad [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int (a+b\arctan(cx))^p\,dx=\int (a+b\,\mathrm{atan}(c\,x))^p\,dx$$

```
[In] int((a + b*atan(c*x))^p,x)
[Out] int((a + b*atan(c*x))^p, x)
```

3.59 $\int (dx)^m(a+b\arctan(cx))^p\,dx$

Optimal result 336
Rubi [N/A] 336
Mathematica [N/A] 337
Maple [N/A] (verified) 337
Fricas [N/A] 337
Sympy [N/A] 337
Maxima [N/A] 338
Giac [N/A] 338
Mupad [N/A] 338

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int (dx)^m(a+b\arctan(cx))^p\,dx=\mathrm{Int}\left((dx)^m(a+b\arctan(cx))^p,x\right)$$

```
[Out] Unintegrable((d*x)^m*(a+b*arctan(c*x))^p,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (dx)^m(a+b\arctan(cx))^p\,dx=\int (dx)^m(a+b\arctan(cx))^p\,dx$$

```
[In] Int[(d*x)^m*(a + b*ArcTan[c*x])^p,x]
```

```
[Out] Defer[Int][(d*x)^m*(a + b*ArcTan[c*x])^p, x]
```

Rubi steps

$$\text{integral}=\int (dx)^m(a+b\arctan(cx))^p\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m(a+b\arctan(cx))^p\,dx=\int (dx)^m(a+b\arctan(cx))^p\,dx$$

```
[In] Integrate[(d*x)^m*(a + b*ArcTan[c*x])^p,x]
[Out] Integrate[(d*x)^m*(a + b*ArcTan[c*x])^p, x]
```

Maple [N/A] (verified)

Not integrable

Time = 4.36 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (dx)^m\left(a+b\arctan\left(cx\right)\right)^p dx$$

```
[In] int((d*x)^m*(a+b*arctan(c*x))^p,x)
[Out] int((d*x)^m*(a+b*arctan(c*x))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m(a+b\arctan(cx))^p\,dx=\int (dx)^m\left(b\arctan\left(cx\right)+a\right)^p dx$$

```
[In] integrate((d*x)^m*(a+b*arctan(c*x))^p,x, algorithm="fricas")
[Out] integral((d*x)^m*(b*arctan(c*x) + a)^p, x)
```

Sympy [N/A]

Not integrable

Time = 155.45 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int (dx)^m(a+b\arctan(cx))^p\,dx=\int (dx)^m\left(a+b\operatorname{atan}\left(cx\right)\right)^p\,dx$$

```
[In] integrate((d*x)**m*(a+b*atan(c*x))**p,x)
[Out] Integral((d*x)**m*(a + b*atan(c*x))**p, x)
```

Maxima [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m(a+b\arctan(cx))^p\,dx=\int (dx)^m\,(b\arctan(cx)+a)^p\,dx$$

```
[In] integrate((d*x)^m*(a+b*arctan(c*x))^p,x, algorithm="maxima")
[Out] integrate((d*x)^m*(b*arctan(c*x) + a)^p, x)
```

Giac [N/A]

Not integrable

Time = 79.27 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.19

$$\int (dx)^m(a+b\arctan(cx))^p\,dx=\int (dx)^m\,(b\arctan(cx)+a)^p\,dx$$

```
[In] integrate((d*x)^m*(a+b*arctan(c*x))^p,x, algorithm="giac")
[Out] sage0*x
```

Mupad [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m(a+b\arctan(cx))^p\,dx=\int (a+b\,\mathrm{atan}(c\,x))^p\,(d\,x)^m\,dx$$

```
[In] int((a + b*atan(c*x))^p*(d*x)^m,x)
[Out] int((a + b*atan(c*x))^p*(d*x)^m, x)
```

3.60 $\int x^7(a+b\arctan(cx^2))\,dx$

Optimal result 339
Rubi [A] (verified) 339
Mathematica [A] (verified) 340
Maple [A] (verified) 341
Fricas [A] (verification not implemented) 341
Sympy [A] (verification not implemented) 341
Maxima [A] (verification not implemented) 342
Giac [A] (verification not implemented) 342
Mupad [B] (verification not implemented) 342

Optimal result

Integrand size = 14, antiderivative size = 54

$$\int x^7(a+b\arctan(cx^2))\,dx=\frac{bx^2}{8c^3}-\frac{bx^6}{24c}-\frac{b\arctan(cx^2)}{8c^4}+\frac{1}{8}x^8(a+b\arctan(cx^2))$$

```
[Out] 1/8*b*x^2/c^3-1/24*b*x^6/c-1/8*b*arctan(c*x^2)/c^4+1/8*x^8*(a+b*arctan(c*x^
2))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4946, 281, 308, 209}

$$\int x^7(a+b\arctan(cx^2))\,dx=\frac{1}{8}x^8(a+b\arctan(cx^2))-\frac{b\arctan(cx^2)}{8c^4}+\frac{bx^2}{8c^3}-\frac{bx^6}{24c}$$

```
[In] Int[x^7*(a + b*ArcTan[c*x^2]),x]
```

```
[Out] (b*x^2)/(8*c^3) - (b*x^6)/(24*c) - (b*ArcTan[c*x^2])/(8*c^4) + (x^8*(a + b*
ArcTan[c*x^2]))/8
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{8}x^8\left(a+b\arctan\left(cx^2\right)\right)-\frac{1}{4}(bc)\int\frac{x^9}{1+c^2x^4}\,dx\\
&= \frac{1}{8}x^8\left(a+b\arctan\left(cx^2\right)\right)-\frac{1}{8}(bc)\text{Subst}\left(\int\frac{x^4}{1+c^2x^2}\,dx,x,x^2\right)\\
&= \frac{1}{8}x^8\left(a+b\arctan\left(cx^2\right)\right)-\frac{1}{8}(bc)\text{Subst}\left(\int\left(-\frac{1}{c^4}+\frac{x^2}{c^2}+\frac{1}{c^4\left(1+c^2x^2\right)}\right)\,dx,x,x^2\right)\\
&= \frac{bx^2}{8c^3}-\frac{bx^6}{24c}+\frac{1}{8}x^8\left(a+b\arctan\left(cx^2\right)\right)-\frac{b\text{Subst}\left(\int\frac{1}{1+c^2x^2}\,dx,x,x^2\right)}{8c^3}\\
&= \frac{bx^2}{8c^3}-\frac{bx^6}{24c}-\frac{b\arctan\left(cx^2\right)}{8c^4}+\frac{1}{8}x^8\left(a+b\arctan\left(cx^2\right)\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.09

$$\int x^7\left(a+b\arctan\left(cx^2\right)\right)\,dx=\frac{bx^2}{8c^3}-\frac{bx^6}{24c}+\frac{ax^8}{8}-\frac{b\arctan\left(cx^2\right)}{8c^4}+\frac{1}{8}bx^8\arctan\left(cx^2\right)$$

```
[In] Integrate[x^7*(a + b*ArcTan[c*x^2]),x]
[Out] (b*x^2)/(8*c^3) - (b*x^6)/(24*c) + (a*x^8)/8 - (b*ArcTan[c*x^2])/(8*c^4) +
(b*x^8*ArcTan[c*x^2])/8
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{a\,x^{8}}{8}+\frac{b\,x^{8}\arctan\left(c\,x^{2}\right)}{8}-\frac{b\,x^{6}}{24c}+\frac{b\,x^{2}}{8c^{3}}-\frac{b\arctan\left(c\,x^{2}\right)}{8c^{4}}$	50
parts	$\frac{a\,x^{8}}{8}+\frac{b\,x^{8}\arctan\left(c\,x^{2}\right)}{8}-\frac{b\,x^{6}}{24c}+\frac{b\,x^{2}}{8c^{3}}-\frac{b\arctan\left(c\,x^{2}\right)}{8c^{4}}$	50
parallelrisch	$\frac{3b\arctan\left(c\,x^{2}\right)x^{8}c^{4}+3a\,c^{4}x^{8}-b\,c^{3}x^{6}+3bc\,x^{2}-3b\arctan\left(c\,x^{2}\right)}{24c^{4}}$	56
risch	$-\frac{ix^{8}b\ln\left(ic\,x^{2}+1\right)}{16}+\frac{ix^{8}b\ln\left(-ic\,x^{2}+1\right)}{16}+\frac{a\,x^{8}}{8}-\frac{b\,x^{6}}{24c}+\frac{b\,x^{2}}{8c^{3}}-\frac{b\arctan\left(c\,x^{2}\right)}{8c^{4}}$	72

```
[In] int(x^7*(a+b*arctan(c*x^2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*a*x^8+1/8*b*x^8*arctan(c*x^2)-1/24*b*x^6/c+1/8*b*x^2/c^3-1/8*b*arctan(c
*x^2)/c^4
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

$$\int x^7\left(a+b\arctan\left(cx^2\right)\right)\,dx=\frac{3\,ac^4x^8-bc^3x^6+3\,bcx^2+3\left(bc^4x^8-b\right)\arctan\left(cx^2\right)}{24\,c^4}$$

```
[In] integrate(x^7*(a+b*arctan(c*x^2)),x, algorithm="fricas")
```

```
[Out] 1/24*(3*a*c^4*x^8 - b*c^3*x^6 + 3*b*c*x^2 + 3*(b*c^4*x^8 - b)*arctan(c*x^2)
)/c^4
```

Sympy [A] (verification not implemented)

Time = 29.73 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.07

$$\int x^7\left(a+b\arctan\left(cx^2\right)\right)\,dx=\begin{cases}\frac{ax^8}{8}+\frac{bx^8\operatorname{atan}\left(cx^2\right)}{8}-\frac{bx^6}{24c}+\frac{bx^2}{8c^3}-\frac{b\operatorname{atan}\left(cx^2\right)}{8c^4} & \text{for}\ c\neq 0\\ \frac{ax^8}{8} & \text{otherwise}\end{cases}$$

```
[In] integrate(x**7*(a+b*atan(c*x**2)),x)
```

```
[Out] Piecewise((a*x**8/8 + b*x**8*atan(c*x**2)/8 - b*x**6/(24*c) + b*x**2/(8*c**
3) - b*atan(c*x**2)/(8*c**4), Ne(c, 0)), (a*x**8/8, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int x^7(a+b\arctan(cx^2))\,dx = \frac{1}{8}ax^8 + \frac{1}{24}\left(3x^8\arctan(cx^2) - c\left(\frac{c^2x^6-3x^2}{c^4} + \frac{3\arctan(cx^2)}{c^5}\right)\right)b$$

```
[In] integrate(x^7*(a+b*arctan(c*x^2)),x, algorithm="maxima")
[Out] 1/8*a*x^8 + 1/24*(3*x^8*arctan(c*x^2) - c*((c^2*x^6 - 3*x^2)/c^4 + 3*arctan
(c*x^2)/c^5))*b
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.11

$$\int x^7(a+b\arctan(cx^2))\,dx = \frac{3acx^8 + \left(3cx^8\arctan(cx^2) - \frac{3\arctan(cx^2)}{c^3} - \frac{c^9x^6-3c^7x^2}{c^9}\right)b}{24c}$$

```
[In] integrate(x^7*(a+b*arctan(c*x^2)),x, algorithm="giac")
[Out] 1/24*(3*a*c*x^8 + (3*c*x^8*arctan(c*x^2) - 3*arctan(c*x^2)/c^3 - (c^9*x^6 -
 3*c^7*x^2)/c^9)*b)/c
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

$$\int x^7(a+b\arctan(cx^2))\,dx = \frac{a\,x^8}{8} + \frac{b\,x^2}{8\,c^3} - \frac{b\,x^6}{24\,c} - \frac{b\,\mathrm{atan}(c\,x^2)}{8\,c^4} + \frac{b\,x^8\,\mathrm{atan}(c\,x^2)}{8}$$

```
[In] int(x^7*(a + b*atan(c*x^2)),x)
[Out] (a*x^8)/8 + (b*x^2)/(8*c^3) - (b*x^6)/(24*c) - (b*atan(c*x^2))/(8*c^4) + (b
*x^8*atan(c*x^2))/8
```

3.61 $\int x^5(a+b\arctan(cx^2))\,dx$

Optimal result 343
Rubi [A] (verified) 343
Mathematica [A] (verified) 344
Maple [A] (verified) 344
Fricas [A] (verification not implemented) 345
Sympy [B] (verification not implemented) 345
Maxima [A] (verification not implemented) 346
Giac [A] (verification not implemented) 346
Mupad [B] (verification not implemented) 346

Optimal result

Integrand size = 14, antiderivative size = 47

$$\int x^5\left(a+b\arctan\left(cx^2\right)\right)\,dx=-\frac{bx^4}{12c}+\frac{1}{6}x^6\left(a+b\arctan\left(cx^2\right)\right)+\frac{b\log\left(1+c^2x^4\right)}{12c^3}$$

```
[Out] -1/12*b*x^4/c+1/6*x^6*(a+b*arctan(c*x^2))+1/12*b*ln(c^2*x^4+1)/c^3
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4946, 272, 45}

$$\int x^5\left(a+b\arctan\left(cx^2\right)\right)\,dx=\frac{1}{6}x^6\left(a+b\arctan\left(cx^2\right)\right)+\frac{b\log\left(c^2x^4+1\right)}{12c^3}-\frac{bx^4}{12c}$$

```
[In] Int[x^5*(a + b*ArcTan[c*x^2]),x]
```

```
[Out] -1/12*(b*x^4)/c + (x^6*(a + b*ArcTan[c*x^2]))/6 + (b*Log[1 + c^2*x^4])/(12*
c^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{6}x^6\left(a+b\arctan\left(cx^2\right)\right)-\frac{1}{3}(bc)\int\frac{x^7}{1+c^2x^4}\,dx\\
&= \frac{1}{6}x^6\left(a+b\arctan\left(cx^2\right)\right)-\frac{1}{12}(bc)\text{Subst}\left(\int\frac{x}{1+c^2x}\,dx,x,x^4\right)\\
&= \frac{1}{6}x^6\left(a+b\arctan\left(cx^2\right)\right)-\frac{1}{12}(bc)\text{Subst}\left(\int\left(\frac{1}{c^2}-\frac{1}{c^2\left(1+c^2x\right)}\right)\,dx,x,x^4\right)\\
&= -\frac{bx^4}{12c}+\frac{1}{6}x^6\left(a+b\arctan\left(cx^2\right)\right)+\frac{b\log\left(1+c^2x^4\right)}{12c^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.11

$$\int x^5\left(a+b\arctan\left(cx^2\right)\right)\,dx=-\frac{bx^4}{12c}+\frac{ax^6}{6}+\frac{1}{6}bx^6\arctan\left(cx^2\right)+\frac{b\log\left(1+c^2x^4\right)}{12c^3}$$

```
[In] Integrate[x^5*(a + b*ArcTan[c*x^2]),x]
[Out] -1/12*(b*x^4)/c + (a*x^6)/6 + (b*x^6*ArcTan[c*x^2])/6 + (b*Log[1 + c^2*x^4]
)/(12*c^3)
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96

method	result	size
default	$\frac{a\,x^{6}}{6}+\frac{b\,x^{6}\arctan\left(c\,x^{2}\right)}{6}-\frac{b\,x^{4}}{12c}+\frac{b\ln\left(c^{2}x^{4}+1\right)}{12c^{3}}$	45
parts	$\frac{a\,x^{6}}{6}+\frac{b\,x^{6}\arctan\left(c\,x^{2}\right)}{6}-\frac{b\,x^{4}}{12c}+\frac{b\ln\left(c^{2}x^{4}+1\right)}{12c^{3}}$	45
parallelrisch	$\frac{2x^{6}\arctan\left(c\,x^{2}\right)b\,c^{3}+2a\,c^{3}x^{6}-b\,c^{2}x^{4}+b\ln\left(c^{2}x^{4}+1\right)}{12c^{3}}$	52
risch	$-\frac{i x^{6}b\ln\left(i c\,x^{2}+1\right)}{12}+\frac{i x^{6}b\ln\left(-i c\,x^{2}+1\right)}{12}+\frac{a\,x^{6}}{6}-\frac{b\,x^{4}}{12c}+\frac{b\ln\left(-c^{2}x^{4}-1\right)}{12c^{3}}$	68

```
[In] int(x^5*(a+b*arctan(c*x^2)),x,method=_RETURNVERBOSE)
[Out] 1/6*a*x^6+1/6*b*x^6*arctan(c*x^2)-1/12*b*x^4/c+1/12*b*ln(c^2*x^4+1)/c^3
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.09

$$\int x^5\left(a+b\arctan\left(cx^2\right)\right)\,dx=\frac{2\,bc^3x^6\arctan\left(cx^2\right)+2\,ac^3x^6-bc^2x^4+b\log\left(c^2x^4+1\right)}{12\,c^3}$$

```
[In] integrate(x^5*(a+b*arctan(c*x^2)),x, algorithm="fricas")
[Out] 1/12*(2*b*c^3*x^6*arctan(c*x^2) + 2*a*c^3*x^6 - b*c^2*x^4 + b*log(c^2*x^4 +
 1))/c^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(39) = 78.

Time = 22.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.70

$$\int x^5\left(a+b\arctan\left(cx^2\right)\right)\,dx$$
$$=\begin{cases}\frac{ax^6}{6}+\frac{bx^6\operatorname{atan}\left(cx^2\right)}{6}-\frac{bx^4}{12c}+\frac{b\sqrt{-\frac{1}{c^2}}\operatorname{atan}\left(cx^2\right)}{6c^2}+\frac{b\log\left(x^2+\sqrt{-\frac{1}{c^2}}\right)}{6c^3} & \text{for } c\neq 0\\ \frac{ax^6}{6} & \text{otherwise}\end{cases}$$

```
[In] integrate(x**5*(a+b*atan(c*x**2)),x)
[Out] Piecewise((a*x**6/6 + b*x**6*atan(c*x**2)/6 - b*x**4/(12*c) + b*sqrt(-1/c**
2)*atan(c*x**2)/(6*c**2) + b*log(x**2 + sqrt(-1/c**2))/(6*c**3), Ne(c, 0)),
 (a*x**6/6, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

$$\int x^5\left(a+b\arctan\left(cx^2\right)\right)\,dx=\frac{1}{6}\,ax^6+\frac{1}{12}\left(2\,x^6\arctan\left(cx^2\right)-\left(\frac{x^4}{c^2}-\frac{\log\left(c^2x^4+1\right)}{c^4}\right)c\right)b$$

```
[In] integrate(x^5*(a+b*arctan(c*x^2)),x, algorithm="maxima")
```

```
[Out] 1/6*a*x^6 + 1/12*(2*x^6*arctan(c*x^2) - (x^4/c^2 - log(c^2*x^4 + 1)/c^4)*c)
*b
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int x^5\left(a+b\arctan\left(cx^2\right)\right)\,dx=\frac{2\,acx^6+\left(2\,cx^6\arctan\left(cx^2\right)-x^4+\frac{\log\left(c^2x^4+1\right)}{c^2}\right)b}{12\,c}$$

```
[In] integrate(x^5*(a+b*arctan(c*x^2)),x, algorithm="giac")
```

```
[Out] 1/12*(2*a*c*x^6 + (2*c*x^6*arctan(c*x^2) - x^4 + log(c^2*x^4 + 1)/c^2)*b)/c
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

$$\int x^5\left(a+b\arctan\left(cx^2\right)\right)\,dx=\frac{a\,x^6}{6}+\frac{b\,\ln\left(c^2\,x^4+1\right)}{12\,c^3}-\frac{b\,x^4}{12\,c}+\frac{b\,x^6\,\mathrm{atan}(c\,x^2)}{6}$$

```
[In] int(x^5*(a + b*atan(c*x^2)),x)
```

```
[Out] (a*x^6)/6 + (b*log(c^2*x^4 + 1))/(12*c^3) - (b*x^4)/(12*c) + (b*x^6*atan(c*
x^2))/6
```

3.62 $\int x^3(a + b\arctan(cx^2))\,dx$

Optimal result 347
Rubi [A] (verified) 347
Mathematica [A] (verified) 348
Maple [A] (verified) 349
Fricas [A] (verification not implemented) 349
Sympy [A] (verification not implemented) 349
Maxima [A] (verification not implemented) 350
Giac [A] (verification not implemented) 350
Mupad [B] (verification not implemented) 350

Optimal result

Integrand size = 14, antiderivative size = 43

$$\int x^3(a + b\arctan(cx^2))\,dx = -\frac{bx^2}{4c} + \frac{b\arctan(cx^2)}{4c^2} + \frac{1}{4}x^4(a + b\arctan(cx^2))$$

```
[Out] -1/4*b*x^2/c+1/4*b*arctan(c*x^2)/c^2+1/4*x^4*(a+b*arctan(c*x^2))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4946, 281, 327, 209}

$$\int x^3(a + b\arctan(cx^2))\,dx = \frac{1}{4}x^4(a + b\arctan(cx^2)) + \frac{b\arctan(cx^2)}{4c^2} - \frac{bx^2}{4c}$$

```
[In] Int[x^3*(a + b*ArcTan[c*x^2]),x]
```

```
[Out] -1/4*(b*x^2)/c + (b*ArcTan[c*x^2])/(4*c^2) + (x^4*(a + b*ArcTan[c*x^2]))/4
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{4}x^4\left(a+b\arctan\left(cx^2\right)\right)-\frac{1}{2}(bc)\int\frac{x^5}{1+c^2x^4}\,dx\\ &= \frac{1}{4}x^4\left(a+b\arctan\left(cx^2\right)\right)-\frac{1}{4}(bc)\text{Subst}\left(\int\frac{x^2}{1+c^2x^2}\,dx,x,x^2\right)\\ &= -\frac{bx^2}{4c}+\frac{1}{4}x^4\left(a+b\arctan\left(cx^2\right)\right)+\frac{b\text{Subst}\left(\int\frac{1}{1+c^2x^2}\,dx,x,x^2\right)}{4c}\\ &= -\frac{bx^2}{4c}+\frac{b\arctan\left(cx^2\right)}{4c^2}+\frac{1}{4}x^4\left(a+b\arctan\left(cx^2\right)\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.12

$$\int x^3\left(a+b\arctan\left(cx^2\right)\right)\,dx=-\frac{bx^2}{4c}+\frac{ax^4}{4}+\frac{b\arctan\left(cx^2\right)}{4c^2}+\frac{1}{4}bx^4\arctan\left(cx^2\right)$$

```
[In] Integrate[x^3*(a + b*ArcTan[c*x^2]),x]
[Out] -1/4*(b*x^2)/c + (a*x^4)/4 + (b*ArcTan[c*x^2])/(4*c^2) + (b*x^4*ArcTan[c*x^
2])/4
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{a\,x^{4}}{4}+\frac{b\,x^{4}\arctan\left(c\,x^{2}\right)}{4}-\frac{b\,x^{2}}{4c}+\frac{b\arctan\left(c\,x^{2}\right)}{4c^{2}}$	41
parts	$\frac{a\,x^{4}}{4}+\frac{b\,x^{4}\arctan\left(c\,x^{2}\right)}{4}-\frac{b\,x^{2}}{4c}+\frac{b\arctan\left(c\,x^{2}\right)}{4c^{2}}$	41
parallelrisch	$\frac{\arctan\left(c\,x^{2}\right)b\,c^{2}x^{4}+a\,c^{2}x^{4}-bc\,x^{2}+b\arctan\left(c\,x^{2}\right)}{4c^{2}}$	44
risch	$-\frac{ix^{4}b\ln\left(ic\,x^{2}+1\right)}{8}+\frac{ix^{4}b\ln\left(-ic\,x^{2}+1\right)}{8}+\frac{a\,x^{4}}{4}-\frac{b\,x^{2}}{4c}+\frac{b\arctan\left(c\,x^{2}\right)}{4c^{2}}+\frac{b^{2}}{16a\,c^{2}}$	74

```
[In] int(x^3*(a+b*arctan(c*x^2)),x,method=_RETURNVERBOSE)
[Out] 1/4*a*x^4+1/4*b*x^4*arctan(c*x^2)-1/4*b*x^2/c+1/4*b*arctan(c*x^2)/c^2
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

$$\int x^3\left(a+b\arctan\left(cx^2\right)\right)\,dx=\frac{ac^2x^4-bcx^2+\left(bc^2x^4+b\right)\arctan\left(cx^2\right)}{4\,c^2}$$

```
[In] integrate(x^3*(a+b*arctan(c*x^2)),x, algorithm="fricas")
[Out] 1/4*(a*c^2*x^4 - b*c*x^2 + (b*c^2*x^4 + b)*arctan(c*x^2))/c^2
```

Sympy [A] (verification not implemented)

Time = 9.85 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.12

$$\int x^3\left(a+b\arctan\left(cx^2\right)\right)\,dx=\begin{cases}\frac{ax^4}{4}+\frac{bx^4\operatorname{atan}\left(cx^2\right)}{4}-\frac{bx^2}{4c}+\frac{b\operatorname{atan}\left(cx^2\right)}{4c^2} & \text{for}\ c\neq 0\\ \frac{ax^4}{4} & \text{otherwise}\end{cases}$$

```
[In] integrate(x**3*(a+b*atan(c*x**2)),x)
[Out] Piecewise((a*x**4/4 + b*x**4*atan(c*x**2)/4 - b*x**2/(4*c) + b*atan(c*x**2)
/(4*c**2), Ne(c, 0)), (a*x**4/4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int x^3(a+b\arctan(cx^2))\,dx = \frac{1}{4}ax^4 + \frac{1}{4}\left(x^4\arctan(cx^2) - c\left(\frac{x^2}{c^2} - \frac{\arctan(cx^2)}{c^3}\right)\right)b$$

```
[In] integrate(x^3*(a+b*arctan(c*x^2)),x, algorithm="maxima")
[Out] 1/4*a*x^4 + 1/4*(x^4*arctan(c*x^2) - c*(x^2/c^2 - arctan(c*x^2)/c^3))*b
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int x^3(a+b\arctan(cx^2))\,dx = \frac{acx^4 + \frac{(c^2x^4\arctan(cx^2) - cx^2 + \arctan(cx^2))b}{c}}{4c}$$

```
[In] integrate(x^3*(a+b*arctan(c*x^2)),x, algorithm="giac")
[Out] 1/4*(a*c*x^4 + (c^2*x^4*arctan(c*x^2) - c*x^2 + arctan(c*x^2))*b/c)/c
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int x^3(a+b\arctan(cx^2))\,dx = \frac{a\,x^4}{4} - \frac{b\,x^2}{4\,c} + \frac{b\,\mathrm{atan}(c\,x^2)}{4\,c^2} + \frac{b\,x^4\,\mathrm{atan}(c\,x^2)}{4}$$

```
[In] int(x^3*(a + b*atan(c*x^2)),x)
[Out] (a*x^4)/4 - (b*x^2)/(4*c) + (b*atan(c*x^2))/(4*c^2) + (b*x^4*atan(c*x^2))/4
```

3.63 $\int x(a+b\arctan(cx^2))\,dx$

Optimal result 351
Rubi [A] (verified) 351
Mathematica [A] (verified) 352
Maple [A] (verified) 352
Fricas [A] (verification not implemented) 353
Sympy [B] (verification not implemented) 353
Maxima [A] (verification not implemented) 353
Giac [A] (verification not implemented) 354
Mupad [B] (verification not implemented) 354

Optimal result

Integrand size = 12, antiderivative size = 36

$$\int x\left(a+b\arctan\left(cx^2\right)\right)\,dx=\frac{1}{2}x^2\left(a+b\arctan\left(cx^2\right)\right)-\frac{b\log\left(1+c^2x^4\right)}{4c}$$

```
[Out] 1/2*x^2*(a+b*arctan(c*x^2))-1/4*b*ln(c^2*x^4+1)/c
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4946, 266}

$$\int x\left(a+b\arctan\left(cx^2\right)\right)\,dx=\frac{1}{2}x^2\left(a+b\arctan\left(cx^2\right)\right)-\frac{b\log\left(c^2x^4+1\right)}{4c}$$

```
[In] Int[x*(a + b*ArcTan[c*x^2]),x]
[Out] (x^2*(a + b*ArcTan[c*x^2]))/2 - (b*Log[1 + c^2*x^4])/(4*c)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
```

```
IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{1}{2}x^2\left(a+b\arctan\left(cx^2\right)\right)-(bc)\int\frac{x^3}{1+c^2x^4}\,dx$$
$$= \frac{1}{2}x^2\left(a+b\arctan\left(cx^2\right)\right)-\frac{b\log\left(1+c^2x^4\right)}{4c}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.14

$$\int x\left(a+b\arctan\left(cx^2\right)\right)\,dx=\frac{ax^2}{2}+\frac{1}{2}bx^2\arctan\left(cx^2\right)-\frac{b\log\left(1+c^2x^4\right)}{4c}$$

```
[In] Integrate[x*(a + b*ArcTan[c*x^2]),x]
[Out] (a*x^2)/2 + (b*x^2*ArcTan[c*x^2])/2 - (b*Log[1 + c^2*x^4])/(4*c)
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

method	result	size
parts	$\frac{a\,x^2}{2}+\frac{b\arctan\left(c\,x^2\right)x^2}{2}-\frac{b\ln\left(c^2x^4+1\right)}{4c}$	36
derivativedivides	$\frac{c\,x^2a+b\left(c\,x^2\arctan\left(c\,x^2\right)-\frac{\ln\left(c^2x^4+1\right)}{2}\right)}{2c}$	39
default	$\frac{c\,x^2a+b\left(c\,x^2\arctan\left(c\,x^2\right)-\frac{\ln\left(c^2x^4+1\right)}{2}\right)}{2c}$	39
parallelrisch	$-\frac{-2x^2\arctan\left(c\,x^2\right)bc-2c\,x^2a+b\ln\left(c^2x^4+1\right)}{4c}$	39
risch	$-\frac{ib\,x^2\ln\left(ic\,x^2+1\right)}{4}+\frac{ib\,x^2\ln\left(-ic\,x^2+1\right)}{4}+\frac{a\,x^2}{2}-\frac{b\ln\left(-c^2x^4-1\right)}{4c}$	59

```
[In] int(x*(a+b*arctan(c*x^2)),x,method=_RETURNVERBOSE)
[Out] 1/2*a*x^2+1/2*b*arctan(c*x^2)*x^2-1/4*b*ln(c^2*x^4+1)/c
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

$$\int x\left(a+b\arctan\left(cx^2\right)\right)\,dx=\frac{2\,bcx^2\arctan\left(cx^2\right)+2\,acx^2-b\log\left(c^2x^4+1\right)}{4\,c}$$

```
[In] integrate(x*(a+b*arctan(c*x^2)),x, algorithm="fricas")
[Out] 1/4*(2*b*c*x^2*arctan(c*x^2) + 2*a*c*x^2 - b*log(c^2*x^4 + 1))/c
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(29) = 58$.

Time = 5.41 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.83

$$\int x\left(a+b\arctan\left(cx^2\right)\right)\,dx$$
$$=\begin{cases}\frac{ax^2}{2}+\frac{bx^2\,\mathrm{atan}\left(cx^2\right)}{2}-\frac{b\sqrt{-\frac{1}{c^2}}\,\mathrm{atan}\left(cx^2\right)}{2}-\frac{b\log\left(x^2+\sqrt{-\frac{1}{c^2}}\right)}{2c} & \text{for } c\neq 0\\ \frac{ax^2}{2} & \text{otherwise}\end{cases}$$

```
[In] integrate(x*(a+b*atan(c*x**2)),x)
[Out] Piecewise((a*x**2/2 + b*x**2*atan(c*x**2)/2 - b*sqrt(-1/c**2)*atan(c*x**2)/
2 - b*log(x**2 + sqrt(-1/c**2))/(2*c), Ne(c, 0)), (a*x**2/2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int x\left(a+b\arctan\left(cx^2\right)\right)\,dx=\frac{1}{2}\,ax^2+\frac{\left(2\,cx^2\arctan\left(cx^2\right)-\log\left(c^2x^4+1\right)\right)b}{4\,c}$$

```
[In] integrate(x*(a+b*arctan(c*x^2)),x, algorithm="maxima")
[Out] 1/2*a*x^2 + 1/4*(2*c*x^2*arctan(c*x^2) - log(c^2*x^4 + 1))*b/c
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11

$$\int x\left(a+b\arctan\left(cx^2\right)\right)\,dx=\frac{2\,acx^2+(2\,cx^2\arctan\left(cx^2\right)-\log\left(c^2x^4+1\right))b}{4\,c}$$

```
[In] integrate(x*(a+b*arctan(c*x^2)),x, algorithm="giac")
[Out] 1/4*(2*a*c*x^2 + (2*c*x^2*arctan(c*x^2) - log(c^2*x^4 + 1))*b)/c
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

$$\int x\left(a+b\arctan\left(cx^2\right)\right)\,dx=\frac{a\,x^2}{2}-\frac{b\,\ln\left(c^2\,x^4+1\right)}{4\,c}+\frac{b\,x^2\,\mathrm{atan}(c\,x^2)}{2}$$

```
[In] int(x*(a + b*atan(c*x^2)),x)
[Out] (a*x^2)/2 - (b*log(c^2*x^4 + 1))/(4*c) + (b*x^2*atan(c*x^2))/2
```

3.64 $\int \frac{a+b\arctan(cx^2)}{x}\,dx$

Optimal result . . . 355
Rubi [A] (verified) . . . 355
Mathematica [A] (verified) . . . 356
Maple [C] (verified) . . . 356
Fricas [F] . . . 357
Sympy [F] . . . 357
Maxima [F] . . . 357
Giac [F] . . . 357
Mupad [B] (verification not implemented) . . . 358

Optimal result

Integrand size = 14, antiderivative size = 39

$$\int \frac{a+b\arctan(cx^2)}{x}\,dx = a\log(x) + \frac{1}{4}ib\,\mathrm{PolyLog}\left(2,-icx^2\right) - \frac{1}{4}ib\,\mathrm{PolyLog}\left(2,icx^2\right)$$

```
[Out] a*ln(x)+1/4*I*b*polylog(2,-I*c*x^2)-1/4*I*b*polylog(2,I*c*x^2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4944, 4940, 2438}

$$\int \frac{a+b\arctan(cx^2)}{x}\,dx = a\log(x) + \frac{1}{4}ib\,\mathrm{PolyLog}\left(2,-icx^2\right) - \frac{1}{4}ib\,\mathrm{PolyLog}\left(2,icx^2\right)$$

```
[In] Int[(a + b*ArcTan[c*x^2])/x,x]
```

```
[Out] a*Log[x] + (I/4)*b*PolyLog[2, (-I)*c*x^2] - (I/4)*b*PolyLog[2, I*c*x^2]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1
/n, Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{a+b\arctan(cx)}{x}\,dx, x, x^2\right)\\ &= a\log(x)+\frac{1}{4}(ib)\text{Subst}\left(\int \frac{\log(1-icx)}{x}\,dx, x, x^2\right)-\frac{1}{4}(ib)\text{Subst}\left(\int \frac{\log(1+icx)}{x}\,dx, x, x^2\right)\\ &= a\log(x)+\frac{1}{4}ib\,\text{PolyLog}\left(2,-icx^2\right)-\frac{1}{4}ib\,\text{PolyLog}\left(2,icx^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{a+b\arctan\left(cx^2\right)}{x}\,dx = a\log(x)+\frac{1}{4}ib\,\text{PolyLog}\left(2,-icx^2\right)-\frac{1}{4}ib\,\text{PolyLog}\left(2,icx^2\right)$$

```
[In] Integrate[(a + b*ArcTan[c*x^2])/x,x]
[Out] a*Log[x] + (I/4)*b*PolyLog[2, (-I)*c*x^2] - (I/4)*b*PolyLog[2, I*c*x^2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.75 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.62

method	result
default	$a\ln(x)+b\ln(x)\arctan\left(c\,x^2\right)-\frac{b\left(\sum_{\_R1=\text{RootOf}\left(c^2\_Z^4+1\right)}\frac{\ln(x)\ln\left(\frac{\_R1-x}{\_R1}\right)+\text{dilog}\left(\frac{\_R1-x}{\_R1}\right)}{\_R1^2}\right)}{2c}$
parts	$a\ln(x)+b\ln(x)\arctan\left(c\,x^2\right)-\frac{b\left(\sum_{\_R1=\text{RootOf}\left(c^2\_Z^4+1\right)}\frac{\ln(x)\ln\left(\frac{\_R1-x}{\_R1}\right)+\text{dilog}\left(\frac{\_R1-x}{\_R1}\right)}{\_R1^2}\right)}{2c}$
risch	$\frac{i\ln\left(-icx^2+1\right)\ln(x)b}{2}-\frac{i\ln(x)\ln\left(1-ix\sqrt{-ic}\right)b}{2}-\frac{i\ln(x)\ln\left(1+ix\sqrt{-ic}\right)b}{2}-\frac{i\,\text{dilog}\left(1-ix\sqrt{-ic}\right)b}{2}-\frac{i\,\text{dilog}\left(1+ix\sqrt{-ic}\right)b}{2}+a\ln$

```
[In] int((a+b*arctan(c*x^2))/x,x,method=_RETURNVERBOSE)
[Out] a*ln(x)+b*ln(x)*arctan(c*x^2)-1/2*b/c*sum(1/_R1^2*(ln(x)*ln((_R1-x)/_R1)+di
log((_R1-x)/_R1)),_R1=RootOf(_Z^4*c^2+1))
```

Fricas [F]

$$\int \frac{a+b\arctan(cx^2)}{x}\,dx=\int \frac{b\arctan(cx^2)+a}{x}\,dx$$

```
[In] integrate((a+b*arctan(c*x^2))/x,x, algorithm="fricas")
[Out] integral((b*arctan(c*x^2) + a)/x, x)
```

Sympy [F]

$$\int \frac{a+b\arctan(cx^2)}{x}\,dx=\int \frac{a+b\,\mathrm{atan}(cx^2)}{x}\,dx$$

```
[In] integrate((a+b*atan(c*x**2))/x,x)
[Out] Integral((a + b*atan(c*x**2))/x, x)
```

Maxima [F]

$$\int \frac{a+b\arctan(cx^2)}{x}\,dx=\int \frac{b\arctan(cx^2)+a}{x}\,dx$$

```
[In] integrate((a+b*arctan(c*x^2))/x,x, algorithm="maxima")
[Out] b*integrate(arctan(c*x^2)/x, x) + a*log(x)
```

Giac [F]

$$\int \frac{a+b\arctan(cx^2)}{x}\,dx=\int \frac{b\arctan(cx^2)+a}{x}\,dx$$

```
[In] integrate((a+b*arctan(c*x^2))/x,x, algorithm="giac")
[Out] integrate((b*arctan(c*x^2) + a)/x, x)
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{a+b\arctan\left(cx^2\right)}{x}\,dx = a\,\ln\left(x\right)-\frac{b\left(\mathrm{Li}_2(1-c\,x^2\,1\mathrm{i})-\mathrm{Li}_2(1\mathrm{i}\,c\,x^2+1)\right)\,1\mathrm{i}}{4}$$

```
[In] int((a + b*atan(c*x^2))/x,x)
[Out] a*log(x) - (b*(dilog(1 - c*x^2*1i) - dilog(c*x^2*1i + 1))*1i)/4
```

3.65 $\int \frac{a+b\arctan(cx^2)}{x^3}\,dx$

Optimal result . . . 359
Rubi [A] (verified) . . . 359
Mathematica [A] (verified) . . . 360
Maple [A] (verified) . . . 361
Fricas [A] (verification not implemented) . . . 361
Sympy [A] (verification not implemented) . . . 361
Maxima [A] (verification not implemented) . . . 362
Giac [A] (verification not implemented) . . . 362
Mupad [B] (verification not implemented) . . . 362

Optimal result

Integrand size = 14, antiderivative size = 39

$$\int \frac{a+b\arctan(cx^2)}{x^3}\,dx = -\frac{a+b\arctan(cx^2)}{2x^2}+bc\log(x)-\frac{1}{4}bc\log\left(1+c^2x^4\right)$$

```
[Out] 1/2*(-a-b*arctan(c*x^2))/x^2+b*c*ln(x)-1/4*b*c*ln(c^2*x^4+1)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {4946, 272, 36, 29, 31}

$$\int \frac{a+b\arctan(cx^2)}{x^3}\,dx = -\frac{a+b\arctan(cx^2)}{2x^2}-\frac{1}{4}bc\log\left(c^2x^4+1\right)+bc\log(x)$$

```
[In] Int[(a + b*ArcTan[c*x^2])/x^3,x]
```

```
[Out] -1/2*(a + b*ArcTan[c*x^2])/x^2 + b*c*Log[x] - (b*c*Log[1 + c^2*x^4])/4
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{a+b\arctan\left(cx^2\right)}{2x^2}+(bc)\int\frac{1}{x\left(1+c^2x^4\right)}\,dx\\ &= -\frac{a+b\arctan\left(cx^2\right)}{2x^2}+\frac{1}{4}(bc)\text{Subst}\left(\int\frac{1}{x\left(1+c^2x\right)}\,dx,x,x^4\right)\\ &= -\frac{a+b\arctan\left(cx^2\right)}{2x^2}+\frac{1}{4}(bc)\text{Subst}\left(\int\frac{1}{x}\,dx,x,x^4\right)-\frac{1}{4}\left(bc^3\right)\text{Subst}\left(\int\frac{1}{1+c^2x}\,dx,x,x^4\right)\\ &= -\frac{a+b\arctan\left(cx^2\right)}{2x^2}+bc\log(x)-\frac{1}{4}bc\log\left(1+c^2x^4\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.13

$$\int\frac{a+b\arctan\left(cx^2\right)}{x^3}\,dx=-\frac{a}{2x^2}-\frac{b\arctan\left(cx^2\right)}{2x^2}+bc\log(x)-\frac{1}{4}bc\log\left(1+c^2x^4\right)$$

```
[In] Integrate[(a + b*ArcTan[c*x^2])/x^3,x]
[Out] -1/2*a/x^2 - (b*ArcTan[c*x^2])/(2*x^2) + b*c*Log[x] - (b*c*Log[1 + c^2*x^4]
)/4
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

method	result	size
default	$-\frac{a}{2x^{2}}+b\left(-\frac{\arctan\left(c\,x^{2}\right)}{2x^{2}}+c\left(\ln\left(x\right)-\frac{\ln\left(c^{2}x^{4}+1\right)}{4}\right)\right)$	39
parts	$-\frac{a}{2x^{2}}+b\left(-\frac{\arctan\left(c\,x^{2}\right)}{2x^{2}}+c\left(\ln\left(x\right)-\frac{\ln\left(c^{2}x^{4}+1\right)}{4}\right)\right)$	39
parallelrisch	$\frac{4bc\ln\left(x\right)x^{2}-bc\ln\left(c^{2}x^{4}+1\right)x^{2}-2b\arctan\left(c\,x^{2}\right)-2a}{4x^{2}}$	45
risch	$\frac{ib\ln\left(ic\,x^{2}+1\right)}{4x^{2}}-\frac{-4bc\ln\left(x\right)x^{2}+bc\ln\left(-c^{2}x^{4}-1\right)x^{2}+ib\ln\left(-ic\,x^{2}+1\right)+2a}{4x^{2}}$	68

```
[In] int((a+b*arctan(c*x^2))/x^3,x,method=_RETURNVERBOSE)
[Out] -1/2*a/x^2+b*(-1/2/x^2*arctan(c*x^2)+c*(ln(x)-1/4*ln(c^2*x^4+1)))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10

$$\int \frac{a+b\arctan\left(cx^{2}\right)}{x^{3}}\,dx=-\frac{bcx^{2}\log\left(c^{2}x^{4}+1\right)-4\,bcx^{2}\log\left(x\right)+2\,b\arctan\left(cx^{2}\right)+2\,a}{4\,x^{2}}$$

```
[In] integrate((a+b*arctan(c*x^2))/x^3,x, algorithm="fricas")
[Out] -1/4*(b*c*x^2*log(c^2*x^4 + 1) - 4*b*c*x^2*log(x) + 2*b*arctan(c*x^2) + 2*a
)/x^2
```

Sympy [A] (verification not implemented)

Time = 12.93 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.92

$$\int \frac{a+b\arctan\left(cx^{2}\right)}{x^{3}}\,dx = \begin{cases} -\frac{a}{2x^{2}}+bc\log\left(x\right)-\frac{bc\log\left(x^{2}+\sqrt{-\frac{1}{c^{2}}}\right)}{2}+\frac{b\operatorname{atan}\left(cx^{2}\right)}{2\sqrt{-\frac{1}{c^{2}}}}-\frac{b\operatorname{atan}\left(cx^{2}\right)}{2x^{2}} & \text{for}\ c\neq 0\\ -\frac{a}{2x^{2}} & \text{otherwise} \end{cases}$$

```
[In] integrate((a+b*atan(c*x**2))/x**3,x)
[Out] Piecewise((-a/(2*x**2) + b*c*log(x) - b*c*log(x**2 + sqrt(-1/c**2))/2 + b*a
tan(c*x**2)/(2*sqrt(-1/c**2)) - b*atan(c*x**2)/(2*x**2), Ne(c, 0)), (-a/(2*
x**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{a + b\arctan\left(cx^2\right)}{x^3}\,dx = -\frac{1}{4}\left(c\left(\log\left(c^2x^4+1\right) - \log\left(x^4\right)\right) + \frac{2\arctan\left(cx^2\right)}{x^2}\right)b - \frac{a}{2\,x^2}$$

```
[In] integrate((a+b*arctan(c*x^2))/x^3,x, algorithm="maxima")
```

```
[Out] -1/4*(c*(log(c^2*x^4 + 1) - log(x^4)) + 2*arctan(c*x^2)/x^2)*b - 1/2*a/x^2
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.54

$$\int \frac{a + b\arctan\left(cx^2\right)}{x^3}\,dx = -\frac{bc^3x^2\log\left(c^2x^4+1\right) - 2\,bc^3x^2\log\left(cx^2\right) + 2\,bc^2\arctan\left(cx^2\right) + 2\,ac^2}{4\,c^2x^2}$$

```
[In] integrate((a+b*arctan(c*x^2))/x^3,x, algorithm="giac")
```

```
[Out] -1/4*(b*c^3*x^2*log(c^2*x^4 + 1) - 2*b*c^3*x^2*log(c*x^2) + 2*b*c^2*arctan(
c*x^2) + 2*a*c^2)/(c^2*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.97

$$\int \frac{a + b\arctan\left(cx^2\right)}{x^3}\,dx = b\,c\,\ln\left(x\right) - \frac{a}{2\,x^2} - \frac{b\,\mathrm{atan}\left(c\,x^2\right)}{2\,x^2} - \frac{b\,c\,\ln\left(c^2\,x^4+1\right)}{4}$$

```
[In] int((a + b*atan(c*x^2))/x^3,x)
```

```
[Out] b*c*log(x) - a/(2*x^2) - (b*atan(c*x^2))/(2*x^2) - (b*c*log(c^2*x^4 + 1))/4
```

3.66 $\int \frac{a+b\arctan(cx^2)}{x^5}\,dx$

Optimal result 363
Rubi [A] (verified) 363
Mathematica [C] (verified) 364
Maple [A] (verified) 365
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Sympy [A] (verification not implemented) 365
Maxima [A] (verification not implemented) 366
Giac [C] (verification not implemented) 366
Mupad [B] (verification not implemented) 366

Optimal result

Integrand size = 14, antiderivative size = 41

$$\int \frac{a+b\arctan(cx^2)}{x^5}\,dx = -\frac{bc}{4x^2} - \frac{1}{4}bc^2\arctan(cx^2) - \frac{a+b\arctan(cx^2)}{4x^4}$$

```
[Out] -1/4*b*c/x^2-1/4*b*c^2*arctan(c*x^2)+1/4*(-a-b*arctan(c*x^2))/x^4
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4946, 281, 331, 209}

$$\int \frac{a+b\arctan(cx^2)}{x^5}\,dx = -\frac{a+b\arctan(cx^2)}{4x^4} - \frac{1}{4}bc^2\arctan(cx^2) - \frac{bc}{4x^2}$$

```
[In] Int[(a + b*ArcTan[c*x^2])/x^5,x]
```

```
[Out] -1/4*(b*c)/x^2 - (b*c^2*ArcTan[c*x^2])/4 - (a + b*ArcTan[c*x^2])/(4*x^4)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a+b\arctan(cx^2)}{4x^4}+\frac{1}{2}(bc)\int\frac{1}{x^3(1+c^2x^4)}\,dx\\
&= -\frac{a+b\arctan(cx^2)}{4x^4}+\frac{1}{4}(bc)\text{Subst}\left(\int\frac{1}{x^2(1+c^2x^2)}\,dx,x,x^2\right)\\
&= -\frac{bc}{4x^2}-\frac{a+b\arctan(cx^2)}{4x^4}-\frac{1}{4}\left(bc^3\right)\text{Subst}\left(\int\frac{1}{1+c^2x^2}\,dx,x,x^2\right)\\
&= -\frac{bc}{4x^2}-\frac{1}{4}bc^2\arctan\left(cx^2\right)-\frac{a+b\arctan(cx^2)}{4x^4}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.17

$$\int\frac{a+b\arctan(cx^2)}{x^5}\,dx=-\frac{a}{4x^4}-\frac{b\arctan(cx^2)}{4x^4}-\frac{bc\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-c^2x^4\right)}{4x^2}$$

```
[In] Integrate[(a + b*ArcTan[c*x^2])/x^5,x]
```

```
[Out] -1/4*a/x^4 - (b*ArcTan[c*x^2])/(4*x^4) - (b*c*Hypergeometric2F1[-1/2, 1, 1/
2, -(c^2*x^4)])/(4*x^2)
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

method	result	size
default	$-\frac{a}{4x^{4}}-\frac{b\arctan\left(c\,x^{2}\right)}{4x^{4}}-\frac{bc}{4x^{2}}-\frac{b\,c^{2}\arctan\left(c\,x^{2}\right)}{4}$	39
parts	$-\frac{a}{4x^{4}}-\frac{b\arctan\left(c\,x^{2}\right)}{4x^{4}}-\frac{bc}{4x^{2}}-\frac{b\,c^{2}\arctan\left(c\,x^{2}\right)}{4}$	39
parallelrisch	$-\frac{\arctan\left(c\,x^{2}\right)b\,c^{2}x^{4}-a\,c^{2}x^{4}+bc\,x^{2}+b\arctan\left(c\,x^{2}\right)+a}{4x^{4}}$	45
risch	$\frac{ib\ln\left(ic\,x^{2}+1\right)}{8x^{4}}-\frac{-ib\,c^{2}\ln\left(c\,x^{2}-i\right)x^{4}+ib\,c^{2}\ln\left(c\,x^{2}+i\right)x^{4}+2bc\,x^{2}+ib\ln\left(-ic\,x^{2}+1\right)+2a}{8x^{4}}$	87

```
[In] int((a+b*arctan(c*x^2))/x^5,x,method=_RETURNVERBOSE)
[Out] -1/4*a/x^4-1/4*b/x^4*arctan(c*x^2)-1/4*b*c/x^2-1/4*b*c^2*arctan(c*x^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.73

$$\int \frac{a+b\arctan\left(cx^2\right)}{x^5}\,dx=-\frac{bcx^2+\left(bc^2x^4+b\right)\arctan\left(cx^2\right)+a}{4\,x^4}$$

```
[In] integrate((a+b*arctan(c*x^2))/x^5,x, algorithm="fricas")
[Out] -1/4*(b*c*x^2 + (b*c^2*x^4 + b)*arctan(c*x^2) + a)/x^4
```

Sympy [A] (verification not implemented)

Time = 12.69 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

$$\int \frac{a+b\arctan\left(cx^2\right)}{x^5}\,dx=-\frac{a}{4x^4}-\frac{bc^2\operatorname{atan}\left(cx^2\right)}{4}-\frac{bc}{4x^2}-\frac{b\operatorname{atan}\left(cx^2\right)}{4x^4}$$

```
[In] integrate((a+b*atan(c*x**2))/x**5,x)
[Out] -a/(4*x**4) - b*c**2*atan(c*x**2)/4 - b*c/(4*x**2) - b*atan(c*x**2)/(4*x**4
)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \frac{a+b\arctan\left(cx^2\right)}{x^5}\,dx=-\frac{1}{4}\left(\left(c\arctan\left(cx^2\right)+\frac{1}{x^2}\right)c+\frac{\arctan\left(cx^2\right)}{x^4}\right)b-\frac{a}{4\,x^4}$$

```
[In] integrate((a+b*arctan(c*x^2))/x^5,x, algorithm="maxima")
[Out] -1/4*((c*arctan(c*x^2) + 1/x^2)*c + arctan(c*x^2)/x^4)*b - 1/4*a/x^4
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.76

$$\int \frac{a+b\arctan\left(cx^2\right)}{x^5}\,dx = \frac{i\,bc^5x^4\log\left(i\,cx^2+1\right)-i\,bc^5x^4\log\left(-i\,cx^2+1\right)-2\,bc^4x^2-2\,bc^3\arctan\left(cx^2\right)-2\,ac^3}{8\,c^3x^4}$$

```
[In] integrate((a+b*arctan(c*x^2))/x^5,x, algorithm="giac")
[Out] 1/8*(I*b*c^5*x^4*log(I*c*x^2 + 1) - I*b*c^5*x^4*log(-I*c*x^2 + 1) - 2*b*c^4
*x^2 - 2*b*c^3*arctan(c*x^2) - 2*a*c^3)/(c^3*x^4)
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{a+b\arctan\left(cx^2\right)}{x^5}\,dx=-\frac{\frac{b\,c\,x^2}{2}+\frac{a}{2}}{2\,x^4}-\frac{b\,c^2\,\mathrm{atan}\left(c\,x^2\right)}{4}-\frac{b\,\mathrm{atan}\left(c\,x^2\right)}{4\,x^4}$$

```
[In] int((a + b*atan(c*x^2))/x^5,x)
[Out] - (a/2 + (b*c*x^2)/2)/(2*x^4) - (b*c^2*atan(c*x^2))/4 - (b*atan(c*x^2))/(4*
x^4)
```

3.67 $\int \frac{a+b\arctan(cx^2)}{x^7}\,dx$

Optimal result 367
Rubi [A] (verified) 367
Mathematica [A] (verified) 368
Maple [A] (verified) 369
Fricas [A] (verification not implemented) 369
Sympy [A] (verification not implemented) 370
Maxima [A] (verification not implemented) 370
Giac [A] (verification not implemented) 370
Mupad [B] (verification not implemented) 371

Optimal result

Integrand size = 14, antiderivative size = 55

$$\int \frac{a+b\arctan(cx^2)}{x^7}\,dx=-\frac{bc}{12x^4}-\frac{a+b\arctan(cx^2)}{6x^6}-\frac{1}{3}bc^3\log(x)+\frac{1}{12}bc^3\log\left(1+c^2x^4\right)$$

```
[Out] -1/12*b*c/x^4+1/6*(-a-b*arctan(c*x^2))/x^6-1/3*b*c^3*ln(x)+1/12*b*c^3*ln(c^
2*x^4+1)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4946, 272, 46}

$$\int \frac{a+b\arctan(cx^2)}{x^7}\,dx=-\frac{a+b\arctan(cx^2)}{6x^6}-\frac{1}{3}bc^3\log(x)+\frac{1}{12}bc^3\log\left(c^2x^4+1\right)-\frac{bc}{12x^4}$$

```
[In] Int[(a + b*ArcTan[c*x^2])/x^7,x]
```

```
[Out] -1/12*(b*c)/x^4 - (a + b*ArcTan[c*x^2])/(6*x^6) - (b*c^3*Log[x])/3 + (b*c^3
*Log[1 + c^2*x^4])/12
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{a+b\arctan(cx^2)}{6x^6}+\frac{1}{3}(bc)\int\frac{1}{x^5(1+c^2x^4)}\,dx\\ &= -\frac{a+b\arctan(cx^2)}{6x^6}+\frac{1}{12}(bc)\text{Subst}\left(\int\frac{1}{x^2(1+c^2x)}\,dx,x,x^4\right)\\ &= -\frac{a+b\arctan(cx^2)}{6x^6}+\frac{1}{12}(bc)\text{Subst}\left(\int\left(\frac{1}{x^2}-\frac{c^2}{x}+\frac{c^4}{1+c^2x}\right)dx,x,x^4\right)\\ &= -\frac{bc}{12x^4}-\frac{a+b\arctan(cx^2)}{6x^6}-\frac{1}{3}bc^3\log(x)+\frac{1}{12}bc^3\log(1+c^2x^4)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.09

$$\int\frac{a+b\arctan(cx^2)}{x^7}\,dx=-\frac{a}{6x^6}-\frac{bc}{12x^4}-\frac{b\arctan(cx^2)}{6x^6}-\frac{1}{3}bc^3\log(x)+\frac{1}{12}bc^3\log(1+c^2x^4)$$

```
[In] Integrate[(a + b*ArcTan[c*x^2])/x^7,x]
[Out] -1/6*a/x^6 - (b*c)/(12*x^4) - (b*ArcTan[c*x^2])/(6*x^6) - (b*c^3*Log[x])/3
+ (b*c^3*Log[1 + c^2*x^4])/12
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

method	result	size
default	$-\frac{a}{6x^{6}}+b\left(-\frac{\arctan\left(c\,x^{2}\right)}{6x^{6}}+\frac{c\left(-\frac{1}{4x^{4}}-c^{2}\ln\left(x\right)+\frac{c^{2}\ln\left(c^{2}x^{4}+1\right)}{4}\right)}{3}\right)$	53
parts	$-\frac{a}{6x^{6}}+b\left(-\frac{\arctan\left(c\,x^{2}\right)}{6x^{6}}+\frac{c\left(-\frac{1}{4x^{4}}-c^{2}\ln\left(x\right)+\frac{c^{2}\ln\left(c^{2}x^{4}+1\right)}{4}\right)}{3}\right)$	53
parallelrisch	$-\frac{4b\,c^{3}\ln\left(x\right)x^{6}-b\,c^{3}\ln\left(c^{2}x^{4}+1\right)x^{6}-b\,c^{3}x^{6}+bc\,x^{2}+2b\arctan\left(c\,x^{2}\right)+2a}{12x^{6}}$	64
risch	$\frac{ib\ln\left(ic\,x^{2}+1\right)}{12x^{6}}-\frac{4b\,c^{3}\ln\left(x\right)x^{6}-b\,c^{3}\ln\left(c^{2}x^{4}+1\right)x^{6}+bc\,x^{2}+ib\ln\left(-ic\,x^{2}+1\right)+2a}{12x^{6}}$	78

```
[In] int((a+b*arctan(c*x^2))/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*a/x^6+b*(-1/6/x^6*arctan(c*x^2)+1/3*c*(-1/4/x^4-c^2*ln(x)+1/4*c^2*ln(c
^2*x^4+1)))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.98

$$\int \frac{a+b\arctan\left(cx^{2}\right)}{x^{7}}\,dx = \frac{bc^{3}x^{6}\log\left(c^{2}x^{4}+1\right)-4\,bc^{3}x^{6}\log\left(x\right)-bcx^{2}-2\,b\arctan\left(cx^{2}\right)-2\,a}{12\,x^{6}}$$

```
[In] integrate((a+b*arctan(c*x^2))/x^7,x, algorithm="fricas")
```

```
[Out] 1/12*(b*c^3*x^6*log(c^2*x^4 + 1) - 4*b*c^3*x^6*log(x) - b*c*x^2 - 2*b*arcta
n(c*x^2) - 2*a)/x^6
```

Sympy [A] (verification not implemented)

Time = 38.96 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.67

$$\int \frac{a+b\arctan(cx^2)}{x^7}\,dx = \begin{cases} -\frac{a}{6x^6} - \frac{bc^3\log(x)}{3} + \frac{bc^3\log\left(x^2+\sqrt{-\frac{1}{c^2}}\right)}{6} - \frac{bc^2\operatorname{atan}(cx^2)}{6\sqrt{-\frac{1}{c^2}}} - \frac{bc}{12x^4} - \frac{b\operatorname{atan}(cx^2)}{6x^6} & \text{for } c \neq 0 \\ -\frac{a}{6x^6} & \text{otherwise} \end{cases}$$

```
[In] integrate((a+b*atan(c*x**2))/x**7,x)
[Out] Piecewise((-a/(6*x**6) - b*c**3*log(x)/3 + b*c**3*log(x**2 + sqrt(-1/c**2))
/6 - b*c**2*atan(c*x**2)/(6*sqrt(-1/c**2)) - b*c/(12*x**4) - b*atan(c*x**2)
/(6*x**6), Ne(c, 0)), (-a/(6*x**6), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int \frac{a+b\arctan(cx^2)}{x^7}\,dx = \frac{1}{12}\left(\left(c^2\log\left(c^2x^4+1\right) - c^2\log\left(x^4\right) - \frac{1}{x^4}\right)c - \frac{2\arctan(cx^2)}{x^6}\right)b - \frac{a}{6x^6}$$

```
[In] integrate((a+b*arctan(c*x^2))/x^7,x, algorithm="maxima")
[Out] 1/12*((c^2*log(c^2*x^4 + 1) - c^2*log(x^4) - 1/x^4)*c - 2*arctan(c*x^2)/x^6
)*b - 1/6*a/x^6
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.25

$$\int \frac{a+b\arctan(cx^2)}{x^7}\,dx = \frac{bc^7x^6\log\left(c^2x^4+1\right) - 2\,bc^7x^6\log\left(cx^2\right) - bc^5x^2 - 2\,bc^4\arctan\left(cx^2\right) - 2\,ac^4}{12\,c^4x^6}$$

```
[In] integrate((a+b*arctan(c*x^2))/x^7,x, algorithm="giac")
[Out] 1/12*(b*c^7*x^6*log(c^2*x^4 + 1) - 2*b*c^7*x^6*log(c*x^2) - b*c^5*x^2 - 2*b
*c^4*arctan(c*x^2) - 2*a*c^4)/(c^4*x^6)
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

$$\int \frac{a+b\arctan\left(cx^2\right)}{x^7}\,dx=\frac{b\,c^3\,\ln\left(c^2\,x^4+1\right)}{12}-\frac{a}{6\,x^6}-\frac{b\,c^3\,\ln\left(x\right)}{3}-\frac{b\,\mathrm{atan}(c\,x^2)}{6\,x^6}-\frac{b\,c}{12\,x^4}$$

```
[In] int((a + b*atan(c*x^2))/x^7,x)
[Out] (b*c^3*log(c^2*x^4 + 1))/12 - a/(6*x^6) - (b*c^3*log(x))/3 - (b*atan(c*x^2)
)/(6*x^6) - (b*c)/(12*x^4)
```

3.68 $\int x^4(a+b\arctan(cx^2))\,dx$

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Optimal result

Integrand size = 14, antiderivative size = 161

$$\int x^4(a+b\arctan(cx^2))\,dx = -\frac{2bx^3}{15c}+\frac{1}{5}x^5(a+b\arctan(cx^2)) - \frac{b\arctan(1-\sqrt{2}\sqrt{c}x)}{5\sqrt{2}c^{5/2}}+\frac{b\arctan(1+\sqrt{2}\sqrt{c}x)}{5\sqrt{2}c^{5/2}} + \frac{b\log(1-\sqrt{2}\sqrt{c}x+cx^2)}{10\sqrt{2}c^{5/2}}-\frac{b\log(1+\sqrt{2}\sqrt{c}x+cx^2)}{10\sqrt{2}c^{5/2}}$$

```
[Out] -2/15*b*x^3/c+1/5*x^5*(a+b*arctan(c*x^2))+1/10*b*arctan(-1+x*2^(1/2)*c^(1/2
))/c^(5/2)*2^(1/2)+1/10*b*arctan(1+x*2^(1/2)*c^(1/2))/c^(5/2)*2^(1/2)+1/20*
b*ln(1+c*x^2-x*2^(1/2)*c^(1/2))/c^(5/2)*2^(1/2)-1/20*b*ln(1+c*x^2+x*2^(1/2)
*c^(1/2))/c^(5/2)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {4946, 327, 303, 1176, 631, 210, 1179, 642}

$$\int x^4(a+b\arctan(cx^2))\,dx = \frac{1}{5}x^5(a+b\arctan(cx^2))-\frac{b\arctan(1-\sqrt{2}\sqrt{c}x)}{5\sqrt{2}c^{5/2}} + \frac{b\arctan(\sqrt{2}\sqrt{c}x+1)}{5\sqrt{2}c^{5/2}}+\frac{b\log(cx^2-\sqrt{2}\sqrt{c}x+1)}{10\sqrt{2}c^{5/2}} - \frac{b\log(cx^2+\sqrt{2}\sqrt{c}x+1)}{10\sqrt{2}c^{5/2}}-\frac{2bx^3}{15c}$$

```
[In] Int[x^4*(a + b*ArcTan[c*x^2]),x]
```

```
[Out] (-2*b*x^3)/(15*c) + (x^5*(a + b*ArcTan[c*x^2]))/5 - (b*ArcTan[1 - Sqrt[2]*S
qrt[c]*x])/(5*Sqrt[2]*c^(5/2)) + (b*ArcTan[1 + Sqrt[2]*Sqrt[c]*x])/(5*Sqrt[
2]*c^(5/2)) + (b*Log[1 - Sqrt[2]*Sqrt[c]*x + c*x^2])/(10*Sqrt[2]*c^(5/2)) -
 (b*Log[1 + Sqrt[2]*Sqrt[c]*x + c*x^2])/(10*Sqrt[2]*c^(5/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{5}x^5\left(a+b\arctan\left(cx^2\right)\right)-\frac{1}{5}(2bc)\int\frac{x^6}{1+c^2x^4}\,dx \\
&= -\frac{2bx^3}{15c}+\frac{1}{5}x^5\left(a+b\arctan\left(cx^2\right)\right)+\frac{(2b)\int\frac{x^2}{1+c^2x^4}\,dx}{5c} \\
&= -\frac{2bx^3}{15c}+\frac{1}{5}x^5\left(a+b\arctan\left(cx^2\right)\right)-\frac{b\int\frac{1-cx^2}{1+c^2x^4}\,dx}{5c^2}+\frac{b\int\frac{1+cx^2}{1+c^2x^4}\,dx}{5c^2} \\
&= -\frac{2bx^3}{15c}+\frac{1}{5}x^5\left(a+b\arctan\left(cx^2\right)\right)+\frac{b\int\frac{1}{\frac{1}{c}-\frac{\sqrt{2}x}{\sqrt{c}}+x^2}\,dx}{10c^3} \\
&\quad+\frac{b\int\frac{1}{\frac{1}{c}+\frac{\sqrt{2}x}{\sqrt{c}}+x^2}\,dx}{10c^3}+\frac{b\int\frac{\frac{\sqrt{2}}{\sqrt{c}}+2x}{-\frac{1}{c}-\frac{\sqrt{2}x}{\sqrt{c}}-x^2}\,dx}{10\sqrt{2}c^{5/2}}+\frac{b\int\frac{\frac{\sqrt{2}}{\sqrt{c}}-2x}{-\frac{1}{c}+\frac{\sqrt{2}x}{\sqrt{c}}-x^2}\,dx}{10\sqrt{2}c^{5/2}} \\
&= -\frac{2bx^3}{15c}+\frac{1}{5}x^5\left(a+b\arctan\left(cx^2\right)\right)+\frac{b\log\left(1-\sqrt{2}\sqrt{c}x+cx^2\right)}{10\sqrt{2}c^{5/2}} \\
&\quad-\frac{b\log\left(1+\sqrt{2}\sqrt{c}x+cx^2\right)}{10\sqrt{2}c^{5/2}}+\frac{b\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}\sqrt{c}x\right)}{5\sqrt{2}c^{5/2}} \\
&\quad-\frac{b\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\sqrt{2}\sqrt{c}x\right)}{5\sqrt{2}c^{5/2}} \\
&= -\frac{2bx^3}{15c}+\frac{1}{5}x^5\left(a+b\arctan\left(cx^2\right)\right)-\frac{b\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{5\sqrt{2}c^{5/2}} \\
&\quad+\frac{b\arctan\left(1+\sqrt{2}\sqrt{c}x\right)}{5\sqrt{2}c^{5/2}}+\frac{b\log\left(1-\sqrt{2}\sqrt{c}x+cx^2\right)}{10\sqrt{2}c^{5/2}}-\frac{b\log\left(1+\sqrt{2}\sqrt{c}x+cx^2\right)}{10\sqrt{2}c^{5/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.11

$$\int x^4(a+b\arctan(cx^2))\,dx=-\frac{2bx^3}{15c}+\frac{ax^5}{5}+\frac{1}{5}bx^5\arctan(cx^2)+\frac{b\arctan\left(\frac{-\sqrt{2}+2\sqrt{c}x}{\sqrt{2}}\right)}{5\sqrt{2}c^{5/2}}+\frac{b\arctan\left(\frac{\sqrt{2}+2\sqrt{c}x}{\sqrt{2}}\right)}{5\sqrt{2}c^{5/2}}+\frac{b\log\left(1-\sqrt{2}\sqrt{c}x+cx^2\right)}{10\sqrt{2}c^{5/2}}-\frac{b\log\left(1+\sqrt{2}\sqrt{c}x+cx^2\right)}{10\sqrt{2}c^{5/2}}$$

```
[In] Integrate[x^4*(a + b*ArcTan[c*x^2]),x]
```

```
[Out] (-2*b*x^3)/(15*c) + (a*x^5)/5 + (b*x^5*ArcTan[c*x^2])/5 + (b*ArcTan[(-Sqrt[
2] + 2*Sqrt[c]*x)/Sqrt[2]])/(5*Sqrt[2]*c^(5/2)) + (b*ArcTan[(Sqrt[2] + 2*Sq
rt[c]*x)/Sqrt[2]])/(5*Sqrt[2]*c^(5/2)) + (b*Log[1 - Sqrt[2]*Sqrt[c]*x + c*x
^2])/(10*Sqrt[2]*c^(5/2)) - (b*Log[1 + Sqrt[2]*Sqrt[c]*x + c*x^2])/(10*Sqrt
[2]*c^(5/2))
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.75

method	result	siz
default	$\frac{a\,x^{5}}{5}+b\left(\frac{x^{5}\arctan\left(c\,x^{2}\right)}{5}-\frac{2c\left(\frac{x^{3}}{3c^{2}}-\frac{\sqrt{2}\left(\ln\left(\frac{x^{2}-\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^{2}}}}{x^{2}+\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^{2}}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}}-1\right)\right)}{8c^{4}\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}}\right)}{5}\right)$	12
parts	$\frac{a\,x^{5}}{5}+b\left(\frac{x^{5}\arctan\left(c\,x^{2}\right)}{5}-\frac{2c\left(\frac{x^{3}}{3c^{2}}-\frac{\sqrt{2}\left(\ln\left(\frac{x^{2}-\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^{2}}}}{x^{2}+\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^{2}}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}}-1\right)\right)}{8c^{4}\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}}\right)}{5}\right)$	12

```
[In] int(x^4*(a+b*arctan(c*x^2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*a*x^5+b*(1/5*x^5*arctan(c*x^2)-2/5*c*(1/3/c^2*x^3-1/8/c^4/(1/c^2)^(1/4)
*2^(1/2)*(ln((x^2-(1/c^2)^(1/4)*x*2^(1/2)+(1/c^2)^(1/2))/(x^2+(1/c^2)^(1/4)
*x*2^(1/2)+(1/c^2)^(1/2)))+2*arctan(2^(1/2)/(1/c^2)^(1/4)*x+1)+2*arctan(2^(
1/2)/(1/c^2)^(1/4)*x-1))))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.06

$$\int x^4\left(a+b\arctan\left(cx^2\right)\right)\,dx$$
$$=\frac{6\,bcx^5\arctan\left(cx^2\right)+6\,acx^5-4\,bx^3+3\,c\left(-\frac{b^4}{c^{10}}\right)^{\frac{1}{4}}\log\left(c^7\left(-\frac{b^4}{c^{10}}\right)^{\frac{3}{4}}+b^3x\right)-3i\,c\left(-\frac{b^4}{c^{10}}\right)^{\frac{1}{4}}\log\left(i\,c^7\left(-\frac{b^4}{c^{10}}\right)^{\frac{3}{4}}\right.}{30\,c}$$

```
[In] integrate(x^4*(a+b*arctan(c*x^2)),x, algorithm="fricas")
```

```
[Out] 1/30*(6*b*c*x^5*arctan(c*x^2) + 6*a*c*x^5 - 4*b*x^3 + 3*c*(-b^4/c^10)^(1/4)
*log(c^7*(-b^4/c^10)^(3/4) + b^3*x) - 3*I*c*(-b^4/c^10)^(1/4)*log(I*c^7*(-b
^4/c^10)^(3/4) + b^3*x) + 3*I*c*(-b^4/c^10)^(1/4)*log(-I*c^7*(-b^4/c^10)^(3
/4) + b^3*x) - 3*c*(-b^4/c^10)^(1/4)*log(-c^7*(-b^4/c^10)^(3/4) + b^3*x))/c
```

Sympy [A] (verification not implemented)

Time = 15.27 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.95

$$\int x^4\left(a+b\arctan\left(cx^2\right)\right)\,dx = \begin{cases} \frac{ax^5}{5} + \frac{bx^5\operatorname{atan}\left(cx^2\right)}{5} - \frac{2bx^3}{15c} + \frac{b\log\left(x-\sqrt[4]{-\frac{1}{c^2}}\right)}{5c^3\sqrt[4]{-\frac{1}{c^2}}} - \frac{b\log\left(x^2+\sqrt{-\frac{1}{c^2}}\right)}{10c^3\sqrt[4]{-\frac{1}{c^2}}} + \frac{b\operatorname{atan}\left(\frac{x}{\sqrt[4]{-\frac{1}{c^2}}}\right)}{5c^3\sqrt[4]{-\frac{1}{c^2}}} - \frac{b\operatorname{atan}\left(cx^2\right)}{5c^6\left(-\frac{1}{c^2}\right)^{\frac{7}{4}}} & \text{for}\ c\neq 0 \\ \frac{ax^5}{5} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**4*(a+b*atan(c*x**2)),x)
```

```
[Out] Piecewise((a*x**5/5 + b*x**5*atan(c*x**2)/5 - 2*b*x**3/(15*c) + b*log(x - (
-1/c**2)**(1/4))/(5*c**3*(-1/c**2)**(1/4)) - b*log(x**2 + sqrt(-1/c**2))/(1
0*c**3*(-1/c**2)**(1/4)) + b*atan(x/(-1/c**2)**(1/4))/(5*c**3*(-1/c**2)**(1
/4)) - b*atan(c*x**2)/(5*c**6*(-1/c**2)**(7/4)), Ne(c, 0)), (a*x**5/5, True
))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.91

$$\int x^4\left(a+b\arctan\left(cx^2\right)\right)\,dx = \frac{1}{5}\,ax^5 + \frac{1}{60}\left(12\,x^5\arctan\left(cx^2\right) - c\left(\frac{8\,x^3}{c^2} - \frac{3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\,cx+\sqrt{2}\sqrt{c}\right)}{2\sqrt{c}}\right)}{c^{\frac{3}{2}}} + \frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\,cx-\sqrt{2}\sqrt{c}\right)}{2\sqrt{c}}\right)}{c^{\frac{3}{2}}} - \frac{\sqrt{2}\log\left(cx^2+\cdots\right.}{c^{\frac{3}{2}}}\cdots\right.}{c^2}\cdots\right.\right.$$

```
[In] integrate(x^4*(a+b*arctan(c*x^2)),x, algorithm="maxima")
```

```
[Out] 1/5*a*x^5 + 1/60*(12*x^5*arctan(c*x^2) - c*(8*x^3/c^2 - 3*(2*sqrt(2)*arctan
(1/2*sqrt(2)*(2*c*x + sqrt(2)*sqrt(c))/sqrt(c))/c^(3/2) + 2*sqrt(2)*arctan(
1/2*sqrt(2)*(2*c*x - sqrt(2)*sqrt(c))/sqrt(c))/c^(3/2) - sqrt(2)*log(c*x^2
+ sqrt(2)*sqrt(c)*x + 1)/c^(3/2) + sqrt(2)*log(c*x^2 - sqrt(2)*sqrt(c)*x +
1)/c^(3/2))/c^2))*b
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.05

$$\int x^4\left(a+b\arctan\left(cx^2\right)\right)\,dx$$
$$=\frac{1}{20}\,bc^9\left(\frac{2\,\sqrt{2}\sqrt{|c|}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x+\frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{c^{12}}+\frac{2\,\sqrt{2}\sqrt{|c|}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(2\,x-\frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{c^{12}}-\frac{\sqrt{2}\log}{}\right.$$
$$+\frac{3\,bcx^5\arctan\left(cx^2\right)+3\,acx^5-2\,bx^3}{15\,c}$$

```
[In] integrate(x^4*(a+b*arctan(c*x^2)),x, algorithm="giac")
```

```
[Out] 1/20*b*c^9*(2*sqrt(2)*sqrt(abs(c))*arctan(1/2*sqrt(2)*(2*x + sqrt(2)/sqrt(a
bs(c)))*sqrt(abs(c)))/c^12 + 2*sqrt(2)*sqrt(abs(c))*arctan(1/2*sqrt(2)*(2*x
 - sqrt(2)/sqrt(abs(c)))*sqrt(abs(c)))/c^12 - sqrt(2)*log(x^2 + sqrt(2)*x/s
qrt(abs(c)) + 1/abs(c))/(c^10*abs(c)^(3/2)) + sqrt(2)*sqrt(abs(c))*log(x^2
- sqrt(2)*x/sqrt(abs(c)) + 1/abs(c))/c^12) + 1/15*(3*b*c*x^5*arctan(c*x^2)
+ 3*a*c*x^5 - 2*b*x^3)/c
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.40

$$\int x^4\left(a+b\arctan\left(cx^2\right)\right)\,dx=\frac{a\,x^5}{5}-\frac{2\,b\,x^3}{15\,c}+\frac{b\,x^5\,\mathrm{atan}(c\,x^2)}{5}+\frac{{(-1)}^{1/4}\,b\,\mathrm{atan}\left({(-1)}^{1/4}\,\sqrt{c}\,x\right)}{5\,c^{5/2}}+\frac{{(-1)}^{1/4}\,b\,\mathrm{atan}\left({(-1)}^{1/4}\,\sqrt{c}\,x\,1\mathrm{i}\right)\,1\mathrm{i}}{5\,c^{5/2}}$$

```
[In] int(x^4*(a + b*atan(c*x^2)),x)
```

```
[Out] (a*x^5)/5 - (2*b*x^3)/(15*c) + (b*x^5*atan(c*x^2))/5 + ((-1)^(1/4)*b*atan((
-1)^(1/4)*c^(1/2)*x))/(5*c^(5/2)) + ((-1)^(1/4)*b*atan((-1)^(1/4)*c^(1/2)*x
*1i)*1i)/(5*c^(5/2))
```

3.69 $\int x^2(a+b\arctan(cx^2))\,dx$

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Optimal result

Integrand size = 14, antiderivative size = 159

$$\int x^2(a+b\arctan(cx^2))\,dx = -\frac{2bx}{3c}+\frac{1}{3}x^3(a+b\arctan(cx^2)) - \frac{b\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{3\sqrt{2}c^{3/2}} + \frac{b\arctan\left(1+\sqrt{2}\sqrt{c}x\right)}{3\sqrt{2}c^{3/2}} - \frac{b\log\left(1-\sqrt{2}\sqrt{c}x+cx^2\right)}{6\sqrt{2}c^{3/2}} + \frac{b\log\left(1+\sqrt{2}\sqrt{c}x+cx^2\right)}{6\sqrt{2}c^{3/2}}$$

```
[Out] -2/3*b*x/c+1/3*x^3*(a+b*arctan(c*x^2))+1/6*b*arctan(-1+x*2^(1/2)*c^(1/2))/c
^(3/2)*2^(1/2)+1/6*b*arctan(1+x*2^(1/2)*c^(1/2))/c^(3/2)*2^(1/2)-1/12*b*ln(
1+c*x^2-x*2^(1/2)*c^(1/2))/c^(3/2)*2^(1/2)+1/12*b*ln(1+c*x^2+x*2^(1/2)*c^(1
/2))/c^(3/2)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {4946, 327, 217, 1179, 642, 1176, 631, 210}

$$\int x^2(a+b\arctan(cx^2))\,dx = \frac{1}{3}x^3(a+b\arctan(cx^2)) - \frac{b\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{3\sqrt{2}c^{3/2}} + \frac{b\arctan\left(\sqrt{2}\sqrt{c}x+1\right)}{3\sqrt{2}c^{3/2}} - \frac{b\log\left(cx^2-\sqrt{2}\sqrt{c}x+1\right)}{6\sqrt{2}c^{3/2}} + \frac{b\log\left(cx^2+\sqrt{2}\sqrt{c}x+1\right)}{6\sqrt{2}c^{3/2}} - \frac{2bx}{3c}$$

```
[In] Int[x^2*(a + b*ArcTan[c*x^2]),x]
```

```
[Out] (-2*b*x)/(3*c) + (x^3*(a + b*ArcTan[c*x^2]))/3 - (b*ArcTan[1 - Sqrt[2]*Sqrt
[c]*x])/(3*Sqrt[2]*c^(3/2)) + (b*ArcTan[1 + Sqrt[2]*Sqrt[c]*x])/(3*Sqrt[2]*
c^(3/2)) - (b*Log[1 - Sqrt[2]*Sqrt[c]*x + c*x^2])/(6*Sqrt[2]*c^(3/2)) + (b*
Log[1 + Sqrt[2]*Sqrt[c]*x + c*x^2])/(6*Sqrt[2]*c^(3/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\left(a+b\arctan\left(cx^2\right)\right)-\frac{1}{3}(2bc)\int\frac{x^4}{1+c^2x^4}\,dx\\
&= -\frac{2bx}{3c}+\frac{1}{3}x^3\left(a+b\arctan\left(cx^2\right)\right)+\frac{(2b)\int\frac{1}{1+c^2x^4}\,dx}{3c}\\
&= -\frac{2bx}{3c}+\frac{1}{3}x^3\left(a+b\arctan\left(cx^2\right)\right)+\frac{b\int\frac{1-cx^2}{1+c^2x^4}\,dx}{3c}+\frac{b\int\frac{1+cx^2}{1+c^2x^4}\,dx}{3c}\\
&= -\frac{2bx}{3c}+\frac{1}{3}x^3\left(a+b\arctan\left(cx^2\right)\right)+\frac{b\int\frac{1}{\frac{1}{c}-\frac{\sqrt{2}x}{\sqrt{c}}+x^2}\,dx}{6c^2}\\
&\quad+\frac{b\int\frac{1}{\frac{1}{c}+\frac{\sqrt{2}x}{\sqrt{c}}+x^2}\,dx}{6c^2}-\frac{b\int\frac{\frac{\sqrt{2}}{\sqrt{c}}+2x}{-\frac{1}{c}-\frac{\sqrt{2}x}{\sqrt{c}}-x^2}\,dx}{6\sqrt{2}c^{3/2}}-\frac{b\int\frac{\frac{\sqrt{2}}{\sqrt{c}}-2x}{-\frac{1}{c}+\frac{\sqrt{2}x}{\sqrt{c}}-x^2}\,dx}{6\sqrt{2}c^{3/2}}\\
&= -\frac{2bx}{3c}+\frac{1}{3}x^3\left(a+b\arctan\left(cx^2\right)\right)-\frac{b\log\left(1-\sqrt{2}\sqrt{c}x+cx^2\right)}{6\sqrt{2}c^{3/2}}\\
&\quad+\frac{b\log\left(1+\sqrt{2}\sqrt{c}x+cx^2\right)}{6\sqrt{2}c^{3/2}}+\frac{b\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}\sqrt{c}x\right)}{3\sqrt{2}c^{3/2}}\\
&\quad-\frac{b\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\sqrt{2}\sqrt{c}x\right)}{3\sqrt{2}c^{3/2}}\\
&= -\frac{2bx}{3c}+\frac{1}{3}x^3\left(a+b\arctan\left(cx^2\right)\right)-\frac{b\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{3\sqrt{2}c^{3/2}}\\
&\quad+\frac{b\arctan\left(1+\sqrt{2}\sqrt{c}x\right)}{3\sqrt{2}c^{3/2}}-\frac{b\log\left(1-\sqrt{2}\sqrt{c}x+cx^2\right)}{6\sqrt{2}c^{3/2}}+\frac{b\log\left(1+\sqrt{2}\sqrt{c}x+cx^2\right)}{6\sqrt{2}c^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.11

$$\int x^2\left(a+b\arctan\left(cx^2\right)\right)\,dx=-\frac{2bx}{3c}+\frac{ax^3}{3}+\frac{1}{3}bx^3\arctan\left(cx^2\right)+\frac{b\arctan\left(\frac{-\sqrt{2}+2\sqrt{c}x}{\sqrt{2}}\right)}{3\sqrt{2}c^{3/2}}+\frac{b\arctan\left(\frac{\sqrt{2}+2\sqrt{c}x}{\sqrt{2}}\right)}{3\sqrt{2}c^{3/2}}-\frac{b\log\left(1-\sqrt{2}\sqrt{c}x+cx^2\right)}{6\sqrt{2}c^{3/2}}+\frac{b\log\left(1+\sqrt{2}\sqrt{c}x+cx^2\right)}{6\sqrt{2}c^{3/2}}$$

```
[In] Integrate[x^2*(a + b*ArcTan[c*x^2]),x]
```

```
[Out] (-2*b*x)/(3*c) + (a*x^3)/3 + (b*x^3*ArcTan[c*x^2])/3 + (b*ArcTan[(-Sqrt[2]
 + 2*Sqrt[c]*x)/Sqrt[2]])/(3*Sqrt[2]*c^(3/2)) + (b*ArcTan[(Sqrt[2] + 2*Sqrt[
c]*x)/Sqrt[2]])/(3*Sqrt[2]*c^(3/2)) - (b*Log[1 - Sqrt[2]*Sqrt[c]*x + c*x^2]
)/(6*Sqrt[2]*c^(3/2)) + (b*Log[1 + Sqrt[2]*Sqrt[c]*x + c*x^2])/(6*Sqrt[2]*c
^(3/2))
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.74

method	result
default	$\frac{x^3 a}{3} + b\left(\frac{x^3\arctan(c x^2)}{3} - \frac{2c\left(\frac{x}{c^2} - \frac{\left(\frac{1}{c^2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{1}{c^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^2}}}{x^2-\left(\frac{1}{c^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}-1\right)\right)}{8c^2}\right)}{3}\right)$
parts	$\frac{x^3 a}{3} + b\left(\frac{x^3\arctan(c x^2)}{3} - \frac{2c\left(\frac{x}{c^2} - \frac{\left(\frac{1}{c^2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{1}{c^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^2}}}{x^2-\left(\frac{1}{c^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}-1\right)\right)}{8c^2}\right)}{3}\right)$

```
[In] int(x^2*(a+b*arctan(c*x^2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^3*a+b*(1/3*x^3*arctan(c*x^2)-2/3*c*(1/c^2*x-1/8/c^2*(1/c^2)^(1/4)*2^(
1/2)*(ln((x^2+(1/c^2)^(1/4)*x*2^(1/2)+(1/c^2)^(1/2))/(x^2-(1/c^2)^(1/4)*x*2
^(1/2)+(1/c^2)^(1/2)))+2*arctan(2^(1/2)/(1/c^2)^(1/4)*x+1)+2*arctan(2^(1/2)
/(1/c^2)^(1/4)*x-1))))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.96

$$\int x^2\left(a+b\arctan\left(cx^2\right)\right)\,dx$$

$$= \frac{2\,bcx^3\arctan\left(cx^2\right)+2\,acx^3+c\left(-\frac{b^4}{c^6}\right)^{\frac{1}{4}}\log\left(bx+c\left(-\frac{b^4}{c^6}\right)^{\frac{1}{4}}\right)+i\,c\left(-\frac{b^4}{c^6}\right)^{\frac{1}{4}}\log\left(bx+i\,c\left(-\frac{b^4}{c^6}\right)^{\frac{1}{4}}\right)-i\,c\left(-\right.}{6\,c}$$

```
[In] integrate(x^2*(a+b*arctan(c*x^2)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b*c*x^3*arctan(c*x^2) + 2*a*c*x^3 + c*(-b^4/c^6)^(1/4)*log(b*x + c*(
-b^4/c^6)^(1/4)) + I*c*(-b^4/c^6)^(1/4)*log(b*x + I*c*(-b^4/c^6)^(1/4)) - I
*c*(-b^4/c^6)^(1/4)*log(b*x - I*c*(-b^4/c^6)^(1/4)) - c*(-b^4/c^6)^(1/4)*lo
g(b*x - c*(-b^4/c^6)^(1/4)) - 4*b*x)/c
```

Sympy [A] (verification not implemented)

Time = 7.49 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.90

$$\int x^2(a+b\arctan(cx^2))\,dx = \begin{cases} \frac{ax^3}{3}+\frac{bx^3\operatorname{atan}(cx^2)}{3}+\frac{b\left(-\frac{1}{c^2}\right)^{\frac{3}{4}}\operatorname{atan}(cx^2)}{3}-\frac{2bx}{3c}-\frac{b\sqrt[4]{-\frac{1}{c^2}}\log\left(x-\sqrt[4]{-\frac{1}{c^2}}\right)}{3c}+\frac{b\sqrt[4]{-\frac{1}{c^2}}\log\left(x^2+\sqrt{-\frac{1}{c^2}}\right)}{6c}+\frac{b\sqrt[4]{-\frac{1}{c^2}}\operatorname{atan}\ldots}{3\ldots} \\ \frac{ax^3}{3} \end{cases}$$

```
[In] integrate(x**2*(a+b*atan(c*x**2)),x)
```

```
[Out] Piecewise((a*x**3/3 + b*x**3*atan(c*x**2)/3 + b*(-1/c**2)**(3/4)*atan(c*x**
2)/3 - 2*b*x/(3*c) - b*(-1/c**2)**(1/4)*log(x - (-1/c**2)**(1/4))/(3*c) + b
*(-1/c**2)**(1/4)*log(x**2 + sqrt(-1/c**2))/(6*c) + b*(-1/c**2)**(1/4)*atan
(x/(-1/c**2)**(1/4))/(3*c), Ne(c, 0)), (a*x**3/3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.91

$$\int x^2(a+b\arctan(cx^2))\,dx = \frac{1}{3}\,ax^3 + \frac{1}{12}\left(4\,x^3\arctan(cx^2) - c\left(\frac{8\,x}{c^2} - \frac{\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}(2\,cx+\sqrt{2}\sqrt{c})}{2\sqrt{c}}\right)}{\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}(2\,cx-\sqrt{2}\sqrt{c})}{2\sqrt{c}}\right)}{\sqrt{c}}+\frac{\sqrt{2}\log\left(cx^2+\sqrt{2}\sqrt{c}x+1\ldots\right)}{\sqrt{c}}\ldots}{c^2}\right)\right)$$

```
[In] integrate(x^2*(a+b*arctan(c*x^2)),x, algorithm="maxima")
```

```
[Out] 1/3*a*x^3 + 1/12*(4*x^3*arctan(c*x^2) - c*(8*x/c^2 - (2*sqrt(2)*arctan(1/2*
sqrt(2)*(2*c*x + sqrt(2)*sqrt(c))/sqrt(c))/sqrt(c) + 2*sqrt(2)*arctan(1/2*s
qrt(2)*(2*c*x - sqrt(2)*sqrt(c))/sqrt(c))/sqrt(c) + sqrt(2)*log(c*x^2 + sqr
t(2)*sqrt(c)*x + 1)/sqrt(c) - sqrt(2)*log(c*x^2 - sqrt(2)*sqrt(c)*x + 1)/sq
rt(c))/c^2))*b
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.04

$$\int x^2 \left(a + b \arctan\left(cx^2\right)\right) \, dx$$
$$= \frac{1}{12} bc^5 \left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x + \frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{c^6\sqrt{|c|}} + \frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x - \frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{c^6\sqrt{|c|}} + \frac{\sqrt{2}\log\left(x^2 + \frac{\sqrt{}}{\sqrt{}}\right.}{c^6\sqrt{|c|}} \right.$$
$$+ \frac{bcx^3 \arctan\left(cx^2\right) + acx^3 - 2bx}{3c}$$

```
[In] integrate(x^2*(a+b*arctan(c*x^2)),x, algorithm="giac")
```

```
[Out] 1/12*b*c^5*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)/sqrt(abs(c)))*sqrt(
abs(c)))/(c^6*sqrt(abs(c))) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)/s
qrt(abs(c)))*sqrt(abs(c)))/(c^6*sqrt(abs(c))) + sqrt(2)*log(x^2 + sqrt(2)*x
/sqrt(abs(c)) + 1/abs(c))/(c^6*sqrt(abs(c))) - sqrt(2)*log(x^2 - sqrt(2)*x/
sqrt(abs(c)) + 1/abs(c))/(c^6*sqrt(abs(c)))) + 1/3*(b*c*x^3*arctan(c*x^2) +
 a*c*x^3 - 2*b*x)/c
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.39

$$\int x^2 \left(a + b \arctan\left(cx^2\right)\right) \, dx = \frac{a\,x^3}{3} + \frac{b\,x^3\,\mathrm{atan}(c\,x^2)}{3} - \frac{2\,b\,x}{3\,c} - \frac{(-1)^{1/4}\,b\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{c}\,x\right)\,1\mathrm{i}}{3\,c^{3/2}} - \frac{(-1)^{1/4}\,b\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{c}\,x\,1\mathrm{i}\right)}{3\,c^{3/2}}$$

```
[In] int(x^2*(a + b*atan(c*x^2)),x)
```

```
[Out] (a*x^3)/3 + (b*x^3*atan(c*x^2))/3 - (2*b*x)/(3*c) - ((-1)^(1/4)*b*atan((-1)
^(1/4)*c^(1/2)*x)*1i)/(3*c^(3/2)) - ((-1)^(1/4)*b*atan((-1)^(1/4)*c^(1/2)*x
*1i))/(3*c^(3/2))
```

3.70 $\int (a+b\arctan(cx^2))\,dx$

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Optimal result

Integrand size = 10, antiderivative size = 140

$$\int (a+b\arctan(cx^2))\,dx = ax+bx\arctan(cx^2)+\frac{b\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}} - \frac{b\arctan\left(1+\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}} - \frac{b\log\left(1-\sqrt{2}\sqrt{c}x+cx^2\right)}{2\sqrt{2}\sqrt{c}} + \frac{b\log\left(1+\sqrt{2}\sqrt{c}x+cx^2\right)}{2\sqrt{2}\sqrt{c}}$$

```
[Out] a*x+b*x*arctan(c*x^2)-1/2*b*arctan(-1+x*2^(1/2)*c^(1/2))*2^(1/2)/c^(1/2)-1/
2*b*arctan(1+x*2^(1/2)*c^(1/2))*2^(1/2)/c^(1/2)-1/4*b*ln(1+c*x^2-x*2^(1/2)*
c^(1/2))*2^(1/2)/c^(1/2)+1/4*b*ln(1+c*x^2+x*2^(1/2)*c^(1/2))*2^(1/2)/c^(1/2
)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {4930, 303, 1176, 631, 210, 1179, 642}

$$\int (a+b\arctan(cx^2))\,dx = ax+bx\arctan(cx^2)+\frac{b\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}} - \frac{b\arctan\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{2}\sqrt{c}} - \frac{b\log\left(cx^2-\sqrt{2}\sqrt{c}x+1\right)}{2\sqrt{2}\sqrt{c}} + \frac{b\log\left(cx^2+\sqrt{2}\sqrt{c}x+1\right)}{2\sqrt{2}\sqrt{c}}$$

```
[In] Int[a + b*ArcTan[c*x^2],x]
```

```
[Out] a*x + b*x*ArcTan[c*x^2] + (b*ArcTan[1 - Sqrt[2]*Sqrt[c]*x])/(Sqrt[2]*Sqrt[c
]) - (b*ArcTan[1 + Sqrt[2]*Sqrt[c]*x])/(Sqrt[2]*Sqrt[c]) - (b*Log[1 - Sqrt[
2]*Sqrt[c]*x + c*x^2])/(2*Sqrt[2]*Sqrt[c]) + (b*Log[1 + Sqrt[2]*Sqrt[c]*x +
 c*x^2])/(2*Sqrt[2]*Sqrt[c])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= ax + b\int \arctan\left(cx^2\right)\,dx\\
&= ax + bx\arctan\left(cx^2\right) - (2bc)\int \frac{x^2}{1+c^2x^4}\,dx\\
&= ax + bx\arctan\left(cx^2\right) + b\int \frac{1-cx^2}{1+c^2x^4}\,dx - b\int \frac{1+cx^2}{1+c^2x^4}\,dx\\
&= ax + bx\arctan\left(cx^2\right) - \frac{b\int \frac{1}{\frac{1}{c}-\frac{\sqrt{2}x}{\sqrt{c}}+x^2}\,dx}{2c} - \frac{b\int \frac{1}{\frac{1}{c}+\frac{\sqrt{2}x}{\sqrt{c}}+x^2}\,dx}{2c}\\
&\quad - \frac{b\int \frac{\frac{\sqrt{2}}{\sqrt{c}}+2x}{-\frac{1}{c}-\frac{\sqrt{2}x}{\sqrt{c}}-x^2}\,dx}{2\sqrt{2}\sqrt{c}} - \frac{b\int \frac{\frac{\sqrt{2}}{\sqrt{c}}-2x}{-\frac{1}{c}+\frac{\sqrt{2}x}{\sqrt{c}}-x^2}\,dx}{2\sqrt{2}\sqrt{c}}\\
&= ax + bx\arctan\left(cx^2\right) - \frac{b\log\left(1-\sqrt{2}\sqrt{c}x+cx^2\right)}{2\sqrt{2}\sqrt{c}} + \frac{b\log\left(1+\sqrt{2}\sqrt{c}x+cx^2\right)}{2\sqrt{2}\sqrt{c}}\\
&\quad - \frac{b\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}} + \frac{b\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}\\
&= ax + bx\arctan\left(cx^2\right) + \frac{b\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}} - \frac{b\arctan\left(1+\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}\sqrt{c}}\\
&\quad - \frac{b\log\left(1-\sqrt{2}\sqrt{c}x+cx^2\right)}{2\sqrt{2}\sqrt{c}} + \frac{b\log\left(1+\sqrt{2}\sqrt{c}x+cx^2\right)}{2\sqrt{2}\sqrt{c}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.76

$$\int \left(a+b\arctan\left(cx^2\right)\right)\,dx = ax + bx\arctan\left(cx^2\right) - \frac{b\left(-2\arctan\left(1-\sqrt{2}\sqrt{c}x\right)+2\arctan\left(1+\sqrt{2}\sqrt{c}x\right)+\log\left(1-\sqrt{2}\sqrt{c}x+cx^2\right)-\log\left(1+\sqrt{2}\sqrt{c}x+cx^2\right)\right)}{2\sqrt{2}\sqrt{c}}$$

```
[In] Integrate[a + b*ArcTan[c*x^2],x]
[Out] a*x + b*x*ArcTan[c*x^2] - (b*(-2*ArcTan[1 - Sqrt[2]*Sqrt[c]*x] + 2*ArcTan[1
 + Sqrt[2]*Sqrt[c]*x] + Log[1 - Sqrt[2]*Sqrt[c]*x + c*x^2] - Log[1 + Sqrt[2
]*Sqrt[c]*x + c*x^2]))/(2*Sqrt[2]*Sqrt[c])
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.74

method	result	siz
default	$ax+b\left(x\arctan\left(c\,x^{2}\right)-\frac{\sqrt{2}\left(\ln\left(\frac{x^{2}-\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^{2}}}}{x^{2}+\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^{2}}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}}-1\right)\right)}{4c\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}}\right)$	103
parts	$ax+b\left(x\arctan\left(c\,x^{2}\right)-\frac{\sqrt{2}\left(\ln\left(\frac{x^{2}-\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^{2}}}}{x^{2}+\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^{2}}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}}-1\right)\right)}{4c\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}}\right)$	103

```
[In] int(a+b*arctan(c*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] a*x+b*(x*arctan(c*x^2)-1/4/c/(1/c^2)^(1/4)*2^(1/2)*(ln((x^2-(1/c^2)^(1/4)*x
*2^(1/2)+(1/c^2)^(1/2))/(x^2+(1/c^2)^(1/4)*x*2^(1/2)+(1/c^2)^(1/2)))+2*arct
an(2^(1/2)/(1/c^2)^(1/4)*x+1)+2*arctan(2^(1/2)/(1/c^2)^(1/4)*x-1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00

$$\begin{aligned}\int \left(a+b\arctan\left(cx^{2}\right)\right)\,dx &= bx\arctan\left(cx^{2}\right)+ax-\frac{1}{2}\left(-\frac{b^{4}}{c^{2}}\right)^{\frac{1}{4}}\log\left(b^{3}x+\left(-\frac{b^{4}}{c^{2}}\right)^{\frac{3}{4}}c\right)\\ &\quad+\frac{1}{2}i\left(-\frac{b^{4}}{c^{2}}\right)^{\frac{1}{4}}\log\left(b^{3}x+i\left(-\frac{b^{4}}{c^{2}}\right)^{\frac{3}{4}}c\right)\\ &\quad-\frac{1}{2}i\left(-\frac{b^{4}}{c^{2}}\right)^{\frac{1}{4}}\log\left(b^{3}x-i\left(-\frac{b^{4}}{c^{2}}\right)^{\frac{3}{4}}c\right)\\ &\quad+\frac{1}{2}\left(-\frac{b^{4}}{c^{2}}\right)^{\frac{1}{4}}\log\left(b^{3}x-\left(-\frac{b^{4}}{c^{2}}\right)^{\frac{3}{4}}c\right)\end{aligned}$$

```
[In] integrate(a+b*arctan(c*x^2),x, algorithm="fricas")
```

```
[Out] b*x*arctan(c*x^2) + a*x - 1/2*(-b^4/c^2)^(1/4)*log(b^3*x + (-b^4/c^2)^(3/4)
*c) + 1/2*I*(-b^4/c^2)^(1/4)*log(b^3*x + I*(-b^4/c^2)^(3/4)*c) - 1/2*I*(-b^
4/c^2)^(1/4)*log(b^3*x - I*(-b^4/c^2)^(3/4)*c) + 1/2*(-b^4/c^2)^(1/4)*log(b
^3*x - (-b^4/c^2)^(3/4)*c)
```

Sympy [A] (verification not implemented)

Time = 4.34 (sec) , antiderivative size = 617, normalized size of antiderivative = 4.41

$$\int \left(a+b\arctan\left(cx^2\right)\right)\,dx=ax+b\left(\begin{cases}0\\ -\infty ix\\ \infty ix\\ \frac{2c^5x^5\left(-\frac{1}{c^2}\right)^{\frac{7}{4}}\operatorname{atan}\left(cx^2\right)}{2c^5x^4\left(-\frac{1}{c^2}\right)^{\frac{7}{4}}+2c^3\left(-\frac{1}{c^2}\right)^{\frac{7}{4}}}-\frac{2c^4x^4\left(-\frac{1}{c^2}\right)^{\frac{3}{2}}\log\left(x-\sqrt[4]{-\frac{1}{c^2}}\right)}{2c^5x^4\left(-\frac{1}{c^2}\right)^{\frac{7}{4}}+2c^3\left(-\frac{1}{c^2}\right)^{\frac{7}{4}}}+\frac{c^4x^4\left(-\frac{1}{c^2}\right)^{\frac{3}{2}}\log\left(x^2+\sqrt{-\frac{1}{c^2}}\right)}{2c^5x^4\left(-\frac{1}{c^2}\right)^{\frac{7}{4}}+2c^3\left(-\frac{1}{c^2}\right)^{\frac{7}{4}}}-\frac{2c^4x^4\left(-\frac{1}{c^2}\right)^{\frac{3}{2}}\operatorname{atan}\left(\frac{\ldots}{\sqrt[4]{\ldots}}\right)}{2c^5x^4\left(-\frac{1}{c^2}\right)^{\frac{7}{4}}+2c^3\left(-\ldots\right.}\end{cases}\right.$$

```
[In] integrate(a+b*atan(c*x**2),x)
```

```
[Out] a*x + b*Piecewise((0, Eq(c, 0)), (-oo*I*x, Eq(c, -I/x**2)), (oo*I*x, Eq(c,
I/x**2)), (2*c**5*x**5*(-1/c**2)**(7/4)*atan(c*x**2)/(2*c**5*x**4*(-1/c**2)
**(7/4) + 2*c**3*(-1/c**2)**(7/4)) - 2*c**4*x**4*(-1/c**2)**(3/2)*log(x - (
-1/c**2)**(1/4))/(2*c**5*x**4*(-1/c**2)**(7/4) + 2*c**3*(-1/c**2)**(7/4)) +
 c**4*x**4*(-1/c**2)**(3/2)*log(x**2 + sqrt(-1/c**2))/(2*c**5*x**4*(-1/c**2
)**(7/4) + 2*c**3*(-1/c**2)**(7/4)) - 2*c**4*x**4*(-1/c**2)**(3/2)*atan(x/(
-1/c**2)**(1/4))/(2*c**5*x**4*(-1/c**2)**(7/4) + 2*c**3*(-1/c**2)**(7/4)) +
 2*c**3*x*(-1/c**2)**(7/4)*atan(c*x**2)/(2*c**5*x**4*(-1/c**2)**(7/4) + 2*c
**3*(-1/c**2)**(7/4)) - 2*c**2*(-1/c**2)**(3/2)*log(x - (-1/c**2)**(1/4))/(
2*c**5*x**4*(-1/c**2)**(7/4) + 2*c**3*(-1/c**2)**(7/4)) + c**2*(-1/c**2)**(
3/2)*log(x**2 + sqrt(-1/c**2))/(2*c**5*x**4*(-1/c**2)**(7/4) + 2*c**3*(-1/c
**2)**(7/4)) - 2*c**2*(-1/c**2)**(3/2)*atan(x/(-1/c**2)**(1/4))/(2*c**5*x**
4*(-1/c**2)**(7/4) + 2*c**3*(-1/c**2)**(7/4)) + 2*c*x**4*atan(c*x**2)/(2*c*
*5*x**4*(-1/c**2)**(7/4) + 2*c**3*(-1/c**2)**(7/4)) + 2*atan(c*x**2)/(2*c**
6*x**4*(-1/c**2)**(7/4) + 2*c**4*(-1/c**2)**(7/4)), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.91

$$\int \left(a+b\arctan\left(cx^2\right)\right)\,dx=-\frac{1}{4}\left(c\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2cx+\sqrt{2}\sqrt{c}\right)}{2\sqrt{c}}\right)}{c^{\frac{3}{2}}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2cx-\sqrt{2}\sqrt{c}\right)}{2\sqrt{c}}\right)}{c^{\frac{3}{2}}}-\frac{\sqrt{2}\log\left(cx^2+\sqrt{2}\sqrt{c}x+1\right)}{c^{\frac{3}{2}}}+\ldots\right.\right.$$
$$+ax$$

```
[In] integrate(a+b*arctan(c*x^2),x, algorithm="maxima")
```

```
[Out] -1/4*(c*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*c*x + sqrt(2)*sqrt(c))/sqrt(c))/c^
(3/2) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*c*x - sqrt(2)*sqrt(c))/sqrt(c))/c^(
3/2) - sqrt(2)*log(c*x^2 + sqrt(2)*sqrt(c)*x + 1)/c^(3/2) + sqrt(2)*log(c*x
^2 - sqrt(2)*sqrt(c)*x + 1)/c^(3/2)) - 4*x*arctan(c*x^2))*b + a*x
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.06

$$\int \left(a+b\arctan\left(cx^2\right)\right)\,dx = -\frac{1}{4}\left(c\left(\frac{2\sqrt{2}\sqrt{|c|}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{c^2}+\frac{2\sqrt{2}\sqrt{|c|}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{c^2}-\frac{\sqrt{2}\sqrt{}\right.\right. + ax$$

```
[In] integrate(a+b*arctan(c*x^2),x, algorithm="giac")
```

```
[Out] -1/4*(c*(2*sqrt(2)*sqrt(abs(c))*arctan(1/2*sqrt(2)*(2*x + sqrt(2)/sqrt(abs(
c)))*sqrt(abs(c)))/c^2 + 2*sqrt(2)*sqrt(abs(c))*arctan(1/2*sqrt(2)*(2*x - s
qrt(2)/sqrt(abs(c)))*sqrt(abs(c)))/c^2 - sqrt(2)*sqrt(abs(c))*log(x^2 + sqr
t(2)*x/sqrt(abs(c)) + 1/abs(c))/c^2 + sqrt(2)*sqrt(abs(c))*log(x^2 - sqrt(2
)*x/sqrt(abs(c)) + 1/abs(c))/c^2) - 4*x*arctan(c*x^2))*b + a*x
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.35

$$\int \left(a+b\arctan\left(cx^2\right)\right)\,dx = a\,x+b\,x\,\mathrm{atan}\left(c\,x^2\right)-\frac{{\left(-1\right)}^{1/4}\,b\,\mathrm{atan}\left({\left(-1\right)}^{1/4}\,\sqrt{c}\,x\right)}{\sqrt{c}}-\frac{{\left(-1\right)}^{1/4}\,b\,\mathrm{atan}\left({\left(-1\right)}^{1/4}\,\sqrt{c}\,x\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{\sqrt{c}}$$

```
[In] int(a + b*atan(c*x^2),x)
```

```
[Out] a*x + b*x*atan(c*x^2) - ((-1)^(1/4)*b*atan((-1)^(1/4)*c^(1/2)*x))/c^(1/2) -
 ((-1)^(1/4)*b*atan((-1)^(1/4)*c^(1/2)*x*1i)*1i)/c^(1/2)
```

3.71 $\int \frac{a+b\arctan(cx^2)}{x^2}\,dx$

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Sympy [A] (verification not implemented) 396
Maxima [A] (verification not implemented) 396
Giac [A] (verification not implemented) 397
Mupad [B] (verification not implemented) 397

Optimal result

Integrand size = 14, antiderivative size = 143

$$\int \frac{a+b\arctan(cx^2)}{x^2}\,dx = -\frac{a+b\arctan(cx^2)}{x} - \frac{b\sqrt{c}\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}} + \frac{b\sqrt{c}\arctan\left(1+\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}} - \frac{b\sqrt{c}\log\left(1-\sqrt{2}\sqrt{c}x+cx^2\right)}{2\sqrt{2}} + \frac{b\sqrt{c}\log\left(1+\sqrt{2}\sqrt{c}x+cx^2\right)}{2\sqrt{2}}$$

```
[Out] (-a-b*arctan(c*x^2))/x+1/2*b*arctan(-1+x*2^(1/2)*c^(1/2))*c^(1/2)*2^(1/2)+1
/2*b*arctan(1+x*2^(1/2)*c^(1/2))*c^(1/2)*2^(1/2)-1/4*b*ln(1+c*x^2-x*2^(1/2)
*c^(1/2))*c^(1/2)*2^(1/2)+1/4*b*ln(1+c*x^2+x*2^(1/2)*c^(1/2))*c^(1/2)*2^(1/
2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4946, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{a+b\arctan(cx^2)}{x^2}\,dx = -\frac{a+b\arctan(cx^2)}{x} - \frac{b\sqrt{c}\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}} + \frac{b\sqrt{c}\arctan\left(\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{2}} - \frac{b\sqrt{c}\log\left(cx^2-\sqrt{2}\sqrt{c}x+1\right)}{2\sqrt{2}} + \frac{b\sqrt{c}\log\left(cx^2+\sqrt{2}\sqrt{c}x+1\right)}{2\sqrt{2}}$$

```
[In] Int[(a + b*ArcTan[c*x^2])/x^2,x]
```

```
[Out] -((a + b*ArcTan[c*x^2])/x) - (b*Sqrt[c]*ArcTan[1 - Sqrt[2]*Sqrt[c]*x])/Sqrt
[2] + (b*Sqrt[c]*ArcTan[1 + Sqrt[2]*Sqrt[c]*x])/Sqrt[2] - (b*Sqrt[c]*Log[1
- Sqrt[2]*Sqrt[c]*x + c*x^2])/(2*Sqrt[2]) + (b*Sqrt[c]*Log[1 + Sqrt[2]*Sqrt
[c]*x + c*x^2])/(2*Sqrt[2])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{a+b\arctan(cx^2)}{x}+(2bc)\int\frac{1}{1+c^2x^4}\,dx\\
&= -\frac{a+b\arctan(cx^2)}{x}+(bc)\int\frac{1-cx^2}{1+c^2x^4}\,dx+(bc)\int\frac{1+cx^2}{1+c^2x^4}\,dx\\
&= -\frac{a+b\arctan(cx^2)}{x}+\frac{1}{2}b\int\frac{1}{\frac{1}{c}-\frac{\sqrt{2}x}{\sqrt{c}}+x^2}\,dx+\frac{1}{2}b\int\frac{1}{\frac{1}{c}+\frac{\sqrt{2}x}{\sqrt{c}}+x^2}\,dx\\
&\quad-\frac{\left(b\sqrt{c}\right)\int\frac{\frac{\sqrt{2}}{\sqrt{c}}+2x}{-\frac{1}{c}-\frac{\sqrt{2}x}{\sqrt{c}}-x^2}\,dx}{2\sqrt{2}}-\frac{\left(b\sqrt{c}\right)\int\frac{\frac{\sqrt{2}}{\sqrt{c}}-2x}{-\frac{1}{c}+\frac{\sqrt{2}x}{\sqrt{c}}-x^2}\,dx}{2\sqrt{2}}\\
&= -\frac{a+b\arctan(cx^2)}{x}-\frac{b\sqrt{c}\log\left(1-\sqrt{2}\sqrt{c}x+cx^2\right)}{2\sqrt{2}}+\frac{b\sqrt{c}\log\left(1+\sqrt{2}\sqrt{c}x+cx^2\right)}{2\sqrt{2}}\\
&\quad+\frac{\left(b\sqrt{c}\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}}-\frac{\left(b\sqrt{c}\right)\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}}\\
&= -\frac{a+b\arctan(cx^2)}{x}-\frac{b\sqrt{c}\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}}+\frac{b\sqrt{c}\arctan\left(1+\sqrt{2}\sqrt{c}x\right)}{\sqrt{2}}\\
&\quad-\frac{b\sqrt{c}\log\left(1-\sqrt{2}\sqrt{c}x+cx^2\right)}{2\sqrt{2}}+\frac{b\sqrt{c}\log\left(1+\sqrt{2}\sqrt{c}x+cx^2\right)}{2\sqrt{2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.10

$$
\begin{aligned}
\int\frac{a+b\arctan(cx^2)}{x^2}\,dx &= -\frac{a}{x}-\frac{b\arctan(cx^2)}{x}+\frac{b\sqrt{c}\arctan\left(\frac{-\sqrt{2}+2\sqrt{c}x}{\sqrt{2}}\right)}{\sqrt{2}}\\
&\quad+\frac{b\sqrt{c}\arctan\left(\frac{\sqrt{2}+2\sqrt{c}x}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{b\sqrt{c}\log\left(1-\sqrt{2}\sqrt{c}x+cx^2\right)}{2\sqrt{2}}\\
&\quad+\frac{b\sqrt{c}\log\left(1+\sqrt{2}\sqrt{c}x+cx^2\right)}{2\sqrt{2}}
\end{aligned}
$$

```
[In] Integrate[(a + b*ArcTan[c*x^2])/x^2,x]
```

```
[Out] -(a/x) - (b*ArcTan[c*x^2])/x + (b*Sqrt[c]*ArcTan[(-Sqrt[2] + 2*Sqrt[c]*x)/S
qrt[2]])/Sqrt[2] + (b*Sqrt[c]*ArcTan[(Sqrt[2] + 2*Sqrt[c]*x)/Sqrt[2]])/Sqrt
[2] - (b*Sqrt[c]*Log[1 - Sqrt[2]*Sqrt[c]*x + c*x^2])/(2*Sqrt[2]) + (b*Sqrt[
c]*Log[1 + Sqrt[2]*Sqrt[c]*x + c*x^2])/(2*Sqrt[2])
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.75

method	result
default	$-\frac{a}{x}+b\left(-\frac{\arctan(c\,x^{2})}{x}+\frac{c\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^{2}}}}{x^{2}-\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^{2}}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}}-1\right)\right)}{4}\right)$
parts	$-\frac{a}{x}+b\left(-\frac{\arctan(c\,x^{2})}{x}+\frac{c\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^{2}}}}{x^{2}-\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^{2}}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}}-1\right)\right)}{4}\right)$

```
[In] int((a+b*arctan(c*x^2))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a/x+b*(-1/x*arctan(c*x^2)+1/4*c*(1/c^2)^(1/4)*2^(1/2)*(ln((x^2+(1/c^2)^(1/
4)*x*2^(1/2)+(1/c^2)^(1/2))/(x^2-(1/c^2)^(1/4)*x*2^(1/2)+(1/c^2)^(1/2)))+2*
arctan(2^(1/2)/(1/c^2)^(1/4)*x+1)+2*arctan(2^(1/2)/(1/c^2)^(1/4)*x-1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.97

$$\int \frac{a+b\arctan\left(cx^{2}\right)}{x^{2}}\,dx$$
$$=\frac{\left(-b^{4}c^{2}\right)^{\frac{1}{4}}x\log\left(bcx+\left(-b^{4}c^{2}\right)^{\frac{1}{4}}\right)+i\left(-b^{4}c^{2}\right)^{\frac{1}{4}}x\log\left(bcx+i\left(-b^{4}c^{2}\right)^{\frac{1}{4}}\right)-i\left(-b^{4}c^{2}\right)^{\frac{1}{4}}x\log\left(bcx-i\left(-b^{4}c^{2}\right.\right.}{2\,x}$$

```
[In] integrate((a+b*arctan(c*x^2))/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*((-b^4*c^2)^(1/4)*x*log(b*c*x + (-b^4*c^2)^(1/4)) + I*(-b^4*c^2)^(1/4)*
x*log(b*c*x + I*(-b^4*c^2)^(1/4)) - I*(-b^4*c^2)^(1/4)*x*log(b*c*x - I*(-b^
4*c^2)^(1/4)) - (-b^4*c^2)^(1/4)*x*log(b*c*x - (-b^4*c^2)^(1/4)) - 2*b*arct
an(c*x^2) - 2*a)/x
```

Sympy [A] (verification not implemented)

Time = 8.53 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.85

$$\int \frac{a + b\arctan(cx^2)}{x^2}\,dx$$
$$= \begin{cases} -\frac{a}{x} - bc\sqrt[4]{-\frac{1}{c^2}}\log\left(x - \sqrt[4]{-\frac{1}{c^2}}\right) + \frac{bc\sqrt[4]{-\frac{1}{c^2}}\log\left(x^2+\sqrt{-\frac{1}{c^2}}\right)}{2} + bc\sqrt[4]{-\frac{1}{c^2}}\operatorname{atan}\left(\frac{x}{\sqrt[4]{-\frac{1}{c^2}}}\right) - \frac{b\operatorname{atan}(cx^2)}{\sqrt[4]{-\frac{1}{c^2}}} - \frac{b\operatorname{atan}}{x} \\ -\frac{a}{x} \end{cases}$$

```
[In] integrate((a+b*atan(c*x**2))/x**2,x)
```

```
[Out] Piecewise((-a/x - b*c*(-1/c**2)**(1/4)*log(x - (-1/c**2)**(1/4)) + b*c*(-1/
c**2)**(1/4)*log(x**2 + sqrt(-1/c**2))/2 + b*c*(-1/c**2)**(1/4)*atan(x/(-1/
c**2)**(1/4)) - b*atan(c*x**2)/(-1/c**2)**(1/4) - b*atan(c*x**2)/x, Ne(c, 0
)), (-a/x, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.92

$$\int \frac{a + b\arctan(cx^2)}{x^2}\,dx$$
$$= \frac{1}{4}\left(c\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2cx+\sqrt{2}\sqrt{c}\right)}{2\sqrt{c}}\right)}{\sqrt{c}} + \frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2cx-\sqrt{2}\sqrt{c}\right)}{2\sqrt{c}}\right)}{\sqrt{c}} + \frac{\sqrt{2}\log\left(cx^2+\sqrt{2}\sqrt{c}x+1\right)}{\sqrt{c}} - \frac{\sqrt{2}}{} \right.\right.$$
$$- \frac{a}{x}$$

```
[In] integrate((a+b*arctan(c*x^2))/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*(c*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*c*x + sqrt(2)*sqrt(c))/sqrt(c))/sqr
t(c) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*c*x - sqrt(2)*sqrt(c))/sqrt(c))/sqrt
(c) + sqrt(2)*log(c*x^2 + sqrt(2)*sqrt(c)*x + 1)/sqrt(c) - sqrt(2)*log(c*x^
2 - sqrt(2)*sqrt(c)*x + 1)/sqrt(c)) - 4*arctan(c*x^2)/x)*b - a/x
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.97

$$\int \frac{a+b\arctan(cx^2)}{x^2}\,dx$$
$$=\frac{1}{4}bc\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{\sqrt{|c|}}+\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{\sqrt{|c|}}+\frac{\sqrt{2}\log\left(x^2+\frac{\sqrt{2}x}{\sqrt{|c|}}\right.}{\sqrt{|c|}}\right.$$
$$-\frac{b\arctan(cx^2)+a}{x}$$

```
[In] integrate((a+b*arctan(c*x^2))/x^2,x, algorithm="giac")
```

```
[Out] 1/4*b*c*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)/sqrt(abs(c)))*sqrt(abs
(c)))/sqrt(abs(c)) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)/sqrt(abs(c
)))*sqrt(abs(c)))/sqrt(abs(c)) + sqrt(2)*log(x^2 + sqrt(2)*x/sqrt(abs(c)) +
 1/abs(c))/sqrt(abs(c)) - sqrt(2)*log(x^2 - sqrt(2)*x/sqrt(abs(c)) + 1/abs(
c))/sqrt(abs(c))) - (b*arctan(c*x^2) + a)/x
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.38

$$\int \frac{a+b\arctan(cx^2)}{x^2}\,dx=-\frac{a}{x}-\frac{b\,\mathrm{atan}(c\,x^2)}{x}-(-1)^{1/4}\,b\,\sqrt{c}\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{c}\,x\right)\,1\mathrm{i}$$
$$-(-1)^{1/4}\,b\,\sqrt{c}\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{c}\,x\,1\mathrm{i}\right)$$

```
[In] int((a + b*atan(c*x^2))/x^2,x)
```

```
[Out] - a/x - (b*atan(c*x^2))/x - (-1)^(1/4)*b*c^(1/2)*atan((-1)^(1/4)*c^(1/2)*x)
*1i - (-1)^(1/4)*b*c^(1/2)*atan((-1)^(1/4)*c^(1/2)*x*1i)
```

3.72 $\int \frac{a+b\arctan(cx^2)}{x^4}\,dx$

Optimal result . . . 398
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Optimal result

Integrand size = 14, antiderivative size = 159

$$\int \frac{a+b\arctan(cx^2)}{x^4}\,dx = -\frac{2bc}{3x}-\frac{a+b\arctan(cx^2)}{3x^3}+\frac{bc^{3/2}\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{3\sqrt{2}}-\frac{bc^{3/2}\arctan\left(1+\sqrt{2}\sqrt{c}x\right)}{3\sqrt{2}}-\frac{bc^{3/2}\log\left(1-\sqrt{2}\sqrt{c}x+cx^2\right)}{6\sqrt{2}}+\frac{bc^{3/2}\log\left(1+\sqrt{2}\sqrt{c}x+cx^2\right)}{6\sqrt{2}}$$

```
[Out] -2/3*b*c/x+1/3*(-a-b*arctan(c*x^2))/x^3-1/6*b*c^(3/2)*arctan(-1+x*2^(1/2)*c
^(1/2))*2^(1/2)-1/6*b*c^(3/2)*arctan(1+x*2^(1/2)*c^(1/2))*2^(1/2)-1/12*b*c^
(3/2)*ln(1+c*x^2-x*2^(1/2)*c^(1/2))*2^(1/2)+1/12*b*c^(3/2)*ln(1+c*x^2+x*2^(
1/2)*c^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {4946, 331, 303, 1176, 631, 210, 1179, 642}

$$\int \frac{a+b\arctan(cx^2)}{x^4}\,dx = -\frac{a+b\arctan(cx^2)}{3x^3}+\frac{bc^{3/2}\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{3\sqrt{2}}-\frac{bc^{3/2}\arctan\left(\sqrt{2}\sqrt{c}x+1\right)}{3\sqrt{2}}-\frac{bc^{3/2}\log\left(cx^2-\sqrt{2}\sqrt{c}x+1\right)}{6\sqrt{2}}+\frac{bc^{3/2}\log\left(cx^2+\sqrt{2}\sqrt{c}x+1\right)}{6\sqrt{2}}-\frac{2bc}{3x}$$

```
[In] Int[(a + b*ArcTan[c*x^2])/x^4,x]
```

```
[Out] (-2*b*c)/(3*x) - (a + b*ArcTan[c*x^2])/(3*x^3) + (b*c^(3/2)*ArcTan[1 - Sqrt
[2]*Sqrt[c]*x])/(3*Sqrt[2]) - (b*c^(3/2)*ArcTan[1 + Sqrt[2]*Sqrt[c]*x])/(3*
Sqrt[2]) - (b*c^(3/2)*Log[1 - Sqrt[2]*Sqrt[c]*x + c*x^2])/(6*Sqrt[2]) + (b*
c^(3/2)*Log[1 + Sqrt[2]*Sqrt[c]*x + c*x^2])/(6*Sqrt[2])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{a+b\arctan\left(cx^2\right)}{3x^3}+\frac{1}{3}(2bc)\int\frac{1}{x^2\left(1+c^2x^4\right)}\,dx\\
&= -\frac{2bc}{3x}-\frac{a+b\arctan\left(cx^2\right)}{3x^3}-\frac{1}{3}\left(2bc^3\right)\int\frac{x^2}{1+c^2x^4}\,dx\\
&= -\frac{2bc}{3x}-\frac{a+b\arctan\left(cx^2\right)}{3x^3}+\frac{1}{3}\left(bc^2\right)\int\frac{1-cx^2}{1+c^2x^4}\,dx-\frac{1}{3}\left(bc^2\right)\int\frac{1+cx^2}{1+c^2x^4}\,dx\\
&= -\frac{2bc}{3x}-\frac{a+b\arctan\left(cx^2\right)}{3x^3}-\frac{1}{6}(bc)\int\frac{1}{\frac{1}{c}-\frac{\sqrt{2}x}{\sqrt{c}}+x^2}\,dx\\
&\quad -\frac{1}{6}(bc)\int\frac{1}{\frac{1}{c}+\frac{\sqrt{2}x}{\sqrt{c}}+x^2}\,dx-\frac{\left(bc^{3/2}\right)\int\frac{\frac{\sqrt{2}}{\sqrt{c}}+2x}{-\frac{1}{c}-\frac{\sqrt{2}x}{\sqrt{c}}-x^2}\,dx}{6\sqrt{2}}-\frac{\left(bc^{3/2}\right)\int\frac{\frac{\sqrt{2}}{\sqrt{c}}-2x}{-\frac{1}{c}+\frac{\sqrt{2}x}{\sqrt{c}}-x^2}\,dx}{6\sqrt{2}}\\
&= -\frac{2bc}{3x}-\frac{a+b\arctan\left(cx^2\right)}{3x^3}-\frac{bc^{3/2}\log\left(1-\sqrt{2}\sqrt{c}x+cx^2\right)}{6\sqrt{2}}+\frac{bc^{3/2}\log\left(1+\sqrt{2}\sqrt{c}x+cx^2\right)}{6\sqrt{2}}\\
&\quad -\frac{\left(bc^{3/2}\right)\operatorname{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}\sqrt{c}x\right)}{3\sqrt{2}}+\frac{\left(bc^{3/2}\right)\operatorname{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\sqrt{2}\sqrt{c}x\right)}{3\sqrt{2}}\\
&= -\frac{2bc}{3x}-\frac{a+b\arctan\left(cx^2\right)}{3x^3}+\frac{bc^{3/2}\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{3\sqrt{2}}-\frac{bc^{3/2}\arctan\left(1+\sqrt{2}\sqrt{c}x\right)}{3\sqrt{2}}\\
&\quad -\frac{bc^{3/2}\log\left(1-\sqrt{2}\sqrt{c}x+cx^2\right)}{6\sqrt{2}}+\frac{bc^{3/2}\log\left(1+\sqrt{2}\sqrt{c}x+cx^2\right)}{6\sqrt{2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.11

$$\int \frac{a+b\arctan(cx^2)}{x^4}\,dx = -\frac{a}{3x^3}-\frac{2bc}{3x}-\frac{b\arctan(cx^2)}{3x^3}-\frac{bc^{3/2}\arctan\left(\frac{-\sqrt{2}+2\sqrt{c}x}{\sqrt{2}}\right)}{3\sqrt{2}} -\frac{bc^{3/2}\arctan\left(\frac{\sqrt{2}+2\sqrt{c}x}{\sqrt{2}}\right)}{3\sqrt{2}}-\frac{bc^{3/2}\log\left(1-\sqrt{2}\sqrt{c}x+cx^2\right)}{6\sqrt{2}} +\frac{bc^{3/2}\log\left(1+\sqrt{2}\sqrt{c}x+cx^2\right)}{6\sqrt{2}}$$

```
[In] Integrate[(a + b*ArcTan[c*x^2])/x^4,x]
```

```
[Out] -1/3*a/x^3 - (2*b*c)/(3*x) - (b*ArcTan[c*x^2])/(3*x^3) - (b*c^(3/2)*ArcTan[
(-Sqrt[2] + 2*Sqrt[c]*x)/Sqrt[2]])/(3*Sqrt[2]) - (b*c^(3/2)*ArcTan[(Sqrt[2]
 + 2*Sqrt[c]*x)/Sqrt[2]])/(3*Sqrt[2]) - (b*c^(3/2)*Log[1 - Sqrt[2]*Sqrt[c]*
x + c*x^2])/(6*Sqrt[2]) + (b*c^(3/2)*Log[1 + Sqrt[2]*Sqrt[c]*x + c*x^2])/(6
*Sqrt[2])
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.72

method	result	s
default	$-\frac{a}{3x^3}+b\left(-\frac{\arctan(c\,x^2)}{3x^3}+\frac{2c\left(-\frac{\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{1}{c^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^2}}}{x^2+\left(\frac{1}{c^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}-1\right)\right)}{8\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}-\frac{1}{x}\right)}{3}\right)$	1
parts	$-\frac{a}{3x^3}+b\left(-\frac{\arctan(c\,x^2)}{3x^3}+\frac{2c\left(-\frac{\sqrt{2}\left(\ln\left(\frac{x^2-\left(\frac{1}{c^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^2}}}{x^2+\left(\frac{1}{c^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}-1\right)\right)}{8\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}-\frac{1}{x}\right)}{3}\right)$	1

```
[In] int((a+b*arctan(c*x^2))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a/x^3+b*(-1/3/x^3*arctan(c*x^2)+2/3*c*(-1/8/(1/c^2)^(1/4)*2^(1/2)*(ln(
(x^2-(1/c^2)^(1/4)*x*2^(1/2)+(1/c^2)^(1/2))/(x^2+(1/c^2)^(1/4)*x*2^(1/2)+(1
/c^2)^(1/2)))+2*arctan(2^(1/2)/(1/c^2)^(1/4)*x+1)+2*arctan(2^(1/2)/(1/c^2)^
(1/4)*x-1))-1/x))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.07

$$\int \frac{a+b\arctan(cx^2)}{x^4}\,dx =$$
$$-\frac{(-b^4c^6)^{\frac{1}{4}}x^3\log\left(b^3c^5x+(-b^4c^6)^{\frac{3}{4}}\right)-i(-b^4c^6)^{\frac{1}{4}}x^3\log\left(b^3c^5x+i(-b^4c^6)^{\frac{3}{4}}\right)+i(-b^4c^6)^{\frac{1}{4}}x^3\log\left(b^3c^5x-\right.}{6x^3}$$

```
[In] integrate((a+b*arctan(c*x^2))/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*((-b^4*c^6)^(1/4)*x^3*log(b^3*c^5*x + (-b^4*c^6)^(3/4)) - I*(-b^4*c^6)
^(1/4)*x^3*log(b^3*c^5*x + I*(-b^4*c^6)^(3/4)) + I*(-b^4*c^6)^(1/4)*x^3*log
```

```
(b^3*c^5*x - I*(-b^4*c^6)^(3/4)) - (-b^4*c^6)^(1/4)*x^3*log(b^3*c^5*x - (-b
^4*c^6)^(3/4)) + 4*b*c*x^2 + 2*b*arctan(c*x^2) + 2*a)/x^3
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 20.04 (sec) , antiderivative size = 529, normalized size of antiderivative = 3.33

$$\int \frac{a+b\arctan\left(cx^2\right)}{x^4}\,dx$$
$$= \begin{cases} -\frac{a}{3x^3} \\ -\frac{a-\infty ib}{3x^3} \\ -\frac{a+\infty ib}{3x^3} \\ -\frac{2ax^4}{6x^7+\frac{6x^3}{c^2}} - \frac{2a}{6c^2x^7+6x^3} + \frac{2bc^3x^7\left(-\frac{1}{c^2}\right)^{\frac{3}{4}}\log\left(x-\sqrt[4]{-\frac{1}{c^2}}\right)}{6x^7+\frac{6x^3}{c^2}} - \frac{bc^3x^7\left(-\frac{1}{c^2}\right)^{\frac{3}{4}}\log\left(x^2+\sqrt{-\frac{1}{c^2}}\right)}{6x^7+\frac{6x^3}{c^2}} + \frac{2bc^3x^7\left(-\frac{1}{c^2}\right)^{\frac{3}{4}}\operatorname{atan}\left(\frac{x}{\sqrt[4]{-\frac{1}{c^2}}}\right)}{6x^7+\frac{6x^3}{c^2}} \end{cases}$$

```
[In] integrate((a+b*atan(c*x**2))/x**4,x)
```

```
[Out] Piecewise((-a/(3*x**3), Eq(c, 0)), (-(a - oo*I*b)/(3*x**3), Eq(c, -I/x**2))
, (-(a + oo*I*b)/(3*x**3), Eq(c, I/x**2)), (-2*a*x**4/(6*x**7 + 6*x**3/c**2
) - 2*a/(6*c**2*x**7 + 6*x**3) + 2*b*c**3*x**7*(-1/c**2)**(3/4)*log(x - (-1
/c**2)**(1/4))/(6*x**7 + 6*x**3/c**2) - b*c**3*x**7*(-1/c**2)**(3/4)*log(x*
*2 + sqrt(-1/c**2))/(6*x**7 + 6*x**3/c**2) + 2*b*c**3*x**7*(-1/c**2)**(3/4)
*atan(x/(-1/c**2)**(1/4))/(6*x**7 + 6*x**3/c**2) + 2*b*c**2*x**7*(-1/c**2)*
*(1/4)*atan(c*x**2)/(6*x**7 + 6*x**3/c**2) - 4*b*c*x**6/(6*x**7 + 6*x**3/c*
*2) + 2*b*c*x**3*(-1/c**2)**(3/4)*log(x - (-1/c**2)**(1/4))/(6*x**7 + 6*x**
3/c**2) - b*c*x**3*(-1/c**2)**(3/4)*log(x**2 + sqrt(-1/c**2))/(6*x**7 + 6*x
**3/c**2) + 2*b*c*x**3*(-1/c**2)**(3/4)*atan(x/(-1/c**2)**(1/4))/(6*x**7 +
6*x**3/c**2) - 2*b*x**4*atan(c*x**2)/(6*x**7 + 6*x**3/c**2) + 2*b*x**3*(-1/
c**2)**(1/4)*atan(c*x**2)/(6*x**7 + 6*x**3/c**2) - 4*b*x**2/(6*c*x**7 + 6*x
**3/c) - 2*b*atan(c*x**2)/(6*c**2*x**7 + 6*x**3), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.89

$$\int \frac{a + b\arctan(cx^2)}{x^4}\,dx =$$

$$-\frac{1}{12}\left(\left(\left(c^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\,cx+\sqrt{2}\sqrt{c}\right)}{2\,\sqrt{c}}\right)}{c^{\frac{3}{2}}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\,cx-\sqrt{2}\sqrt{c}\right)}{2\,\sqrt{c}}\right)}{c^{\frac{3}{2}}}-\frac{\sqrt{2}\log\left(cx^2+\sqrt{2}\sqrt{c}x+1\right)}{c^{\frac{3}{2}}}\right.\right.\right.\right.$$

$$-\frac{a}{3\,x^3}$$

```
[In] integrate((a+b*arctan(c*x^2))/x^4,x, algorithm="maxima")
```

```
[Out] -1/12*((c^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*c*x + sqrt(2)*sqrt(c))/sqrt(c)
)/c^(3/2) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*c*x - sqrt(2)*sqrt(c))/sqrt(c))
/c^(3/2) - sqrt(2)*log(c*x^2 + sqrt(2)*sqrt(c)*x + 1)/c^(3/2) + sqrt(2)*log
(c*x^2 - sqrt(2)*sqrt(c)*x + 1)/c^(3/2)) + 8/x)*c + 4*arctan(c*x^2)/x^3)*b
- 1/3*a/x^3
```

Giac [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00

$$\int \frac{a + b\arctan(cx^2)}{x^4}\,dx =$$

$$-\frac{1}{12}\,bc^3\left(\frac{2\sqrt{2}\sqrt{|c|}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x+\frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{c^2}+\frac{2\sqrt{2}\sqrt{|c|}\arctan\left(\frac{1}{2}\sqrt{2}\left(2\,x-\frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{c^2}-\frac{\sqrt{2}\sqrt{}}{}\right.$$

$$-\frac{2\,bcx^2+b\arctan(cx^2)+a}{3\,x^3}$$

```
[In] integrate((a+b*arctan(c*x^2))/x^4,x, algorithm="giac")
```

```
[Out] -1/12*b*c^3*(2*sqrt(2)*sqrt(abs(c))*arctan(1/2*sqrt(2)*(2*x + sqrt(2)/sqrt(
abs(c)))*sqrt(abs(c)))/c^2 + 2*sqrt(2)*sqrt(abs(c))*arctan(1/2*sqrt(2)*(2*x
 - sqrt(2)/sqrt(abs(c)))*sqrt(abs(c)))/c^2 - sqrt(2)*sqrt(abs(c))*log(x^2 +
 sqrt(2)*x/sqrt(abs(c)) + 1/abs(c))/c^2 + sqrt(2)*sqrt(abs(c))*log(x^2 - sq
rt(2)*x/sqrt(abs(c)) + 1/abs(c))/c^2) - 1/3*(2*b*c*x^2 + b*arctan(c*x^2) +
a)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.40

$$\int \frac{a+b\arctan(cx^2)}{x^4}\,dx = -\frac{2\,b\,c\,x^2+a}{3\,x^3} - \frac{b\,\mathrm{atan}(c\,x^2)}{3\,x^3} - \frac{(-1)^{1/4}\,b\,c^{3/2}\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{c}\,x\right)}{3} - \frac{(-1)^{1/4}\,b\,c^{3/2}\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{c}\,x\,1\mathrm{i}\right)\,1\mathrm{i}}{3}$$

```
[In] int((a + b*atan(c*x^2))/x^4,x)
```

```
[Out] - (a + 2*b*c*x^2)/(3*x^3) - (b*atan(c*x^2))/(3*x^3) - ((-1)^(1/4)*b*c^(3/2)
*atan((-1)^(1/4)*c^(1/2)*x))/3 - ((-1)^(1/4)*b*c^(3/2)*atan((-1)^(1/4)*c^(1
/2)*x*1i)*1i)/3
```

3.73 $\int \frac{a+b\arctan(cx^2)}{x^6}\,dx$

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Optimal result

Integrand size = 14, antiderivative size = 159

$$\int \frac{a+b\arctan(cx^2)}{x^6}\,dx = -\frac{2bc}{15x^3}-\frac{a+b\arctan(cx^2)}{5x^5}+\frac{bc^{5/2}\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{5\sqrt{2}} - \frac{bc^{5/2}\arctan\left(1+\sqrt{2}\sqrt{c}x\right)}{5\sqrt{2}}+\frac{bc^{5/2}\log\left(1-\sqrt{2}\sqrt{c}x+cx^2\right)}{10\sqrt{2}} - \frac{bc^{5/2}\log\left(1+\sqrt{2}\sqrt{c}x+cx^2\right)}{10\sqrt{2}}$$

```
[Out] -2/15*b*c/x^3+1/5*(-a-b*arctan(c*x^2))/x^5-1/10*b*c^(5/2)*arctan(-1+x*2^(1/
2)*c^(1/2))*2^(1/2)-1/10*b*c^(5/2)*arctan(1+x*2^(1/2)*c^(1/2))*2^(1/2)+1/20
*b*c^(5/2)*ln(1+c*x^2-x*2^(1/2)*c^(1/2))*2^(1/2)-1/20*b*c^(5/2)*ln(1+c*x^2+
x*2^(1/2)*c^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {4946, 331, 217, 1179, 642, 1176, 631, 210}

$$\int \frac{a+b\arctan(cx^2)}{x^6}\,dx = -\frac{a+b\arctan(cx^2)}{5x^5}+\frac{bc^{5/2}\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{5\sqrt{2}} - \frac{bc^{5/2}\arctan\left(\sqrt{2}\sqrt{c}x+1\right)}{5\sqrt{2}}+\frac{bc^{5/2}\log\left(cx^2-\sqrt{2}\sqrt{c}x+1\right)}{10\sqrt{2}} - \frac{bc^{5/2}\log\left(cx^2+\sqrt{2}\sqrt{c}x+1\right)}{10\sqrt{2}}-\frac{2bc}{15x^3}$$

```
[In] Int[(a + b*ArcTan[c*x^2])/x^6,x]
```

```
[Out] (-2*b*c)/(15*x^3) - (a + b*ArcTan[c*x^2])/(5*x^5) + (b*c^(5/2)*ArcTan[1 - S
qrt[2]*Sqrt[c]*x])/(5*Sqrt[2]) - (b*c^(5/2)*ArcTan[1 + Sqrt[2]*Sqrt[c]*x])/
(5*Sqrt[2]) + (b*c^(5/2)*Log[1 - Sqrt[2]*Sqrt[c]*x + c*x^2])/(10*Sqrt[2]) -
 (b*c^(5/2)*Log[1 + Sqrt[2]*Sqrt[c]*x + c*x^2])/(10*Sqrt[2])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
```

```
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{a+b\arctan(cx^2)}{5x^5}+\frac{1}{5}(2bc)\int\frac{1}{x^4(1+c^2x^4)}\,dx\\
&= -\frac{2bc}{15x^3}-\frac{a+b\arctan(cx^2)}{5x^5}-\frac{1}{5}(2bc^3)\int\frac{1}{1+c^2x^4}\,dx\\
&= -\frac{2bc}{15x^3}-\frac{a+b\arctan(cx^2)}{5x^5}-\frac{1}{5}(bc^3)\int\frac{1-cx^2}{1+c^2x^4}\,dx-\frac{1}{5}(bc^3)\int\frac{1+cx^2}{1+c^2x^4}\,dx\\
&= -\frac{2bc}{15x^3}-\frac{a+b\arctan(cx^2)}{5x^5}-\frac{1}{10}(bc^2)\int\frac{1}{\frac{1}{c}-\frac{\sqrt{2}x}{\sqrt{c}}+x^2}\,dx\\
&\quad -\frac{1}{10}(bc^2)\int\frac{1}{\frac{1}{c}+\frac{\sqrt{2}x}{\sqrt{c}}+x^2}\,dx+\frac{(bc^{5/2})\int\frac{\frac{\sqrt{2}}{\sqrt{c}}+2x}{-\frac{1}{c}-\frac{\sqrt{2}x}{\sqrt{c}}-x^2}\,dx}{10\sqrt{2}}+\frac{(bc^{5/2})\int\frac{\frac{\sqrt{2}}{\sqrt{c}}-2x}{-\frac{1}{c}+\frac{\sqrt{2}x}{\sqrt{c}}-x^2}\,dx}{10\sqrt{2}}\\
&= -\frac{2bc}{15x^3}-\frac{a+b\arctan(cx^2)}{5x^5}+\frac{bc^{5/2}\log\left(1-\sqrt{2}\sqrt{c}x+cx^2\right)}{10\sqrt{2}}-\frac{bc^{5/2}\log\left(1+\sqrt{2}\sqrt{c}x+cx^2\right)}{10\sqrt{2}}\\
&\quad -\frac{(bc^{5/2})\,\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}\sqrt{c}x\right)}{5\sqrt{2}}+\frac{(bc^{5/2})\,\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx,x,1+\sqrt{2}\sqrt{c}x\right)}{5\sqrt{2}}\\
&= -\frac{2bc}{15x^3}-\frac{a+b\arctan(cx^2)}{5x^5}+\frac{bc^{5/2}\arctan\left(1-\sqrt{2}\sqrt{c}x\right)}{5\sqrt{2}}\\
&\quad -\frac{bc^{5/2}\arctan\left(1+\sqrt{2}\sqrt{c}x\right)}{5\sqrt{2}}+\frac{bc^{5/2}\log\left(1-\sqrt{2}\sqrt{c}x+cx^2\right)}{10\sqrt{2}}\\
&\quad -\frac{bc^{5/2}\log\left(1+\sqrt{2}\sqrt{c}x+cx^2\right)}{10\sqrt{2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.11

$$\int \frac{a+b\arctan(cx^2)}{x^6}\,dx = -\frac{a}{5x^5}-\frac{2bc}{15x^3}-\frac{b\arctan(cx^2)}{5x^5}-\frac{bc^{5/2}\arctan\left(\frac{-\sqrt{2}+2\sqrt{c}x}{\sqrt{2}}\right)}{5\sqrt{2}}-\frac{bc^{5/2}\arctan\left(\frac{\sqrt{2}+2\sqrt{c}x}{\sqrt{2}}\right)}{5\sqrt{2}}+\frac{bc^{5/2}\log\left(1-\sqrt{2}\sqrt{c}x+cx^2\right)}{10\sqrt{2}}-\frac{bc^{5/2}\log\left(1+\sqrt{2}\sqrt{c}x+cx^2\right)}{10\sqrt{2}}$$

```
[In] Integrate[(a + b*ArcTan[c*x^2])/x^6,x]
```

```
[Out] -1/5*a/x^5 - (2*b*c)/(15*x^3) - (b*ArcTan[c*x^2])/(5*x^5) - (b*c^(5/2)*ArcT
an[(-Sqrt[2] + 2*Sqrt[c]*x)/Sqrt[2]])/(5*Sqrt[2]) - (b*c^(5/2)*ArcTan[(Sqrt
[2] + 2*Sqrt[c]*x)/Sqrt[2]])/(5*Sqrt[2]) + (b*c^(5/2)*Log[1 - Sqrt[2]*Sqrt[
c]*x + c*x^2])/(10*Sqrt[2]) - (b*c^(5/2)*Log[1 + Sqrt[2]*Sqrt[c]*x + c*x^2]
)/(10*Sqrt[2])
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.74

method	result
default	$-\frac{a}{5x^5}+b\left(-\frac{\arctan(cx^2)}{5x^5}+\frac{2c\left(-\frac{c^2\left(\frac{1}{c^2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{1}{c^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^2}}}{x^2-\left(\frac{1}{c^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}-1\right)\right)}{8}-\frac{1}{3x}\right.}{5}\right.$
parts	$-\frac{a}{5x^5}+b\left(-\frac{\arctan(cx^2)}{5x^5}+\frac{2c\left(-\frac{c^2\left(\frac{1}{c^2}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{1}{c^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^2}}}{x^2-\left(\frac{1}{c^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^2}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}-1\right)\right)}{8}-\frac{1}{3x}\right.}{5}\right.$

```
[In] int((a+b*arctan(c*x^2))/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*a/x^5+b*(-1/5/x^5*arctan(c*x^2)+2/5*c*(-1/8*c^2*(1/c^2)^(1/4)*2^(1/2)*
(ln((x^2+(1/c^2)^(1/4)*x*2^(1/2)+(1/c^2)^(1/2))/(x^2-(1/c^2)^(1/4)*x*2^(1/2
)+(1/c^2)^(1/2)))+2*arctan(2^(1/2)/(1/c^2)^(1/4)*x+1)+2*arctan(2^(1/2)/(1/c
^2)^(1/4)*x-1))-1/3/x^3))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.03

$$\int \frac{a+b\arctan(cx^2)}{x^6}\,dx = -\frac{3\left(-b^4c^{10}\right)^{\frac{1}{4}}x^5\log\left(bc^3x+\left(-b^4c^{10}\right)^{\frac{1}{4}}\right)+3i\left(-b^4c^{10}\right)^{\frac{1}{4}}x^5\log\left(bc^3x+i\left(-b^4c^{10}\right)^{\frac{1}{4}}\right)-3i\left(-b^4c^{10}\right)^{\frac{1}{4}}x^5\log\left(\right.}{30\,x^5}$$

```
[In] integrate((a+b*arctan(c*x^2))/x^6,x, algorithm="fricas")
```

```
[Out] -1/30*(3*(-b^4*c^10)^(1/4)*x^5*log(b*c^3*x + (-b^4*c^10)^(1/4)) + 3*I*(-b^4
*c^10)^(1/4)*x^5*log(b*c^3*x + I*(-b^4*c^10)^(1/4)) - 3*I*(-b^4*c^10)^(1/4)
```

```
*x^5*log(b*c^3*x - I*(-b^4*c^10)^(1/4)) - 3*(-b^4*c^10)^(1/4)*x^5*log(b*c^3
*x - (-b^4*c^10)^(1/4)) + 4*b*c*x^2 + 6*b*arctan(c*x^2) + 6*a)/x^5
```

Sympy [A] (verification not implemented)

Time = 29.70 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.97

$$\int \frac{a+b\arctan(cx^2)}{x^6}\,dx = \begin{cases} -\frac{a}{5x^5} + \frac{bc^3\sqrt[4]{-\frac{1}{c^2}}\log\left(x-\sqrt[4]{-\frac{1}{c^2}}\right)}{5} - \frac{bc^3\sqrt[4]{-\frac{1}{c^2}}\log\left(x^2+\sqrt{-\frac{1}{c^2}}\right)}{10} - \frac{bc^3\sqrt[4]{-\frac{1}{c^2}}\operatorname{atan}\left(\frac{x}{\sqrt[4]{-\frac{1}{c^2}}}\right)}{5} + \frac{bc^2\operatorname{atan}(cx^2)}{5\sqrt[4]{-\frac{1}{c^2}}} - \frac{2bc}{15x^3} \\ -\frac{a}{5x^5} \end{cases}$$

```
[In] integrate((a+b*atan(c*x**2))/x**6,x)
```

```
[Out] Piecewise((-a/(5*x**5) + b*c**3*(-1/c**2)**(1/4)*log(x - (-1/c**2)**(1/4))/
5 - b*c**3*(-1/c**2)**(1/4)*log(x**2 + sqrt(-1/c**2))/10 - b*c**3*(-1/c**2)
**(1/4)*atan(x/(-1/c**2)**(1/4))/5 + b*c**2*atan(c*x**2)/(5*(-1/c**2)**(1/4
)) - 2*b*c/(15*x**3) - b*atan(c*x**2)/(5*x**5), Ne(c, 0)), (-a/(5*x**5), Tr
ue))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.87

$$\int \frac{a+b\arctan(cx^2)}{x^6}\,dx = -\frac{1}{60}\left(\left(6\sqrt{2}c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}(2cx+\sqrt{2}\sqrt{c})}{2\sqrt{c}}\right) + 6\sqrt{2}c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}(2cx-\sqrt{2}\sqrt{c})}{2\sqrt{c}}\right) + 3\sqrt{2}c^{\frac{3}{2}}\log\left(cx^2\right.\right.\right. - \frac{a}{5x^5}$$

```
[In] integrate((a+b*arctan(c*x^2))/x^6,x, algorithm="maxima")
```

```
[Out] -1/60*((6*sqrt(2)*c^(3/2)*arctan(1/2*sqrt(2)*(2*c*x + sqrt(2)*sqrt(c))/sqrt
(c)) + 6*sqrt(2)*c^(3/2)*arctan(1/2*sqrt(2)*(2*c*x - sqrt(2)*sqrt(c))/sqrt(
c)) + 3*sqrt(2)*c^(3/2)*log(c*x^2 + sqrt(2)*sqrt(c)*x + 1) - 3*sqrt(2)*c^(3
/2)*log(c*x^2 - sqrt(2)*sqrt(c)*x + 1) + 8/x^3)*c + 12*arctan(c*x^2)/x^5)*b
 - 1/5*a/x^5
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.94

$$\int \frac{a+b\arctan(cx^2)}{x^6}\,dx = -\frac{1}{20}\,bc^3\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{\sqrt{|c|}}+\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{\sqrt{|c|}}+\frac{\sqrt{2}\log\left(x^2+\right.}{\sqrt{|c|}}\right. - \frac{2bcx^2+3b\arctan(cx^2)+3a}{15x^5}$$

```
[In] integrate((a+b*arctan(c*x^2))/x^6,x, algorithm="giac")
```

```
[Out] -1/20*b*c^3*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)/sqrt(abs(c)))*sqrt
(abs(c)))/sqrt(abs(c)) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)/sqrt(a
bs(c)))*sqrt(abs(c)))/sqrt(abs(c)) + sqrt(2)*log(x^2 + sqrt(2)*x/sqrt(abs(c
)) + 1/abs(c))/sqrt(abs(c)) - sqrt(2)*log(x^2 - sqrt(2)*x/sqrt(abs(c)) + 1/
abs(c))/sqrt(abs(c))) - 1/15*(2*b*c*x^2 + 3*b*arctan(c*x^2) + 3*a)/x^5
```

Mupad [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.40

$$\int \frac{a+b\arctan(cx^2)}{x^6}\,dx = -\frac{\frac{2bcx^2}{3}+a}{5x^5}-\frac{b\,\mathrm{atan}(cx^2)}{5x^5}+\frac{(-1)^{1/4}\,b\,c^{5/2}\,\mathrm{atan}\left((-1)^{1/4}\sqrt{c}\,x\right)\,1\mathrm{i}}{5} + \frac{(-1)^{1/4}\,b\,c^{5/2}\,\mathrm{atan}\left((-1)^{1/4}\sqrt{c}\,x\,1\mathrm{i}\right)}{5}$$

```
[In] int((a + b*atan(c*x^2))/x^6,x)
```

```
[Out] ((-1)^(1/4)*b*c^(5/2)*atan((-1)^(1/4)*c^(1/2)*x)*1i)/5 - (b*atan(c*x^2))/(5
*x^5) - (a + (2*b*c*x^2)/3)/(5*x^5) + ((-1)^(1/4)*b*c^(5/2)*atan((-1)^(1/4)
*c^(1/2)*x*1i))/5
```

3.74 $\int x^7(a+b\arctan(cx^2))^2\,dx$

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Optimal result

Integrand size = 16, antiderivative size = 124

$$\int x^7\left(a+b\arctan\left(cx^2\right)\right)^2\,dx=\frac{abx^2}{4c^3}+\frac{b^2x^4}{24c^2}+\frac{b^2x^2\arctan(cx^2)}{4c^3}-\frac{bx^6(a+b\arctan(cx^2))}{12c}-\frac{(a+b\arctan(cx^2))^2}{8c^4}+\frac{1}{8}x^8\left(a+b\arctan\left(cx^2\right)\right)^2-\frac{b^2\log(1+c^2x^4)}{6c^4}$$

```
[Out] 1/4*a*b*x^2/c^3+1/24*b^2*x^4/c^2+1/4*b^2*x^2*arctan(c*x^2)/c^3-1/12*b*x^6*(
a+b*arctan(c*x^2))/c-1/8*(a+b*arctan(c*x^2))^2/c^4+1/8*x^8*(a+b*arctan(c*x^
2))^2-1/6*b^2*ln(c^2*x^4+1)/c^4
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4948, 4946, 5036, 272, 45, 4930, 266, 5004}

$$\int x^7\left(a+b\arctan\left(cx^2\right)\right)^2\,dx=-\frac{(a+b\arctan(cx^2))^2}{8c^4}+\frac{1}{8}x^8\left(a+b\arctan\left(cx^2\right)\right)^2-\frac{bx^6(a+b\arctan(cx^2))}{12c}+\frac{abx^2}{4c^3}+\frac{b^2x^2\arctan(cx^2)}{4c^3}+\frac{b^2x^4}{24c^2}-\frac{b^2\log(c^2x^4+1)}{6c^4}$$

```
[In] Int[x^7*(a + b*ArcTan[c*x^2])^2,x]
```

```
[Out] (a*b*x^2)/(4*c^3) + (b^2*x^4)/(24*c^2) + (b^2*x^2*ArcTan[c*x^2])/(4*c^3) -
(b*x^6*(a + b*ArcTan[c*x^2]))/(12*c) - (a + b*ArcTan[c*x^2])^2/(8*c^4) + (x
^8*(a + b*ArcTan[c*x^2])^2)/8 - (b^2*Log[1 + c^2*x^4])/(6*c^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplify
[(m + 1)/n]]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
```

```
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int x^3(a+b\arctan(cx))^2\,dx,x,x^2\right)\\
&= \frac{1}{8}x^8\left(a+b\arctan\left(cx^2\right)\right)^2-\frac{1}{4}(bc)\text{Subst}\left(\int \frac{x^4(a+b\arctan(cx))}{1+c^2x^2}\,dx,x,x^2\right)\\
&= \frac{1}{8}x^8\left(a+b\arctan\left(cx^2\right)\right)^2-\frac{b\text{Subst}\left(\int x^2(a+b\arctan(cx))\,dx,x,x^2\right)}{4c}\\
&\quad+\frac{b\text{Subst}\left(\int \frac{x^2(a+b\arctan(cx))}{1+c^2x^2}\,dx,x,x^2\right)}{4c}\\
&= -\frac{bx^6\left(a+b\arctan\left(cx^2\right)\right)}{12c}+\frac{1}{8}x^8\left(a+b\arctan\left(cx^2\right)\right)^2+\frac{1}{12}b^2\text{Subst}\left(\int \frac{x^3}{1+c^2x^2}\,dx,x,x^2\right)\\
&\quad+\frac{b\text{Subst}\left(\int(a+b\arctan(cx))\,dx,x,x^2\right)}{4c^3}-\frac{b\text{Subst}\left(\int \frac{a+b\arctan(cx)}{1+c^2x^2}\,dx,x,x^2\right)}{4c^3}\\
&= \frac{abx^2}{4c^3}-\frac{bx^6\left(a+b\arctan\left(cx^2\right)\right)}{12c}-\frac{\left(a+b\arctan\left(cx^2\right)\right)^2}{8c^4}+\frac{1}{8}x^8\left(a+b\arctan\left(cx^2\right)\right)^2\\
&\quad+\frac{1}{24}b^2\text{Subst}\left(\int \frac{x}{1+c^2x}\,dx,x,x^4\right)+\frac{b^2\text{Subst}\left(\int \arctan(cx)\,dx,x,x^2\right)}{4c^3}\\
&= \frac{abx^2}{4c^3}+\frac{b^2x^2\arctan\left(cx^2\right)}{4c^3}-\frac{bx^6\left(a+b\arctan\left(cx^2\right)\right)}{12c}\\
&\quad-\frac{\left(a+b\arctan\left(cx^2\right)\right)^2}{8c^4}+\frac{1}{8}x^8\left(a+b\arctan\left(cx^2\right)\right)^2\\
&\quad+\frac{1}{24}b^2\text{Subst}\left(\int\left(\frac{1}{c^2}-\frac{1}{c^2\left(1+c^2x\right)}\right)dx,x,x^4\right)-\frac{b^2\text{Subst}\left(\int \frac{x}{1+c^2x^2}\,dx,x,x^2\right)}{4c^2}\\
&= \frac{abx^2}{4c^3}+\frac{b^2x^4}{24c^2}+\frac{b^2x^2\arctan\left(cx^2\right)}{4c^3}-\frac{bx^6\left(a+b\arctan\left(cx^2\right)\right)}{12c}\\
&\quad-\frac{\left(a+b\arctan\left(cx^2\right)\right)^2}{8c^4}+\frac{1}{8}x^8\left(a+b\arctan\left(cx^2\right)\right)^2-\frac{b^2\log\left(1+c^2x^4\right)}{6c^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.98

$$\int x^7\left(a+b\arctan\left(cx^2\right)\right)^2\,dx$$
$$=\frac{cx^2(6ab+b^2cx^2-2abc^2x^4+3a^2c^3x^6)-2b(bcx^2(-3+c^2x^4)+a(3-3c^4x^8))\arctan\left(cx^2\right)+3b^2(-1+c^4x^8}{24c^4}$$

[In] Integrate[x^7*(a + b*ArcTan[c*x^2])^2,x]

[Out] (c*x^2*(6*a*b + b^2*c*x^2 - 2*a*b*c^2*x^4 + 3*a^2*c^3*x^6) - 2*b*(b*c*x^2*(-3 + c^2*x^4) + a*(3 - 3*c^4*x^8))*ArcTan[c*x^2] + 3*b^2*(-1 + c^4*x^8)*ArcTan[c*x^2]^2 - 4*b^2*Log[1 + c^2*x^4])/(24*c^4)

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.22

method	result
default	$\frac{a^2x^8}{8}+\frac{b^2x^8\arctan(cx^2)^2}{8}-\frac{b^2\arctan(cx^2)x^6}{12c}+\frac{b^2x^2\arctan(cx^2)}{4c^3}-\frac{b^2\arctan(cx^2)^2}{8c^4}+\frac{b^2x^4}{24c^2}-\frac{b^2\ln(c^2x^4+1)}{6c^4}+$
parts	$\frac{a^2x^8}{8}+\frac{b^2x^8\arctan(cx^2)^2}{8}-\frac{b^2\arctan(cx^2)x^6}{12c}+\frac{b^2x^2\arctan(cx^2)}{4c^3}-\frac{b^2\arctan(cx^2)^2}{8c^4}+\frac{b^2x^4}{24c^2}-\frac{b^2\ln(c^2x^4+1)}{6c^4}+$
parallelrisch	$-\frac{-3b^2\arctan(cx^2)^2x^8c^4-6ab\arctan(cx^2)x^8c^4-3c^4a^2x^8+2b^2\arctan(cx^2)x^6c^3+2abc^3x^6-x^4b^2c^2-6b^2\arctan(cx^2)x^2c-6a}{24c^4}$
risch	$-\frac{b^2(c^4x^8-1)\ln(icx^2+1)^2}{32c^4}-\frac{ib(6ac^4x^8+3ibc^4x^8\ln(-icx^2+1)-2bc^3x^6+6bcx^2-3ib\ln(-icx^2+1))\ln(icx^2+1)}{48c^4}-\frac{b^2x^8\ln}{}$

[In] int(x^7*(a+b*arctan(c*x^2))^2,x,method=_RETURNVERBOSE)

[Out] 1/8*a^2*x^8+1/8*b^2*x^8*arctan(c*x^2)^2-1/12*b^2*arctan(c*x^2)/c*x^6+1/4*b^2*x^2*arctan(c*x^2)/c^3-1/8*b^2/c^4*arctan(c*x^2)^2+1/24*b^2*x^4/c^2-1/6*b^2*ln(c^2*x^4+1)/c^4+1/4*a*b*x^8*arctan(c*x^2)-1/12*a*b/c*x^6+1/4*a*b*x^2/c^3-1/4*a*b/c^4*arctan(c*x^2)

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.10

$$\int x^7\left(a+b\arctan\left(cx^2\right)\right)^2\,dx$$
$$=\frac{3\,a^2c^4x^8-2\,abc^3x^6+b^2c^2x^4+6\,abcx^2+3\,(b^2c^4x^8-b^2)\arctan\left(cx^2\right)^2+6\,ab\arctan\left(\frac{1}{cx^2}\right)-4\,b^2\log\left(c^2x^4+\right.}{24\,c^4}$$

```
[In] integrate(x^7*(a+b*arctan(c*x^2))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(3*a^2*c^4*x^8 - 2*a*b*c^3*x^6 + b^2*c^2*x^4 + 6*a*b*c*x^2 + 3*(b^2*c^
4*x^8 - b^2)*arctan(c*x^2)^2 + 6*a*b*arctan(1/(c*x^2)) - 4*b^2*log(c^2*x^4
+ 1) + 2*(3*a*b*c^4*x^8 - b^2*c^3*x^6 + 3*b^2*c*x^2)*arctan(c*x^2))/c^4
```

Sympy [A] (verification not implemented)

Time = 44.49 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.60

$$\int x^7 \left(a + b \arctan\left(cx^2\right)\right)^2 \, dx = \begin{cases} \frac{a^2x^8}{8} + \frac{abx^8 \operatorname{atan}\left(cx^2\right)}{4} - \frac{abx^6}{12c} + \frac{abx^2}{4c^3} - \frac{ab \operatorname{atan}\left(cx^2\right)}{4c^4} + \frac{b^2x^8 \operatorname{atan}^2\left(cx^2\right)}{8} - \frac{b^2x^6 \operatorname{atan}\left(cx^2\right)}{12c} + \frac{b^2x^4}{24c^2} + \frac{b^2x^2 \operatorname{atan}\left(cx^2\right)}{4c^3} - \frac{b^2 \sqrt{}}{} \\ \frac{a^2x^8}{8} \end{cases}$$

```
[In] integrate(x**7*(a+b*atan(c*x**2))**2,x)
```

```
[Out] Piecewise((a**2*x**8/8 + a*b*x**8*atan(c*x**2)/4 - a*b*x**6/(12*c) + a*b*x*
*2/(4*c**3) - a*b*atan(c*x**2)/(4*c**4) + b**2*x**8*atan(c*x**2)**2/8 - b**
2*x**6*atan(c*x**2)/(12*c) + b**2*x**4/(24*c**2) + b**2*x**2*atan(c*x**2)/(
4*c**3) - b**2*sqrt(-1/c**2)*atan(c*x**2)/(3*c**3) - b**2*log(x**2 + sqrt(-
1/c**2))/(3*c**4) - b**2*atan(c*x**2)**2/(8*c**4), Ne(c, 0)), (a**2*x**8/8,
 True))
```

Maxima [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.36

$$\begin{aligned} \int x^7 \left(a + b \arctan\left(cx^2\right)\right)^2 \, dx &= \frac{1}{8} b^2 x^8 \arctan\left(cx^2\right)^2 + \frac{1}{8} a^2 x^8 \\ &+ \frac{1}{12} \left(3 x^8 \arctan\left(cx^2\right) - c\left(\frac{c^2x^6 - 3x^2}{c^4} + \frac{3 \arctan\left(cx^2\right)}{c^5}\right)\right) ab \\ &- \frac{1}{24} \left(2c\left(\frac{c^2x^6 - 3x^2}{c^4} + \frac{3 \arctan\left(cx^2\right)}{c^5}\right) \arctan\left(cx^2\right) - \frac{c^2x^4 + 3 \arctan\left(cx^2\right)^2 - 3 \log\left(12c^7x^4 + 12c^5\right)}{c^4}\right. \end{aligned}$$

```
[In] integrate(x^7*(a+b*arctan(c*x^2))^2,x, algorithm="maxima")
```

```
[Out] 1/8*b^2*x^8*arctan(c*x^2)^2 + 1/8*a^2*x^8 + 1/12*(3*x^8*arctan(c*x^2) - c*(
(c^2*x^6 - 3*x^2)/c^4 + 3*arctan(c*x^2)/c^5))*a*b - 1/24*(2*c*((c^2*x^6 - 3
*x^2)/c^4 + 3*arctan(c*x^2)/c^5)*arctan(c*x^2) - (c^2*x^4 + 3*arctan(c*x^2)
^2 - 3*log(12*c^7*x^4 + 12*c^5) - log(c^2*x^4 + 1))/c^4)*b^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.17

$$\int x^7 \left(a + b \arctan\left(cx^2\right)\right)^2 dx$$
$$= \frac{3\,a^2cx^8 + 2\left(3\,cx^8\arctan\left(cx^2\right) - \frac{3\arctan(cx^2)}{c^3} - \frac{c^9x^6 - 3\,c^7x^2}{c^9}\right)ab + \left(3\,cx^8\arctan\left(cx^2\right)^2 - \frac{2\,c^3x^6\arctan(cx^2) - c^2x^4 - 6 \cdots}{\cdots}\right.}{24\,c}$$

```
[In] integrate(x^7*(a+b*arctan(c*x^2))^2,x, algorithm="giac")
```

```
[Out] 1/24*(3*a^2*c*x^8 + 2*(3*c*x^8*arctan(c*x^2) - 3*arctan(c*x^2)/c^3 - (c^9*x
^6 - 3*c^7*x^2)/c^9)*a*b + (3*c*x^8*arctan(c*x^2)^2 - (2*c^3*x^6*arctan(c*x
^2) - c^2*x^4 - 6*c*x^2*arctan(c*x^2) + 3*arctan(c*x^2)^2 + 4*log(c^2*x^4 +
 1))/c^3)*b^2)/c
```

Mupad [B] (verification not implemented)

Time = 1.17 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.21

$$\int x^7 \left(a + b \arctan\left(cx^2\right)\right)^2 dx = \frac{a^2\,x^8}{8} - \frac{b^2\,\mathrm{atan}(c\,x^2)^2}{8\,c^4} + \frac{b^2\,x^8\,\mathrm{atan}(c\,x^2)^2}{8} - \frac{b^2\,\ln\left(c^2\,x^4+1\right)}{6\,c^4} + \frac{b^2\,x^4}{24\,c^2} + \frac{b^2\,x^2\,\mathrm{atan}(c\,x^2)}{4\,c^3} - \frac{b^2\,x^6\,\mathrm{atan}(c\,x^2)}{12\,c} + \frac{a\,b\,x^2}{4\,c^3} - \frac{a\,b\,x^6}{12\,c} - \frac{a\,b\,\mathrm{atan}(c\,x^2)}{4\,c^4} + \frac{a\,b\,x^8\,\mathrm{atan}(c\,x^2)}{4}$$

```
[In] int(x^7*(a + b*atan(c*x^2))^2,x)
```

```
[Out] (a^2*x^8)/8 - (b^2*atan(c*x^2)^2)/(8*c^4) + (b^2*x^8*atan(c*x^2)^2)/8 - (b^
2*log(c^2*x^4 + 1))/(6*c^4) + (b^2*x^4)/(24*c^2) + (b^2*x^2*atan(c*x^2))/(4
*c^3) - (b^2*x^6*atan(c*x^2))/(12*c) + (a*b*x^2)/(4*c^3) - (a*b*x^6)/(12*c)
 - (a*b*atan(c*x^2))/(4*c^4) + (a*b*x^8*atan(c*x^2))/4
```

3.75 $\int x^5(a+b\arctan(cx^2))^2\,dx$

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Rubi [A] (verified) 419
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Optimal result

Integrand size = 16, antiderivative size = 154

$$\int x^5\left(a+b\arctan\left(cx^2\right)\right)^2\,dx=\frac{b^2x^2}{6c^2}-\frac{b^2\arctan(cx^2)}{6c^3}-\frac{bx^4(a+b\arctan(cx^2))}{6c}-\frac{i(a+b\arctan(cx^2))^2}{6c^3}+\frac{1}{6}x^6\left(a+b\arctan\left(cx^2\right)\right)^2-\frac{b(a+b\arctan(cx^2))\log\left(\frac{2}{1+icx^2}\right)}{3c^3}-\frac{ib^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx^2}\right)}{6c^3}$$

```
[Out] 1/6*b^2*x^2/c^2-1/6*b^2*arctan(c*x^2)/c^3-1/6*b*x^4*(a+b*arctan(c*x^2))/c-1
/6*I*(a+b*arctan(c*x^2))^2/c^3+1/6*x^6*(a+b*arctan(c*x^2))^2-1/3*b*(a+b*arc
tan(c*x^2))*ln(2/(1+I*c*x^2))/c^3-1/6*I*b^2*polylog(2,1-2/(1+I*c*x^2))/c^3
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {4948, 4946, 5036, 327, 209, 5040, 4964, 2449, 2352}

$$\int x^5\left(a+b\arctan\left(cx^2\right)\right)^2\,dx=-\frac{i(a+b\arctan(cx^2))^2}{6c^3}-\frac{b\log\left(\frac{2}{1+icx^2}\right)(a+b\arctan(cx^2))}{3c^3}+\frac{1}{6}x^6\left(a+b\arctan\left(cx^2\right)\right)^2-\frac{bx^4(a+b\arctan(cx^2))}{6c}-\frac{b^2\arctan(cx^2)}{6c^3}-\frac{ib^2\operatorname{PolyLog}\left(2,1-\frac{2}{icx^2+1}\right)}{6c^3}+\frac{b^2x^2}{6c^2}$$

```
[In] Int[x^5*(a + b*ArcTan[c*x^2])^2,x]
```

```
[Out] (b^2*x^2)/(6*c^2) - (b^2*ArcTan[c*x^2])/(6*c^3) - (b*x^4*(a + b*ArcTan[c*x^
2]))/(6*c) - ((I/6)*(a + b*ArcTan[c*x^2])^2)/c^3 + (x^6*(a + b*ArcTan[c*x^2
])^2)/6 - (b*(a + b*ArcTan[c*x^2])*Log[2/(1 + I*c*x^2)])/(3*c^3) - ((I/6)*b
^2*PolyLog[2, 1 - 2/(1 + I*c*x^2)])/c^3
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplify
[(m + 1)/n]]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int x^2(a+b\arctan(cx))^2\,dx, x, x^2\right) \\
&= \frac{1}{6}x^6\left(a+b\arctan\left(cx^2\right)\right)^2 - \frac{1}{3}(bc)\text{Subst}\left(\int \frac{x^3(a+b\arctan(cx))}{1+c^2x^2}\,dx, x, x^2\right) \\
&= \frac{1}{6}x^6\left(a+b\arctan\left(cx^2\right)\right)^2 - \frac{b\text{Subst}\left(\int x(a+b\arctan(cx))\,dx, x, x^2\right)}{3c} \\
&\quad + \frac{b\text{Subst}\left(\int \frac{x(a+b\arctan(cx))}{1+c^2x^2}\,dx, x, x^2\right)}{3c} \\
&= -\frac{bx^4\left(a+b\arctan\left(cx^2\right)\right)}{6c} - \frac{i\left(a+b\arctan\left(cx^2\right)\right)^2}{6c^3} + \frac{1}{6}x^6\left(a+b\arctan\left(cx^2\right)\right)^2 \\
&\quad + \frac{1}{6}b^2\text{Subst}\left(\int \frac{x^2}{1+c^2x^2}\,dx, x, x^2\right) - \frac{b\text{Subst}\left(\int \frac{a+b\arctan(cx)}{i-cx}\,dx, x, x^2\right)}{3c^2} \\
&= \frac{b^2x^2}{6c^2} - \frac{bx^4\left(a+b\arctan\left(cx^2\right)\right)}{6c} - \frac{i\left(a+b\arctan\left(cx^2\right)\right)^2}{6c^3} \\
&\quad + \frac{1}{6}x^6\left(a+b\arctan\left(cx^2\right)\right)^2 - \frac{b\left(a+b\arctan\left(cx^2\right)\right)\log\left(\frac{2}{1+icx^2}\right)}{3c^3} \\
&\quad - \frac{b^2\text{Subst}\left(\int \frac{1}{1+c^2x^2}\,dx, x, x^2\right)}{6c^2} + \frac{b^2\text{Subst}\left(\int \frac{\log\left(\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx, x, x^2\right)}{3c^2}
\end{aligned}
$$

$$= \frac{b^2x^2}{6c^2} - \frac{b^2\arctan(cx^2)}{6c^3} - \frac{bx^4(a+b\arctan(cx^2))}{6c} - \frac{i(a+b\arctan(cx^2))^2}{6c^3} + \frac{1}{6}x^6(a+b\arctan(cx^2))^2 - \frac{b(a+b\arctan(cx^2))\log\left(\frac{2}{1+icx^2}\right)}{3c^3} - \frac{(ib^2)\operatorname{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1+icx^2}\right)}{3c^3}$$

$$= \frac{b^2x^2}{6c^2} - \frac{b^2\arctan(cx^2)}{6c^3} - \frac{bx^4(a+b\arctan(cx^2))}{6c} - \frac{i(a+b\arctan(cx^2))^2}{6c^3} + \frac{1}{6}x^6(a+b\arctan(cx^2))^2 - \frac{b(a+b\arctan(cx^2))\log\left(\frac{2}{1+icx^2}\right)}{3c^3} - \frac{ib^2\operatorname{PolyLog}\left(2, 1-\frac{2}{1+icx^2}\right)}{6c^3}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.92

$$\int x^5(a+b\arctan(cx^2))^2\,dx$$

$$= \frac{b^2cx^2 - abc^2x^4 + a^2c^3x^6 + b^2(i+c^3x^6)\arctan(cx^2)^2 - b\arctan(cx^2)\left(b + bc^2x^4 - 2ac^3x^6 + 2b\log\left(1+e^{2i\right.\right.}}{6c^3}$$

```
[In] Integrate[x^5*(a + b*ArcTan[c*x^2])^2,x]
```

```
[Out] (b^2*c*x^2 - a*b*c^2*x^4 + a^2*c^3*x^6 + b^2*(I + c^3*x^6)*ArcTan[c*x^2]^2
- b*ArcTan[c*x^2]*(b + b*c^2*x^4 - 2*a*c^3*x^6 + 2*b*Log[1 + E^((2*I)*ArcTa
n[c*x^2])]) + a*b*Log[1 + c^2*x^4] + I*b^2*PolyLog[2, -E^((2*I)*ArcTan[c*x^
2])])/(6*c^3)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.56 (sec) , antiderivative size = 333, normalized size of antiderivative = 2.16

method	result
default	$\frac{x^6 a^2}{6}+\frac{b^2 x^6 \arctan(c x^2)^2}{6}-\frac{b^2 \arctan(c x^2) x^4}{6c}+\frac{b^2 \arctan(c x^2)\ln(c^2 x^4+1)}{6c^3}+\frac{b^2 x^2}{6c^2}-\frac{b^2 \arctan(c x^2)}{6c^3}-\frac{b^2\left(\sum_{\_\alpha=\mathrm{RootOf}} \ldots\right.}{}$
parts	$\frac{x^6 a^2}{6}+\frac{b^2 x^6 \arctan(c x^2)^2}{6}-\frac{b^2 \arctan(c x^2) x^4}{6c}+\frac{b^2 \arctan(c x^2)\ln(c^2 x^4+1)}{6c^3}+\frac{b^2 x^2}{6c^2}-\frac{b^2 \arctan(c x^2)}{6c^3}-\frac{b^2\left(\sum_{\_\alpha=\mathrm{RootOf}} \ldots\right.}{}$
risch	$-\frac{i b^2 \ln\left(\frac{1}{2}+\frac{i c x^2}{2}\right)\ln\left(\frac{1}{2}-\frac{i c x^2}{2}\right)}{6c^3}+\frac{a b \ln(c^2 x^4+1)}{6c^3}-\frac{a b x^4}{6c}+\frac{i b^2 x^4 \ln(i c x^2+1)}{12c}-\frac{b^2 x^6 \ln(i c x^2+1)^2}{24}-\frac{b^2 \arctan(c x^2)}{12c^3}+\ldots$

```
[In] int(x^5*(a+b*arctan(c*x^2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*x^6*a^2+1/6*b^2*x^6*arctan(c*x^2)^2-1/6*b^2*arctan(c*x^2)/c*x^4+1/6*b^2
*arctan(c*x^2)/c^3*ln(c^2*x^4+1)+1/6*b^2*x^2/c^2-1/6*b^2*arctan(c*x^2)/c^3-
1/24*b^2/c^4*sum(1/_alpha^2*(2*ln(x-_alpha)*ln(c^2*x^4+1)-c*(1/c/_alpha^3*l
n(x-_alpha)^2+2/_alpha*ln(x-_alpha)*(_alpha^2*ln(1/2*(x+_alpha)/_alpha)*c-l
n((_alpha^3*c+x)/_alpha/(_alpha^2*c+1))+ln((_alpha^3*c-x)/_alpha/(_alpha^2*
c-1)))+2/_alpha*(_alpha^2*dilog(1/2*(x+_alpha)/_alpha)*c-dilog((_alpha^3*c+
x)/_alpha/(_alpha^2*c+1))+dilog((_alpha^3*c-x)/_alpha/(_alpha^2*c-1))))),_a
lpha=RootOf(_Z^4*c^2+1))+1/3*a*b*x^6*arctan(c*x^2)-1/6/c*a*b*x^4+1/6*a*b/c^
3*ln(c^2*x^4+1)
```

Fricas [F]

$$\int x^5 \left(a+b\arctan\left(cx^2\right)\right)^2\,dx=\int \left(b\arctan\left(cx^2\right)+a\right)^2 x^5\,dx$$

```
[In] integrate(x^5*(a+b*arctan(c*x^2))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^5*arctan(c*x^2)^2 + 2*a*b*x^5*arctan(c*x^2) + a^2*x^5, x)
```

Sympy [F]

$$\int x^5 \left(a + b \arctan\left(cx^2\right)\right)^2 \, dx = \int x^5 \left(a + b \operatorname{atan}\left(cx^2\right)\right)^2 \, dx$$

```
[In] integrate(x**5*(a+b*atan(c*x**2))**2,x)
[Out] Integral(x**5*(a + b*atan(c*x**2))**2, x)
```

Maxima [F]

$$\int x^5 \left(a + b \arctan\left(cx^2\right)\right)^2 \, dx = \int \left(b \arctan\left(cx^2\right) + a\right)^2 x^5 \, dx$$

```
[In] integrate(x^5*(a+b*arctan(c*x^2))^2,x, algorithm="maxima")
[Out] 1/6*a^2*x^6 + 1/6*(2*x^6*arctan(c*x^2) - (x^4/c^2 - log(c^2*x^4 + 1)/c^4)*c
)*a*b + 1/96*(4*x^6*arctan(c*x^2)^2 - x^6*log(c^2*x^4 + 1)^2 + 96*integrate
(1/48*(4*c^2*x^9*log(c^2*x^4 + 1) - 8*c*x^7*arctan(c*x^2) + 36*(c^2*x^9 + x
^5)*arctan(c*x^2)^2 + 3*(c^2*x^9 + x^5)*log(c^2*x^4 + 1)^2)/(c^2*x^4 + 1),
x))*b^2
```

Giac [F]

$$\int x^5 \left(a + b \arctan\left(cx^2\right)\right)^2 \, dx = \int \left(b \arctan\left(cx^2\right) + a\right)^2 x^5 \, dx$$

```
[In] integrate(x^5*(a+b*arctan(c*x^2))^2,x, algorithm="giac")
[Out] integrate((b*arctan(c*x^2) + a)^2*x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^5 \left(a + b \arctan\left(cx^2\right)\right)^2 \, dx = \int x^5 \left(a + b \operatorname{atan}\left(c\,x^2\right)\right)^2 \, dx$$

```
[In] int(x^5*(a + b*atan(c*x^2))^2,x)
[Out] int(x^5*(a + b*atan(c*x^2))^2, x)
```

3.76 $\int x^3(a+b\arctan(cx^2))^2\,dx$

Optimal result . . . 425
Rubi [A] (verified) . . . 425
Mathematica [A] (verified) . . . 427
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Giac [A] (verification not implemented) . . . 429
Mupad [B] (verification not implemented) . . . 429

Optimal result

Integrand size = 16, antiderivative size = 90

$$\int x^3\left(a+b\arctan\left(cx^2\right)\right)^2\,dx=-\frac{abx^2}{2c}-\frac{b^2x^2\arctan\left(cx^2\right)}{2c}+\frac{\left(a+b\arctan\left(cx^2\right)\right)^2}{4c^2}+\frac{1}{4}x^4\left(a+b\arctan\left(cx^2\right)\right)^2+\frac{b^2\log\left(1+c^2x^4\right)}{4c^2}$$

```
[Out] -1/2*a*b*x^2/c-1/2*b^2*x^2*arctan(c*x^2)/c+1/4*(a+b*arctan(c*x^2))^2/c^2+1/
4*x^4*(a+b*arctan(c*x^2))^2+1/4*b^2*ln(c^2*x^4+1)/c^2
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4948, 4946, 5036, 4930, 266, 5004}

$$\int x^3\left(a+b\arctan\left(cx^2\right)\right)^2\,dx=\frac{\left(a+b\arctan\left(cx^2\right)\right)^2}{4c^2}+\frac{1}{4}x^4\left(a+b\arctan\left(cx^2\right)\right)^2-\frac{abx^2}{2c}-\frac{b^2x^2\arctan\left(cx^2\right)}{2c}+\frac{b^2\log\left(c^2x^4+1\right)}{4c^2}$$

```
[In] Int[x^3*(a + b*ArcTan[c*x^2])^2,x]
```

```
[Out] -1/2*(a*b*x^2)/c - (b^2*x^2*ArcTan[c*x^2])/(2*c) + (a + b*ArcTan[c*x^2])^2/
(4*c^2) + (x^4*(a + b*ArcTan[c*x^2])^2)/4 + (b^2*Log[1 + c^2*x^4])/(4*c^2)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplify
[(m + 1)/n]]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int x(a+b\arctan(cx))^2\,dx,x,x^2\right)\\
&= \frac{1}{4}x^4\left(a+b\arctan\left(cx^2\right)\right)^2-\frac{1}{2}(bc)\text{Subst}\left(\int\frac{x^2(a+b\arctan(cx))}{1+c^2x^2}\,dx,x,x^2\right)\\
&= \frac{1}{4}x^4\left(a+b\arctan\left(cx^2\right)\right)^2-\frac{b\text{Subst}\left(\int(a+b\arctan(cx))\,dx,x,x^2\right)}{2c}\\
&\quad+\frac{b\text{Subst}\left(\int\frac{a+b\arctan(cx)}{1+c^2x^2}\,dx,x,x^2\right)}{2c}
\end{aligned}$$

$$= -\frac{abx^2}{2c} + \frac{(a + b\arctan(cx^2))^2}{4c^2} + \frac{1}{4}x^4(a+b\arctan(cx^2))^2 - \frac{b^2\text{Subst}\left(\int \arctan(cx)\,dx, x, x^2\right)}{2c}$$

$$= -\frac{abx^2}{2c} - \frac{b^2x^2\arctan(cx^2)}{2c} + \frac{(a + b\arctan(cx^2))^2}{4c^2} + \frac{1}{4}x^4(a + b\arctan(cx^2))^2 + \frac{1}{2}b^2\text{Subst}\left(\int \frac{x}{1 + c^2x^2}\,dx, x, x^2\right)$$

$$= -\frac{abx^2}{2c} - \frac{b^2x^2\arctan(cx^2)}{2c} + \frac{(a + b\arctan(cx^2))^2}{4c^2} + \frac{1}{4}x^4(a + b\arctan(cx^2))^2 + \frac{b^2\log(1 + c^2x^4)}{4c^2}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.94

$$\int x^3(a + b\arctan(cx^2))^2\,dx = \frac{acx^2(-2b + acx^2) + 2b(a - bcx^2 + ac^2x^4)\arctan(cx^2) + b^2(1 + c^2x^4)\arctan(cx^2)^2 + b^2\log(1 + c^2x^4)}{4c^2}$$

```
[In] Integrate[x^3*(a + b*ArcTan[c*x^2])^2,x]
```

```
[Out] (a*c*x^2*(-2*b + a*c*x^2) + 2*b*(a - b*c*x^2 + a*c^2*x^4)*ArcTan[c*x^2] + b
^2*(1 + c^2*x^4)*ArcTan[c*x^2]^2 + b^2*Log[1 + c^2*x^4])/(4*c^2)
```

Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.22

method	result
parallelrisch	$\frac{b^2x^4\arctan(cx^2)^2c^2+2abx^4\arctan(cx^2)c^2+a^2c^2x^4-2b^2\arctan(cx^2)x^2c-2abcx^2+b^2\arctan(cx^2)^2+b^2\ln(c^2x^4+1)+2ab\,\mathrm{ar}}{4c^2}$
default	$\frac{a^2x^4}{4}+\frac{b^2x^4\arctan(cx^2)^2}{4}-\frac{b^2x^2\arctan(cx^2)}{2c}+\frac{b^2\arctan(cx^2)^2}{4c^2}+\frac{b^2\ln(c^2x^4+1)}{4c^2}+\frac{abx^4\arctan(cx^2)}{2}-\frac{abx^2}{2c}$
parts	$\frac{a^2x^4}{4}+\frac{b^2x^4\arctan(cx^2)^2}{4}-\frac{b^2x^2\arctan(cx^2)}{2c}+\frac{b^2\arctan(cx^2)^2}{4c^2}+\frac{b^2\ln(c^2x^4+1)}{4c^2}+\frac{abx^4\arctan(cx^2)}{2}-\frac{abx^2}{2c}$
risch	$-\frac{b^2(c^2x^4+1)\ln(icx^2+1)^2}{16c^2}-\frac{ib(4a^2c^2x^4+2ix^4b\ln(-icx^2+1)ac^2-4abcx^2+b^2+2ib\ln(-icx^2+1)a)\ln(icx^2+1)}{16ac^2}+$

```
[In] int(x^3*(a+b*arctan(c*x^2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(b^2*x^4*arctan(c*x^2)^2*c^2+2*a*b*x^4*arctan(c*x^2)*c^2+a^2*c^2*x^4-2*
b^2*arctan(c*x^2)*x^2*c-2*a*b*c*x^2+b^2*arctan(c*x^2)^2+b^2*ln(c^2*x^4+1)+2
*a*b*arctan(c*x^2))/c^2
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.11

$$\int x^3 \left(a + b \arctan\left(cx^2\right)\right)^2 \, dx$$
$$= \frac{a^2c^2x^4 - 2\,abcx^2 + \left(b^2c^2x^4 + b^2\right)\arctan\left(cx^2\right)^2 - 2\,ab\arctan\left(\frac{1}{cx^2}\right) + b^2\log\left(c^2x^4 + 1\right) + 2\left(abc^2x^4 - b^2cx^2\right)\,\mathrm{a}}{4\,c^2}$$

```
[In] integrate(x^3*(a+b*arctan(c*x^2))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(a^2*c^2*x^4 - 2*a*b*c*x^2 + (b^2*c^2*x^4 + b^2)*arctan(c*x^2)^2 - 2*a*
b*arctan(1/(c*x^2)) + b^2*log(c^2*x^4 + 1) + 2*(a*b*c^2*x^4 - b^2*c*x^2)*ar
ctan(c*x^2))/c^2
```

Sympy [A] (verification not implemented)

Time = 16.83 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.72

$$\int x^3 \left(a + b \arctan\left(cx^2\right)\right)^2 \, dx$$
$$= \begin{cases} \frac{a^2x^4}{4} + \frac{abx^4\operatorname{atan}\left(cx^2\right)}{2} - \frac{abx^2}{2c} + \frac{ab\operatorname{atan}\left(cx^2\right)}{2c^2} + \frac{b^2x^4\operatorname{atan}^2\left(cx^2\right)}{4} - \frac{b^2x^2\operatorname{atan}\left(cx^2\right)}{2c} + \frac{b^2\sqrt{-\frac{1}{c^2}}\operatorname{atan}\left(cx^2\right)}{2c} + \frac{b^2\log\left(x^2+\sqrt{-\frac{1}{c^2}}\right)}{2c^2} \\ \frac{a^2x^4}{4} \end{cases}$$

```
[In] integrate(x**3*(a+b*atan(c*x**2))**2,x)
```

```
[Out] Piecewise((a**2*x**4/4 + a*b*x**4*atan(c*x**2)/2 - a*b*x**2/(2*c) + a*b*ata
n(c*x**2)/(2*c**2) + b**2*x**4*atan(c*x**2)**2/4 - b**2*x**2*atan(c*x**2)/(
2*c) + b**2*sqrt(-1/c**2)*atan(c*x**2)/(2*c) + b**2*log(x**2 + sqrt(-1/c**2
))/(2*c**2) + b**2*atan(c*x**2)**2/(4*c**2), Ne(c, 0)), (a**2*x**4/4, True)
)
```

Maxima [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.40

$$\int x^3 \left(a + b \arctan\left(cx^2\right)\right)^2 \, dx$$
$$= \frac{1}{4}\,b^2x^4\arctan\left(cx^2\right)^2 + \frac{1}{4}\,a^2x^4 + \frac{1}{2}\left(x^4\arctan\left(cx^2\right) - c\left(\frac{x^2}{c^2} - \frac{\arctan\left(cx^2\right)}{c^3}\right)\right)ab$$
$$- \frac{1}{4}\left(2\,c\left(\frac{x^2}{c^2} - \frac{\arctan\left(cx^2\right)}{c^3}\right)\arctan\left(cx^2\right) + \frac{\arctan\left(cx^2\right)^2 - \log\left(4\,c^5x^4 + 4\,c^3\right)}{c^2}\right)b^2$$

```
[In] integrate(x^3*(a+b*arctan(c*x^2))^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*x^4*arctan(c*x^2)^2 + 1/4*a^2*x^4 + 1/2*(x^4*arctan(c*x^2) - c*(x^2
/c^2 - arctan(c*x^2)/c^3))*a*b - 1/4*(2*c*(x^2/c^2 - arctan(c*x^2)/c^3)*arc
tan(c*x^2) + (arctan(c*x^2)^2 - log(4*c^5*x^4 + 4*c^3))/c^2)*b^2
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.11

$$\int x^3 \left(a + b \arctan\left(cx^2\right)\right)^2 \, dx = \frac{a^2cx^4 + \frac{2\left(c^2x^4 \arctan(cx^2) - cx^2 + \arctan(cx^2)\right)ab}{c} + \frac{\left(c^2x^4 \arctan(cx^2)^2 - 2\,cx^2 \arctan(cx^2) + \arctan(cx^2)^2 + \log(c^2x^4+1)\right)b^2}{c}}{4\,c}$$

```
[In] integrate(x^3*(a+b*arctan(c*x^2))^2,x, algorithm="giac")
```

```
[Out] 1/4*(a^2*c*x^4 + 2*(c^2*x^4*arctan(c*x^2) - c*x^2 + arctan(c*x^2))*a*b/c +
(c^2*x^4*arctan(c*x^2)^2 - 2*c*x^2*arctan(c*x^2) + arctan(c*x^2)^2 + log(c^
2*x^4 + 1))*b^2/c)/c
```

Mupad [B] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.24

$$\int x^3 \left(a + b \arctan\left(cx^2\right)\right)^2 \, dx = \frac{a^2\,x^4}{4} + \frac{b^2\,\mathrm{atan}(c\,x^2)^2}{4\,c^2} + \frac{b^2\,x^4\,\mathrm{atan}(c\,x^2)^2}{4} + \frac{b^2\,\ln\left(c^2\,x^4+1\right)}{4\,c^2} - \frac{b^2\,x^2\,\mathrm{atan}(c\,x^2)}{2\,c} - \frac{a\,b\,x^2}{2\,c} + \frac{a\,b\,\mathrm{atan}(c\,x^2)}{2\,c^2} + \frac{a\,b\,x^4\,\mathrm{atan}(c\,x^2)}{2}$$

```
[In] int(x^3*(a + b*atan(c*x^2))^2,x)
```

```
[Out] (a^2*x^4)/4 + (b^2*atan(c*x^2)^2)/(4*c^2) + (b^2*x^4*atan(c*x^2)^2)/4 + (b^
2*log(c^2*x^4 + 1))/(4*c^2) - (b^2*x^2*atan(c*x^2))/(2*c) - (a*b*x^2)/(2*c)
 + (a*b*atan(c*x^2))/(2*c^2) + (a*b*x^4*atan(c*x^2))/2
```

3.77 $\int x(a+b\arctan(cx^2))^2\,dx$

Optimal result . . . 430
Rubi [A] (verified) . . . 430
Mathematica [A] (verified) . . . 432
Maple [A] (verified) . . . 432
Fricas [F] . . . 433
Sympy [F] . . . 433
Maxima [F] . . . 434
Giac [F] . . . 434
Mupad [F(-1)] . . . 434

Optimal result

Integrand size = 14, antiderivative size = 101

$$\int x\left(a+b\arctan\left(cx^2\right)\right)^2\,dx=\frac{i(a+b\arctan(cx^2))^2}{2c}+\frac{1}{2}x^2\left(a+b\arctan\left(cx^2\right)\right)^2+\frac{b(a+b\arctan(cx^2))\log\left(\frac{2}{1+icx^2}\right)}{c}+\frac{ib^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx^2}\right)}{2c}$$

```
[Out] 1/2*I*(a+b*arctan(c*x^2))^2/c+1/2*x^2*(a+b*arctan(c*x^2))^2+b*(a+b*arctan(c
*x^2))*ln(2/(1+I*c*x^2))/c+1/2*I*b^2*polylog(2,1-2/(1+I*c*x^2))/c
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4948, 4930, 5040, 4964, 2449, 2352}

$$\int x\left(a+b\arctan\left(cx^2\right)\right)^2\,dx=\frac{1}{2}x^2\left(a+b\arctan\left(cx^2\right)\right)^2+\frac{i(a+b\arctan(cx^2))^2}{2c}+\frac{b\log\left(\frac{2}{1+icx^2}\right)(a+b\arctan(cx^2))}{c}+\frac{ib^2\operatorname{PolyLog}\left(2,1-\frac{2}{icx^2+1}\right)}{2c}$$

```
[In] Int[x*(a + b*ArcTan[c*x^2])^2,x]
```

```
[Out] ((I/2)*(a + b*ArcTan[c*x^2])^2)/c + (x^2*(a + b*ArcTan[c*x^2])^2)/2 + (b*(a
 + b*ArcTan[c*x^2])*Log[2/(1 + I*c*x^2)])/c + ((I/2)*b^2*PolyLog[2, 1 - 2/(
1 + I*c*x^2)])/c
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplify
[(m + 1)/n]]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}\text{Subst}\left(\int (a+b\arctan(cx))^2\,dx, x, x^2\right)\\ &= \frac{1}{2}x^2\left(a+b\arctan\left(cx^2\right)\right)^2-(bc)\text{Subst}\left(\int \frac{x(a+b\arctan(cx))}{1+c^2x^2}\,dx, x, x^2\right)\end{aligned}$$

$$= \frac{i(a+b\arctan(cx^2))^2}{2c} + \frac{1}{2}x^2(a+b\arctan(cx^2))^2 + b\text{Subst}\left(\int \frac{a+b\arctan(cx)}{i-cx}\,dx, x, x^2\right)$$

$$= \frac{i(a+b\arctan(cx^2))^2}{2c} + \frac{1}{2}x^2(a+b\arctan(cx^2))^2 + \frac{b(a+b\arctan(cx^2))\log\left(\frac{2}{1+icx^2}\right)}{c} - b^2\text{Subst}\left(\int \frac{\log\left(\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx, x, x^2\right)$$

$$= \frac{i(a+b\arctan(cx^2))^2}{2c} + \frac{1}{2}x^2(a+b\arctan(cx^2))^2 + \frac{b(a+b\arctan(cx^2))\log\left(\frac{2}{1+icx^2}\right)}{c} + \frac{(ib^2)\,\text{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1+icx^2}\right)}{c}$$

$$= \frac{i(a+b\arctan(cx^2))^2}{2c} + \frac{1}{2}x^2(a+b\arctan(cx^2))^2 + \frac{b(a+b\arctan(cx^2))\log\left(\frac{2}{1+icx^2}\right)}{c} + \frac{ib^2\,\text{PolyLog}\left(2, 1-\frac{2}{1+icx^2}\right)}{2c}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.06

$$\int x(a+b\arctan(cx^2))^2\,dx$$
$$= \frac{b^2(-i+cx^2)\arctan(cx^2)^2 + 2b\arctan(cx^2)\left(acx^2 + b\log\left(1+e^{2i\arctan(cx^2)}\right)\right) + a(acx^2 - b\log(1+c^2x^4))}{2c}$$

```
[In] Integrate[x*(a + b*ArcTan[c*x^2])^2,x]
```

```
[Out] (b^2*(-I + c*x^2)*ArcTan[c*x^2]^2 + 2*b*ArcTan[c*x^2]*(a*c*x^2 + b*Log[1 +
E^((2*I)*ArcTan[c*x^2])]) + a*(a*c*x^2 - b*Log[1 + c^2*x^4]) - I*b^2*PolyLo
g[2, -E^((2*I)*ArcTan[c*x^2])])/(2*c)
```

Maple [A] (verified)

Time = 4.44 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.39

method	result
parts	$\frac{a^2x^2}{2}+\frac{b^2\left(\arctan(c\,x^2)^2(c\,x^2+i)+2\arctan(c\,x^2)\ln\left(1+\frac{(ic\,x^2+1)^2}{c^2x^4+1}\right)-2i\arctan(c\,x^2)^2-i\operatorname{polylog}\left(2,-\frac{(ic\,x^2+1)^2}{c^2x^4+1}\right)\right.}{2c}$
derivativedivides	$\frac{a^2c\,x^2-i\arctan(c\,x^2)^2b^2+\arctan(c\,x^2)^2b^2c\,x^2-i\operatorname{polylog}\left(2,-\frac{(ic\,x^2+1)^2}{c^2x^4+1}\right)b^2+2\arctan(c\,x^2)\ln\left(1+\frac{(ic\,x^2+1)^2}{c^2x^4+1}\right)b^2+2}{2c}$
default	$\frac{a^2c\,x^2-i\arctan(c\,x^2)^2b^2+\arctan(c\,x^2)^2b^2c\,x^2-i\operatorname{polylog}\left(2,-\frac{(ic\,x^2+1)^2}{c^2x^4+1}\right)b^2+2\arctan(c\,x^2)\ln\left(1+\frac{(ic\,x^2+1)^2}{c^2x^4+1}\right)b^2+2}{2c}$
risch	$\frac{ia^2}{2c}-\frac{b^2\arctan(c\,x^2)}{2c}+\frac{i\ln(-ic\,x^2+1)ab\,x^2}{2}-\frac{\ln(-ic\,x^2+1)ab}{2c}-\frac{i\ln(-ic\,x^2+1)^2b^2}{8c}+\frac{ib^2\ln(ic\,x^2+1)^2}{8c}-\frac{ba}{}$

[In] int(x*(a+b*arctan(c*x^2))^2,x,method=_RETURNVERBOSE)

[Out] 1/2*a^2*x^2+1/2*b^2/c*(arctan(c*x^2)^2*(c*x^2+I)+2*arctan(c*x^2)*ln(1+(1+I*
c*x^2)^2/(c^2*x^4+1))-2*I*arctan(c*x^2)^2-I*polylog(2,-(1+I*c*x^2)^2/(c^2*x
^4+1)))+a*b/c*(c*x^2*arctan(c*x^2)-1/2*ln(c^2*x^4+1))

Fricas [F]

$$\int x\left(a+b\arctan\left(cx^2\right)\right)^2\,dx=\int\left(b\arctan\left(cx^2\right)+a\right)^2x\,dx$$

[In] integrate(x*(a+b*arctan(c*x^2))^2,x, algorithm="fricas")

[Out] integral(b^2*x*arctan(c*x^2)^2 + 2*a*b*x*arctan(c*x^2) + a^2*x, x)

Sympy [F]

$$\int x\left(a+b\arctan\left(cx^2\right)\right)^2\,dx=\int x\left(a+b\operatorname{atan}\left(cx^2\right)\right)^2\,dx$$

[In] integrate(x*(a+b*atan(c*x**2))**2,x)

[Out] Integral(x*(a + b*atan(c*x**2))**2, x)

Maxima [F]

$$\int x(a+b\arctan(cx^2))^2\,dx = \int (b\arctan(cx^2)+a)^2 x\,dx$$

```
[In] integrate(x*(a+b*arctan(c*x^2))^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*x^2 + 1/32*(4*x^2*arctan(c*x^2)^2 - x^2*log(c^2*x^4 + 1)^2 + 384*c^
2*integrate(1/16*x^5*arctan(c*x^2)^2/(c^2*x^4 + 1), x) + 32*c^2*integrate(1
/16*x^5*log(c^2*x^4 + 1)^2/(c^2*x^4 + 1), x) + 128*c^2*integrate(1/16*x^5*l
og(c^2*x^4 + 1)/(c^2*x^4 + 1), x) + 4*arctan(c*x^2)^3/c - 256*c*integrate(1
/16*x^3*arctan(c*x^2)/(c^2*x^4 + 1), x) + 32*integrate(1/16*x*log(c^2*x^4 +
 1)^2/(c^2*x^4 + 1), x))*b^2 + 1/2*(2*c*x^2*arctan(c*x^2) - log(c^2*x^4 + 1
))*a*b/c
```

Giac [F]

$$\int x(a+b\arctan(cx^2))^2\,dx = \int (b\arctan(cx^2)+a)^2 x\,dx$$

```
[In] integrate(x*(a+b*arctan(c*x^2))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^2) + a)^2*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x(a+b\arctan(cx^2))^2\,dx = \int x\,(a+b\,\mathrm{atan}(c\,x^2))^2\,dx$$

```
[In] int(x*(a + b*atan(c*x^2))^2,x)
```

```
[Out] int(x*(a + b*atan(c*x^2))^2, x)
```

3.78 $\int \frac{(a+b\arctan(cx^2))^2}{x}\,dx$

Optimal result 435
Rubi [A] (verified) 435
Mathematica [A] (verified) 438
Maple [F] 439
Fricas [F] 439
Sympy [F] 439
Maxima [F] 439
Giac [F] 440
Mupad [F(-1)] 440

Optimal result

Integrand size = 16, antiderivative size = 151

$$\begin{aligned}\int \frac{(a+b\arctan(cx^2))^2}{x}\,dx = {} & (a+b\arctan(cx^2))^2\operatorname{arctanh}\left(1-\frac{2}{1+icx^2}\right) \\ & -\frac{1}{2}ib(a+b\arctan(cx^2))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx^2}\right) \\ & +\frac{1}{2}ib(a+b\arctan(cx^2))\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx^2}\right) \\ & -\frac{1}{4}b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx^2}\right) \\ & +\frac{1}{4}b^2\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx^2}\right)\end{aligned}$$

```
[Out] -(a+b*arctan(c*x^2))^2*arctanh(-1+2/(1+I*c*x^2))-1/2*I*b*(a+b*arctan(c*x^2)
)*polylog(2,1-2/(1+I*c*x^2))+1/2*I*b*(a+b*arctan(c*x^2))*polylog(2,-1+2/(1+
I*c*x^2))-1/4*b^2*polylog(3,1-2/(1+I*c*x^2))+1/4*b^2*polylog(3,-1+2/(1+I*c*
x^2))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used

$= \{4944, 4942, 5108, 5004, 5114, 6745\}$

$$\int \frac{(a+b\arctan(cx^2))^2}{x}\,dx = \operatorname{arctanh}\left(1-\frac{2}{1+icx^2}\right)(a+b\arctan(cx^2))^2 - \frac{1}{2}ib\operatorname{PolyLog}\left(2,1-\frac{2}{icx^2+1}\right)(a+b\arctan(cx^2)) + \frac{1}{2}ib\operatorname{PolyLog}\left(2,\frac{2}{icx^2+1}-1\right)(a+b\arctan(cx^2)) - \frac{1}{4}b^2\operatorname{PolyLog}\left(3,1-\frac{2}{icx^2+1}\right) + \frac{1}{4}b^2\operatorname{PolyLog}\left(3,\frac{2}{icx^2+1}-1\right)$$

```
[In] Int[(a + b*ArcTan[c*x^2])^2/x,x]
```

```
[Out] (a + b*ArcTan[c*x^2])^2*ArcTanh[1 - 2/(1 + I*c*x^2)] - (I/2)*b*(a + b*ArcTa
n[c*x^2])*PolyLog[2, 1 - 2/(1 + I*c*x^2)] + (I/2)*b*(a + b*ArcTan[c*x^2])*P
olyLog[2, -1 + 2/(1 + I*c*x^2)] - (b^2*PolyLog[3, 1 - 2/(1 + I*c*x^2)])/4 +
 (b^2*PolyLog[3, -1 + 2/(1 + I*c*x^2)])/4
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1
/n, Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[p, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5108

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e*
x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+b\arctan(cx))^2}{x}\,dx, x, x^2\right)\\
&= \left(a+b\arctan\left(cx^2\right)\right)^2 \operatorname{arctanh}\left(1-\frac{2}{1+icx^2}\right)\\
&\quad -(2bc)\text{Subst}\left(\int \frac{(a+b\arctan(cx))\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx, x, x^2\right)\\
&= \left(a+b\arctan\left(cx^2\right)\right)^2 \operatorname{arctanh}\left(1-\frac{2}{1+icx^2}\right)\\
&\quad +(bc)\text{Subst}\left(\int \frac{(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx, x, x^2\right)\\
&\quad -(bc)\text{Subst}\left(\int \frac{(a+b\arctan(cx))\log\left(2-\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx, x, x^2\right)\\
&= \left(a+b\arctan\left(cx^2\right)\right)^2 \operatorname{arctanh}\left(1-\frac{2}{1+icx^2}\right)\\
&\quad -\frac{1}{2}ib\left(a+b\arctan\left(cx^2\right)\right)\text{PolyLog}\left(2,1-\frac{2}{1+icx^2}\right)\\
&\quad +\frac{1}{2}ib\left(a+b\arctan\left(cx^2\right)\right)\text{PolyLog}\left(2,-1+\frac{2}{1+icx^2}\right)\\
&\quad +\frac{1}{2}\left(ib^2c\right)\text{Subst}\left(\int \frac{\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx, x, x^2\right)\\
&\quad -\frac{1}{2}\left(ib^2c\right)\text{Subst}\left(\int \frac{\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx, x, x^2\right)
\end{aligned}
$$

$$= \left(a + b\arctan\left(cx^2\right)\right)^2 \operatorname{arctanh}\left(1 - \frac{2}{1+icx^2}\right) - \frac{1}{2}ib\left(a + b\arctan\left(cx^2\right)\right)\operatorname{PolyLog}\left(2, 1 - \frac{2}{1+icx^2}\right) + \frac{1}{2}ib\left(a + b\arctan\left(cx^2\right)\right)\operatorname{PolyLog}\left(2, -1 + \frac{2}{1+icx^2}\right) - \frac{1}{4}b^2\operatorname{PolyLog}\left(3, 1 - \frac{2}{1+icx^2}\right) + \frac{1}{4}b^2\operatorname{PolyLog}\left(3, -1 + \frac{2}{1+icx^2}\right)$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.33

$$\int \frac{\left(a + b\arctan\left(cx^2\right)\right)^2}{x} dx = a^2 \log(x) + \frac{1}{2}iab\left(\operatorname{PolyLog}\left(2, -icx^2\right) - \operatorname{PolyLog}\left(2, icx^2\right)\right) + \frac{1}{48}b^2\left(-i\pi^3 + 16i\arctan\left(cx^2\right)^3 + 24\arctan\left(cx^2\right)^2\log\left(1 - e^{-2i\arctan(cx^2)}\right) - 24\arctan\left(cx^2\right)^2\log\left(1 + e^{2i\arctan(cx^2)}\right) + 24i\arctan\left(cx^2\right)\operatorname{PolyLog}\left(2, e^{-2i\arctan(cx^2)}\right) + 24i\arctan\left(cx^2\right)\operatorname{PolyLog}\left(2, -e^{2i\arctan(cx^2)}\right) + 12\operatorname{PolyLog}\left(3, e^{-2i\arctan(cx^2)}\right) - 12\operatorname{PolyLog}\left(3, -e^{2i\arctan(cx^2)}\right)\right)$$

```
[In] Integrate[(a + b*ArcTan[c*x^2])^2/x,x]
```

```
[Out] a^2*Log[x] + (I/2)*a*b*(PolyLog[2, (-I)*c*x^2] - PolyLog[2, I*c*x^2]) + (b^
2*((-I)*Pi^3 + (16*I)*ArcTan[c*x^2]^3 + 24*ArcTan[c*x^2]^2*Log[1 - E^((-2*I
)*ArcTan[c*x^2])] - 24*ArcTan[c*x^2]^2*Log[1 + E^((2*I)*ArcTan[c*x^2])] + (
24*I)*ArcTan[c*x^2]*PolyLog[2, E^((-2*I)*ArcTan[c*x^2])] + (24*I)*ArcTan[c*
x^2]*PolyLog[2, -E^((2*I)*ArcTan[c*x^2])] + 12*PolyLog[3, E^((-2*I)*ArcTan[
c*x^2])] - 12*PolyLog[3, -E^((2*I)*ArcTan[c*x^2])]))/48
```

Maple [F]

$$\int \frac{(a+b\arctan(c\,x^2))^2}{x}dx$$

```
[In] int((a+b*arctan(c*x^2))^2/x,x)
[Out] int((a+b*arctan(c*x^2))^2/x,x)
```

Fricas [F]

$$\int \frac{(a+b\arctan(cx^2))^2}{x}\,dx = \int \frac{(b\arctan(cx^2)+a)^2}{x}\,dx$$

```
[In] integrate((a+b*arctan(c*x^2))^2/x,x, algorithm="fricas")
[Out] integral((b^2*arctan(c*x^2)^2 + 2*a*b*arctan(c*x^2) + a^2)/x, x)
```

Sympy [F]

$$\int \frac{(a+b\arctan(cx^2))^2}{x}\,dx = \int \frac{(a+b\,\mathrm{atan}(cx^2))^2}{x}\,dx$$

```
[In] integrate((a+b*atan(c*x**2))**2/x,x)
[Out] Integral((a + b*atan(c*x**2))**2/x, x)
```

Maxima [F]

$$\int \frac{(a+b\arctan(cx^2))^2}{x}\,dx = \int \frac{(b\arctan(cx^2)+a)^2}{x}\,dx$$

```
[In] integrate((a+b*arctan(c*x^2))^2/x,x, algorithm="maxima")
[Out] a^2*log(x) + 1/16*integrate((12*b^2*arctan(c*x^2)^2 + b^2*log(c^2*x^4 + 1)^
2 + 32*a*b*arctan(c*x^2))/x, x)
```

Giac [F]

$$\int \frac{(a + b\arctan(cx^2))^2}{x} \, dx = \int \frac{(b\arctan(cx^2) + a)^2}{x} \, dx$$

```
[In] integrate((a+b*arctan(c*x^2))^2/x,x, algorithm="giac")
[Out] integrate((b*arctan(c*x^2) + a)^2/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\arctan(cx^2))^2}{x} \, dx = \int \frac{(a + b\,\mathrm{atan}(c\,x^2))^2}{x} \, dx$$

```
[In] int((a + b*atan(c*x^2))^2/x,x)
[Out] int((a + b*atan(c*x^2))^2/x, x)
```

3.79 $\int \frac{(a+b\arctan(cx^2))^2}{x^3}\,dx$

Optimal result . . . 441
Rubi [A] (verified) . . . 441
Mathematica [A] (verified) . . . 443
Maple [C] (warning: unable to verify) . . . 444
Fricas [F] . . . 444
Sympy [F] . . . 445
Maxima [F] . . . 445
Giac [F] . . . 445
Mupad [F(-1)] . . . 445

Optimal result

Integrand size = 16, antiderivative size = 97

$$\int \frac{(a+b\arctan(cx^2))^2}{x^3}\,dx = -\frac{1}{2}ic\big(a+b\arctan(cx^2)\big)^2 - \frac{(a+b\arctan(cx^2))^2}{2x^2} + bc\big(a+b\arctan(cx^2)\big)\log\left(2-\frac{2}{1-icx^2}\right) - \frac{1}{2}ib^2c\,\text{PolyLog}\left(2,-1+\frac{2}{1-icx^2}\right)$$

```
[Out] -1/2*I*c*(a+b*arctan(c*x^2))^2-1/2*(a+b*arctan(c*x^2))^2/x^2+b*c*(a+b*arcta
n(c*x^2))*ln(2-2/(1-I*c*x^2))-1/2*I*b^2*c*polylog(2,-1+2/(1-I*c*x^2))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {4948, 4946, 5044, 4988, 2497}

$$\int \frac{(a+b\arctan(cx^2))^2}{x^3}\,dx = -\frac{1}{2}ic\big(a+b\arctan(cx^2)\big)^2 - \frac{(a+b\arctan(cx^2))^2}{2x^2} + bc\log\left(2-\frac{2}{1-icx^2}\right)\big(a+b\arctan(cx^2)\big) - \frac{1}{2}ib^2c\,\text{PolyLog}\left(2,\frac{2}{1-icx^2}-1\right)$$

```
[In] Int[(a + b*ArcTan[c*x^2])^2/x^3,x]
```

```
[Out] (-1/2*I)*c*(a + b*ArcTan[c*x^2])^2 - (a + b*ArcTan[c*x^2])^2/(2*x^2) + b*c*
(a + b*ArcTan[c*x^2])*Log[2 - 2/(1 - I*c*x^2)] - (I/2)*b^2*c*PolyLog[2, -1
+ 2/(1 - I*c*x^2)]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplify
[(m + 1)/n]]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{(a+b\arctan(cx))^2}{x^2}\,dx, x, x^2\right)$$

$$= -\frac{\left(a+b\arctan\left(cx^2\right)\right)^2}{2x^2} + (bc)\text{Subst}\left(\int \frac{a+b\arctan(cx)}{x\left(1+c^2x^2\right)}\,dx, x, x^2\right)$$

$$= -\frac{1}{2}ic\left(a+b\arctan\left(cx^2\right)\right)^2 - \frac{\left(a+b\arctan\left(cx^2\right)\right)^2}{2x^2} + (ibc)\text{Subst}\left(\int \frac{a+b\arctan(cx)}{x(i+cx)}\,dx, x, x^2\right)$$

$$= -\frac{1}{2}ic\left(a+b\arctan\left(cx^2\right)\right)^2 - \frac{\left(a+b\arctan\left(cx^2\right)\right)^2}{2x^2} + bc\left(a+b\arctan\left(cx^2\right)\right)\log\left(2-\frac{2}{1-icx^2}\right) - \left(b^2c^2\right)\text{Subst}\left(\int \frac{\log\left(2-\frac{2}{1-icx}\right)}{1+c^2x^2}\,dx, x, x^2\right)$$

$$= -\frac{1}{2}ic\left(a+b\arctan\left(cx^2\right)\right)^2 - \frac{\left(a+b\arctan\left(cx^2\right)\right)^2}{2x^2} + bc\left(a+b\arctan\left(cx^2\right)\right)\log\left(2-\frac{2}{1-icx^2}\right) - \frac{1}{2}ib^2c\,\text{PolyLog}\left(2,-1+\frac{2}{1-icx^2}\right)$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.31

$$\int \frac{\left(a+b\arctan\left(cx^2\right)\right)^2}{x^3}\,dx = -\frac{a^2}{2x^2} + abc\left(-\frac{\arctan\left(cx^2\right)}{cx^2} + \log\left(cx^2\right) - \frac{1}{2}\log\left(1+c^2x^4\right)\right) + \frac{1}{2}b^2c\left(-\frac{\arctan\left(cx^2\right)^2}{cx^2} + 2\arctan\left(cx^2\right)\log\left(1-e^{2i\arctan\left(cx^2\right)}\right) - i\left(\arctan\left(cx^2\right)^2 + \text{PolyLog}\left(2,e^{2i\arctan\left(cx^2\right)}\right)\right)\right)$$

```
[In] Integrate[(a + b*ArcTan[c*x^2])^2/x^3,x]
```

```
[Out] -1/2*a^2/x^2 + a*b*c*(-(ArcTan[c*x^2]/(c*x^2)) + Log[c*x^2] - Log[1 + c^2*x
^4]/2) + (b^2*c*(-(ArcTan[c*x^2]^2/(c*x^2)) + 2*ArcTan[c*x^2]*Log[1 - E^((2
*I)*ArcTan[c*x^2])] - I*(ArcTan[c*x^2]^2 + PolyLog[2, E^((2*I)*ArcTan[c*x^2
])])))/2
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.33 (sec) , antiderivative size = 339, normalized size of antiderivative = 3.49

method	result
default	$-\frac{a^2}{2x^2}-\frac{b^2\arctan(c\,x^2)^2}{2x^2}+2b^2c\arctan\left(c\,x^2\right)\ln\left(x\right)-\frac{b^2\arctan(c\,x^2)\ln(c^2x^4+1)c}{2}+\frac{b^2\left(\sum_{\_\alpha=\mathrm{RootOf}\left(c^2\_Z^4+1\right)}\frac{2\ln(x-}{}\right.}{}$
parts	$-\frac{a^2}{2x^2}-\frac{b^2\arctan(c\,x^2)^2}{2x^2}+2b^2c\arctan\left(c\,x^2\right)\ln\left(x\right)-\frac{b^2\arctan(c\,x^2)\ln(c^2x^4+1)c}{2}+\frac{b^2\left(\sum_{\_\alpha=\mathrm{RootOf}\left(c^2\_Z^4+1\right)}\frac{2\ln(x-}{}\right.}{}$

```
[In] int((a+b*arctan(c*x^2))^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a^2/x^2-1/2*b^2/x^2*arctan(c*x^2)^2+2*b^2*c*arctan(c*x^2)*ln(x)-1/2*b^
2*arctan(c*x^2)*ln(c^2*x^4+1)*c+1/8*b^2*sum(1/_alpha^2*(2*ln(x-_alpha)*ln(c
^2*x^4+1)-c*(1/c/_alpha^3*ln(x-_alpha)^2+2/_alpha*ln(x-_alpha)*(_alpha^2*ln
(1/2*(x+_alpha)/_alpha)*c-ln((_alpha^3*c+x)/_alpha/(_alpha^2*c+1))+ln((_alp
ha^3*c-x)/_alpha/(_alpha^2*c-1)))+2/_alpha*(_alpha^2*dilog(1/2*(x+_alpha)/_
alpha)*c-dilog((_alpha^3*c+x)/_alpha/(_alpha^2*c+1))+dilog((_alpha^3*c-x)/_
alpha/(_alpha^2*c-1))))),_alpha=RootOf(_Z^4*c^2+1))-b^2*sum(1/_R1^2*(ln(x)*
ln((_R1-x)/_R1)+dilog((_R1-x)/_R1)),_R1=RootOf(_Z^4*c^2+1))+2*a*b*(-1/2/x^2
*arctan(c*x^2)+c*(ln(x)-1/4*ln(c^2*x^4+1)))
```

Fricas [F]

$$\int \frac{(a+b\arctan(cx^2))^2}{x^3}\,dx=\int \frac{(b\arctan(cx^2)+a)^2}{x^3}\,dx$$

```
[In] integrate((a+b*arctan(c*x^2))^2/x^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c*x^2)^2 + 2*a*b*arctan(c*x^2) + a^2)/x^3, x)
```

Sympy [F]

$$\int \frac{(a+b\arctan(cx^2))^2}{x^3}\,dx = \int \frac{(a+b\,\mathrm{atan}(cx^2))^2}{x^3}\,dx$$

```
[In] integrate((a+b*atan(c*x**2))**2/x**3,x)
[Out] Integral((a + b*atan(c*x**2))**2/x**3, x)
```

Maxima [F]

$$\int \frac{(a+b\arctan(cx^2))^2}{x^3}\,dx = \int \frac{(b\arctan(cx^2)+a)^2}{x^3}\,dx$$

```
[In] integrate((a+b*arctan(c*x^2))^2/x^3,x, algorithm="maxima")
[Out] -1/2*(c*(log(c^2*x^4 + 1) - log(x^4)) + 2*arctan(c*x^2)/x^2)*a*b + 1/32*(32
*x^2*integrate(-1/16*(4*c^2*x^4*log(c^2*x^4 + 1) - 8*c*x^2*arctan(c*x^2) -
12*(c^2*x^4 + 1)*arctan(c*x^2)^2 - (c^2*x^4 + 1)*log(c^2*x^4 + 1)^2)/(c^2*x
^7 + x^3), x) - 4*arctan(c*x^2)^2 + log(c^2*x^4 + 1)^2)*b^2/x^2 - 1/2*a^2/x
^2
```

Giac [F]

$$\int \frac{(a+b\arctan(cx^2))^2}{x^3}\,dx = \int \frac{(b\arctan(cx^2)+a)^2}{x^3}\,dx$$

```
[In] integrate((a+b*arctan(c*x^2))^2/x^3,x, algorithm="giac")
[Out] integrate((b*arctan(c*x^2) + a)^2/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx^2))^2}{x^3}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x^2))^2}{x^3}\,dx$$

```
[In] int((a + b*atan(c*x^2))^2/x^3,x)
[Out] int((a + b*atan(c*x^2))^2/x^3, x)
```

3.80 $\int \frac{(a+b\arctan(cx^2))^2}{x^5} \, dx$

Optimal result 446
Rubi [A] (verified) 446
Mathematica [A] (verified) 448
Maple [A] (verified) 449
Fricas [A] (verification not implemented) 449
Sympy [B] (verification not implemented) 449
Maxima [A] (verification not implemented) 450
Giac [F] 450
Mupad [B] (verification not implemented) 451

Optimal result

Integrand size = 16, antiderivative size = 87

$$\int \frac{(a+b\arctan(cx^2))^2}{x^5} \, dx = -\frac{bc(a+b\arctan(cx^2))}{2x^2} - \frac{1}{4}c^2\left(a+b\arctan\left(cx^2\right)\right)^2 - \frac{(a+b\arctan(cx^2))^2}{4x^4} + b^2c^2\log(x) - \frac{1}{4}b^2c^2\log\left(1+c^2x^4\right)$$

```
[Out] -1/2*b*c*(a+b*arctan(c*x^2))/x^2-1/4*c^2*(a+b*arctan(c*x^2))^2-1/4*(a+b*arc
tan(c*x^2))^2/x^4+b^2*c^2*ln(x)-1/4*b^2*c^2*ln(c^2*x^4+1)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4948, 4946, 5038, 272, 36, 29, 31, 5004}

$$\int \frac{(a+b\arctan(cx^2))^2}{x^5} \, dx = -\frac{1}{4}c^2\left(a+b\arctan\left(cx^2\right)\right)^2 - \frac{bc(a+b\arctan(cx^2))}{2x^2} - \frac{(a+b\arctan(cx^2))^2}{4x^4} - \frac{1}{4}b^2c^2\log\left(c^2x^4+1\right) + b^2c^2\log(x)$$

```
[In] Int[(a + b*ArcTan[c*x^2])^2/x^5,x]
```

```
[Out] -1/2*(b*c*(a + b*ArcTan[c*x^2]))/x^2 - (c^2*(a + b*ArcTan[c*x^2])^2)/4 - (a
 + b*ArcTan[c*x^2])^2/(4*x^4) + b^2*c^2*Log[x] - (b^2*c^2*Log[1 + c^2*x^4])
/4
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplify
[(m + 1)/n]]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+b\arctan(cx))^2}{x^3}\,dx, x, x^2\right)\\
&= -\frac{(a+b\arctan(cx^2))^2}{4x^4} + \frac{1}{2}(bc)\text{Subst}\left(\int \frac{a+b\arctan(cx)}{x^2(1+c^2x^2)}\,dx, x, x^2\right)\\
&= -\frac{(a+b\arctan(cx^2))^2}{4x^4} + \frac{1}{2}(bc)\text{Subst}\left(\int \frac{a+b\arctan(cx)}{x^2}\,dx, x, x^2\right)\\
&\quad - \frac{1}{2}\left(bc^3\right)\text{Subst}\left(\int \frac{a+b\arctan(cx)}{1+c^2x^2}\,dx, x, x^2\right)\\
&= -\frac{bc(a+b\arctan(cx^2))}{2x^2} - \frac{1}{4}c^2\left(a+b\arctan\left(cx^2\right)\right)^2\\
&\quad - \frac{(a+b\arctan(cx^2))^2}{4x^4} + \frac{1}{2}\left(b^2c^2\right)\text{Subst}\left(\int \frac{1}{x(1+c^2x^2)}\,dx, x, x^2\right)\\
&= -\frac{bc(a+b\arctan(cx^2))}{2x^2} - \frac{1}{4}c^2\left(a+b\arctan\left(cx^2\right)\right)^2\\
&\quad - \frac{(a+b\arctan(cx^2))^2}{4x^4} + \frac{1}{4}\left(b^2c^2\right)\text{Subst}\left(\int \frac{1}{x(1+c^2x)}\,dx, x, x^4\right)\\
&= -\frac{bc(a+b\arctan(cx^2))}{2x^2} - \frac{1}{4}c^2\left(a+b\arctan\left(cx^2\right)\right)^2 - \frac{(a+b\arctan(cx^2))^2}{4x^4}\\
&\quad + \frac{1}{4}\left(b^2c^2\right)\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^4\right) - \frac{1}{4}\left(b^2c^4\right)\text{Subst}\left(\int \frac{1}{1+c^2x}\,dx, x, x^4\right)\\
&= -\frac{bc(a+b\arctan(cx^2))}{2x^2} - \frac{1}{4}c^2\left(a+b\arctan\left(cx^2\right)\right)^2\\
&\quad - \frac{(a+b\arctan(cx^2))^2}{4x^4} + b^2c^2\log(x) - \frac{1}{4}b^2c^2\log\left(1+c^2x^4\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.13

$$\int \frac{(a+b\arctan(cx^2))^2}{x^5}\,dx = -\frac{a^2+2abcx^2+2b(a+bcx^2+ac^2x^4)\arctan(cx^2)+b^2(1+c^2x^4)\arctan(cx^2)^2-4b^2c^2x^4\log(x)+b^2c^2x^4\,\mathrm{l}}{4x^4}$$

```
[In] Integrate[(a + b*ArcTan[c*x^2])^2/x^5,x]
```

```
[Out] -1/4*(a^2 + 2*a*b*c*x^2 + 2*b*(a + b*c*x^2 + a*c^2*x^4)*ArcTan[c*x^2] + b^2
*(1 + c^2*x^4)*ArcTan[c*x^2]^2 - 4*b^2*c^2*x^4*Log[x] + b^2*c^2*x^4*Log[1 +
 c^2*x^4])/x^4
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.36

method	result
default	$-\frac{a^2}{4x^4}-\frac{b^2\arctan(cx^2)^2}{4x^4}-\frac{b^2c\arctan(cx^2)}{2x^2}-\frac{b^2\arctan(cx^2)^2c^2}{4}+b^2c^2\ln(x)-\frac{b^2c^2\ln(c^2x^4+1)}{4}-\frac{ab\arctan(}{2x^4}$
parts	$-\frac{a^2}{4x^4}-\frac{b^2\arctan(cx^2)^2}{4x^4}-\frac{b^2c\arctan(cx^2)}{2x^2}-\frac{b^2\arctan(cx^2)^2c^2}{4}+b^2c^2\ln(x)-\frac{b^2c^2\ln(c^2x^4+1)}{4}-\frac{ab\arctan(}{2x^4}$
parallelrisch	$\frac{-b^2x^4\arctan(cx^2)^2c^2+4b^2c^2\ln(x)x^4-b^2c^2\ln(c^2x^4+1)x^4-2abx^4\arctan(cx^2)c^2+a^2c^2x^4-2b^2\arctan(cx^2)x^2c-2abcx^2-b^2}{4x^4}$
risch	$\frac{b^2(c^2x^4+1)\ln(icx^2+1)^2}{16x^4}+\frac{ib(ibc^2x^4\ln(-icx^2+1)+2bcx^2+2a+ib\ln(-icx^2+1))\ln(icx^2+1)}{8x^4}-\frac{-4i\ln((5ibc+ac)x^2+5b}{}$

```
[In] int((a+b*arctan(c*x^2))^2/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/x^4*a^2-1/4*b^2/x^4*arctan(c*x^2)^2-1/2*b^2*c*arctan(c*x^2)/x^2-1/4*b^
2*arctan(c*x^2)^2*c^2+b^2*c^2*ln(x)-1/4*b^2*c^2*ln(c^2*x^4+1)-1/2*a*b/x^4*a
rctan(c*x^2)-1/2*a*b*c/x^2-1/2*a*b*arctan(c*x^2)*c^2
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.32

$$\int \frac{(a+b\arctan(cx^2))^2}{x^5}\,dx$$
$$=\frac{2\,abc^2x^4\arctan\left(\frac{1}{cx^2}\right)-b^2c^2x^4\log(c^2x^4+1)+4\,b^2c^2x^4\log(x)-2\,abcx^2-(b^2c^2x^4+b^2)\arctan(cx^2)^2-a}{4\,x^4}$$

```
[In] integrate((a+b*arctan(c*x^2))^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*b*c^2*x^4*arctan(1/(c*x^2)) - b^2*c^2*x^4*log(c^2*x^4 + 1) + 4*b^2
*c^2*x^4*log(x) - 2*a*b*c*x^2 - (b^2*c^2*x^4 + b^2)*arctan(c*x^2)^2 - a^2 -
 2*(b^2*c*x^2 + a*b)*arctan(c*x^2))/x^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. 2(80) = 160.

Time = 22.23 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.92

$$\int \frac{(a+b\arctan(cx^2))^2}{x^5}\,dx$$
$$=\begin{cases}-\frac{a^2}{4x^4}-\frac{abc^2\operatorname{atan}(cx^2)}{2}-\frac{abc}{2x^2}-\frac{ab\operatorname{atan}(cx^2)}{2x^4}+b^2c^2\log(x)-\frac{b^2c^2\log\left(x^2+\sqrt{-\frac{1}{c^2}}\right)}{2}-\frac{b^2c^2\operatorname{atan}^2(cx^2)}{4}+\frac{b^2c\operatorname{atan}(cx^2)}{2\sqrt{-\frac{1}{c^2}}}- \\ -\frac{a^2}{4x^4}\end{cases}$$

```
[In] integrate((a+b*atan(c*x**2))**2/x**5,x)
```

```
[Out] Piecewise((-a**2/(4*x**4) - a*b*c**2*atan(c*x**2)/2 - a*b*c/(2*x**2) - a*b*
atan(c*x**2)/(2*x**4) + b**2*c**2*log(x) - b**2*c**2*log(x**2 + sqrt(-1/c**
2))/2 - b**2*c**2*atan(c*x**2)**2/4 + b**2*c*atan(c*x**2)/(2*sqrt(-1/c**2))
 - b**2*c*atan(c*x**2)/(2*x**2) - b**2*atan(c*x**2)**2/(4*x**4), Ne(c, 0)),
 (-a**2/(4*x**4), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.26

$$\int \frac{(a+b\arctan(cx^2))^2}{x^5}\,dx = -\frac{1}{2}\left(\left(c\arctan\left(cx^2\right)+\frac{1}{x^2}\right)c+\frac{\arctan\left(cx^2\right)}{x^4}\right)ab + \frac{1}{4}\left(\left(\arctan\left(cx^2\right)^2-\log\left(c^2x^4+1\right)+4\,\log\left(x\right)\right)c^2-2\left(c\arctan\left(cx^2\right)+\frac{1}{x^2}\right)c\arctan\left(cx^2\right)\right)b^2 - \frac{b^2\arctan\left(cx^2\right)^2}{4\,x^4}-\frac{a^2}{4\,x^4}$$

```
[In] integrate((a+b*arctan(c*x^2))^2/x^5,x, algorithm="maxima")
```

```
[Out] -1/2*((c*arctan(c*x^2) + 1/x^2)*c + arctan(c*x^2)/x^4)*a*b + 1/4*((arctan(c
*x^2)^2 - log(c^2*x^4 + 1) + 4*log(x))*c^2 - 2*(c*arctan(c*x^2) + 1/x^2)*c*
arctan(c*x^2))*b^2 - 1/4*b^2*arctan(c*x^2)^2/x^4 - 1/4*a^2/x^4
```

Giac [F]

$$\int \frac{(a+b\arctan(cx^2))^2}{x^5}\,dx = \int \frac{(b\arctan(cx^2)+a)^2}{x^5}\,dx$$

```
[In] integrate((a+b*arctan(c*x^2))^2/x^5,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^2) + a)^2/x^5, x)
```

Mupad [B] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.75

$$\int \frac{(a+b\arctan(cx^2))^2}{x^5}\,dx = b^2\,c^2\,\ln(x) - \frac{b^2\,c^2\,\mathrm{atan}(c\,x^2)^2}{4} - \frac{b^2\,\mathrm{atan}(c\,x^2)^2}{4\,x^4} - \frac{b^2\,c^2\,\ln(c^2\,x^4+1)}{4} - \frac{a^2}{4\,x^4} - \frac{b^2\,c\,\mathrm{atan}(c\,x^2)}{2\,x^2} - \frac{a\,b\,c}{2\,x^2} - \frac{a\,b\,c^2\,\mathrm{atan}\left(\frac{a^2\,c\,x^2}{a^2+25\,b^2}+\frac{25\,b^2\,c\,x^2}{a^2+25\,b^2}\right)}{2} - \frac{a\,b\,\mathrm{atan}(c\,x^2)}{2\,x^4}$$

```
[In] int((a + b*atan(c*x^2))^2/x^5,x)
```

```
[Out] b^2*c^2*log(x) - (b^2*c^2*atan(c*x^2)^2)/4 - (b^2*atan(c*x^2)^2)/(4*x^4) -
(b^2*c^2*log(c^2*x^4 + 1))/4 - a^2/(4*x^4) - (b^2*c*atan(c*x^2))/(2*x^2) -
(a*b*c)/(2*x^2) - (a*b*c^2*atan((a^2*c*x^2)/(a^2 + 25*b^2) + (25*b^2*c*x^2)
/(a^2 + 25*b^2)))/2 - (a*b*atan(c*x^2))/(2*x^4)
```

3.81 $\int x^2(a+b\arctan(cx^2))^2\,dx$

Optimal result 453
Rubi [A] (verified) 454
Mathematica [F] 464
Maple [F] 464
Fricas [F] 465
Sympy [F] 465
Maxima [F] 465
Giac [F] 466
Mupad [F(-1)] 466

Optimal result

Integrand size = 16, antiderivative size = 1393

$$\int x^2 \left(a+b\arctan\left(cx^2\right)\right)^2\,dx$$

$$= -\frac{4abx}{3c}+\frac{2}{9}iabx^3+\frac{4(-1)^{3/4}b^2\arctan\left((-1)^{3/4}\sqrt{c}x\right)}{3c^{3/2}}+\frac{\sqrt[4]{-1}b^2\arctan\left((-1)^{3/4}\sqrt{c}x\right)^2}{3c^{3/2}}$$
$$-\frac{2\sqrt[4]{-1}ab\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)}{3c^{3/2}}-\frac{4(-1)^{3/4}b^2\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)}{3c^{3/2}}$$
$$-\frac{(-1)^{3/4}b^2\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)^2}{3c^{3/2}}-\frac{2(-1)^{3/4}b^2\arctan\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{2}{1-\sqrt[4]{-1}\sqrt{c}x}\right)}{3c^{3/2}}$$
$$+\frac{2(-1)^{3/4}b^2\arctan\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{2}{1+\sqrt[4]{-1}\sqrt{c}x}\right)}{3c^{3/2}}$$
$$-\frac{(-1)^{3/4}b^2\arctan\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{\sqrt{2}\left(\sqrt[4]{-1}+\sqrt{c}x\right)}{1+\sqrt[4]{-1}\sqrt{c}x}\right)}{3c^{3/2}}$$
$$+\frac{2(-1)^{3/4}b^2\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{2}{1-(-1)^{3/4}\sqrt{c}x}\right)}{3c^{3/2}}$$
$$-\frac{2(-1)^{3/4}b^2\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{2}{1+(-1)^{3/4}\sqrt{c}x}\right)}{3c^{3/2}}$$
$$+\frac{(-1)^{3/4}b^2\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)\log\left(-\frac{\sqrt{2}\left((-1)^{3/4}+\sqrt{c}x\right)}{1+(-1)^{3/4}\sqrt{c}x}\right)}{3c^{3/2}}$$
$$+\frac{(-1)^{3/4}b^2\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{(1+i)\left(1+\sqrt[4]{-1}\sqrt{c}x\right)}{1+(-1)^{3/4}\sqrt{c}x}\right)}{3c^{3/2}}$$
$$-\frac{(-1)^{3/4}b^2\arctan\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{(1-i)\left(1+(-1)^{3/4}\sqrt{c}x\right)}{1+\sqrt[4]{-1}\sqrt{c}x}\right)}{3c^{3/2}}-\frac{2ib^2x\log\left(1-icx^2\right)}{3c}$$

$$-\frac{1}{9}b^2x^3\log\left(1-icx^2\right)-\frac{(-1)^{3/4}b^2\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)\log\left(1-icx^2\right)}{3c^{3/2}}-\frac{1}{9}ibx^3\left(2a+ib\log\left(1-icx^2\right)\right)-$$

```
[Out] 1/12*x^3*(2*a+I*b*ln(1-I*c*x^2))^2+4/3*(-1)^(3/4)*b^2*arctan((-1)^(3/4)*x*c
^(1/2))/c^(3/2)+1/3*(-1)^(1/4)*b^2*arctan((-1)^(3/4)*x*c^(1/2))^2/c^(3/2)-4
/3*(-1)^(3/4)*b^2*arctanh((-1)^(3/4)*x*c^(1/2))/c^(3/2)-1/3*(-1)^(3/4)*b^2*
arctanh((-1)^(3/4)*x*c^(1/2))^2/c^(3/2)+1/6*b^2*x^3*ln(1-I*c*x^2)*ln(1+I*c*
x^2)+1/3*(-1)^(1/4)*b^2*polylog(2,1-2/(1-(-1)^(1/4)*x*c^(1/2)))/c^(3/2)+1/3
*(-1)^(1/4)*b^2*polylog(2,1-2/(1+(-1)^(1/4)*x*c^(1/2)))/c^(3/2)-1/6*(-1)^(1
```

```
/4)*b^2*polylog(2,1-2^(1/2)*((-1)^(1/4)+x*c^(1/2))/(1+(-1)^(1/4)*x*c^(1/2))
)/c^(3/2)+1/3*(-1)^(3/4)*b^2*polylog(2,1-2/(1-(-1)^(3/4)*x*c^(1/2)))/c^(3/2
)+1/3*(-1)^(3/4)*b^2*polylog(2,1-2/(1+(-1)^(3/4)*x*c^(1/2)))/c^(3/2)-1/6*(-
1)^(3/4)*b^2*polylog(2,1+2^(1/2)*((-1)^(3/4)+x*c^(1/2))/(1+(-1)^(3/4)*x*c^(
1/2)))/c^(3/2)-1/6*(-1)^(3/4)*b^2*polylog(2,1-(1+I)*(1+(-1)^(1/4)*x*c^(1/2)
)/(1+(-1)^(3/4)*x*c^(1/2)))/c^(3/2)-1/6*(-1)^(1/4)*b^2*polylog(2,1+(-1+I)*(
1+(-1)^(3/4)*x*c^(1/2))/(1+(-1)^(1/4)*x*c^(1/2)))/c^(3/2)-1/9*I*b*x^3*(2*a+
I*b*ln(1-I*c*x^2))-2/3*(-1)^(1/4)*a*b*arctanh((-1)^(3/4)*x*c^(1/2))/c^(3/2)
-1/3*(-1)^(3/4)*b^2*arctanh((-1)^(3/4)*x*c^(1/2))*ln(1-I*c*x^2)/c^(3/2)-1/3
*(-1)^(1/4)*b*arctan((-1)^(3/4)*x*c^(1/2))*(2*a+I*b*ln(1-I*c*x^2))/c^(3/2)+
1/3*(-1)^(3/4)*b^2*arctan((-1)^(3/4)*x*c^(1/2))*ln(1+I*c*x^2)/c^(3/2)+1/3*(
-1)^(3/4)*b^2*arctanh((-1)^(3/4)*x*c^(1/2))*ln(1+I*c*x^2)/c^(3/2)-2/3*(-1)^
(3/4)*b^2*arctan((-1)^(3/4)*x*c^(1/2))*ln(2/(1-(-1)^(1/4)*x*c^(1/2)))/c^(3/
2)+2/3*(-1)^(3/4)*b^2*arctan((-1)^(3/4)*x*c^(1/2))*ln(2/(1+(-1)^(1/4)*x*c^(
1/2)))/c^(3/2)-1/3*(-1)^(3/4)*b^2*arctan((-1)^(3/4)*x*c^(1/2))*ln(2^(1/2)*(
(-1)^(1/4)+x*c^(1/2))/(1+(-1)^(1/4)*x*c^(1/2)))/c^(3/2)+2/3*(-1)^(3/4)*b^2*
arctanh((-1)^(3/4)*x*c^(1/2))*ln(2/(1-(-1)^(3/4)*x*c^(1/2)))/c^(3/2)-2/3*(-
1)^(3/4)*b^2*arctanh((-1)^(3/4)*x*c^(1/2))*ln(2/(1+(-1)^(3/4)*x*c^(1/2)))/c
^(3/2)+1/3*(-1)^(3/4)*b^2*arctanh((-1)^(3/4)*x*c^(1/2))*ln(-2^(1/2)*((-1)^(
3/4)+x*c^(1/2))/(1+(-1)^(3/4)*x*c^(1/2)))/c^(3/2)+1/3*(-1)^(3/4)*b^2*arctan
h((-1)^(3/4)*x*c^(1/2))*ln((1+I)*(1+(-1)^(1/4)*x*c^(1/2))/(1+(-1)^(3/4)*x*c
^(1/2)))/c^(3/2)-1/3*(-1)^(3/4)*b^2*arctan((-1)^(3/4)*x*c^(1/2))*ln((1-I)*(
1+(-1)^(3/4)*x*c^(1/2))/(1+(-1)^(1/4)*x*c^(1/2)))/c^(3/2)-2/3*I*b^2*x*ln(1-
I*c*x^2)/c-1/3*I*a*b*x^3*ln(1+I*c*x^2)+2/3*I*b^2*x*ln(1+I*c*x^2)/c+2/9*I*a*
b*x^3-4/3*a*b*x/c-1/9*b^2*x^3*ln(1-I*c*x^2)-1/12*b^2*x^3*ln(1+I*c*x^2)^2
```

Rubi [A] (verified)

Time = 1.88 (sec) , antiderivative size = 1393, normalized size of antiderivative = 1.00, number of steps used = 86, number of rules used = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 1.688$, Rules used = {4950, 2507, 2526, 2498, 327, 209, 2505, 308, 2520, 12, 5040, 4964, 2449, 2352, 6874, 212, 30, 2637, 211, 5048, 4966, 2497, 214, 6139, 6057, 6131, 6055}

$$\int x^2 \left(a + b \arctan\left(cx^2\right)\right)^2 dx = \frac{1}{12}\left(2a + ib\log\left(1 - icx^2\right)\right)^2 x^3 - \frac{1}{12}b^2\log^2\left(icx^2 + 1\right)x^3 + \frac{2}{9}iabx^3 - \frac{1}{9}b^2\log\left(1 - icx^2\right)x^3 - \frac{1}{9}ib\left(2a + ib\log\left(1 - icx^2\right)\right)x^3 - \frac{1}{3}iab\log\left(icx^2 + 1\right)x^3 + \frac{1}{6}b^2\log\left(1 - icx^2\right)\log\left(icx^2 + 1\right)x^3 - \frac{2ib^2\log\left(1 - icx^2\right)x}{3c} + \frac{2ib^2\log\left(icx^2 + 1\right)x}{3c} - \frac{4abx}{3c} + \frac{\sqrt[4]{-1}b^2\arctan\left((-1)^{3/4}\sqrt{c}x\right)^2}{3c^{3/2}} - \frac{(-1)^{3/4}b^2\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)^2}{3c^{3/2}} + \frac{4(-1)^{3/4}b^2\arctan\left((-1)^{3/4}\sqrt{c}x\right)}{3c^{3/2}} - \frac{4(-1)^{3/4}b^2\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)}{3c^{3/2}} - \frac{2\sqrt[4]{-1}ab\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)}{3c^{3/2}} - \frac{2(-1)^{3/4}b^2\arctan\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{2}{1-\sqrt[4]{-1}\sqrt{c}x}\right)}{3c^{3/2}} + \frac{2(-1)^{3/4}b^2\arctan\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{2}{\sqrt[4]{-1}\sqrt{c}x+1}\right)}{3c^{3/2}} - \frac{(-1)^{3/4}b^2\arctan\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{\sqrt{2}\left(\sqrt{c}x+\sqrt[4]{-1}\right)}{\sqrt[4]{-1}\sqrt{c}x+1}\right)}{3c^{3/2}} + \frac{2(-1)^{3/4}b^2\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{2}{1-(-1)^{3/4}\sqrt{c}x}\right)}{3c^{3/2}} - \frac{2(-1)^{3/4}b^2\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{2}{(-1)^{3/4}\sqrt{c}x+1}\right)}{3c^{3/2}} + \frac{(-1)^{3/4}b^2\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)\log\left(-\frac{\sqrt{2}\left(\sqrt{c}x+(-1)^{3/4}\right)}{(-1)^{3/4}\sqrt{c}x+1}\right)}{3c^{3/2}} + \frac{(-1)^{3/4}b^2\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{(1+i)\left(\sqrt[4]{-1}\sqrt{c}x+1\right)}{(-1)^{3/4}\sqrt{c}x+1}\right)}{3c^{3/2}} - \frac{(-1)^{3/4}b^2\arctan\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{(1-i)\left((-1)^{3/4}\sqrt{c}x+1\right)}{\sqrt[4]{-1}\sqrt{c}x+1}\right)}{3c^{3/2}} - \frac{(-1)^{3/4}b^2\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)\log\left(1 - icx^2\right)}{3c^{3/2}}$$

$$\sqrt[4]{-1}b\arctan\left((-1)^{3/4}\sqrt{c}x\right)\left(2a + ib\log\left(1 - icx^2\right)\right)$$

```
[In] Int[x^2*(a + b*ArcTan[c*x^2])^2,x]
```

```
[Out] (-4*a*b*x)/(3*c) + ((2*I)/9)*a*b*x^3 + (4*(-1)^(3/4)*b^2*ArcTan[(-1)^(3/4)*
Sqrt[c]*x])/(3*c^(3/2)) + ((-1)^(1/4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[c]*x]^2)/(
3*c^(3/2)) - (2*(-1)^(1/4)*a*b*ArcTanh[(-1)^(3/4)*Sqrt[c]*x])/(3*c^(3/2)) -
 (4*(-1)^(3/4)*b^2*ArcTanh[(-1)^(3/4)*Sqrt[c]*x])/(3*c^(3/2)) - ((-1)^(3/4)
*b^2*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]^2)/(3*c^(3/2)) - (2*(-1)^(3/4)*b^2*ArcTa
n[(-1)^(3/4)*Sqrt[c]*x]*Log[2/(1 - (-1)^(1/4)*Sqrt[c]*x)])/(3*c^(3/2)) + (2
*(-1)^(3/4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[2/(1 + (-1)^(1/4)*Sqrt[c]*
x)])/(3*c^(3/2)) - ((-1)^(3/4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[(Sqrt[2
]*((-1)^(1/4) + Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*x)])/(3*c^(3/2)) + (2*(
-1)^(3/4)*b^2*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[2/(1 - (-1)^(3/4)*Sqrt[c]*x
)])/(3*c^(3/2)) - (2*(-1)^(3/4)*b^2*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[2/(1
+ (-1)^(3/4)*Sqrt[c]*x)])/(3*c^(3/2)) + ((-1)^(3/4)*b^2*ArcTanh[(-1)^(3/4)*
Sqrt[c]*x]*Log[-((Sqrt[2]*((-1)^(3/4) + Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqrt[c]
*x))])/(3*c^(3/2)) + ((-1)^(3/4)*b^2*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[((1
+ I)*(1 + (-1)^(1/4)*Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x)])/(3*c^(3/2)) -
 ((-1)^(3/4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[((1 - I)*(1 + (-1)^(3/4)*
Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*x)])/(3*c^(3/2)) - (((2*I)/3)*b^2*x*Log
[1 - I*c*x^2])/c - (b^2*x^3*Log[1 - I*c*x^2])/9 - ((-1)^(3/4)*b^2*ArcTanh[(
-1)^(3/4)*Sqrt[c]*x]*Log[1 - I*c*x^2])/(3*c^(3/2)) - (I/9)*b*x^3*(2*a + I*b
*Log[1 - I*c*x^2]) - ((-1)^(1/4)*b*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*(2*a + I*b*
Log[1 - I*c*x^2]))/(3*c^(3/2)) + (x^3*(2*a + I*b*Log[1 - I*c*x^2])^2)/12 +
(((2*I)/3)*b^2*x*Log[1 + I*c*x^2])/c - (I/3)*a*b*x^3*Log[1 + I*c*x^2] + ((-
1)^(3/4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[1 + I*c*x^2])/(3*c^(3/2)) + (
(-1)^(3/4)*b^2*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[1 + I*c*x^2])/(3*c^(3/2))
+ (b^2*x^3*Log[1 - I*c*x^2]*Log[1 + I*c*x^2])/6 - (b^2*x^3*Log[1 + I*c*x^2]
^2)/12 + ((-1)^(1/4)*b^2*PolyLog[2, 1 - 2/(1 - (-1)^(1/4)*Sqrt[c]*x)])/(3*c
^(3/2)) + ((-1)^(1/4)*b^2*PolyLog[2, 1 - 2/(1 + (-1)^(1/4)*Sqrt[c]*x)])/(3*
c^(3/2)) - ((-1)^(1/4)*b^2*PolyLog[2, 1 - (Sqrt[2]*((-1)^(1/4) + Sqrt[c]*x)
)/(1 + (-1)^(1/4)*Sqrt[c]*x)])/(6*c^(3/2)) + ((-1)^(3/4)*b^2*PolyLog[2, 1 -
 2/(1 - (-1)^(3/4)*Sqrt[c]*x)])/(3*c^(3/2)) + ((-1)^(3/4)*b^2*PolyLog[2, 1
- 2/(1 + (-1)^(3/4)*Sqrt[c]*x)])/(3*c^(3/2)) - ((-1)^(3/4)*b^2*PolyLog[2, 1
 + (Sqrt[2]*((-1)^(3/4) + Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x)])/(6*c^(3/
2)) - ((-1)^(3/4)*b^2*PolyLog[2, 1 - ((1 + I)*(1 + (-1)^(1/4)*Sqrt[c]*x))/(
1 + (-1)^(3/4)*Sqrt[c]*x)])/(6*c^(3/2)) - ((-1)^(1/4)*b^2*PolyLog[2, 1 - ((
1 - I)*(1 + (-1)^(3/4)*Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*x)])/(6*c^(3/2))
```

<u>Rule 12</u>

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

<u>Rule 30</u>

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
```

```
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2507

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Dist[b*e*n*p*(q/(f^n*(m + 1))), Int[(f*x)^(m + n)*((a +
 b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2637

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[z*Log[w]*(D[v, x]/v), x], x] - In
t[SimplifyIntegrand[z*Log[v]*(D[w, x]/w), x], x]) /; InverseFunctionFreeQ[z
, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> I
nt[ExpandIntegrand[x^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*Log[1 + I*c*x^n
])/2)^p, x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1] && IGtQ[n, 0] && Integ
erQ[m]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5048

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
```

```
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6057

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcTanh[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x
)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcTanh[c*x])
*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x]) /; FreeQ[{a, b, c, d,
e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6139

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = \int \left(\frac{1}{4}x^2\left(2a+ib\log\left(1-icx^2\right)\right)^2+\frac{1}{2}bx^2\left(-2ia+b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right) - \frac{1}{4}b^2x^2\log^2\left(1+icx^2\right)\right)dx$$

$$= \frac{1}{4}\int x^2\left(2a+ib\log\left(1-icx^2\right)\right)^2 dx + \frac{1}{2}b\int x^2\left(-2ia+b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right)dx - \frac{1}{4}b^2\int x^2\log^2\left(1+icx^2\right)dx$$

$$= \frac{1}{12}x^3\left(2a+ib\log\left(1-icx^2\right)\right)^2 - \frac{1}{12}b^2x^3\log^2\left(1+icx^2\right) + \frac{1}{2}b\int\left(-2iax^2\log\left(1+icx^2\right)+bx^2\log\left(1-icx^2\right)\log\left(1+icx^2\right)\right)dx - \frac{1}{3}(bc)\int\frac{x^4(2a+ib\log(1-icx^2))}{1-icx^2}dx + \frac{1}{3}\left(ib^2c\right)\int\frac{x^4\log(1+icx^2)}{1+icx^2}dx$$

$$= \frac{1}{12}x^3\left(2a+ib\log\left(1-icx^2\right)\right)^2 - \frac{1}{12}b^2x^3\log^2\left(1+icx^2\right) - (iab)\int x^2\log\left(1+icx^2\right)dx + \frac{1}{2}b^2\int x^2\log\left(1-icx^2\right)\log\left(1+icx^2\right)dx - \frac{1}{3}(bc)\int\left(\frac{2a+ib\log(1-icx^2)}{c^2} + \frac{ix^2(2a+ib\log(1-icx^2))}{c} - \frac{2a+ib\log(1-icx^2)}{c^2(1-icx^2)}\right)dx + \frac{1}{3}\left(ib^2c\right)\int\left(\frac{\log(1+icx^2)}{c^2} - \frac{ix^2\log(1+icx^2)}{c} - \frac{\log(1+icx^2)}{c^2(1+icx^2)}\right)dx$$

$$= \frac{1}{12}x^3\left(2a+ib\log\left(1-icx^2\right)\right)^2 - \frac{1}{3}iabx^3\log\left(1+icx^2\right) + \frac{1}{6}b^2x^3\log\left(1-icx^2\right)\log\left(1+icx^2\right) - \frac{1}{12}b^2x^3\log^2\left(1+icx^2\right) - \frac{1}{3}(ib)\int x^2\left(2a+ib\log\left(1-icx^2\right)\right)dx + \frac{1}{3}b^2\int x^2\log\left(1+icx^2\right)dx - \frac{1}{2}b^2\int\frac{2cx^4\log(1-icx^2)}{-3i+3cx^2}dx - \frac{1}{2}b^2\int\frac{2cx^4\log(1+icx^2)}{3i+3cx^2}dx - \frac{b\int(2a+ib\log(1-icx^2))\,dx}{3c} + \frac{b\int\frac{2a+ib\log(1-icx^2)}{1-icx^2}dx}{3c} + \frac{(ib^2)\int\log(1+icx^2)\,dx}{3c} - \frac{(ib^2)\int\frac{\log(1+icx^2)}{1+icx^2}dx}{3c} - \frac{1}{3}(2abc)\int\frac{x^4}{1+icx^2}dx$$

$$= -\frac{2abx}{3c} - \frac{1}{9}ibx^3\left(2a+ib\log\left(1-icx^2\right)\right) - \frac{\sqrt[4]{-1}b\arctan\left((-1)^{3/4}\sqrt{c}x\right)(2a+ib\log(1-icx^2))}{3c^{3/2}} + \frac{1}{12}x^3\left(2a+ib\log\left(1-icx^2\right)\right)^2 + \frac{ib^2x\log(1+icx^2)}{3c} - \frac{1}{3}iabx^3\log\left(1+icx^2\right) + \frac{1}{9}b^2x^3\log\left(1+icx^2\right) + \frac{(-1)^{3/4}b^2\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)\log(1+icx^2)}{3c^{3/2}} + \frac{1}{6}b^2x^3\log\left(1-icx^2\right)\log\left(1+icx^2\right) - \frac{1}{12}b^2x^3\log^2\left(1+icx^2\right) + \frac{1}{3}\left(2b^2\right)\int\frac{x^2}{1+icx^2}dx + \frac{1}{3}\left(2b^2\right)\int\frac{\sqrt[4]{}}{}$$

$$= -\frac{4abx}{3c} - \frac{2ib^2x}{3c} + \frac{2}{9}iabx^3 - \frac{ib^2x\log(1-icx^2)}{3c} - \frac{1}{9}ibx^3\left(2a+ib\log(1-icx^2)\right)$$
$$- \frac{\sqrt[4]{-1}b\arctan\left((-1)^{3/4}\sqrt{c}x\right)\left(2a+ib\log(1-icx^2)\right)}{3c^{3/2}}$$
$$+ \frac{1}{12}x^3\left(2a+ib\log(1-icx^2)\right)^2 + \frac{ib^2x\log(1+icx^2)}{3c} - \frac{1}{3}iabx^3\log(1+icx^2)$$
$$+ \frac{1}{9}b^2x^3\log(1+icx^2) + \frac{(-1)^{3/4}b^2\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)\log(1+icx^2)}{3c^{3/2}}$$
$$+ \frac{1}{6}b^2x^3\log(1-icx^2)\log(1+icx^2) - \frac{1}{12}b^2x^3\log^2(1+icx^2) + \frac{1}{3}(2b^2)\int\frac{x^2}{1-icx^2}dx + \frac{(2ab)\int\frac{1}{1+icx^2}}{3c}$$

$$= -\frac{4abx}{3c} + \frac{2}{9}iabx^3 - \frac{4b^2x^3}{27} + \frac{\sqrt[4]{-1}b^2\arctan\left((-1)^{3/4}\sqrt{c}x\right)^2}{3c^{3/2}} - \frac{2\sqrt[4]{-1}ab\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)}{3c^{3/2}}$$
$$- \frac{2(-1)^{3/4}b^2\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)}{3c^{3/2}} - \frac{(-1)^{3/4}b^2\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)^2}{3c^{3/2}} - \frac{ib^2x\log(1-icx^2)}{3c}$$
$$- \frac{1}{9}ibx^3\left(2a+ib\log(1-icx^2)\right) - \frac{\sqrt[4]{-1}b\arctan\left((-1)^{3/4}\sqrt{c}x\right)\left(2a+ib\log(1-icx^2)\right)}{3c^{3/2}} + \frac{1}{12}x^3(2a+ib$$

$$= -\frac{4abx}{3c} + \frac{2}{9}iabx^3 - \frac{4b^2x^3}{27} + \frac{8(-1)^{3/4}b^2\arctan\left((-1)^{3/4}\sqrt{c}x\right)}{9c^{3/2}}$$
$$+ \frac{\sqrt[4]{-1}b^2\arctan\left((-1)^{3/4}\sqrt{c}x\right)^2}{3c^{3/2}} - \frac{2\sqrt[4]{-1}ab\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)}{3c^{3/2}}$$
$$- \frac{8(-1)^{3/4}b^2\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)}{9c^{3/2}} - \frac{(-1)^{3/4}b^2\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)^2}{3c^{3/2}}$$
$$- \frac{2(-1)^{3/4}b^2\arctan\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{2}{1-\sqrt[4]{-1}\sqrt{c}x}\right)}{3c^{3/2}}$$
$$+ \frac{2(-1)^{3/4}b^2\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{2}{1-(-1)^{3/4}\sqrt{c}x}\right)}{3c^{3/2}} - \frac{2ib^2x\log(1-icx^2)}{3c}$$
$$- \frac{1}{9}b^2x^3\log(1-icx^2) - \frac{(-1)^{3/4}b^2\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)\log(1-icx^2)}{3c^{3/2}} - \frac{1}{9}ibx^3\left(2a+ib\log(1-icx^2)\right)$$

$$= -\frac{4abx}{3c} + \frac{2}{9}iabx^3 - \frac{4b^2x^3}{27} + \frac{8(-1)^{3/4}b^2\arctan\left((-1)^{3/4}\sqrt{c}x\right)}{9c^{3/2}}$$
$$+ \frac{\sqrt[4]{-1}b^2\arctan\left((-1)^{3/4}\sqrt{c}x\right)^2}{3c^{3/2}} - \frac{2\sqrt[4]{-1}ab\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)}{3c^{3/2}}$$
$$- \frac{8(-1)^{3/4}b^2\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)}{9c^{3/2}} - \frac{(-1)^{3/4}b^2\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)^2}{3c^{3/2}}$$
$$- \frac{2(-1)^{3/4}b^2\arctan\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{2}{1-\sqrt[4]{-1}\sqrt{c}x}\right)}{3c^{3/2}}$$
$$+ \frac{2(-1)^{3/4}b^2\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{2}{1-(-1)^{3/4}\sqrt{c}x}\right)}{3c^{3/2}} - \frac{2ib^2x\log(1-icx^2)}{3c}$$
$$- \frac{1}{9}b^2x^3\log(1-icx^2) - \frac{(-1)^{3/4}b^2\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)\log(1-icx^2)}{3c^{3/2}} - \frac{1}{9}ibx^3\left(2a+ib\log(1-icx^2\right.$$

$$= -\frac{4abx}{3c} + \frac{2}{9}iabx^3 + \frac{14(-1)^{3/4}b^2\arctan\left((-1)^{3/4}\sqrt{c}x\right)}{9c^{3/2}}$$
$$+ \frac{\sqrt[4]{-1}b^2\arctan\left((-1)^{3/4}\sqrt{c}x\right)^2}{3c^{3/2}} - \frac{2\sqrt[4]{-1}ab\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)}{3c^{3/2}}$$
$$- \frac{14(-1)^{3/4}b^2\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)}{9c^{3/2}} - \frac{(-1)^{3/4}b^2\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)^2}{3c^{3/2}}$$
$$- \frac{2(-1)^{3/4}b^2\arctan\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{2}{1-\sqrt[4]{-1}\sqrt{c}x}\right)}{3c^{3/2}}$$
$$+ \frac{2(-1)^{3/4}b^2\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{2}{1-(-1)^{3/4}\sqrt{c}x}\right)}{3c^{3/2}} - \frac{2ib^2x\log(1-icx^2)}{3c}$$
$$- \frac{1}{9}b^2x^3\log(1-icx^2) - \frac{(-1)^{3/4}b^2\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)\log(1-icx^2)}{3c^{3/2}} - \frac{1}{9}ibx^3\left(2a+ib\log(1-icx^2\right.$$

$$= -\frac{4abx}{3c} + \frac{2}{9}iabx^3 + \frac{4(-1)^{3/4}b^2\arctan\left((-1)^{3/4}\sqrt{c}x\right)}{3c^{3/2}} + \frac{\sqrt[4]{-1}b^2\arctan\left((-1)^{3/4}\sqrt{c}x\right)^2}{3c^{3/2}} - \frac{2\sqrt[4]{-1}ab\operatorname{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)}{3c^{3/2}} - \frac{4(-1)^{3/4}b^2\operatorname{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)}{3c^{3/2}} - \frac{(-1)^{3/4}b^2\operatorname{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)^2}{3c^{3/2}} - \frac{2(-1)^{3/4}b^2\arctan\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{2}{1-\sqrt[4]{-1}\sqrt{c}x}\right)}{3c^{3/2}} + \frac{2(-1)^{3/4}b^2\operatorname{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{2}{1-(-1)^{3/4}\sqrt{c}x}\right)}{3c^{3/2}} - \frac{2ib^2x\log\left(1-icx^2\right)}{3c} - \frac{1}{9}b^2x^3\log\left(1-icx^2\right) - \frac{(-1)^{3/4}b^2\operatorname{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)\log\left(1-icx^2\right)}{3c^{3/2}} - \frac{1}{9}ibx^3\left(2a+ib\log\left(1-icx^2\right)\right)$$

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Mathematica [F]

$$\int x^2\left(a+b\arctan\left(cx^2\right)\right)^2\,dx = \int x^2\left(a+b\arctan\left(cx^2\right)\right)^2\,dx$$

```
[In] Integrate[x^2*(a + b*ArcTan[c*x^2])^2,x]
[Out] Integrate[x^2*(a + b*ArcTan[c*x^2])^2, x]
```

Maple [F]

$$\int x^2\left(a+b\arctan\left(c\,x^2\right)\right)^2dx$$

```
[In] int(x^2*(a+b*arctan(c*x^2))^2,x)
[Out] int(x^2*(a+b*arctan(c*x^2))^2,x)
```

Fricas [F]

$$\int x^2\left(a+b\arctan\left(cx^2\right)\right)^2\,dx=\int\left(b\arctan\left(cx^2\right)+a\right)^2x^2\,dx$$

[In] integrate(x^2*(a+b*arctan(c*x^2))^2,x, algorithm="fricas")

[Out] integral(b^2*x^2*arctan(c*x^2)^2 + 2*a*b*x^2*arctan(c*x^2) + a^2*x^2, x)

Sympy [F]

$$\int x^2\left(a+b\arctan\left(cx^2\right)\right)^2\,dx=\int x^2\left(a+b\operatorname{atan}\left(cx^2\right)\right)^2\,dx$$

[In] integrate(x**2*(a+b*atan(c*x**2))**2,x)

[Out] Integral(x**2*(a + b*atan(c*x**2))**2, x)

Maxima [F]

$$\int x^2\left(a+b\arctan\left(cx^2\right)\right)^2\,dx=\int\left(b\arctan\left(cx^2\right)+a\right)^2x^2\,dx$$

[In] integrate(x^2*(a+b*arctan(c*x^2))^2,x, algorithm="maxima")

[Out] 1/3*a^2*x^3 + 1/6*(4*x^3*arctan(c*x^2) - c*(8*x/c^2 - (2*sqrt(2)*arctan(1/2*sqrt(2)*(2*c*x + sqrt(2)*sqrt(c))/sqrt(c))/sqrt(c) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*c*x - sqrt(2)*sqrt(c))/sqrt(c))/sqrt(c) + sqrt(2)*log(c*x^2 + sqrt(2)*sqrt(c)*x + 1)/sqrt(c) - sqrt(2)*log(c*x^2 - sqrt(2)*sqrt(c)*x + 1)/sqrt(c))/c^2))*a*b + 1/48*(4*x^3*arctan(c*x^2)^2 - x^3*log(c^2*x^4 + 1)^2 + 48*integrate(1/48*(8*c^2*x^6*log(c^2*x^4 + 1) - 16*c*x^4*arctan(c*x^2) + 36*(c^2*x^6 + x^2)*arctan(c*x^2)^2 + 3*(c^2*x^6 + x^2)*log(c^2*x^4 + 1)^2)/(c^2*x^4 + 1), x))*b^2

Giac [F]

$$\int x^2 \left(a+b\arctan\left(cx^2\right)\right)^2\,dx=\int \left(b\arctan\left(cx^2\right)+a\right)^2 x^2\,dx$$

```
[In] integrate(x^2*(a+b*arctan(c*x^2))^2,x, algorithm="giac")
[Out] integrate((b*arctan(c*x^2) + a)^2*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \left(a+b\arctan\left(cx^2\right)\right)^2\,dx=\int x^2\,\left(a+b\,\mathrm{atan}\left(c\,x^2\right)\right)^2\,dx$$

```
[In] int(x^2*(a + b*atan(c*x^2))^2,x)
[Out] int(x^2*(a + b*atan(c*x^2))^2, x)
```

3.82 $\int (a + b \arctan(cx^2))^2 \, dx$

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Mathematica [B] (warning: unable to verify) 476
Maple [F] 479
Fricas [F] 479
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Giac [F] 480
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Optimal result

Integrand size = 12, antiderivative size = 1191

$$\int \left(a+b\arctan\left(cx^2\right)\right)^2\,dx = a^2x-\frac{2(-1)^{3/4}ab\arctan\left((-1)^{3/4}\sqrt{c}x\right)}{\sqrt{c}}$$
$$+\frac{(-1)^{3/4}b^2\arctan\left((-1)^{3/4}\sqrt{c}x\right)^2}{\sqrt{c}}+\frac{2(-1)^{3/4}ab\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)}{\sqrt{c}}$$
$$-\frac{\sqrt[4]{-1}b^2\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)^2}{\sqrt{c}}+\frac{2\sqrt[4]{-1}b^2\arctan\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{2}{1-\sqrt[4]{-1}\sqrt{c}x}\right)}{\sqrt{c}}$$
$$-\frac{2\sqrt[4]{-1}b^2\arctan\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{2}{1+\sqrt[4]{-1}\sqrt{c}x}\right)}{\sqrt{c}}$$
$$+\frac{\sqrt[4]{-1}b^2\arctan\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{\sqrt{2}\left(\sqrt[4]{-1}+\sqrt{c}x\right)}{1+\sqrt[4]{-1}\sqrt{c}x}\right)}{\sqrt{c}}$$
$$+\frac{2\sqrt[4]{-1}b^2\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{2}{1-(-1)^{3/4}\sqrt{c}x}\right)}{\sqrt{c}}$$
$$-\frac{2\sqrt[4]{-1}b^2\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{2}{1+(-1)^{3/4}\sqrt{c}x}\right)}{\sqrt{c}}$$
$$+\frac{\sqrt[4]{-1}b^2\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)\log\left(-\frac{\sqrt{2}\left((-1)^{3/4}+\sqrt{c}x\right)}{1+(-1)^{3/4}\sqrt{c}x}\right)}{\sqrt{c}}$$
$$+\frac{\sqrt[4]{-1}b^2\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{(1+i)\left(1+\sqrt[4]{-1}\sqrt{c}x\right)}{1+(-1)^{3/4}\sqrt{c}x}\right)}{\sqrt{c}}$$
$$+\frac{\sqrt[4]{-1}b^2\arctan\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{(1-i)\left(1+(-1)^{3/4}\sqrt{c}x\right)}{1+\sqrt[4]{-1}\sqrt{c}x}\right)}{\sqrt{c}}$$

$$+iabx\log\left(1-icx^2\right)+\frac{\sqrt[4]{-1}b^2\arctan\left((-1)^{3/4}\sqrt{c}x\right)\log\left(1-icx^2\right)}{\sqrt{c}}-\frac{\sqrt[4]{-1}b^2\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)\log\left(1-i\right.}{\sqrt{c}}$$

```
[Out] -1/2*(-1)^(3/4)*b^2*polylog(2,1+(-1+I)*(1+(-1)^(3/4)*x*c^(1/2))/(1+(-1)^(1/
4)*x*c^(1/2)))/c^(1/2)+1/2*b^2*x*ln(1-I*c*x^2)*ln(1+I*c*x^2)-1/2*(-1)^(3/4)
*b^2*polylog(2,1-2^(1/2)*((-1)^(1/4)+x*c^(1/2))/(1+(-1)^(1/4)*x*c^(1/2)))/c
^(1/2)-1/2*(-1)^(1/4)*b^2*polylog(2,1+2^(1/2)*((-1)^(3/4)+x*c^(1/2))/(1+(-1
)^(3/4)*x*c^(1/2)))/c^(1/2)-1/2*(-1)^(1/4)*b^2*polylog(2,1-(1+I)*(1+(-1)^(1
```

```
/4)*x*c^(1/2))/(1+(-1)^(3/4)*x*c^(1/2)))/c^(1/2)-2*(-1)^(3/4)*a*b*arctan((-
1)^(3/4)*x*c^(1/2))/c^(1/2)+2*(-1)^(3/4)*a*b*arctanh((-1)^(3/4)*x*c^(1/2))/
c^(1/2)+2*(-1)^(1/4)*b^2*arctan((-1)^(3/4)*x*c^(1/2))*ln(2/(1-(-1)^(1/4)*x*
c^(1/2)))/c^(1/2)-2*(-1)^(1/4)*b^2*arctan((-1)^(3/4)*x*c^(1/2))*ln(2/(1+(-1
)^(1/4)*x*c^(1/2)))/c^(1/2)+2*(-1)^(1/4)*b^2*arctanh((-1)^(3/4)*x*c^(1/2))*
ln(2/(1-(-1)^(3/4)*x*c^(1/2)))/c^(1/2)-2*(-1)^(1/4)*b^2*arctanh((-1)^(3/4)*
x*c^(1/2))*ln(2/(1+(-1)^(3/4)*x*c^(1/2)))/c^(1/2)-I*a*b*x*ln(1+I*c*x^2)+(-1
)^(1/4)*b^2*arctan((-1)^(3/4)*x*c^(1/2))*ln(1-I*c*x^2)/c^(1/2)-(-1)^(1/4)*b
^2*arctanh((-1)^(3/4)*x*c^(1/2))*ln(1-I*c*x^2)/c^(1/2)-(-1)^(1/4)*b^2*arcta
n((-1)^(3/4)*x*c^(1/2))*ln(1+I*c*x^2)/c^(1/2)+(-1)^(1/4)*b^2*arctanh((-1)^(
3/4)*x*c^(1/2))*ln(1+I*c*x^2)/c^(1/2)+(-1)^(1/4)*b^2*arctan((-1)^(3/4)*x*c^
(1/2))*ln(2^(1/2)*((-1)^(1/4)+x*c^(1/2))/(1+(-1)^(1/4)*x*c^(1/2)))/c^(1/2)+
(-1)^(1/4)*b^2*arctanh((-1)^(3/4)*x*c^(1/2))*ln(-2^(1/2)*((-1)^(3/4)+x*c^(1
/2))/(1+(-1)^(3/4)*x*c^(1/2)))/c^(1/2)+(-1)^(1/4)*b^2*arctanh((-1)^(3/4)*x*
c^(1/2))*ln((1+I)*(1+(-1)^(1/4)*x*c^(1/2))/(1+(-1)^(3/4)*x*c^(1/2)))/c^(1/2
)+(-1)^(1/4)*b^2*arctan((-1)^(3/4)*x*c^(1/2))*ln((1-I)*(1+(-1)^(3/4)*x*c^(1
/2))/(1+(-1)^(1/4)*x*c^(1/2)))/c^(1/2)+I*a*b*x*ln(1-I*c*x^2)+(-1)^(3/4)*b^2
*polylog(2,1-2/(1-(-1)^(1/4)*x*c^(1/2)))/c^(1/2)+(-1)^(3/4)*b^2*polylog(2,1
-2/(1+(-1)^(1/4)*x*c^(1/2)))/c^(1/2)+(-1)^(1/4)*b^2*polylog(2,1-2/(1-(-1)^(
3/4)*x*c^(1/2)))/c^(1/2)+(-1)^(1/4)*b^2*polylog(2,1-2/(1+(-1)^(3/4)*x*c^(1/
2)))/c^(1/2)+(-1)^(3/4)*b^2*arctan((-1)^(3/4)*x*c^(1/2))^2/c^(1/2)-(-1)^(1/
4)*b^2*arctanh((-1)^(3/4)*x*c^(1/2))^2/c^(1/2)-1/4*b^2*x*ln(1-I*c*x^2)^2-1/
4*b^2*x*ln(1+I*c*x^2)^2+a^2*x
```

Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 1191, normalized size of antiderivative = 1.00, number of steps used = 69, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.917, Rules used = {4932, 2498, 327, 209, 2500, 2526, 2520, 12, 5040, 4964, 2449, 2352, 212, 2636, 211, 5048, 4966, 2497, 214, 6139, 6057, 6131, 6055}

$$\int \left(a+b\arctan\left(cx^2\right)\right)^2\,dx$$

$$=xa^2-\frac{2(-1)^{3/4}b\arctan\left((-1)^{3/4}\sqrt{c}x\right)a}{\sqrt{c}}+\frac{2(-1)^{3/4}b\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)a}{\sqrt{c}}$$

$$+ibx\log\left(1-icx^2\right)a-ibx\log\left(icx^2+1\right)a+\frac{(-1)^{3/4}b^2\arctan\left((-1)^{3/4}\sqrt{c}x\right)^2}{\sqrt{c}}-\frac{\sqrt[4]{-1}b^2\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right.}{\sqrt{c}}$$

```
[In] Int[(a + b*ArcTan[c*x^2])^2,x]
```

```
[Out] a^2*x - (2*(-1)^(3/4)*a*b*ArcTan[(-1)^(3/4)*Sqrt[c]*x])/Sqrt[c] + ((-1)^(3/
4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[c]*x]^2)/Sqrt[c] + (2*(-1)^(3/4)*a*b*ArcTanh[
(-1)^(3/4)*Sqrt[c]*x])/Sqrt[c] - ((-1)^(1/4)*b^2*ArcTanh[(-1)^(3/4)*Sqrt[c]
```

```
*x]^2)/Sqrt[c] + (2*(-1)^(1/4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[2/(1 -
(-1)^(1/4)*Sqrt[c]*x)])/Sqrt[c] - (2*(-1)^(1/4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[
c]*x]*Log[2/(1 + (-1)^(1/4)*Sqrt[c]*x)])/Sqrt[c] + ((-1)^(1/4)*b^2*ArcTan[(
-1)^(3/4)*Sqrt[c]*x]*Log[(Sqrt[2]*((-1)^(1/4) + Sqrt[c]*x))/(1 + (-1)^(1/4)
*Sqrt[c]*x)])/Sqrt[c] + (2*(-1)^(1/4)*b^2*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log
[2/(1 - (-1)^(3/4)*Sqrt[c]*x)])/Sqrt[c] - (2*(-1)^(1/4)*b^2*ArcTanh[(-1)^(3
/4)*Sqrt[c]*x]*Log[2/(1 + (-1)^(3/4)*Sqrt[c]*x)])/Sqrt[c] + ((-1)^(1/4)*b^2
*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[-((Sqrt[2]*((-1)^(3/4) + Sqrt[c]*x))/(1
+ (-1)^(3/4)*Sqrt[c]*x))])/Sqrt[c] + ((-1)^(1/4)*b^2*ArcTanh[(-1)^(3/4)*Sqr
t[c]*x]*Log[((1 + I)*(1 + (-1)^(1/4)*Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x)
])/Sqrt[c] + ((-1)^(1/4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[((1 - I)*(1 +
 (-1)^(3/4)*Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*x)])/Sqrt[c] + I*a*b*x*Log[
1 - I*c*x^2] + ((-1)^(1/4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[1 - I*c*x^2
])/Sqrt[c] - ((-1)^(1/4)*b^2*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[1 - I*c*x^2]
)/Sqrt[c] - (b^2*x*Log[1 - I*c*x^2]^2)/4 - I*a*b*x*Log[1 + I*c*x^2] - ((-1)
^(1/4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[1 + I*c*x^2])/Sqrt[c] + ((-1)^(
1/4)*b^2*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[1 + I*c*x^2])/Sqrt[c] + (b^2*x*L
og[1 - I*c*x^2]*Log[1 + I*c*x^2])/2 - (b^2*x*Log[1 + I*c*x^2]^2)/4 + ((-1)^
(3/4)*b^2*PolyLog[2, 1 - 2/(1 - (-1)^(1/4)*Sqrt[c]*x)])/Sqrt[c] + ((-1)^(3/
4)*b^2*PolyLog[2, 1 - 2/(1 + (-1)^(1/4)*Sqrt[c]*x)])/Sqrt[c] - ((-1)^(3/4)*
b^2*PolyLog[2, 1 - (Sqrt[2]*((-1)^(1/4) + Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[
c]*x)])/(2*Sqrt[c]) + ((-1)^(1/4)*b^2*PolyLog[2, 1 - 2/(1 - (-1)^(3/4)*Sqrt
[c]*x)])/Sqrt[c] + ((-1)^(1/4)*b^2*PolyLog[2, 1 - 2/(1 + (-1)^(3/4)*Sqrt[c]
*x)])/Sqrt[c] - ((-1)^(1/4)*b^2*PolyLog[2, 1 + (Sqrt[2]*((-1)^(3/4) + Sqrt[
c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x)])/(2*Sqrt[c]) - ((-1)^(1/4)*b^2*PolyLog[2
, 1 - ((1 + I)*(1 + (-1)^(1/4)*Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x)])/(2*
Sqrt[c]) - ((-1)^(3/4)*b^2*PolyLog[2, 1 - ((1 - I)*(1 + (-1)^(3/4)*Sqrt[c]*
x))/(1 + (-1)^(1/4)*Sqrt[c]*x)])/(2*Sqrt[c])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2500

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> Simp[x*(a + b*Log[c*(d + e*x^n)^p])^q, x] - Dist[b*e*n*p*q, Int[x^n*(
(a + b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c
```

```
, d, e, n, p}, x] && IGtQ[q, 0] && (EqQ[q, 1] || IntegerQ[n])
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2636

```
Int[Log[v_]*Log[w_], x_Symbol] :> Simp[x*Log[v]*Log[w], x] + (-Int[Simplify
Integrand[x*Log[w]*(D[v, x]/v), x], x] - Int[SimplifyIntegrand[x*Log[v]*(D[
w, x]/w), x], x]) /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w,
x]
```

Rule 4932

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_), x_Symbol] :> Int[ExpandIn
tegrand[(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*Log[1 + I*c*x^n])/2)^p, x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5048

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6057

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcTanh[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x
)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcTanh[c*x])
*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x]) /; FreeQ[{a, b, c, d,
e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6139

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rubi steps

$$\text{integral} = \int \left(a^2 + iab\log\left(1-icx^2\right) - \frac{1}{4}b^2\log^2\left(1-icx^2\right) - iab\log\left(1+icx^2\right) + \frac{1}{2}b^2\log\left(1-icx^2\right)\log\left(1+icx^2\right) - \frac{1}{4}b^2\log^2\left(1+icx^2\right)\right)\,dx$$

$$= a^2x + (iab)\int \log\left(1-icx^2\right)\,dx - (iab)\int \log\left(1+icx^2\right)\,dx - \frac{1}{4}b^2\int \log^2\left(1-icx^2\right)\,dx - \frac{1}{4}b^2\int \log^2\left(1+icx^2\right)\,dx + \frac{1}{2}b^2\int \log\left(1-icx^2\right)\log\left(1+icx^2\right)\,dx$$

$$= a^2x + iabx\log\left(1-icx^2\right) - \frac{1}{4}b^2x\log^2\left(1-icx^2\right) - iabx\log\left(1+icx^2\right) + \frac{1}{2}b^2x\log\left(1-icx^2\right)\log\left(1+icx^2\right) - \frac{1}{4}b^2x\log^2\left(1+icx^2\right) - \frac{1}{2}b^2\int \frac{2cx^2\log\left(1-icx^2\right)}{-i+cx^2}\,dx - \frac{1}{2}b^2\int \frac{2cx^2\log\left(1+icx^2\right)}{i+cx^2}\,dx - (2abc)\int \frac{x^2}{1-icx^2}\,dx - (2abc)\int \frac{x^2}{1+icx^2}\,dx - \left(ib^2c\right)\int \frac{x^2\log\left(1-icx^2\right)}{1-icx^2}\,dx + \left(ib^2c\right)\int \frac{x^2\log\left(1+icx^2\right)}{1+icx^2}\,dx$$

$$= a^2x + iabx\log\left(1-icx^2\right) - \frac{1}{4}b^2x\log^2\left(1-icx^2\right) - iabx\log\left(1+icx^2\right) + \frac{1}{2}b^2x\log\left(1-icx^2\right)\log\left(1+icx^2\right) - \frac{1}{4}b^2x\log^2\left(1+icx^2\right) + (2iab)\int \frac{1}{1-icx^2}\,dx - (2iab)\int \frac{1}{1+icx^2}\,dx - \left(ib^2c\right)\int \left(\frac{i\log\left(1-icx^2\right)}{c} - \frac{i\log\left(1-icx^2\right)}{c\left(1-icx^2\right)}\right)dx + \left(ib^2c\right)\int \left(-\frac{i\log\left(1+icx^2\right)}{c} + \frac{i\log\left(1+icx^2\right)}{c\left(1+icx^2\right)}\right)dx - \left(b^2c\right)\int \frac{x^2\log\left(1-icx^2\right)}{-i+cx^2}\,dx - \left(b^2c\right)\int \frac{x^2\log\left(1+icx^2\right)}{i+cx^2}\,dx$$

$$= a^2x - \frac{2(-1)^{3/4}ab\arctan\left((-1)^{3/4}\sqrt{c}x\right)}{\sqrt{c}} + \frac{2(-1)^{3/4}ab\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)}{\sqrt{c}} + iabx\log\left(1-icx^2\right) - \frac{1}{4}b^2x\log^2\left(1-icx^2\right) - iabx\log\left(1+icx^2\right) + \frac{1}{2}b^2x\log\left(1-icx^2\right)\log\left(1+icx^2\right) - \frac{1}{4}$$

$$= a^2x - \frac{2(-1)^{3/4}ab\arctan\left((-1)^{3/4}\sqrt{c}x\right)}{\sqrt{c}} + \frac{2(-1)^{3/4}ab\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)}{\sqrt{c}} + iabx\log\left(1-icx^2\right) + b^2x\log\left(1-icx^2\right) + \frac{\sqrt[4]{-1}b^2\arctan\left((-1)^{3/4}\sqrt{c}x\right)\log\left(1-icx^2\right)}{\sqrt{c}} - \frac{1}{4}b^2x\log^2\left(1-\right.$$

$$= a^2x - 4b^2x - \frac{2(-1)^{3/4}ab\arctan\left((-1)^{3/4}\sqrt{c}x\right)}{\sqrt{c}} + \frac{2(-1)^{3/4}ab\,\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)}{\sqrt{c}}$$

$$+iabx\log\left(1-icx^2\right)+\frac{\sqrt[4]{-1}b^2\arctan\left((-1)^{3/4}\sqrt{c}x\right)\log\left(1-icx^2\right)}{\sqrt{c}}-\frac{\sqrt[4]{-1}b^2\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)\text{l}}{\sqrt{c}}$$

$$= a^2x - \frac{2(-1)^{3/4}ab\arctan\left((-1)^{3/4}\sqrt{c}x\right)}{\sqrt{c}} - \frac{2\sqrt[4]{-1}b^2\arctan\left((-1)^{3/4}\sqrt{c}x\right)}{\sqrt{c}}$$

$$+\frac{(-1)^{3/4}b^2\arctan\left((-1)^{3/4}\sqrt{c}x\right)^2}{\sqrt{c}}+\frac{2(-1)^{3/4}ab\,\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)}{\sqrt{c}}$$

$$-\frac{2\sqrt[4]{-1}b^2\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)}{\sqrt{c}}-\frac{\sqrt[4]{-1}b^2\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)^2}{\sqrt{c}}$$

$$+iabx\log\left(1-icx^2\right)+\frac{\sqrt[4]{-1}b^2\arctan\left((-1)^{3/4}\sqrt{c}x\right)\log\left(1-icx^2\right)}{\sqrt{c}}-\frac{\sqrt[4]{-1}b^2\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)\text{l}}{\sqrt{c}}$$

$$= a^2x - \frac{2(-1)^{3/4}ab\arctan\left((-1)^{3/4}\sqrt{c}x\right)}{\sqrt{c}} + \frac{(-1)^{3/4}b^2\arctan\left((-1)^{3/4}\sqrt{c}x\right)^2}{\sqrt{c}}$$

$$+\frac{2(-1)^{3/4}ab\,\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)}{\sqrt{c}}-\frac{\sqrt[4]{-1}b^2\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)^2}{\sqrt{c}}$$

$$+\frac{2\sqrt[4]{-1}b^2\arctan\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{2}{1-\sqrt[4]{-1}\sqrt{c}x}\right)}{\sqrt{c}}$$

$$+\frac{2\sqrt[4]{-1}b^2\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{2}{1-(-1)^{3/4}\sqrt{c}x}\right)}{\sqrt{c}}$$

$$+iabx\log\left(1-icx^2\right)+\frac{\sqrt[4]{-1}b^2\arctan\left((-1)^{3/4}\sqrt{c}x\right)\log\left(1-icx^2\right)}{\sqrt{c}}-\frac{\sqrt[4]{-1}b^2\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)\text{l}}{\sqrt{c}}$$

$$= a^2x - \frac{2(-1)^{3/4}ab\arctan\left((-1)^{3/4}\sqrt{c}x\right)}{\sqrt{c}} + \frac{(-1)^{3/4}b^2\arctan\left((-1)^{3/4}\sqrt{c}x\right)^2}{\sqrt{c}}$$

$$+\frac{2(-1)^{3/4}ab\,\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)}{\sqrt{c}}-\frac{\sqrt[4]{-1}b^2\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)^2}{\sqrt{c}}$$

$$+\frac{2\sqrt[4]{-1}b^2\arctan\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{2}{1-\sqrt[4]{-1}\sqrt{c}x}\right)}{\sqrt{c}}$$

$$+\frac{2\sqrt[4]{-1}b^2\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{2}{1-(-1)^{3/4}\sqrt{c}x}\right)}{\sqrt{c}}$$

$$+iabx\log\left(1-icx^2\right)+\frac{\sqrt[4]{-1}b^2\arctan\left((-1)^{3/4}\sqrt{c}x\right)\log\left(1-icx^2\right)}{\sqrt{c}}-\frac{\sqrt[4]{-1}b^2\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)\text{l}}{\sqrt{c}}$$

= Too large to display

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 4697 vs. 2(1191) = 2382.

Time = 38.10 (sec) , antiderivative size = 4697, normalized size of antiderivative = 3.94

$$\int \left(a+b\arctan\left(cx^2\right)\right)^2\,dx=\text{Result too large to show}$$

```
[In] Integrate[(a + b*ArcTan[c*x^2])^2,x]
```

```
[Out] a^2*x + (a*b*Sqrt[c*x^2]*(2*Sqrt[c*x^2]*ArcTan[c*x^2] - Sqrt[2]*(ArcTan[(-1
 + c*x^2)/(Sqrt[2]*Sqrt[c*x^2])] - ArcTanh[(Sqrt[2]*Sqrt[c*x^2])/(1 + c*x^2
)])))/(c*x) + (b^2*Sqrt[c*x^2]*(2*Sqrt[c*x^2]*ArcTan[c*x^2]^2 - 4*((ArcTan[
c*x^2]*(-2*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]] + 2*ArcTan[1 + Sqrt[2]*Sqrt[c*x^
2]] + Log[1 + c*x^2 - Sqrt[2]*Sqrt[c*x^2]] - Log[1 + c*x^2 + Sqrt[2]*Sqrt[c
*x^2]]))/(2*Sqrt[2]) - (-((ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]] + ArcTan[1 + Sqr
t[2]*Sqrt[c*x^2]])*Log[1 + c*x^2 - Sqrt[2]*Sqrt[c*x^2]]) + (ArcTan[1 - Sqrt
[2]*Sqrt[c*x^2]] + ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]])*Log[1 + c*x^2 + Sqrt[2]
*Sqrt[c*x^2]] - (Sqrt[c*x^2]*(1 + (1 - Sqrt[2]*Sqrt[c*x^2])^2)^(3/2)*(2*(-5
*ArcTan[2 + I]*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]] + 4*ArcTan[1 - Sqrt[2]*Sqrt[
c*x^2]]^2 + ((1 + 2*I)*Sqrt[1 + I]*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]^2)/E^(I*
ArcTan[2 + I]) + ((1 - 2*I)*Sqrt[1 - I]*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]^2)/
E^ArcTanh[1 + 2*I] - (5*I)*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]*ArcTanh[1 + 2*I]
 + (5*I)*(-ArcTan[2 + I] + ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]])*Log[1 - E^((2*I
)*(-ArcTan[2 + I] + ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]))] + 5*((-I)*ArcTan[1 -
 Sqrt[2]*Sqrt[c*x^2]] + ArcTanh[1 + 2*I])*Log[1 - E^((2*I)*ArcTan[1 - Sqrt[
2]*Sqrt[c*x^2]] - 2*ArcTanh[1 + 2*I])] + (5*I)*ArcTan[2 + I]*Log[-Sin[ArcTa
n[2 + I] - ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]]] - 5*ArcTanh[1 + 2*I]*Log[Sin[A
rcTan[1 - Sqrt[2]*Sqrt[c*x^2]] + I*ArcTanh[1 + 2*I]]]) + 5*PolyLog[2, E^((2
*I)*(-ArcTan[2 + I] + ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]))] - 5*PolyLog[2, E^(
(2*I)*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]] - 2*ArcTanh[1 + 2*I])])*(3 + 2*Cos[2*
ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]] - 2*Sin[2*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]]
))/(20*Sqrt[2]*(-1 - c*x^2 + Sqrt[2]*Sqrt[c*x^2])*(1 + c*x^2 + Sqrt[2]*Sqrt
[c*x^2])*(1/Sqrt[1 + (1 - Sqrt[2]*Sqrt[c*x^2])^2] - (1 - Sqrt[2]*Sqrt[c*x^2
])/Sqrt[1 + (1 - Sqrt[2]*Sqrt[c*x^2])^2])) + ((1/40 + I/40)*c*x^2*(1 + (1 -
 Sqrt[2]*Sqrt[c*x^2])^2)*((5 + 5*I)*Pi*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]] + 10
*ArcTan[2 + I]*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]] + (4 - 4*I)*ArcTan[1 - Sqrt[
2]*Sqrt[c*x^2]]^2 - ((2 + 4*I)*Sqrt[1 + I]*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]^
2)/E^(I*ArcTan[2 + I]) + ((4 + 2*I)*Sqrt[1 - I]*ArcTan[1 - Sqrt[2]*Sqrt[c*x
^2]]^2)/E^ArcTanh[1 + 2*I] + 10*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]*ArcTanh[1 +
 2*I] + (5 - 5*I)*Pi*Log[1 + E^((-2*I)*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]])] +
(10*I)*ArcTan[2 + I]*Log[1 - E^((2*I)*(-ArcTan[2 + I] + ArcTan[1 - Sqrt[2]*
```

Sqrt[c*x^2]]))] - (10*I)*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]*Log[1 - E^((2*I)*(
-ArcTan[2 + I] + ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]))] + 10*ArcTan[1 - Sqrt[2]
*Sqrt[c*x^2]]*Log[1 - E^((2*I)*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]] - 2*ArcTanh[
1 + 2*I])] + (10*I)*ArcTanh[1 + 2*I]*Log[1 - E^((2*I)*ArcTan[1 - Sqrt[2]*Sq
rt[c*x^2]] - 2*ArcTanh[1 + 2*I])] - (5 - 5*I)*Pi*Log[1/Sqrt[1 + (1 - Sqrt[2
]*Sqrt[c*x^2])^2]] - (10*I)*ArcTan[2 + I]*Log[-Sin[ArcTan[2 + I] - ArcTan[1
 - Sqrt[2]*Sqrt[c*x^2]]]] - (10*I)*ArcTanh[1 + 2*I]*Log[Sin[ArcTan[1 - Sqrt
[2]*Sqrt[c*x^2]] + I*ArcTanh[1 + 2*I]]] - 5*PolyLog[2, E^((2*I)*(-ArcTan[2
+ I] + ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]))] - (5*I)*PolyLog[2, E^((2*I)*ArcTa
n[1 - Sqrt[2]*Sqrt[c*x^2]] - 2*ArcTanh[1 + 2*I])])*(3 + 2*Cos[2*ArcTan[1 -
Sqrt[2]*Sqrt[c*x^2]]] - 2*Sin[2*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]]))/((-1 - c
*x^2 + Sqrt[2]*Sqrt[c*x^2])*(1 + c*x^2 + Sqrt[2]*Sqrt[c*x^2])*(1/Sqrt[1 + (
1 - Sqrt[2]*Sqrt[c*x^2])^2] - (1 - Sqrt[2]*Sqrt[c*x^2])/Sqrt[1 + (1 - Sqrt[
2]*Sqrt[c*x^2])^2])^2) + ((1/80 + I/80)*(2 + 2*c*x^2 - 2*Sqrt[2]*Sqrt[c*x^2
])^2*((-5 - 5*I)*Pi*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]] - (10*I)*ArcTan[2 + I]*
ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]] + (8 - 8*I)*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]
^2 - ((4 - 2*I)*Sqrt[1 + I]*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]^2)/E^(I*ArcTan[
2 + I]) - ((2 - 4*I)*Sqrt[1 - I]*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]^2)/E^ArcTa
nh[1 + 2*I] + (10*I)*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]*ArcTanh[1 + 2*I] - (5
- 5*I)*Pi*Log[1 + E^((-2*I)*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]])] + 10*ArcTan[2
 + I]*Log[1 - E^((2*I)*(-ArcTan[2 + I] + ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]))]
 - 10*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]*Log[1 - E^((2*I)*(-ArcTan[2 + I] + Ar
cTan[1 - Sqrt[2]*Sqrt[c*x^2]]))] + (10*I)*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]*L
og[1 - E^((2*I)*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]] - 2*ArcTanh[1 + 2*I])] - 10
*ArcTanh[1 + 2*I]*Log[1 - E^((2*I)*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]] - 2*ArcT
anh[1 + 2*I])] + (5 - 5*I)*Pi*Log[1/Sqrt[2 + 2*c*x^2 - 2*Sqrt[2]*Sqrt[c*x^2
]]] - 10*ArcTan[2 + I]*Log[-Sin[ArcTan[2 + I] - ArcTan[1 - Sqrt[2]*Sqrt[c*x
^2]]]] + 10*ArcTanh[1 + 2*I]*Log[Sin[ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]] + I*Ar
cTanh[1 + 2*I]]] + (5*I)*PolyLog[2, E^((2*I)*(-ArcTan[2 + I] + ArcTan[1 - S
qrt[2]*Sqrt[c*x^2]]))] + 5*PolyLog[2, E^((2*I)*ArcTan[1 - Sqrt[2]*Sqrt[c*x^
2]] - 2*ArcTanh[1 + 2*I])])*(3 + 2*Cos[2*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]] -
 2*Sin[2*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]]))/(1 + c^2*x^4) - (Sqrt[c*x^2]*(1
 + (1 + Sqrt[2]*Sqrt[c*x^2])^2)^(3/2)*(2*(-5*ArcTan[2 + I]*ArcTan[1 + Sqrt[
2]*Sqrt[c*x^2]] + 4*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]]^2 + ((1 + 2*I)*Sqrt[1 +
 I]*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]]^2)/E^(I*ArcTan[2 + I]) + ((1 - 2*I)*Sqr
t[1 - I]*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]]^2)/E^ArcTanh[1 + 2*I] - (5*I)*ArcT
an[1 + Sqrt[2]*Sqrt[c*x^2]]*ArcTanh[1 + 2*I] + (5*I)*(-ArcTan[2 + I] + ArcT
an[1 + Sqrt[2]*Sqrt[c*x^2]])*Log[1 - E^((2*I)*(-ArcTan[2 + I] + ArcTan[1 +
Sqrt[2]*Sqrt[c*x^2]]))] + 5*((-I)*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]] + ArcTanh
[1 + 2*I])*Log[1 - E^((2*I)*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]] - 2*ArcTanh[1 +
 2*I])] + (5*I)*ArcTan[2 + I]*Log[-Sin[ArcTan[2 + I] - ArcTan[1 + Sqrt[2]*S
qrt[c*x^2]]]] - 5*ArcTanh[1 + 2*I]*Log[Sin[ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]]
+ I*ArcTanh[1 + 2*I]]]) + 5*PolyLog[2, E^((2*I)*(-ArcTan[2 + I] + ArcTan[1
+ Sqrt[2]*Sqrt[c*x^2]]))] - 5*PolyLog[2, E^((2*I)*ArcTan[1 + Sqrt[2]*Sqrt[c
*x^2]] - 2*ArcTanh[1 + 2*I])])*(3 + 2*Cos[2*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]]

```
] - 2*Sin[2*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]]]))/(20*Sqrt[2]*(-1 - c*x^2 + Sq
rt[2]*Sqrt[c*x^2])*(1 + c*x^2 + Sqrt[2]*Sqrt[c*x^2])*(1/Sqrt[1 + (1 + Sqrt[
2]*Sqrt[c*x^2])^2] - (1 + Sqrt[2]*Sqrt[c*x^2])/Sqrt[1 + (1 + Sqrt[2]*Sqrt[c
*x^2])^2])) - ((1/40 + I/40)*c*x^2*(1 + (1 + Sqrt[2]*Sqrt[c*x^2])^2)*((5 +
5*I)*Pi*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]] + 10*ArcTan[2 + I]*ArcTan[1 + Sqrt[
2]*Sqrt[c*x^2]] + (4 - 4*I)*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]]^2 - ((2 + 4*I)*
Sqrt[1 + I]*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]]^2)/E^(I*ArcTan[2 + I]) + ((4 +
2*I)*Sqrt[1 - I]*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]]^2)/E^ArcTanh[1 + 2*I] + 10
*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]]*ArcTanh[1 + 2*I] + (5 - 5*I)*Pi*Log[1 + E^
((-2*I)*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]])] + (10*I)*ArcTan[2 + I]*Log[1 - E^
((2*I)*(-ArcTan[2 + I] + ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]]))] - (10*I)*ArcTan
[1 + Sqrt[2]*Sqrt[c*x^2]]*Log[1 - E^((2*I)*(-ArcTan[2 + I] + ArcTan[1 + Sqr
t[2]*Sqrt[c*x^2]]))] + 10*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]]*Log[1 - E^((2*I)*
ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]] - 2*ArcTanh[1 + 2*I])] + (10*I)*ArcTanh[1 +
 2*I]*Log[1 - E^((2*I)*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]] - 2*ArcTanh[1 + 2*I]
)] - (5 - 5*I)*Pi*Log[1/Sqrt[1 + (1 + Sqrt[2]*Sqrt[c*x^2])^2]] - (10*I)*Arc
Tan[2 + I]*Log[-Sin[ArcTan[2 + I] - ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]]]] - (10
*I)*ArcTanh[1 + 2*I]*Log[Sin[ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]] + I*ArcTanh[1
+ 2*I]]] - 5*PolyLog[2, E^((2*I)*(-ArcTan[2 + I] + ArcTan[1 + Sqrt[2]*Sqrt[
c*x^2]]))] - (5*I)*PolyLog[2, E^((2*I)*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]] - 2*
ArcTanh[1 + 2*I])])*(3 + 2*Cos[2*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]]] - 2*Sin[2
*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]]]))/((-1 - c*x^2 + Sqrt[2]*Sqrt[c*x^2])*(1
+ c*x^2 + Sqrt[2]*Sqrt[c*x^2])*(1/Sqrt[1 + (1 + Sqrt[2]*Sqrt[c*x^2])^2] - (
1 + Sqrt[2]*Sqrt[c*x^2])/Sqrt[1 + (1 + Sqrt[2]*Sqrt[c*x^2])^2])^2) + ((1/80
 + I/80)*(1 + (1 + Sqrt[2]*Sqrt[c*x^2])^2)^2*((-5 - 5*I)*Pi*ArcTan[1 + Sqrt
[2]*Sqrt[c*x^2]] - (10*I)*ArcTan[2 + I]*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]] + (
8 - 8*I)*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]]^2 - ((4 - 2*I)*Sqrt[1 + I]*ArcTan[
1 + Sqrt[2]*Sqrt[c*x^2]]^2)/E^(I*ArcTan[2 + I]) - ((2 - 4*I)*Sqrt[1 - I]*Ar
cTan[1 + Sqrt[2]*Sqrt[c*x^2]]^2)/E^ArcTanh[1 + 2*I] + (10*I)*ArcTan[1 + Sqr
t[2]*Sqrt[c*x^2]]*ArcTanh[1 + 2*I] - (5 - 5*I)*Pi*Log[1 + E^((-2*I)*ArcTan[
1 + Sqrt[2]*Sqrt[c*x^2]])] + 10*ArcTan[2 + I]*Log[1 - E^((2*I)*(-ArcTan[2 +
 I] + ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]]))] - 10*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2
]]*Log[1 - E^((2*I)*(-ArcTan[2 + I] + ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]]))] +
(10*I)*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]]*Log[1 - E^((2*I)*ArcTan[1 + Sqrt[2]*
Sqrt[c*x^2]] - 2*ArcTanh[1 + 2*I])] - 10*ArcTanh[1 + 2*I]*Log[1 - E^((2*I)*
ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]] - 2*ArcTanh[1 + 2*I])] + (5 - 5*I)*Pi*Log[1
/Sqrt[1 + (1 + Sqrt[2]*Sqrt[c*x^2])^2]] - 10*ArcTan[2 + I]*Log[-Sin[ArcTan[
2 + I] - ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]]]] + 10*ArcTanh[1 + 2*I]*Log[Sin[Ar
cTan[1 + Sqrt[2]*Sqrt[c*x^2]] + I*ArcTanh[1 + 2*I]]] + (5*I)*PolyLog[2, E^(
(2*I)*(-ArcTan[2 + I] + ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]]))] + 5*PolyLog[2, E
^((2*I)*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]] - 2*ArcTanh[1 + 2*I])])*(3 + 2*Cos[
2*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]]] - 2*Sin[2*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]
]]))/((-1 - c*x^2 + Sqrt[2]*Sqrt[c*x^2])*(1 + c*x^2 + Sqrt[2]*Sqrt[c*x^2]))
)/(2*Sqrt[2]))))/(2*c*x)
```

Maple [F]

$$\int \left(a + b\arctan\left(c\,x^2\right)\right)^2 dx$$

```
[In] int((a+b*arctan(c*x^2))^2,x)
[Out] int((a+b*arctan(c*x^2))^2,x)
```

Fricas [F]

$$\int \left(a + b\arctan\left(cx^2\right)\right)^2 \, dx = \int \left(b\arctan\left(cx^2\right) + a\right)^2 dx$$

```
[In] integrate((a+b*arctan(c*x^2))^2,x, algorithm="fricas")
[Out] integral(b^2*arctan(c*x^2)^2 + 2*a*b*arctan(c*x^2) + a^2, x)
```

Sympy [F]

$$\int \left(a + b\arctan\left(cx^2\right)\right)^2 \, dx = \int \left(a + b\,\mathrm{atan}\left(cx^2\right)\right)^2 \, dx$$

```
[In] integrate((a+b*atan(c*x**2))**2,x)
[Out] Integral((a + b*atan(c*x**2))**2, x)
```

Maxima [F]

$$\int \left(a + b\arctan\left(cx^2\right)\right)^2 \, dx = \int \left(b\arctan\left(cx^2\right) + a\right)^2 dx$$

```
[In] integrate((a+b*arctan(c*x^2))^2,x, algorithm="maxima")
[Out] -1/2*(c*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*c*x + sqrt(2)*sqrt(c))/sqrt(c))/c^
(3/2) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*c*x - sqrt(2)*sqrt(c))/sqrt(c))/c^(
3/2) - sqrt(2)*log(c*x^2 + sqrt(2)*sqrt(c)*x + 1)/c^(3/2) + sqrt(2)*log(c*x
^2 - sqrt(2)*sqrt(c)*x + 1)/c^(3/2)) - 4*x*arctan(c*x^2))*a*b + 1/16*(4*x*a
rctan(c*x^2)^2 - x*log(c^2*x^4 + 1)^2 + 16*integrate(1/16*(8*c^2*x^4*log(c^
2*x^4 + 1) - 16*c*x^2*arctan(c*x^2) + 12*(c^2*x^4 + 1)*arctan(c*x^2)^2 + (c
^2*x^4 + 1)*log(c^2*x^4 + 1)^2)/(c^2*x^4 + 1), x))*b^2 + a^2*x
```

Giac [F]

$$\int \left(a+b\arctan\left(cx^2\right)\right)^2\,dx=\int \left(b\arctan\left(cx^2\right)+a\right)^2\,dx$$

```
[In] integrate((a+b*arctan(c*x^2))^2,x, algorithm="giac")
[Out] integrate((b*arctan(c*x^2) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\arctan\left(cx^2\right)\right)^2\,dx=\int \left(a+b\,\mathrm{atan}\left(c\,x^2\right)\right)^2\,dx$$

```
[In] int((a + b*atan(c*x^2))^2,x)
[Out] int((a + b*atan(c*x^2))^2, x)
```

3.83 $\int \frac{(a+b\arctan(cx^2))^2}{x^2}\,dx$

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Optimal result

Integrand size = 16, antiderivative size = 1164

$$\int \frac{(a+b\arctan(cx^2))^2}{x^2}\,dx$$
$$= \sqrt[4]{-1}b^2\sqrt{c}\arctan\left((-1)^{3/4}\sqrt{c}x\right)^2 - 2\sqrt[4]{-1}ab\sqrt{c}\operatorname{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)$$
$$-(-1)^{3/4}b^2\sqrt{c}\operatorname{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)^2 - 2(-1)^{3/4}b^2\sqrt{c}\arctan\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{2}{1-\sqrt[4]{-1}\sqrt{c}x}\right) + 2(-1)^{3}$$

```
[Out] -1/4*(2*a+I*b*ln(1-I*c*x^2))^2/x-1/2*b^2*ln(1-I*c*x^2)*ln(1+I*c*x^2)/x-1/2*
(-1)^(1/4)*b^2*polylog(2,1-2^(1/2)*((-1)^(1/4)+x*c^(1/2))/(1+(-1)^(1/4)*x*c
^(1/2)))*c^(1/2)-1/2*(-1)^(3/4)*b^2*polylog(2,1+2^(1/2)*((-1)^(3/4)+x*c^(1/
2))/(1+(-1)^(3/4)*x*c^(1/2)))*c^(1/2)-1/2*(-1)^(3/4)*b^2*polylog(2,1-(1+I)*
(1+(-1)^(1/4)*x*c^(1/2))/(1+(-1)^(3/4)*x*c^(1/2)))*c^(1/2)-1/2*(-1)^(1/4)*b
^2*polylog(2,1+(-1+I)*(1+(-1)^(3/4)*x*c^(1/2))/(1+(-1)^(1/4)*x*c^(1/2)))*c^
(1/2)-2*(-1)^(3/4)*b^2*arctanh((-1)^(3/4)*x*c^(1/2))*ln(2/(1+(-1)^(3/4)*x*c
^(1/2)))*c^(1/2)-2*(-1)^(1/4)*a*b*arctanh((-1)^(3/4)*x*c^(1/2))*c^(1/2)-2*(
-1)^(3/4)*b^2*arctan((-1)^(3/4)*x*c^(1/2))*ln(2/(1-(-1)^(1/4)*x*c^(1/2)))*c
^(1/2)+2*(-1)^(3/4)*b^2*arctan((-1)^(3/4)*x*c^(1/2))*ln(2/(1+(-1)^(1/4)*x*c
^(1/2)))*c^(1/2)+2*(-1)^(3/4)*b^2*arctanh((-1)^(3/4)*x*c^(1/2))*ln(2/(1-(-1
)^(3/4)*x*c^(1/2)))*c^(1/2)+I*a*b*ln(1+I*c*x^2)/x-(-1)^(3/4)*b^2*arctanh((-
1)^(3/4)*x*c^(1/2))*ln(1-I*c*x^2)*c^(1/2)-(-1)^(1/4)*b*arctan((-1)^(3/4)*x*
c^(1/2))*(2*a+I*b*ln(1-I*c*x^2))*c^(1/2)+(-1)^(3/4)*b^2*arctan((-1)^(3/4)*x
*c^(1/2))*ln(1+I*c*x^2)*c^(1/2)+(-1)^(3/4)*b^2*arctanh((-1)^(3/4)*x*c^(1/2)
)*ln(1+I*c*x^2)*c^(1/2)-(-1)^(3/4)*b^2*arctan((-1)^(3/4)*x*c^(1/2))*ln(2^(1
/2)*((-1)^(1/4)+x*c^(1/2))/(1+(-1)^(1/4)*x*c^(1/2)))*c^(1/2)+(-1)^(3/4)*b^2
*arctanh((-1)^(3/4)*x*c^(1/2))*ln(-2^(1/2)*((-1)^(3/4)+x*c^(1/2))/(1+(-1)^(
3/4)*x*c^(1/2)))*c^(1/2)+(-1)^(3/4)*b^2*arctanh((-1)^(3/4)*x*c^(1/2))*ln((1
+I)*(1+(-1)^(1/4)*x*c^(1/2))/(1+(-1)^(3/4)*x*c^(1/2)))*c^(1/2)-(-1)^(3/4)*b
```

```
^2*arctan((-1)^(3/4)*x*c^(1/2))*ln((1-I)*(1+(-1)^(3/4)*x*c^(1/2))/(1+(-1)^(
1/4)*x*c^(1/2)))*c^(1/2)+(-1)^(1/4)*b^2*polylog(2,1-2/(1-(-1)^(1/4)*x*c^(1/
2)))*c^(1/2)+(-1)^(1/4)*b^2*polylog(2,1-2/(1+(-1)^(1/4)*x*c^(1/2)))*c^(1/2)
+(-1)^(3/4)*b^2*polylog(2,1-2/(1-(-1)^(3/4)*x*c^(1/2)))*c^(1/2)+(-1)^(3/4)*
b^2*polylog(2,1-2/(1+(-1)^(3/4)*x*c^(1/2)))*c^(1/2)+(-1)^(1/4)*b^2*arctan((
-1)^(3/4)*x*c^(1/2))^2*c^(1/2)-(-1)^(3/4)*b^2*arctanh((-1)^(3/4)*x*c^(1/2))
^2*c^(1/2)+1/4*b^2*ln(1+I*c*x^2)^2/x
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 1164, normalized size of antiderivative = 1.00, number of steps used = 47, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 1.438$, Rules used = {4950, 2507, 209, 2520, 12, 5040, 4964, 2449, 2352, 2505, 6874, 212, 30, 2637, 211, 5048, 4966, 2497, 214, 6139, 6057, 6131, 6055}

$$\int \frac{(a+b\arctan(cx^2))^2}{x^2}\,dx = \sqrt[4]{-1}\sqrt{c}\arctan\left((-1)^{3/4}\sqrt{c}x\right)^2 b^2$$
$$-(-1)^{3/4}\sqrt{c}\operatorname{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)^2 b^2+\frac{\log^2(icx^2+1)\,b^2}{4x}-2(-1)^{3/4}\sqrt{c}\arctan\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{2}{1-\sqrt[4]{-}}\right.$$

```
[In] Int[(a + b*ArcTan[c*x^2])^2/x^2,x]
```

```
[Out] (-1)^(1/4)*b^2*Sqrt[c]*ArcTan[(-1)^(3/4)*Sqrt[c]*x]^2 - 2*(-1)^(1/4)*a*b*Sq
rt[c]*ArcTanh[(-1)^(3/4)*Sqrt[c]*x] - (-1)^(3/4)*b^2*Sqrt[c]*ArcTanh[(-1)^(
3/4)*Sqrt[c]*x]^2 - 2*(-1)^(3/4)*b^2*Sqrt[c]*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*L
og[2/(1 - (-1)^(1/4)*Sqrt[c]*x)] + 2*(-1)^(3/4)*b^2*Sqrt[c]*ArcTan[(-1)^(3/
4)*Sqrt[c]*x]*Log[2/(1 + (-1)^(1/4)*Sqrt[c]*x)] - (-1)^(3/4)*b^2*Sqrt[c]*Ar
cTan[(-1)^(3/4)*Sqrt[c]*x]*Log[(Sqrt[2]*((-1)^(1/4) + Sqrt[c]*x))/(1 + (-1)
^(1/4)*Sqrt[c]*x)] + 2*(-1)^(3/4)*b^2*Sqrt[c]*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]
*Log[2/(1 - (-1)^(3/4)*Sqrt[c]*x)] - 2*(-1)^(3/4)*b^2*Sqrt[c]*ArcTanh[(-1)^
(3/4)*Sqrt[c]*x]*Log[2/(1 + (-1)^(3/4)*Sqrt[c]*x)] + (-1)^(3/4)*b^2*Sqrt[c]
*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[-((Sqrt[2]*((-1)^(3/4) + Sqrt[c]*x))/(1
+ (-1)^(3/4)*Sqrt[c]*x))] + (-1)^(3/4)*b^2*Sqrt[c]*ArcTanh[(-1)^(3/4)*Sqrt[
c]*x]*Log[((1 + I)*(1 + (-1)^(1/4)*Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x)]
- (-1)^(3/4)*b^2*Sqrt[c]*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[((1 - I)*(1 + (-1
)^(3/4)*Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*x)] - (-1)^(3/4)*b^2*Sqrt[c]*Ar
cTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[1 - I*c*x^2] - (-1)^(1/4)*b*Sqrt[c]*ArcTan[
(-1)^(3/4)*Sqrt[c]*x]*(2*a + I*b*Log[1 - I*c*x^2]) - (2*a + I*b*Log[1 - I*c
*x^2])^2/(4*x) + (I*a*b*Log[1 + I*c*x^2])/x + (-1)^(3/4)*b^2*Sqrt[c]*ArcTan
[(-1)^(3/4)*Sqrt[c]*x]*Log[1 + I*c*x^2] + (-1)^(3/4)*b^2*Sqrt[c]*ArcTanh[(-
1)^(3/4)*Sqrt[c]*x]*Log[1 + I*c*x^2] - (b^2*Log[1 - I*c*x^2]*Log[1 + I*c*x^
2])/(2*x) + (b^2*Log[1 + I*c*x^2]^2)/(4*x) + (-1)^(1/4)*b^2*Sqrt[c]*PolyLog
[2, 1 - 2/(1 - (-1)^(1/4)*Sqrt[c]*x)] + (-1)^(1/4)*b^2*Sqrt[c]*PolyLog[2, 1
 - 2/(1 + (-1)^(1/4)*Sqrt[c]*x)] - ((-1)^(1/4)*b^2*Sqrt[c]*PolyLog[2, 1 - (
```

```
Sqrt[2]*((-1)^(1/4) + Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*x)])/2 + (-1)^(3/
4)*b^2*Sqrt[c]*PolyLog[2, 1 - 2/(1 - (-1)^(3/4)*Sqrt[c]*x)] + (-1)^(3/4)*b^
2*Sqrt[c]*PolyLog[2, 1 - 2/(1 + (-1)^(3/4)*Sqrt[c]*x)] - ((-1)^(3/4)*b^2*Sq
rt[c]*PolyLog[2, 1 + (Sqrt[2]*((-1)^(3/4) + Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqr
t[c]*x)])/2 - ((-1)^(3/4)*b^2*Sqrt[c]*PolyLog[2, 1 - ((1 + I)*(1 + (-1)^(1/
4)*Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x)])/2 - ((-1)^(1/4)*b^2*Sqrt[c]*Pol
yLog[2, 1 - ((1 - I)*(1 + (-1)^(3/4)*Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*x)
])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2507

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Dist[b*e*n*p*(q/(f^n*(m + 1))), Int[(f*x)^(m + n)*((a +
 b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2637

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[z*Log[w]*(D[v, x]/v), x], x] - In
t[SimplifyIntegrand[z*Log[v]*(D[w, x]/w), x], x]) /; InverseFunctionFreeQ[z
, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> I
nt[ExpandIntegrand[x^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*Log[1 + I*c*x^n
])/2)^p, x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1] && IGtQ[n, 0] && Integ
erQ[m]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5048

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6057

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcTanh[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x
)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcTanh[c*x])
*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x]) /; FreeQ[{a, b, c, d,
e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6139

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = \int \left(\frac{(2a+ib\log(1-icx^2))^2}{4x^2}+\frac{b(-2ia+b\log(1-icx^2))\log(1+icx^2)}{2x^2}-\frac{b^2\log^2(1+icx^2)}{4x^2}\right)dx$$

$$=\frac{1}{4}\int\frac{(2a+ib\log(1-icx^2))^2}{x^2}\,dx+\frac{1}{2}b\int\frac{(-2ia+b\log(1-icx^2))\log(1+icx^2)}{x^2}\,dx-\frac{1}{4}b^2\int\frac{\log^2(1+icx^2)}{x^2}\,dx$$

$$=-\frac{(2a+ib\log(1-icx^2))^2}{4x}+\frac{b^2\log^2(1+icx^2)}{4x}+\frac{1}{2}b\int\left(-\frac{2ia\log(1+icx^2)}{x^2}+\frac{b\log(1-icx^2)\log(1+icx^2)}{x^2}\right)dx+(bc)\int\frac{2a+ib\log(1-icx^2)}{1-icx^2}\,dx-\left(ib^2c\right)\int\frac{\log(1+icx^2)}{1+icx^2}\,dx$$

$$=-\sqrt[4]{-1}b\sqrt{c}\arctan\left((-1)^{3/4}\sqrt{c}x\right)\left(2a+ib\log\left(1-icx^2\right)\right)-\frac{(2a+ib\log(1-icx^2))^2}{4x}+(-1)^{3/4}b^2\sqrt{c}\operatorname{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)\log\left(1+icx^2\right)+\frac{b^2\log^2(1+icx^2)}{4x}-(iab)\int\frac{\log(1+icx^2)}{x^2}\,dx+$$

$$= -\sqrt[4]{-1}b\sqrt{c}\arctan\left((-1)^{3/4}\sqrt{c}x\right)\left(2a+ib\log\left(1-icx^2\right)\right) - \frac{\left(2a+ib\log\left(1-icx^2\right)\right)^2}{4x} + \frac{iab\log\left(1+icx^2\right)}{x} + (-1)^{3/4}b^2\sqrt{c}\operatorname{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)\log\left(1+icx^2\right) - \frac{b^2\log\left(1-icx^2\right)\log\left(1+icx^2\right)}{2x} + \frac{b^2\log^2\left(1+i\right.}{4x}$$

$$= \sqrt[4]{-1}b^2\sqrt{c}\arctan\left((-1)^{3/4}\sqrt{c}x\right)^2 - 2\sqrt[4]{-1}ab\sqrt{c}\operatorname{arctanh}\left((-1)^{3/4}\sqrt{c}x\right) - (-1)^{3/4}b^2\sqrt{c}\operatorname{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)^2 - \sqrt[4]{-1}b\sqrt{c}\arctan\left((-1)^{3/4}\sqrt{c}x\right)\left(2a+ib\log\left(1-icx^2\right)\right) - \frac{(2}{}$$

$$= \sqrt[4]{-1}b^2\sqrt{c}\arctan\left((-1)^{3/4}\sqrt{c}x\right)^2 - 2\sqrt[4]{-1}ab\sqrt{c}\operatorname{arctanh}\left((-1)^{3/4}\sqrt{c}x\right) - (-1)^{3/4}b^2\sqrt{c}\operatorname{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)^2 - 2(-1)^{3/4}b^2\sqrt{c}\arctan\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{2}{1-\sqrt[4]{-1}\sqrt{c}x}\right) -$$

$$= \sqrt[4]{-1}b^2\sqrt{c}\arctan\left((-1)^{3/4}\sqrt{c}x\right)^2 - 2\sqrt[4]{-1}ab\sqrt{c}\operatorname{arctanh}\left((-1)^{3/4}\sqrt{c}x\right) - (-1)^{3/4}b^2\sqrt{c}\operatorname{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)^2 - 2(-1)^{3/4}b^2\sqrt{c}\arctan\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{2}{1-\sqrt[4]{-1}\sqrt{c}x}\right) -$$

$$= \sqrt[4]{-1}b^2\sqrt{c}\arctan\left((-1)^{3/4}\sqrt{c}x\right)^2 - 2\sqrt[4]{-1}ab\sqrt{c}\operatorname{arctanh}\left((-1)^{3/4}\sqrt{c}x\right) - (-1)^{3/4}b^2\sqrt{c}\operatorname{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)^2 - 2(-1)^{3/4}b^2\sqrt{c}\arctan\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{2}{1-\sqrt[4]{-1}\sqrt{c}x}\right) -$$

$$= \sqrt[4]{-1}b^2\sqrt{c}\arctan\left((-1)^{3/4}\sqrt{c}x\right)^2 - 2\sqrt[4]{-1}ab\sqrt{c}\operatorname{arctanh}\left((-1)^{3/4}\sqrt{c}x\right) - (-1)^{3/4}b^2\sqrt{c}\operatorname{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)^2 - 2(-1)^{3/4}b^2\sqrt{c}\arctan\left((-1)^{3/4}\sqrt{c}x\right)\log\left(\frac{2}{1-\sqrt[4]{-1}\sqrt{c}x}\right) -$$

$$= \text{Too large to display}$$

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 4697 vs. $2(1164) = 2328$.

Time = 35.92 (sec) , antiderivative size = 4697, normalized size of antiderivative = 4.04

$$\int \frac{\left(a+b\arctan\left(cx^2\right)\right)^2}{x^2}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(a + b*ArcTan[c*x^2])^2/x^2,x]
```

```
[Out] -(a^2/x) + (a*b*(c*x^2)^(3/2)*((-2*ArcTan[c*x^2])/Sqrt[c*x^2] + Sqrt[2]*(Ar
cTan[(-1 + c*x^2)/(Sqrt[2]*Sqrt[c*x^2])] + ArcTanh[(Sqrt[2]*Sqrt[c*x^2])/(1
 + c*x^2)])))/(c*x^3) + (b^2*(c*x^2)^(3/2)*((-2*ArcTan[c*x^2]^2)/Sqrt[c*x^2
```

] + 4*((ArcTan[c*x^2]*(-2*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]] + 2*ArcTan[1 + Sq rt[2]*Sqrt[c*x^2]] - Log[1 + c*x^2 - Sqrt[2]*Sqrt[c*x^2]] + Log[1 + c*x^2 + Sqrt[2]*Sqrt[c*x^2]]))/(2*Sqrt[2]) - ((ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]] + A rcTan[1 + Sqrt[2]*Sqrt[c*x^2]])*Log[1 + c*x^2 - Sqrt[2]*Sqrt[c*x^2]] - (Arc Tan[1 - Sqrt[2]*Sqrt[c*x^2]] + ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]])*Log[1 + c*x ^2 + Sqrt[2]*Sqrt[c*x^2]] - (Sqrt[c*x^2]*(1 + (1 - Sqrt[2]*Sqrt[c*x^2])^2)^ (3/2)*(2*(-5*ArcTan[2 + I]*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]] + 4*ArcTan[1 - S qrt[2]*Sqrt[c*x^2]]^2 + ((1 + 2*I)*Sqrt[1 + I]*ArcTan[1 - Sqrt[2]*Sqrt[c*x^ 2]]^2)/E^(I*ArcTan[2 + I]) + ((1 - 2*I)*Sqrt[1 - I]*ArcTan[1 - Sqrt[2]*Sqrt [c*x^2]]^2)/E^ArcTanh[1 + 2*I] - (5*I)*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]*ArcT anh[1 + 2*I] + (5*I)*(-ArcTan[2 + I] + ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]])*Log [1 - E^((2*I)*(-ArcTan[2 + I] + ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]))] + 5*((-I)*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]] + ArcTanh[1 + 2*I])*Log[1 - E^((2*I)*ArcT an[1 - Sqrt[2]*Sqrt[c*x^2]] - 2*ArcTanh[1 + 2*I])] + (5*I)*ArcTan[2 + I]*Lo g[-Sin[ArcTan[2 + I] - ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]]] - 5*ArcTanh[1 + 2* I]*Log[Sin[ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]] + I*ArcTanh[1 + 2*I]]]) + 5*Poly Log[2, E^((2*I)*(-ArcTan[2 + I] + ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]))] - 5*Po lyLog[2, E^((2*I)*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]] - 2*ArcTanh[1 + 2*I])])*(3 + 2*Cos[2*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]] - 2*Sin[2*ArcTan[1 - Sqrt[2]*S qrt[c*x^2]]]))/(20*Sqrt[2]*(-1 - c*x^2 + Sqrt[2]*Sqrt[c*x^2])*(1 + c*x^2 + Sqrt[2]*Sqrt[c*x^2])*(1/Sqrt[1 + (1 - Sqrt[2]*Sqrt[c*x^2])^2] - (1 - Sqrt[2]*Sqrt[c*x^2])/Sqrt[1 + (1 - Sqrt[2]*Sqrt[c*x^2])^2])) - ((1/40 + I/40)*c*x ^2*(1 + (1 - Sqrt[2]*Sqrt[c*x^2])^2)*((5 + 5*I)*Pi*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]] + 10*ArcTan[2 + I]*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]] + (4 - 4*I)*ArcT an[1 - Sqrt[2]*Sqrt[c*x^2]]^2 - ((2 + 4*I)*Sqrt[1 + I]*ArcTan[1 - Sqrt[2]*S qrt[c*x^2]]^2)/E^(I*ArcTan[2 + I]) + ((4 + 2*I)*Sqrt[1 - I]*ArcTan[1 - Sqrt [2]*Sqrt[c*x^2]]^2)/E^ArcTanh[1 + 2*I] + 10*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]] *ArcTanh[1 + 2*I] + (5 - 5*I)*Pi*Log[1 + E^((-2*I)*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]])] + (10*I)*ArcTan[2 + I]*Log[1 - E^((2*I)*(-ArcTan[2 + I] + ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]))] - (10*I)*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]*Log[1 - E^((2*I)*(-ArcTan[2 + I] + ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]))] + 10*ArcTan [1 - Sqrt[2]*Sqrt[c*x^2]]*Log[1 - E^((2*I)*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]] - 2*ArcTanh[1 + 2*I])] + (10*I)*ArcTanh[1 + 2*I]*Log[1 - E^((2*I)*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]] - 2*ArcTanh[1 + 2*I])] - (5 - 5*I)*Pi*Log[1/Sqrt[1 + (1 - Sqrt[2]*Sqrt[c*x^2])^2]] - (10*I)*ArcTan[2 + I]*Log[-Sin[ArcTan[2 + I] - ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]]] - (10*I)*ArcTanh[1 + 2*I]*Log[Sin[Arc Tan[1 - Sqrt[2]*Sqrt[c*x^2]] + I*ArcTanh[1 + 2*I]]] - 5*PolyLog[2, E^((2*I) *(-ArcTan[2 + I] + ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]))] - (5*I)*PolyLog[2, E^ ((2*I)*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]] - 2*ArcTanh[1 + 2*I])])*(3 + 2*Cos[2 *ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]] - 2*Sin[2*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]]))/((-1 - c*x^2 + Sqrt[2]*Sqrt[c*x^2])*(1 + c*x^2 + Sqrt[2]*Sqrt[c*x^2])*(1/Sqrt[1 + (1 - Sqrt[2]*Sqrt[c*x^2])^2] - (1 - Sqrt[2]*Sqrt[c*x^2])/Sqrt[1 + (1 - Sqrt[2]*Sqrt[c*x^2])^2])^2) - ((1/80 + I/80)*(2 + 2*c*x^2 - 2*Sqrt[2]*Sqrt[c*x^2])^2*((-5 - 5*I)*Pi*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]] - (10*I)*Ar cTan[2 + I]*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]] + (8 - 8*I)*ArcTan[1 - Sqrt[2]*

```
Sqrt[c*x^2]]^2 - ((4 - 2*I)*Sqrt[1 + I]*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]^2)/
E^(I*ArcTan[2 + I]) - ((2 - 4*I)*Sqrt[1 - I]*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]
]^2)/E^ArcTanh[1 + 2*I] + (10*I)*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]*ArcTanh[1
+ 2*I] - (5 - 5*I)*Pi*Log[1 + E^((-2*I)*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]])] +
 10*ArcTan[2 + I]*Log[1 - E^((2*I)*(-ArcTan[2 + I] + ArcTan[1 - Sqrt[2]*Sqr
t[c*x^2]]))] - 10*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]*Log[1 - E^((2*I)*(-ArcTan
[2 + I] + ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]))] + (10*I)*ArcTan[1 - Sqrt[2]*Sq
rt[c*x^2]]*Log[1 - E^((2*I)*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]] - 2*ArcTanh[1 +
 2*I])] - 10*ArcTanh[1 + 2*I]*Log[1 - E^((2*I)*ArcTan[1 - Sqrt[2]*Sqrt[c*x^
2]] - 2*ArcTanh[1 + 2*I])] + (5 - 5*I)*Pi*Log[1/Sqrt[2 + 2*c*x^2 - 2*Sqrt[2
]*Sqrt[c*x^2]]] - 10*ArcTan[2 + I]*Log[-Sin[ArcTan[2 + I] - ArcTan[1 - Sqrt
[2]*Sqrt[c*x^2]]]] + 10*ArcTanh[1 + 2*I]*Log[Sin[ArcTan[1 - Sqrt[2]*Sqrt[c*
x^2]] + I*ArcTanh[1 + 2*I]]] + (5*I)*PolyLog[2, E^((2*I)*(-ArcTan[2 + I] +
ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]))] + 5*PolyLog[2, E^((2*I)*ArcTan[1 - Sqrt[
2]*Sqrt[c*x^2]] - 2*ArcTanh[1 + 2*I])])*(3 + 2*Cos[2*ArcTan[1 - Sqrt[2]*Sqr
t[c*x^2]]] - 2*Sin[2*ArcTan[1 - Sqrt[2]*Sqrt[c*x^2]]]))/(1 + c^2*x^4) - (Sq
rt[c*x^2]*(1 + (1 + Sqrt[2]*Sqrt[c*x^2])^2)^(3/2)*(2*(-5*ArcTan[2 + I]*ArcT
an[1 + Sqrt[2]*Sqrt[c*x^2]] + 4*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]]^2 + ((1 + 2
*I)*Sqrt[1 + I]*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]]^2)/E^(I*ArcTan[2 + I]) + ((
1 - 2*I)*Sqrt[1 - I]*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]]^2)/E^ArcTanh[1 + 2*I]
- (5*I)*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]]*ArcTanh[1 + 2*I] + (5*I)*(-ArcTan[2
 + I] + ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]])*Log[1 - E^((2*I)*(-ArcTan[2 + I] +
 ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]]))] + 5*((-I)*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2
]] + ArcTanh[1 + 2*I])*Log[1 - E^((2*I)*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]] - 2
*ArcTanh[1 + 2*I])] + (5*I)*ArcTan[2 + I]*Log[-Sin[ArcTan[2 + I] - ArcTan[1
 + Sqrt[2]*Sqrt[c*x^2]]]] - 5*ArcTanh[1 + 2*I]*Log[Sin[ArcTan[1 + Sqrt[2]*S
qrt[c*x^2]] + I*ArcTanh[1 + 2*I]]]) + 5*PolyLog[2, E^((2*I)*(-ArcTan[2 + I]
 + ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]]))] - 5*PolyLog[2, E^((2*I)*ArcTan[1 + Sq
rt[2]*Sqrt[c*x^2]] - 2*ArcTanh[1 + 2*I])])*(3 + 2*Cos[2*ArcTan[1 + Sqrt[2]*
Sqrt[c*x^2]]] - 2*Sin[2*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]]]))/(20*Sqrt[2]*(-1
- c*x^2 + Sqrt[2]*Sqrt[c*x^2])*(1 + c*x^2 + Sqrt[2]*Sqrt[c*x^2])*(1/Sqrt[1
+ (1 + Sqrt[2]*Sqrt[c*x^2])^2] - (1 + Sqrt[2]*Sqrt[c*x^2])/Sqrt[1 + (1 + Sq
rt[2]*Sqrt[c*x^2])^2])) + ((1/40 + I/40)*c*x^2*(1 + (1 + Sqrt[2]*Sqrt[c*x^2
])^2)*((5 + 5*I)*Pi*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]] + 10*ArcTan[2 + I]*ArcT
an[1 + Sqrt[2]*Sqrt[c*x^2]] + (4 - 4*I)*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]]^2 -
 ((2 + 4*I)*Sqrt[1 + I]*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]]^2)/E^(I*ArcTan[2 +
I]) + ((4 + 2*I)*Sqrt[1 - I]*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]]^2)/E^ArcTanh[1
 + 2*I] + 10*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]]*ArcTanh[1 + 2*I] + (5 - 5*I)*P
i*Log[1 + E^((-2*I)*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]])] + (10*I)*ArcTan[2 + I
]*Log[1 - E^((2*I)*(-ArcTan[2 + I] + ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]]))] - (
10*I)*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]]*Log[1 - E^((2*I)*(-ArcTan[2 + I] + Ar
cTan[1 + Sqrt[2]*Sqrt[c*x^2]]))] + 10*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]]*Log[1
 - E^((2*I)*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]] - 2*ArcTanh[1 + 2*I])] + (10*I)
*ArcTanh[1 + 2*I]*Log[1 - E^((2*I)*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]] - 2*ArcT
anh[1 + 2*I])] - (5 - 5*I)*Pi*Log[1/Sqrt[1 + (1 + Sqrt[2]*Sqrt[c*x^2])^2]]
```

```
- (10*I)*ArcTan[2 + I]*Log[-Sin[ArcTan[2 + I] - ArcTan[1 + Sqrt[2]*Sqrt[c*x
^2]]]] - (10*I)*ArcTanh[1 + 2*I]*Log[Sin[ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]] +
I*ArcTanh[1 + 2*I]]] - 5*PolyLog[2, E^((2*I)*(-ArcTan[2 + I] + ArcTan[1 + S
qrt[2]*Sqrt[c*x^2]]))] - (5*I)*PolyLog[2, E^((2*I)*ArcTan[1 + Sqrt[2]*Sqrt[
c*x^2]] - 2*ArcTanh[1 + 2*I])])*(3 + 2*Cos[2*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]
]] - 2*Sin[2*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]]]))/((-1 - c*x^2 + Sqrt[2]*Sqrt
[c*x^2])*(1 + c*x^2 + Sqrt[2]*Sqrt[c*x^2])*(1/Sqrt[1 + (1 + Sqrt[2]*Sqrt[c*
x^2])^2] - (1 + Sqrt[2]*Sqrt[c*x^2])/Sqrt[1 + (1 + Sqrt[2]*Sqrt[c*x^2])^2])
^2) - ((1/80 + I/80)*(1 + (1 + Sqrt[2]*Sqrt[c*x^2])^2)^2*((-5 - 5*I)*Pi*Arc
Tan[1 + Sqrt[2]*Sqrt[c*x^2]] - (10*I)*ArcTan[2 + I]*ArcTan[1 + Sqrt[2]*Sqrt
[c*x^2]] + (8 - 8*I)*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]]^2 - ((4 - 2*I)*Sqrt[1
+ I]*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]]^2)/E^(I*ArcTan[2 + I]) - ((2 - 4*I)*Sq
rt[1 - I]*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]]^2)/E^ArcTanh[1 + 2*I] + (10*I)*Ar
cTan[1 + Sqrt[2]*Sqrt[c*x^2]]*ArcTanh[1 + 2*I] - (5 - 5*I)*Pi*Log[1 + E^((-
2*I)*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]])] + 10*ArcTan[2 + I]*Log[1 - E^((2*I)*
(-ArcTan[2 + I] + ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]]))] - 10*ArcTan[1 + Sqrt[2
]*Sqrt[c*x^2]]*Log[1 - E^((2*I)*(-ArcTan[2 + I] + ArcTan[1 + Sqrt[2]*Sqrt[c
*x^2]]))] + (10*I)*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]]*Log[1 - E^((2*I)*ArcTan[
1 + Sqrt[2]*Sqrt[c*x^2]] - 2*ArcTanh[1 + 2*I])] - 10*ArcTanh[1 + 2*I]*Log[1
 - E^((2*I)*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]] - 2*ArcTanh[1 + 2*I])] + (5 - 5
*I)*Pi*Log[1/Sqrt[1 + (1 + Sqrt[2]*Sqrt[c*x^2])^2]] - 10*ArcTan[2 + I]*Log[
-Sin[ArcTan[2 + I] - ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]]]] + 10*ArcTanh[1 + 2*I
]*Log[Sin[ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]] + I*ArcTanh[1 + 2*I]]] + (5*I)*Po
lyLog[2, E^((2*I)*(-ArcTan[2 + I] + ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]]))] + 5*
PolyLog[2, E^((2*I)*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]] - 2*ArcTanh[1 + 2*I])])
*(3 + 2*Cos[2*ArcTan[1 + Sqrt[2]*Sqrt[c*x^2]]] - 2*Sin[2*ArcTan[1 + Sqrt[2]
*Sqrt[c*x^2]]]))/((-1 - c*x^2 + Sqrt[2]*Sqrt[c*x^2])*(1 + c*x^2 + Sqrt[2]*S
qrt[c*x^2]))/(2*Sqrt[2]))))/(2*c*x^3)
```

Maple [F]

$$\int \frac{(a + b \arctan(c x^2))^2}{x^2} dx$$

[In] int((a+b*arctan(c*x^2))^2/x^2,x)

[Out] int((a+b*arctan(c*x^2))^2/x^2,x)

Fricas [F]

$$\int \frac{(a+b\arctan(cx^2))^2}{x^2}\,dx = \int \frac{(b\arctan(cx^2)+a)^2}{x^2}\,dx$$

```
[In] integrate((a+b*arctan(c*x^2))^2/x^2,x, algorithm="fricas")
[Out] integral((b^2*arctan(c*x^2)^2 + 2*a*b*arctan(c*x^2) + a^2)/x^2, x)
```

Sympy [F]

$$\int \frac{(a+b\arctan(cx^2))^2}{x^2}\,dx = \int \frac{(a+b\,\mathrm{atan}(cx^2))^2}{x^2}\,dx$$

```
[In] integrate((a+b*atan(c*x**2))**2/x**2,x)
[Out] Integral((a + b*atan(c*x**2))**2/x**2, x)
```

Maxima [F]

$$\int \frac{(a+b\arctan(cx^2))^2}{x^2}\,dx = \int \frac{(b\arctan(cx^2)+a)^2}{x^2}\,dx$$

```
[In] integrate((a+b*arctan(c*x^2))^2/x^2,x, algorithm="maxima")
[Out] 1/2*(c*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*c*x + sqrt(2)*sqrt(c))/sqrt(c))/sqr
t(c) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*c*x - sqrt(2)*sqrt(c))/sqrt(c))/sqrt
(c) + sqrt(2)*log(c*x^2 + sqrt(2)*sqrt(c)*x + 1)/sqrt(c) - sqrt(2)*log(c*x^
2 - sqrt(2)*sqrt(c)*x + 1)/sqrt(c)) - 4*arctan(c*x^2)/x)*a*b - 1/16*(4*arct
an(c*x^2)^2 - 16*x*integrate(-1/16*(8*c^2*x^4*log(c^2*x^4 + 1) - 16*c*x^2*a
rctan(c*x^2) - 12*(c^2*x^4 + 1)*arctan(c*x^2)^2 - (c^2*x^4 + 1)*log(c^2*x^4
 + 1)^2)/(c^2*x^6 + x^2), x) - log(c^2*x^4 + 1)^2)*b^2/x - a^2/x
```

Giac [F]

$$\int \frac{(a+b\arctan(cx^2))^2}{x^2}\,dx = \int \frac{(b\arctan(cx^2)+a)^2}{x^2}\,dx$$

```
[In] integrate((a+b*arctan(c*x^2))^2/x^2,x, algorithm="giac")
[Out] integrate((b*arctan(c*x^2) + a)^2/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx^2))^2}{x^2}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x^2))^2}{x^2}\,dx$$

```
[In] int((a + b*atan(c*x^2))^2/x^2,x)
[Out] int((a + b*atan(c*x^2))^2/x^2, x)
```

3.84 $\int \frac{(a+b\arctan(cx^2))^2}{x^4}\,dx$

Optimal result 493
Rubi [A] (verified) 494
Mathematica [F] 500
Maple [F] 501
Fricas [F] 501
Sympy [F] 501
Maxima [F] 501
Giac [F] 502
Mupad [F(-1)] 502

Optimal result

Integrand size = 16, antiderivative size = 1360

$$\int \frac{(a+b\arctan(cx^2))^2}{x^4}\,dx = -\frac{2abc}{3x} - \frac{4}{3}\sqrt[4]{-1}b^2c^{3/2}\arctan\left((-1)^{3/4}\sqrt{c}x\right)$$
$$+\frac{1}{3}(-1)^{3/4}b^2c^{3/2}\arctan\left((-1)^{3/4}\sqrt{c}x\right)^2+\frac{2}{3}(-1)^{3/4}abc^{3/2}\operatorname{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)-\frac{4}{3}\sqrt[4]{-1}b^2c^{3/2}\operatorname{arctanh}\left((-\right.$$

```
[Out] -1/12*(2*a+I*b*ln(1-I*c*x^2))^2/x^3-2/3*a*b*c/x-4/3*(-1)^(1/4)*b^2*c^(3/2)*
arctan((-1)^(3/4)*x*c^(1/2))+1/3*(-1)^(3/4)*b^2*c^(3/2)*arctan((-1)^(3/4)*x
*c^(1/2))^2-4/3*(-1)^(1/4)*b^2*c^(3/2)*arctanh((-1)^(3/4)*x*c^(1/2))-1/3*(-
1)^(1/4)*b^2*c^(3/2)*arctanh((-1)^(3/4)*x*c^(1/2))^2-1/3*b*c*(2*a+I*b*ln(1-
I*c*x^2))/x-1/6*b^2*ln(1-I*c*x^2)*ln(1+I*c*x^2)/x^3+1/3*(-1)^(3/4)*b^2*c^(3
/2)*polylog(2,1-2/(1-(-1)^(1/4)*x*c^(1/2)))+1/3*(-1)^(3/4)*b^2*c^(3/2)*poly
log(2,1-2/(1+(-1)^(1/4)*x*c^(1/2)))-1/6*(-1)^(3/4)*b^2*c^(3/2)*polylog(2,1-
2^(1/2)*((-1)^(1/4)+x*c^(1/2))/(1+(-1)^(1/4)*x*c^(1/2)))+1/3*(-1)^(1/4)*b^2
*c^(3/2)*polylog(2,1-2/(1-(-1)^(3/4)*x*c^(1/2)))+1/3*(-1)^(1/4)*b^2*c^(3/2)
*polylog(2,1-2/(1+(-1)^(3/4)*x*c^(1/2)))-1/6*(-1)^(1/4)*b^2*c^(3/2)*polylog
(2,1+2^(1/2)*((-1)^(3/4)+x*c^(1/2))/(1+(-1)^(3/4)*x*c^(1/2)))-1/6*(-1)^(1/4
)*b^2*c^(3/2)*polylog(2,1-(1+I)*(1+(-1)^(1/4)*x*c^(1/2))/(1+(-1)^(3/4)*x*c^
(1/2)))-1/6*(-1)^(3/4)*b^2*c^(3/2)*polylog(2,1+(-1+I)*(1+(-1)^(3/4)*x*c^(1/
2))/(1+(-1)^(1/4)*x*c^(1/2)))+1/3*(-1)^(1/4)*b^2*c^(3/2)*arctan((-1)^(3/4)*
x*c^(1/2))*ln(2^(1/2)*((-1)^(1/4)+x*c^(1/2))/(1+(-1)^(1/4)*x*c^(1/2)))+2/3*
(-1)^(1/4)*b^2*c^(3/2)*arctanh((-1)^(3/4)*x*c^(1/2))*ln(2/(1-(-1)^(3/4)*x*c
^(1/2)))-2/3*(-1)^(1/4)*b^2*c^(3/2)*arctanh((-1)^(3/4)*x*c^(1/2))*ln(2/(1+(
-1)^(3/4)*x*c^(1/2)))+1/3*(-1)^(1/4)*b^2*c^(3/2)*arctanh((-1)^(3/4)*x*c^(1/
2))*ln(-2^(1/2)*((-1)^(3/4)+x*c^(1/2))/(1+(-1)^(3/4)*x*c^(1/2)))+1/3*(-1)^(
1/4)*b^2*c^(3/2)*arctanh((-1)^(3/4)*x*c^(1/2))*ln((1+I)*(1+(-1)^(1/4)*x*c^(
1/2))/(1+(-1)^(3/4)*x*c^(1/2)))+1/3*(-1)^(1/4)*b^2*c^(3/2)*arctan((-1)^(3/4
```

```
)*x*c^(1/2))*ln((1-I)*(1+(-1)^(3/4)*x*c^(1/2))/(1+(-1)^(1/4)*x*c^(1/2)))+2/
3*(-1)^(3/4)*a*b*c^(3/2)*arctanh((-1)^(3/4)*x*c^(1/2))-1/3*(-1)^(1/4)*b^2*c
^(3/2)*arctanh((-1)^(3/4)*x*c^(1/2))*ln(1-I*c*x^2)-1/3*(-1)^(3/4)*b*c^(3/2)
*arctan((-1)^(3/4)*x*c^(1/2))*(2*a+I*b*ln(1-I*c*x^2))-1/3*(-1)^(1/4)*b^2*c^
(3/2)*arctan((-1)^(3/4)*x*c^(1/2))*ln(1+I*c*x^2)+1/3*(-1)^(1/4)*b^2*c^(3/2)
*arctanh((-1)^(3/4)*x*c^(1/2))*ln(1+I*c*x^2)+2/3*(-1)^(1/4)*b^2*c^(3/2)*arc
tan((-1)^(3/4)*x*c^(1/2))*ln(2/(1-(-1)^(1/4)*x*c^(1/2)))-2/3*(-1)^(1/4)*b^2
*c^(3/2)*arctan((-1)^(3/4)*x*c^(1/2))*ln(2/(1+(-1)^(1/4)*x*c^(1/2)))-1/3*I*
b^2*c*ln(1-I*c*x^2)/x+1/3*I*a*b*ln(1+I*c*x^2)/x^3+2/3*I*b^2*c*ln(1+I*c*x^2)
/x+1/12*b^2*ln(1+I*c*x^2)^2/x^3
```

Rubi [A] (verified)

Time = 1.52 (sec) , antiderivative size = 1360, normalized size of antiderivative = 1.00, number of steps used = 64, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 1.562$, Rules used = {4950, 2507, 2526, 2505, 209, 211, 2520, 12, 5040, 4964, 2449, 2352, 331, 6874, 212, 30, 2637, 5048, 4966, 2497, 214, 6139, 6057, 6131, 6055}

$$\int \frac{(a+b\arctan(cx^2))^2}{x^4}\,dx$$

$$=\frac{1}{3}(-1)^{3/4}c^{3/2}\arctan\left((-1)^{3/4}\sqrt{c}x\right)^2b^2-\frac{1}{3}\sqrt[4]{-1}c^{3/2}\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)^2b^2+\frac{\log^2(icx^2+1)\,b^2}{12x^3}-\frac{4}{3}\sqrt[4]{-1}c^{3/}$$

```
[In] Int[(a + b*ArcTan[c*x^2])^2/x^4,x]
```

```
[Out] (-2*a*b*c)/(3*x) - (4*(-1)^(1/4)*b^2*c^(3/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x])/
3 + ((-1)^(3/4)*b^2*c^(3/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x]^2)/3 + (2*(-1)^(3/
4)*a*b*c^(3/2)*ArcTanh[(-1)^(3/4)*Sqrt[c]*x])/3 - (4*(-1)^(1/4)*b^2*c^(3/2)
*ArcTanh[(-1)^(3/4)*Sqrt[c]*x])/3 - ((-1)^(1/4)*b^2*c^(3/2)*ArcTanh[(-1)^(3
/4)*Sqrt[c]*x]^2)/3 + (2*(-1)^(1/4)*b^2*c^(3/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x
]*Log[2/(1 - (-1)^(1/4)*Sqrt[c]*x)])/3 - (2*(-1)^(1/4)*b^2*c^(3/2)*ArcTan[(
-1)^(3/4)*Sqrt[c]*x]*Log[2/(1 + (-1)^(1/4)*Sqrt[c]*x)])/3 + ((-1)^(1/4)*b^2
*c^(3/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[(Sqrt[2]*((-1)^(1/4) + Sqrt[c]*x)
)/(1 + (-1)^(1/4)*Sqrt[c]*x)])/3 + (2*(-1)^(1/4)*b^2*c^(3/2)*ArcTanh[(-1)^(
3/4)*Sqrt[c]*x]*Log[2/(1 - (-1)^(3/4)*Sqrt[c]*x)])/3 - (2*(-1)^(1/4)*b^2*c^
(3/2)*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[2/(1 + (-1)^(3/4)*Sqrt[c]*x)])/3 +
((-1)^(1/4)*b^2*c^(3/2)*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[-((Sqrt[2]*((-1)^
(3/4) + Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x))])/3 + ((-1)^(1/4)*b^2*c^(3/
2)*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[((1 + I)*(1 + (-1)^(1/4)*Sqrt[c]*x))/(
1 + (-1)^(3/4)*Sqrt[c]*x)])/3 + ((-1)^(1/4)*b^2*c^(3/2)*ArcTan[(-1)^(3/4)*S
qrt[c]*x]*Log[((1 - I)*(1 + (-1)^(3/4)*Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*
x)])/3 - ((I/3)*b^2*c*Log[1 - I*c*x^2])/x - ((-1)^(1/4)*b^2*c^(3/2)*ArcTanh
[(-1)^(3/4)*Sqrt[c]*x]*Log[1 - I*c*x^2])/3 - (b*c*(2*a + I*b*Log[1 - I*c*x^
2]))/(3*x) - ((-1)^(3/4)*b*c^(3/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*(2*a + I*b*
```

```
Log[1 - I*c*x^2]))/3 - (2*a + I*b*Log[1 - I*c*x^2])^2/(12*x^3) + ((I/3)*a*b
*Log[1 + I*c*x^2])/x^3 + (((2*I)/3)*b^2*c*Log[1 + I*c*x^2])/x - ((-1)^(1/4)
*b^2*c^(3/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[1 + I*c*x^2])/3 + ((-1)^(1/4)
*b^2*c^(3/2)*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[1 + I*c*x^2])/3 - (b^2*Log[1
 - I*c*x^2]*Log[1 + I*c*x^2])/(6*x^3) + (b^2*Log[1 + I*c*x^2]^2)/(12*x^3) +
 ((-1)^(3/4)*b^2*c^(3/2)*PolyLog[2, 1 - 2/(1 - (-1)^(1/4)*Sqrt[c]*x)])/3 +
((-1)^(3/4)*b^2*c^(3/2)*PolyLog[2, 1 - 2/(1 + (-1)^(1/4)*Sqrt[c]*x)])/3 - (
(-1)^(3/4)*b^2*c^(3/2)*PolyLog[2, 1 - (Sqrt[2]*((-1)^(1/4) + Sqrt[c]*x))/(1
 + (-1)^(1/4)*Sqrt[c]*x)])/6 + ((-1)^(1/4)*b^2*c^(3/2)*PolyLog[2, 1 - 2/(1
- (-1)^(3/4)*Sqrt[c]*x)])/3 + ((-1)^(1/4)*b^2*c^(3/2)*PolyLog[2, 1 - 2/(1 +
 (-1)^(3/4)*Sqrt[c]*x)])/3 - ((-1)^(1/4)*b^2*c^(3/2)*PolyLog[2, 1 + (Sqrt[2
]*((-1)^(3/4) + Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x)])/6 - ((-1)^(1/4)*b^
2*c^(3/2)*PolyLog[2, 1 - ((1 + I)*(1 + (-1)^(1/4)*Sqrt[c]*x))/(1 + (-1)^(3/
4)*Sqrt[c]*x)])/6 - ((-1)^(3/4)*b^2*c^(3/2)*PolyLog[2, 1 - ((1 - I)*(1 + (-
1)^(3/4)*Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*x)])/6
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2507

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Dist[b*e*n*p*(q/(f^n*(m + 1))), Int[(f*x)^(m + n)*((a +
 b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
```

```
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2637

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[z*Log[w]*(D[v, x]/v), x], x] - In
t[SimplifyIntegrand[z*Log[v]*(D[w, x]/w), x], x]) /; InverseFunctionFreeQ[z
, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> I
nt[ExpandIntegrand[x^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*Log[1 + I*c*x^n
])/2)^p, x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1] && IGtQ[n, 0] && Integ
erQ[m]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5048

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6057

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcTanh[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x
)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcTanh[c*x])
*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x]) /; FreeQ[{a, b, c, d,
e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6139

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = \int \left(\frac{(2a+ib\log(1-icx^2))^2}{4x^4}+\frac{b(-2ia+b\log(1-icx^2))\log(1+icx^2)}{2x^4}-\frac{b^2\log^2(1+icx^2)}{4x^4}\right)dx$$

$$= \frac{1}{4}\int\frac{(2a+ib\log(1-icx^2))^2}{x^4}\,dx+\frac{1}{2}b\int\frac{(-2ia+b\log(1-icx^2))\log(1+icx^2)}{x^4}\,dx-\frac{1}{4}b^2\int\frac{\log^2(1+icx^2)}{x^4}\,dx$$

$$= -\frac{(2a+ib\log(1-icx^2))^2}{12x^3}+\frac{b^2\log^2(1+icx^2)}{12x^3}+\frac{1}{2}b\int\left(-\frac{2ia\log(1+icx^2)}{x^4}+\frac{b\log(1-icx^2)\log(1+icx^2)}{x^4}\right)dx+\frac{1}{3}(bc)\int\frac{2a+ib\log(1-icx^2)}{x^2(1-icx^2)}\,dx-\frac{1}{3}(ib^2c)\int\frac{\log(1+icx^2)}{x^2(1+icx^2)}\,dx$$

$$= -\frac{(2a+ib\log(1-icx^2))^2}{12x^3}+\frac{b^2\log^2(1+icx^2)}{12x^3}-(iab)\int\frac{\log(1+icx^2)}{x^4}\,dx+\frac{1}{2}b^2\int\frac{\log(1-icx^2)\log(1+icx^2)}{x^4}\,dx+\frac{1}{3}(bc)\int\left(\frac{2a+ib\log(1-icx^2)}{x^2}-\frac{c(2a+ib\log(1-icx^2))}{i+cx^2}\right)dx-\frac{1}{3}(ib^2c)\int\left(\frac{\log(1+icx^2)}{x^2}-\frac{c\log(1+icx^2)}{-i+cx^2}\right)dx$$

$$= -\frac{(2a+ib\log(1-icx^2))^2}{12x^3}+\frac{iab\log(1+icx^2)}{3x^3}-\frac{b^2\log(1-icx^2)\log(1+icx^2)}{6x^3}+\frac{b^2\log^2(1+icx^2)}{12x^3}-\frac{1}{2}b^2\int\frac{2c\log(1-icx^2)}{x^2(3i-3cx^2)}\,dx-\frac{1}{2}b^2\int\frac{2c\log(1+icx^2)}{x^2(-3i-3cx^2)}\,dx+\frac{1}{3}(bc)\int\frac{2a+ib\log(1-icx^2)}{x^2}\,dx+\frac{1}{3}(2abc)\int\frac{1}{x^2(1+icx^2)}\,dx-\frac{1}{3}(ib^2c)\int\frac{\log(1+icx^2)}{x^2}\,dx-\frac{1}{3}(bc^2)\int\frac{2a+ib\log(1-icx^2)}{i+cx^2}\,dx+\frac{1}{3}(ib^2c^2)\int\frac{\log(1+icx^2)}{-i+cx^2}\,dx$$

$$= -\frac{2abc}{3x}-\frac{bc(2a+ib\log(1-icx^2))}{3x}-\frac{1}{3}(-1)^{3/4}bc^{3/2}\arctan\left((-1)^{3/4}\sqrt{c}x\right)\left(2a+ib\log(1-icx^2)\right)-\frac{(2a+ib\log(1-icx^2))^2}{12x^3}+\frac{iab\log(1 \ldots}{3x \ldots}$$

$$= -\frac{2abc}{3x} - \frac{2}{3}\sqrt[4]{-1}b^2c^{3/2}\arctan\left((-1)^{3/4}\sqrt{c}x\right) + \frac{2}{3}(-1)^{3/4}abc^{3/2}\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right) - \frac{2}{3}\sqrt[4]{-1}b^2c^{3/2}\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right) - \frac{bc(2a + ib\log(1 - ic}{3x}$$

$$= -\frac{2abc}{3x} - \frac{2}{3}\sqrt[4]{-1}b^2c^{3/2}\arctan\left((-1)^{3/4}\sqrt{c}x\right) + \frac{1}{3}(-1)^{3/4}b^2c^{3/2}\arctan\left((-1)^{3/4}\sqrt{c}x\right)^2 + \frac{2}{3}(-1)^{3/4}abc^{3/2}\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right) - \frac{2}{3}\sqrt[4]{-1}b^2c^{3/2}\text{arctar}$$

$$= -\frac{2abc}{3x} - \frac{2}{3}\sqrt[4]{-1}b^2c^{3/2}\arctan\left((-1)^{3/4}\sqrt{c}x\right) + \frac{1}{3}(-1)^{3/4}b^2c^{3/2}\arctan\left((-1)^{3/4}\sqrt{c}x\right)^2 + \frac{2}{3}(-1)^{3/4}abc^{3/2}\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right) - \frac{2}{3}\sqrt[4]{-1}b^2c^{3/2}\text{arctar}$$

$$= -\frac{2abc}{3x} - \frac{4}{3}\sqrt[4]{-1}b^2c^{3/2}\arctan\left((-1)^{3/4}\sqrt{c}x\right) + \frac{1}{3}(-1)^{3/4}b^2c^{3/2}\arctan\left((-1)^{3/4}\sqrt{c}x\right)^2 + \frac{2}{3}(-1)^{3/4}abc^{3/2}\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right) - \frac{4}{3}\sqrt[4]{-1}b^2c^{3/2}\text{arctar}$$

$$= -\frac{2abc}{3x} - \frac{4}{3}\sqrt[4]{-1}b^2c^{3/2}\arctan\left((-1)^{3/4}\sqrt{c}x\right) + \frac{1}{3}(-1)^{3/4}b^2c^{3/2}\arctan\left((-1)^{3/4}\sqrt{c}x\right)^2 + \frac{2}{3}(-1)^{3/4}abc^{3/2}\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right) - \frac{4}{3}\sqrt[4]{-1}b^2c^{3/2}\text{arctar}$$

$$= -\frac{2abc}{3x} - \frac{4}{3}\sqrt[4]{-1}b^2c^{3/2}\arctan\left((-1)^{3/4}\sqrt{c}x\right) + \frac{1}{3}(-1)^{3/4}b^2c^{3/2}\arctan\left((-1)^{3/4}\sqrt{c}x\right)^2 + \frac{2}{3}(-1)^{3/4}abc^{3/2}\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right) - \frac{4}{3}\sqrt[4]{-1}b^2c^{3/2}\text{arctar}$$

$= \text{Too large to display}$

Mathematica [F]

$$\int \frac{(a + b\arctan(cx^2))^2}{x^4}\,dx = \int \frac{(a + b\arctan(cx^2))^2}{x^4}\,dx$$

```
[In] Integrate[(a + b*ArcTan[c*x^2])^2/x^4,x]
[Out] Integrate[(a + b*ArcTan[c*x^2])^2/x^4, x]
```

Maple [F]

$$\int \frac{(a + b \arctan(c x^2))^2}{x^4} dx$$

```
[In] int((a+b*arctan(c*x^2))^2/x^4,x)
[Out] int((a+b*arctan(c*x^2))^2/x^4,x)
```

Fricas [F]

$$\int \frac{(a + b \arctan(cx^2))^2}{x^4} dx = \int \frac{(b \arctan(cx^2) + a)^2}{x^4} dx$$

```
[In] integrate((a+b*arctan(c*x^2))^2/x^4,x, algorithm="fricas")
[Out] integral((b^2*arctan(c*x^2)^2 + 2*a*b*arctan(c*x^2) + a^2)/x^4, x)
```

Sympy [F]

$$\int \frac{(a + b \arctan(cx^2))^2}{x^4} dx = \int \frac{(a + b \operatorname{atan}(cx^2))^2}{x^4} dx$$

```
[In] integrate((a+b*atan(c*x**2))**2/x**4,x)
[Out] Integral((a + b*atan(c*x**2))**2/x**4, x)
```

Maxima [F]

$$\int \frac{(a + b \arctan(cx^2))^2}{x^4} dx = \int \frac{(b \arctan(cx^2) + a)^2}{x^4} dx$$

```
[In] integrate((a+b*arctan(c*x^2))^2/x^4,x, algorithm="maxima")
[Out] -1/6*((c^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*c*x + sqrt(2)*sqrt(c))/sqrt(c))
/c^(3/2) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*c*x - sqrt(2)*sqrt(c))/sqrt(c))/
c^(3/2) - sqrt(2)*log(c*x^2 + sqrt(2)*sqrt(c)*x + 1)/c^(3/2) + sqrt(2)*log(
c*x^2 - sqrt(2)*sqrt(c)*x + 1)/c^(3/2)) + 8/x)*c + 4*arctan(c*x^2)/x^3)*a*b
 + 1/48*(48*x^3*integrate(-1/48*(8*c^2*x^4*log(c^2*x^4 + 1) - 16*c*x^2*arct
an(c*x^2) - 36*(c^2*x^4 + 1)*arctan(c*x^2)^2 - 3*(c^2*x^4 + 1)*log(c^2*x^4
+ 1)^2)/(c^2*x^8 + x^4), x) - 4*arctan(c*x^2)^2 + log(c^2*x^4 + 1)^2)*b^2/x
^3 - 1/3*a^2/x^3
```

Giac [F]

$$\int \frac{(a + b \arctan(cx^2))^2}{x^4} dx = \int \frac{(b \arctan(cx^2) + a)^2}{x^4} dx$$

```
[In] integrate((a+b*arctan(c*x^2))^2/x^4,x, algorithm="giac")
[Out] integrate((b*arctan(c*x^2) + a)^2/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b \arctan(cx^2))^2}{x^4} dx = \int \frac{(a + b \operatorname{atan}(c x^2))^2}{x^4} dx$$

```
[In] int((a + b*atan(c*x^2))^2/x^4,x)
[Out] int((a + b*atan(c*x^2))^2/x^4, x)
```

3.85 $\int \frac{\left(a+b\arctan\left(cx^2\right)\right)^2}{x^6}\,dx$

Optimal result 503
Rubi [A] (verified) 504
Mathematica [F] 511
Maple [F] 511
Fricas [F] 511
Sympy [F] 511
Maxima [F] 512
Giac [F] 512
Mupad [F(-1)] 512

Optimal result

Integrand size = 16, antiderivative size = 1444

$$\int \frac{\left(a+b\arctan\left(cx^2\right)\right)^2}{x^6}\,dx = -\frac{2abc}{15x^3}+\frac{2iabc^2}{5x}-\frac{8b^2c^2}{15x}$$
$$-\frac{4}{15}(-1)^{3/4}b^2c^{5/2}\arctan\left((-1)^{3/4}\sqrt{c}x\right)-\frac{1}{5}\sqrt[4]{-1}b^2c^{5/2}\arctan\left((-1)^{3/4}\sqrt{c}x\right)^2+\frac{2}{5}\sqrt[4]{-1}abc^{5/2}\operatorname{arctanh}((-1)$$

```
[Out] -8/15*b^2*c^2/x-1/20*(2*a+I*b*ln(1-I*c*x^2))^2/x^5-2/15*a*b*c/x^3-4/15*(-1)
^(3/4)*b^2*c^(5/2)*arctan((-1)^(3/4)*x*c^(1/2))-1/5*(-1)^(1/4)*b^2*c^(5/2)*
arctan((-1)^(3/4)*x*c^(1/2))^2+4/15*(-1)^(3/4)*b^2*c^(5/2)*arctanh((-1)^(3/
4)*x*c^(1/2))+1/5*(-1)^(3/4)*b^2*c^(5/2)*arctanh((-1)^(3/4)*x*c^(1/2))^2-1/
5*b^2*c^2*ln(1-I*c*x^2)/x-1/15*b*c*(2*a+I*b*ln(1-I*c*x^2))/x^3-1/10*b^2*ln(
1-I*c*x^2)*ln(1+I*c*x^2)/x^5-1/5*(-1)^(1/4)*b^2*c^(5/2)*polylog(2,1-2/(1-(-
1)^(1/4)*x*c^(1/2)))-1/5*(-1)^(1/4)*b^2*c^(5/2)*polylog(2,1-2/(1+(-1)^(1/4)
*x*c^(1/2)))+1/10*(-1)^(1/4)*b^2*c^(5/2)*polylog(2,1-2^(1/2)*((-1)^(1/4)+x*
c^(1/2))/(1+(-1)^(1/4)*x*c^(1/2)))-1/5*(-1)^(3/4)*b^2*c^(5/2)*polylog(2,1-2
/(1-(-1)^(3/4)*x*c^(1/2)))-1/5*(-1)^(3/4)*b^2*c^(5/2)*polylog(2,1-2/(1+(-1)
^(3/4)*x*c^(1/2)))+1/10*(-1)^(3/4)*b^2*c^(5/2)*polylog(2,1+2^(1/2)*((-1)^(3
/4)+x*c^(1/2))/(1+(-1)^(3/4)*x*c^(1/2)))+1/10*(-1)^(3/4)*b^2*c^(5/2)*polylo
g(2,1-(1+I)*(1+(-1)^(1/4)*x*c^(1/2))/(1+(-1)^(3/4)*x*c^(1/2)))+1/10*(-1)^(1
/4)*b^2*c^(5/2)*polylog(2,1+(-1+I)*(1+(-1)^(3/4)*x*c^(1/2))/(1+(-1)^(1/4)*x
*c^(1/2)))-1/5*(-1)^(3/4)*b^2*c^(5/2)*arctanh((-1)^(3/4)*x*c^(1/2))*ln(-2^(
1/2)*((-1)^(3/4)+x*c^(1/2))/(1+(-1)^(3/4)*x*c^(1/2)))-1/5*(-1)^(3/4)*b^2*c^
(5/2)*arctanh((-1)^(3/4)*x*c^(1/2))*ln((1+I)*(1+(-1)^(1/4)*x*c^(1/2))/(1+(-
1)^(3/4)*x*c^(1/2)))+1/5*(-1)^(3/4)*b^2*c^(5/2)*arctan((-1)^(3/4)*x*c^(1/2)
)*ln((1-I)*(1+(-1)^(3/4)*x*c^(1/2))/(1+(-1)^(1/4)*x*c^(1/2)))+1/5*(-1)^(3/4
)*b^2*c^(5/2)*arctanh((-1)^(3/4)*x*c^(1/2))*ln(1-I*c*x^2)+1/5*(-1)^(1/4)*b*
c^(5/2)*arctan((-1)^(3/4)*x*c^(1/2))*(2*a+I*b*ln(1-I*c*x^2))-1/5*(-1)^(3/4)
```

```
*b^2*c^(5/2)*arctan((-1)^(3/4)*x*c^(1/2))*ln(1+I*c*x^2)-1/5*(-1)^(3/4)*b^2*
c^(5/2)*arctanh((-1)^(3/4)*x*c^(1/2))*ln(1+I*c*x^2)+2/5*(-1)^(3/4)*b^2*c^(5
/2)*arctan((-1)^(3/4)*x*c^(1/2))*ln(2/(1-(-1)^(1/4)*x*c^(1/2)))-2/5*(-1)^(3
/4)*b^2*c^(5/2)*arctan((-1)^(3/4)*x*c^(1/2))*ln(2/(1+(-1)^(1/4)*x*c^(1/2)))
+1/5*(-1)^(3/4)*b^2*c^(5/2)*arctan((-1)^(3/4)*x*c^(1/2))*ln(2^(1/2)*((-1)^(
1/4)+x*c^(1/2))/(1+(-1)^(1/4)*x*c^(1/2)))-2/5*(-1)^(3/4)*b^2*c^(5/2)*arctan
h((-1)^(3/4)*x*c^(1/2))*ln(2/(1-(-1)^(3/4)*x*c^(1/2)))+2/5*(-1)^(3/4)*b^2*c
^(5/2)*arctanh((-1)^(3/4)*x*c^(1/2))*ln(2/(1+(-1)^(3/4)*x*c^(1/2)))+2/5*(-1
)^(1/4)*a*b*c^(5/2)*arctanh((-1)^(3/4)*x*c^(1/2))-1/15*I*b^2*c*ln(1-I*c*x^2
)/x^3-1/5*I*b*c^2*(2*a+I*b*ln(1-I*c*x^2))/x+1/20*b^2*ln(1+I*c*x^2)^2/x^5+1/
5*I*a*b*ln(1+I*c*x^2)/x^5+2/15*I*b^2*c*ln(1+I*c*x^2)/x^3+2/5*I*a*b*c^2/x
```

Rubi [A] (verified)

Time = 1.68 (sec) , antiderivative size = 1444, normalized size of antiderivative = 1.00, number of steps used = 77, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 1.562$, Rules used = {4950, 2507, 2526, 2505, 331, 209, 211, 2520, 12, 5040, 4964, 2449, 2352, 6874, 212, 30, 2637, 5048, 4966, 2497, 214, 6139, 6057, 6131, 6055}

$$\int \frac{(a+b\arctan(cx^2))^2}{x^6}\,dx = -\frac{1}{5}\sqrt[4]{-1}b^2\arctan\left((-1)^{3/4}\sqrt{c}x\right)^2c^{5/2}$$
$$+\frac{1}{5}(-1)^{3/4}b^2\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)^2c^{5/2}-\frac{4}{15}(-1)^{3/4}b^2\arctan\left((-1)^{3/4}\sqrt{c}x\right)c^{5/2}+\frac{4}{15}(-1)^{3/4}b^2\text{arctanh}\left((-\right.$$

```
[In] Int[(a + b*ArcTan[c*x^2])^2/x^6,x]
```

```
[Out] (-2*a*b*c)/(15*x^3) + (((2*I)/5)*a*b*c^2)/x - (8*b^2*c^2)/(15*x) - (4*(-1)^
(3/4)*b^2*c^(5/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x])/15 - ((-1)^(1/4)*b^2*c^(5/2
)*ArcTan[(-1)^(3/4)*Sqrt[c]*x]^2)/5 + (2*(-1)^(1/4)*a*b*c^(5/2)*ArcTanh[(-1
)^(3/4)*Sqrt[c]*x])/5 + (4*(-1)^(3/4)*b^2*c^(5/2)*ArcTanh[(-1)^(3/4)*Sqrt[c
]*x])/15 + ((-1)^(3/4)*b^2*c^(5/2)*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]^2)/5 + (2*
(-1)^(3/4)*b^2*c^(5/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[2/(1 - (-1)^(1/4)*S
qrt[c]*x)])/5 - (2*(-1)^(3/4)*b^2*c^(5/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[
2/(1 + (-1)^(1/4)*Sqrt[c]*x)])/5 + ((-1)^(3/4)*b^2*c^(5/2)*ArcTan[(-1)^(3/4
)*Sqrt[c]*x]*Log[(Sqrt[2]*((-1)^(1/4) + Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]
*x)])/5 - (2*(-1)^(3/4)*b^2*c^(5/2)*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[2/(1
- (-1)^(3/4)*Sqrt[c]*x)])/5 + (2*(-1)^(3/4)*b^2*c^(5/2)*ArcTanh[(-1)^(3/4)*
Sqrt[c]*x]*Log[2/(1 + (-1)^(3/4)*Sqrt[c]*x)])/5 - ((-1)^(3/4)*b^2*c^(5/2)*A
rcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[-((Sqrt[2]*((-1)^(3/4) + Sqrt[c]*x))/(1 +
(-1)^(3/4)*Sqrt[c]*x))])/5 - ((-1)^(3/4)*b^2*c^(5/2)*ArcTanh[(-1)^(3/4)*Sqr
t[c]*x]*Log[((1 + I)*(1 + (-1)^(1/4)*Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x)
])/5 + ((-1)^(3/4)*b^2*c^(5/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[((1 - I)*(1
 + (-1)^(3/4)*Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*x)])/5 - ((I/15)*b^2*c*Lo
g[1 - I*c*x^2])/x^3 - (b^2*c^2*Log[1 - I*c*x^2])/(5*x) + ((-1)^(3/4)*b^2*c^
```

```
(5/2)*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[1 - I*c*x^2])/5 - (b*c*(2*a + I*b*L
og[1 - I*c*x^2]))/(15*x^3) - ((I/5)*b*c^2*(2*a + I*b*Log[1 - I*c*x^2]))/x +
 ((-1)^(1/4)*b*c^(5/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*(2*a + I*b*Log[1 - I*c*
x^2]))/5 - (2*a + I*b*Log[1 - I*c*x^2])^2/(20*x^5) + ((I/5)*a*b*Log[1 + I*c
*x^2])/x^5 + (((2*I)/15)*b^2*c*Log[1 + I*c*x^2])/x^3 - ((-1)^(3/4)*b^2*c^(5
/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[1 + I*c*x^2])/5 - ((-1)^(3/4)*b^2*c^(5
/2)*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[1 + I*c*x^2])/5 - (b^2*Log[1 - I*c*x^
2]*Log[1 + I*c*x^2])/(10*x^5) + (b^2*Log[1 + I*c*x^2]^2)/(20*x^5) - ((-1)^(
1/4)*b^2*c^(5/2)*PolyLog[2, 1 - 2/(1 - (-1)^(1/4)*Sqrt[c]*x)])/5 - ((-1)^(1
/4)*b^2*c^(5/2)*PolyLog[2, 1 - 2/(1 + (-1)^(1/4)*Sqrt[c]*x)])/5 + ((-1)^(1/
4)*b^2*c^(5/2)*PolyLog[2, 1 - (Sqrt[2]*((-1)^(1/4) + Sqrt[c]*x))/(1 + (-1)^
(1/4)*Sqrt[c]*x)])/10 - ((-1)^(3/4)*b^2*c^(5/2)*PolyLog[2, 1 - 2/(1 - (-1)^
(3/4)*Sqrt[c]*x)])/5 - ((-1)^(3/4)*b^2*c^(5/2)*PolyLog[2, 1 - 2/(1 + (-1)^(
3/4)*Sqrt[c]*x)])/5 + ((-1)^(3/4)*b^2*c^(5/2)*PolyLog[2, 1 + (Sqrt[2]*((-1)
^(3/4) + Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x)])/10 + ((-1)^(3/4)*b^2*c^(5
/2)*PolyLog[2, 1 - ((1 + I)*(1 + (-1)^(1/4)*Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqr
t[c]*x)])/10 + ((-1)^(1/4)*b^2*c^(5/2)*PolyLog[2, 1 - ((1 - I)*(1 + (-1)^(3
/4)*Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*x)])/10
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2507

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Dist[b*e*n*p*(q/(f^n*(m + 1))), Int[(f*x)^(m + n)*((a +
 b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2637

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[z*Log[w]*(D[v, x]/v), x], x] - In
t[SimplifyIntegrand[z*Log[v]*(D[w, x]/w), x], x]) /; InverseFunctionFreeQ[z
, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> I
nt[ExpandIntegrand[x^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*Log[1 + I*c*x^n
])/2)^p, x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1] && IGtQ[n, 0] && Integ
erQ[m]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
```

```
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5048

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6057

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcTanh[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x
)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcTanh[c*x])
*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x]) /; FreeQ[{a, b, c, d,
e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6139

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\text{integral} = \int \left(\frac{(2a + ib\log(1 - icx^2))^2}{4x^6} + \frac{b(-2ia + b\log(1 - icx^2))\log(1 + icx^2)}{2x^6} - \frac{b^2\log^2(1 + icx^2)}{4x^6} \right) dx$$

$$= \frac{1}{4}\int \frac{(2a + ib\log(1 - icx^2))^2}{x^6}\,dx + \frac{1}{2}b\int \frac{(-2ia + b\log(1 - icx^2))\log(1 + icx^2)}{x^6}\,dx - \frac{1}{4}b^2\int \frac{\log^2(1 + icx^2)}{x^6}\,dx$$

$$= -\frac{(2a + ib\log(1 - icx^2))^2}{20x^5} + \frac{b^2\log^2(1 + icx^2)}{20x^5} + \frac{1}{2}b\int \left(-\frac{2ia\log(1 + icx^2)}{x^6} + \frac{b\log(1 - icx^2)\log(1 + icx^2)}{x^6} \right) dx + \frac{1}{5}(bc)\int \frac{2a + ib\log(1 - icx^2)}{x^4(1 - icx^2)}\,dx - \frac{1}{5}(ib^2c)\int \frac{\log(1 + icx^2)}{x^4(1 + icx^2)}\,dx$$

$$= -\frac{(2a + ib\log(1 - icx^2))^2}{20x^5} + \frac{b^2\log^2(1 + icx^2)}{20x^5} - (iab)\int \frac{\log(1 + icx^2)}{x^6}\,dx + \frac{1}{2}b^2\int \frac{\log(1 - icx^2)\log(1 + icx^2)}{x^6}\,dx + \frac{1}{5}(bc)\int \left(\frac{2a + ib\log(1 - icx^2)}{x^4} + \frac{ic(2a + ib\log(1 - icx^2))}{x^2} - \frac{ic^2(2a + ib\log(1 - icx^2))}{i + cx^2} \right) dx - \frac{1}{5}(ib^2c)\int \left(\frac{\log(1 + icx^2)}{x^4} - \frac{ic\log(1 + icx^2)}{x^2} + \frac{ic^2\log(1 + icx^2)}{-i + cx^2} \right) dx$$

$$= -\frac{(2a + ib\log(1 - icx^2))^2}{20x^5} + \frac{iab\log(1 + icx^2)}{5x^5} - \frac{b^2\log(1 - icx^2)\log(1 + icx^2)}{10x^5} + \frac{b^2\log^2(1 + icx^2)}{20x^5} - \frac{1}{2}b^2\int \frac{2c\log(1 - icx^2)}{5x^4(i - cx^2)}\,dx - \frac{1}{2}b^2\int \frac{2c\log(1 + icx^2)}{5x^4(-i - cx^2)}\,dx + \frac{1}{5}(bc)\int \frac{2a + ib\log(1 - icx^2)}{x^4}\,dx + \frac{1}{5}(2abc)\int \frac{1}{x^4(1 + icx^2)}\,dx - \frac{1}{5}(ib^2c)\int \frac{\log(1 + icx^2)}{x^4}\,dx + \frac{1}{5}(ibc^2)\int \frac{2a + ib\log(1 - icx^2)}{x^2}\,dx - \frac{1}{5}(b^2c^2)\int \frac{\log(1 + icx^2)}{x^2}\,dx - \frac{1}{5}(ibc^3)\int \frac{2a + ib\log(1 - icx^2)}{i + cx^2}\,dx + \frac{1}{5}(b^2c^3)\int \frac{\log(1 + icx^2)}{-i + cx^2}\,dx$$

$$= -\frac{2abc}{15x^3} - \frac{bc(2a + ib\log(1 - icx^2))}{15x^3} - \frac{ibc^2(2a + ib\log(1 - icx^2))}{5x} + \frac{1}{5}\sqrt[4]{-1}bc^{5/2}\arctan\left((-1)^{3/4}\sqrt{c}x\right)\left(2a + ib\log(1 - icx^2)\right) - \frac{(2a + ib\log(1 - icx^2))^2}{20x^5} + \frac{iab\log(1 + icx^2)}{5x^5} + \frac{ib^2c\log(1 + icx^2)}{15x^3} + \frac{b^2c^2\log(1 + icx^2)}{5x} - \frac{1}{5}(-1)^{3/4}b^2c^{5/2}\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right)\log\left(1 + icx^2\right) - \frac{b^2\log(1 - icx^2)\log(1 + icx^2)}{10x^5} + \frac{b^2\log^2(1 +}{20x^5}$$

$$= -\frac{2abc}{15x^3} + \frac{2iabc^2}{5x} - \frac{4b^2c^2}{15x} - \frac{2}{5}(-1)^{3/4}b^2c^{5/2}\arctan\left((-1)^{3/4}\sqrt{c}x\right) + \frac{2}{5}(-1)^{3/4}b^2c^{5/2}\text{arctanh}\left((-1)^{3/4}\sqrt{c}x\right) - \frac{bc(2a + ib\log(1 -}{15x^3}$$

$$= -\frac{2abc}{15x^3} + \frac{2iabc^2}{5x} - \frac{4b^2c^2}{15x} - \frac{8}{15}(-1)^{3/4}b^2c^{5/2}\arctan\left((-1)^{3/4}\sqrt{c}x\right) - \frac{1}{5}\sqrt[4]{-1}b^2c^{5/2}\arctan\left((-1)^{3/4}\sqrt{c}x\right)^2 + \frac{2}{5}\sqrt[4]{-1}abc^{5/2}\text{arctanh}$$

$$= -\frac{2abc}{15x^3} + \frac{2iabc^2}{5x} - \frac{4b^2c^2}{15x} - \frac{8}{15}(-1)^{3/4}b^2c^{5/2}\arctan\left((-1)^{3/4}\sqrt{c}x\right) - \frac{1}{5}\sqrt[4]{-1}b^2c^{5/2}\arctan\left((-1)^{3/4}\sqrt{c}x\right)^2 + \frac{2}{5}\sqrt[4]{-1}abc^{5/2}\text{arctanh}$$

$$= -\frac{2abc}{15x^3} + \frac{2iabc^2}{5x} - \frac{8b^2c^2}{15x} - \frac{2}{15}(-1)^{3/4}b^2c^{5/2}\arctan\left((-1)^{3/4}\sqrt{c}x\right) - \frac{1}{5}\sqrt[4]{-1}b^2c^{5/2}\arctan\left((-1)^{3/4}\sqrt{c}x\right)^2 + \frac{2}{5}\sqrt[4]{-1}abc^{5/2}\text{arctanh}$$

$$= -\frac{2abc}{15x^3} + \frac{2iabc^2}{5x} - \frac{8b^2c^2}{15x} - \frac{4}{15}(-1)^{3/4}b^2c^{5/2}\arctan\left((-1)^{3/4}\sqrt{c}x\right) - \frac{1}{5}\sqrt[4]{-1}b^2c^{5/2}\arctan\left((-1)^{3/4}\sqrt{c}x\right)^2 + \frac{2}{5}\sqrt[4]{-1}abc^{5/2}\text{arctanh}$$

$$= -\frac{2abc}{15x^3} + \frac{2iabc^2}{5x} - \frac{8b^2c^2}{15x} - \frac{4}{15}(-1)^{3/4}b^2c^{5/2}\arctan\left((-1)^{3/4}\sqrt{c}x\right) - \frac{1}{5}\sqrt[4]{-1}b^2c^{5/2}\arctan\left((-1)^{3/4}\sqrt{c}x\right)^2 + \frac{2}{5}\sqrt[4]{-1}abc^{5/2}\text{arctanh}$$

$$= \text{Too large to display}$$

Mathematica [F]

$$\int \frac{(a+b\arctan(cx^2))^2}{x^6}\,dx = \int \frac{(a+b\arctan(cx^2))^2}{x^6}\,dx$$

```
[In] Integrate[(a + b*ArcTan[c*x^2])^2/x^6,x]
[Out] Integrate[(a + b*ArcTan[c*x^2])^2/x^6, x]
```

Maple [F]

$$\int \frac{(a+b\arctan(c\,x^2))^2}{x^6}dx$$

```
[In] int((a+b*arctan(c*x^2))^2/x^6,x)
[Out] int((a+b*arctan(c*x^2))^2/x^6,x)
```

Fricas [F]

$$\int \frac{(a+b\arctan(cx^2))^2}{x^6}\,dx = \int \frac{(b\arctan(cx^2)+a)^2}{x^6}\,dx$$

```
[In] integrate((a+b*arctan(c*x^2))^2/x^6,x, algorithm="fricas")
[Out] integral((b^2*arctan(c*x^2)^2 + 2*a*b*arctan(c*x^2) + a^2)/x^6, x)
```

Sympy [F]

$$\int \frac{(a+b\arctan(cx^2))^2}{x^6}\,dx = \int \frac{(a+b\operatorname{atan}(cx^2))^2}{x^6}\,dx$$

```
[In] integrate((a+b*atan(c*x**2))**2/x**6,x)
[Out] Integral((a + b*atan(c*x**2))**2/x**6, x)
```

Maxima [F]

$$\int \frac{(a + b \arctan(cx^2))^2}{x^6} \, dx = \int \frac{(b \arctan(cx^2) + a)^2}{x^6} \, dx$$

```
[In] integrate((a+b*arctan(c*x^2))^2/x^6,x, algorithm="maxima")
```

```
[Out] -1/30*((6*sqrt(2)*c^(3/2)*arctan(1/2*sqrt(2)*(2*c*x + sqrt(2)*sqrt(c))/sqrt
(c)) + 6*sqrt(2)*c^(3/2)*arctan(1/2*sqrt(2)*(2*c*x - sqrt(2)*sqrt(c))/sqrt(
c)) + 3*sqrt(2)*c^(3/2)*log(c*x^2 + sqrt(2)*sqrt(c)*x + 1) - 3*sqrt(2)*c^(3
/2)*log(c*x^2 - sqrt(2)*sqrt(c)*x + 1) + 8/x^3)*c + 12*arctan(c*x^2)/x^5)*a
*b + 1/80*(80*x^5*integrate(-1/80*(8*c^2*x^4*log(c^2*x^4 + 1) - 16*c*x^2*ar
ctan(c*x^2) - 60*(c^2*x^4 + 1)*arctan(c*x^2)^2 - 5*(c^2*x^4 + 1)*log(c^2*x^
4 + 1)^2)/(c^2*x^10 + x^6), x) - 4*arctan(c*x^2)^2 + log(c^2*x^4 + 1)^2)*b^
2/x^5 - 1/5*a^2/x^5
```

Giac [F]

$$\int \frac{(a + b \arctan(cx^2))^2}{x^6} \, dx = \int \frac{(b \arctan(cx^2) + a)^2}{x^6} \, dx$$

```
[In] integrate((a+b*arctan(c*x^2))^2/x^6,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^2) + a)^2/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b \arctan(cx^2))^2}{x^6} \, dx = \int \frac{(a + b \operatorname{atan}(c x^2))^2}{x^6} \, dx$$

```
[In] int((a + b*atan(c*x^2))^2/x^6,x)
```

```
[Out] int((a + b*atan(c*x^2))^2/x^6, x)
```

3.86 $\int x^3(a+b\arctan(cx^2))^3\,dx$

Optimal result 513
Rubi [A] (verified) 513
Mathematica [A] (verified) 516
Maple [C] (warning: unable to verify) 517
Fricas [F] 518
Sympy [F] 518
Maxima [F] 518
Giac [F] 519
Mupad [F(-1)] 519

Optimal result

Integrand size = 16, antiderivative size = 149

$$\int x^3(a+b\arctan(cx^2))^3\,dx=-\frac{3ib(a+b\arctan(cx^2))^2}{4c^2}-\frac{3bx^2(a+b\arctan(cx^2))^2}{4c}+\frac{(a+b\arctan(cx^2))^3}{4c^2}+\frac{1}{4}x^4(a+b\arctan(cx^2))^3-\frac{3b^2(a+b\arctan(cx^2))\log\left(\frac{2}{1+icx^2}\right)}{2c^2}-\frac{3ib^3\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx^2}\right)}{4c^2}$$

```
[Out] -3/4*I*b*(a+b*arctan(c*x^2))^2/c^2-3/4*b*x^2*(a+b*arctan(c*x^2))^2/c+1/4*(a
+b*arctan(c*x^2))^3/c^2+1/4*x^4*(a+b*arctan(c*x^2))^3-3/2*b^2*(a+b*arctan(c
*x^2))*ln(2/(1+I*c*x^2))/c^2-3/4*I*b^3*polylog(2,1-2/(1+I*c*x^2))/c^2
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}=0.562$, Rules used = {4948, 4946, 5036, 4930, 5040, 4964, 2449, 2352, 5004}

$$\int x^3(a+b\arctan(cx^2))^3\,dx=-\frac{3b^2\log\left(\frac{2}{1+icx^2}\right)(a+b\arctan(cx^2))}{2c^2}+\frac{(a+b\arctan(cx^2))^3}{4c^2}-\frac{3ib(a+b\arctan(cx^2))^2}{4c^2}-\frac{3bx^2(a+b\arctan(cx^2))^2}{4c}+\frac{1}{4}x^4(a+b\arctan(cx^2))^3-\frac{3ib^3\operatorname{PolyLog}\left(2,1-\frac{2}{icx^2+1}\right)}{4c^2}$$

```
[In] Int[x^3*(a + b*ArcTan[c*x^2])^3,x]
```

```
[Out] (((-3*I)/4)*b*(a + b*ArcTan[c*x^2])^2)/c^2 - (3*b*x^2*(a + b*ArcTan[c*x^2])
^2)/(4*c) + (a + b*ArcTan[c*x^2])^3/(4*c^2) + (x^4*(a + b*ArcTan[c*x^2])^3)
/4 - (3*b^2*(a + b*ArcTan[c*x^2])*Log[2/(1 + I*c*x^2)])/(2*c^2) - (((3*I)/4
)*b^3*PolyLog[2, 1 - 2/(1 + I*c*x^2)])/c^2
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplify
[(m + 1)/n]]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int x(a+b\arctan(cx))^3\,dx,x,x^2\right)\\
&= \frac{1}{4}x^4\left(a+b\arctan\left(cx^2\right)\right)^3-\frac{1}{4}(3bc)\text{Subst}\left(\int\frac{x^2(a+b\arctan(cx))^2}{1+c^2x^2}\,dx,x,x^2\right)\\
&= \frac{1}{4}x^4\left(a+b\arctan\left(cx^2\right)\right)^3-\frac{(3b)\text{Subst}\left(\int(a+b\arctan(cx))^2\,dx,x,x^2\right)}{4c}\\
&\quad+\frac{(3b)\text{Subst}\left(\int\frac{(a+b\arctan(cx))^2}{1+c^2x^2}\,dx,x,x^2\right)}{4c}\\
&= -\frac{3bx^2\left(a+b\arctan\left(cx^2\right)\right)^2}{4c}+\frac{\left(a+b\arctan\left(cx^2\right)\right)^3}{4c^2}+\frac{1}{4}x^4\left(a+b\arctan\left(cx^2\right)\right)^3\\
&\quad+\frac{1}{2}\left(3b^2\right)\text{Subst}\left(\int\frac{x(a+b\arctan(cx))}{1+c^2x^2}\,dx,x,x^2\right)\\
&= -\frac{3ib\left(a+b\arctan\left(cx^2\right)\right)^2}{4c^2}-\frac{3bx^2\left(a+b\arctan\left(cx^2\right)\right)^2}{4c}+\frac{\left(a+b\arctan\left(cx^2\right)\right)^3}{4c^2}\\
&\quad+\frac{1}{4}x^4\left(a+b\arctan\left(cx^2\right)\right)^3-\frac{\left(3b^2\right)\text{Subst}\left(\int\frac{a+b\arctan(cx)}{i-cx}\,dx,x,x^2\right)}{2c}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{3ib(a+b\arctan(cx^2))^2}{4c^2} - \frac{3bx^2(a+b\arctan(cx^2))^2}{4c} \\
&\quad + \frac{(a+b\arctan(cx^2))^3}{4c^2} + \frac{1}{4}x^4(a+b\arctan(cx^2))^3 \\
&\quad - \frac{3b^2(a+b\arctan(cx^2))\log\left(\frac{2}{1+icx^2}\right)}{2c^2} + \frac{(3b^3)\,\text{Subst}\left(\int \frac{\log\left(\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx, x, x^2\right)}{2c} \\
&= -\frac{3ib(a+b\arctan(cx^2))^2}{4c^2} - \frac{3bx^2(a+b\arctan(cx^2))^2}{4c} \\
&\quad + \frac{(a+b\arctan(cx^2))^3}{4c^2} + \frac{1}{4}x^4(a+b\arctan(cx^2))^3 \\
&\quad - \frac{3b^2(a+b\arctan(cx^2))\log\left(\frac{2}{1+icx^2}\right)}{2c^2} - \frac{(3ib^3)\,\text{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1+icx^2}\right)}{2c^2} \\
&= -\frac{3ib(a+b\arctan(cx^2))^2}{4c^2} - \frac{3bx^2(a+b\arctan(cx^2))^2}{4c} \\
&\quad + \frac{(a+b\arctan(cx^2))^3}{4c^2} + \frac{1}{4}x^4(a+b\arctan(cx^2))^3 \\
&\quad - \frac{3b^2(a+b\arctan(cx^2))\log\left(\frac{2}{1+icx^2}\right)}{2c^2} - \frac{3ib^3\,\text{PolyLog}\left(2, 1-\frac{2}{1+icx^2}\right)}{4c^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.14

$$
\int x^3(a+b\arctan(cx^2))^3\,dx = \frac{3b^2(a+ac^2x^4+b(i-cx^2))\arctan(cx^2)^2 + b^3(1+c^2x^4)\arctan(cx^2)^3 + 3b\arctan(cx^2)\Big(a(a-2bcx^2+ac^2
$$

```
[In] Integrate[x^3*(a + b*ArcTan[c*x^2])^3,x]
```

```
[Out] (3*b^2*(a + a*c^2*x^4 + b*(I - c*x^2))*ArcTan[c*x^2]^2 + b^3*(1 + c^2*x^4)*
ArcTan[c*x^2]^3 + 3*b*ArcTan[c*x^2]*(a*(a - 2*b*c*x^2 + a*c^2*x^4) - 2*b^2*
Log[1 + E^((2*I)*ArcTan[c*x^2])]) + a*(a*c*x^2*(-3*b + a*c*x^2) + 3*b^2*Log
[1 + c^2*x^4]) + (3*I)*b^3*PolyLog[2, -E^((2*I)*ArcTan[c*x^2])])/(4*c^2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.66 (sec) , antiderivative size = 399, normalized size of antiderivative = 2.68

method	result
default	$\frac{a^3x^4}{4}+\frac{b^3x^4\arctan(cx^2)^3}{4}-\frac{3b^3\arctan(cx^2)^2x^2}{4c}+\frac{b^3\arctan(cx^2)^3}{4c^2}+\frac{3b^3\ln(c^2x^4+1)\arctan(cx^2)}{4c^2}-\frac{3b^3\left(\sum_{\_\alpha=\mathrm{RootOf}(c^2\ldots}\ldots\right)}{\ldots}$
parts	$\frac{a^3x^4}{4}+\frac{b^3x^4\arctan(cx^2)^3}{4}-\frac{3b^3\arctan(cx^2)^2x^2}{4c}+\frac{b^3\arctan(cx^2)^3}{4c^2}+\frac{3b^3\ln(c^2x^4+1)\arctan(cx^2)}{4c^2}-\frac{3b^3\left(\sum_{\_\alpha=\mathrm{RootOf}(c^2\ldots}\ldots\right)}{\ldots}$
risch	$\frac{3ab^2\ln(c^2x^4+1)}{4c^2}+\frac{3ib^2\left(\sum_{\_\alpha=\mathrm{RootOf}\left(c\_Z^2-\mathrm{RootOf}(\_Z^2+1,\mathrm{index}=1)\right)}\frac{\ln(x-\_\alpha)\ln(-icx^2+1)+2c\left(-\frac{\ln(x-\_\alpha)\left(\ln\left(\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\left(i\sqrt{\frac{i}{c}}\ldots\right.}{\ldots}\right.\right.}{\ldots}\ldots\right.}{\ldots}\right)}{\ldots}$

```
[In] int(x^3*(a+b*arctan(c*x^2))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a^3*x^4+1/4*b^3*x^4*arctan(c*x^2)^3-3/4*b^3*arctan(c*x^2)^2/c*x^2+1/4*b
^3*arctan(c*x^2)^3/c^2+3/4*b^3/c^2*ln(c^2*x^4+1)*arctan(c*x^2)-3/16*b^3/c^3
*sum(1/_alpha^2*(2*ln(x-_alpha)*ln(c^2*x^4+1)-c*(1/c/_alpha^3*ln(x-_alpha)^
2+2/_alpha*ln(x-_alpha)*(_alpha^2*ln(1/2*(x+_alpha)/_alpha)*c-ln((_alpha^3*
c+x)/_alpha/(_alpha^2*c+1))+ln((_alpha^3*c-x)/_alpha/(_alpha^2*c-1)))+2/_al
pha*(_alpha^2*dilog(1/2*(x+_alpha)/_alpha)*c-dilog((_alpha^3*c+x)/_alpha/(_
alpha^2*c+1))+dilog((_alpha^3*c-x)/_alpha/(_alpha^2*c-1))))),_alpha=RootOf(
_Z^4*c^2+1))+3/4*a*b^2*x^4*arctan(c*x^2)^2-3/2*a*b^2*arctan(c*x^2)/c*x^2+3/
4*a*b^2/c^2*arctan(c*x^2)^2+3/4*a*b^2/c^2*ln(c^2*x^4+1)+3/4*a^2*b*x^4*arcta
n(c*x^2)-3/4*a^2*b/c*x^2+3/4*a^2*b/c^2*arctan(c*x^2)
```

Fricas [F]

$$\int x^3 (a + b \arctan(cx^2))^3 \, dx = \int (b \arctan(cx^2) + a)^3 x^3 \, dx$$

```
[In] integrate(x^3*(a+b*arctan(c*x^2))^3,x, algorithm="fricas")
[Out] integral(b^3*x^3*arctan(c*x^2)^3 + 3*a*b^2*x^3*arctan(c*x^2)^2 + 3*a^2*b*x^
3*arctan(c*x^2) + a^3*x^3, x)
```

Sympy [F]

$$\int x^3 (a + b \arctan(cx^2))^3 \, dx = \int x^3 (a + b \operatorname{atan}(cx^2))^3 \, dx$$

```
[In] integrate(x**3*(a+b*atan(c*x**2))**3,x)
[Out] Integral(x**3*(a + b*atan(c*x**2))**3, x)
```

Maxima [F]

$$\int x^3 (a + b \arctan(cx^2))^3 \, dx = \int (b \arctan(cx^2) + a)^3 x^3 \, dx$$

```
[In] integrate(x^3*(a+b*arctan(c*x^2))^3,x, algorithm="maxima")
[Out] 3/4*a*b^2*x^4*arctan(c*x^2)^2 + 1/4*a^3*x^4 + 3/4*(x^4*arctan(c*x^2) - c*(x
^2/c^2 - arctan(c*x^2)/c^3))*a^2*b - 3/4*(2*c*(x^2/c^2 - arctan(c*x^2)/c^3)
*arctan(c*x^2) + (arctan(c*x^2)^2 - log(4*c^5*x^4 + 4*c^3))/c^2)*a*b^2 + 1/
128*(4*x^4*arctan(c*x^2)^3 - 3*x^4*arctan(c*x^2)*log(c^2*x^4 + 1)^2 + 128*i
ntegrate(1/64*(12*c^2*x^7*arctan(c*x^2)*log(c^2*x^4 + 1) - 12*c*x^5*arctan(
c*x^2)^2 + 56*(c^2*x^7 + x^3)*arctan(c*x^2)^3 + 3*(c*x^5 + 2*(c^2*x^7 + x^3
)*arctan(c*x^2))*log(c^2*x^4 + 1)^2)/(c^2*x^4 + 1), x))*b^3
```

Giac [F]

$$\int x^3 \left(a + b \arctan\left(cx^2\right)\right)^3 \, dx = \int \left(b \arctan\left(cx^2\right) + a\right)^3 x^3 \, dx$$

```
[In] integrate(x^3*(a+b*arctan(c*x^2))^3,x, algorithm="giac")
[Out] integrate((b*arctan(c*x^2) + a)^3*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \left(a + b \arctan\left(cx^2\right)\right)^3 \, dx = \int x^3 \left(a + b \operatorname{atan}\left(c x^2\right)\right)^3 \, dx$$

```
[In] int(x^3*(a + b*atan(c*x^2))^3,x)
[Out] int(x^3*(a + b*atan(c*x^2))^3, x)
```

3.87 $\int x(a+b\arctan(cx^2))^3\,dx$

Optimal result 520
Rubi [A] (verified) 520
Mathematica [A] (verified) 523
Maple [B] (verified) 523
Fricas [F] 524
Sympy [F] 524
Maxima [F] 524
Giac [F] 525
Mupad [F(-1)] 525

Optimal result

Integrand size = 14, antiderivative size = 144

$$\int x(a+b\arctan(cx^2))^3\,dx = \frac{i(a+b\arctan(cx^2))^3}{2c} + \frac{1}{2}x^2(a+b\arctan(cx^2))^3 + \frac{3b(a+b\arctan(cx^2))^2\log\left(\frac{2}{1+icx^2}\right)}{2c} + \frac{3ib^2(a+b\arctan(cx^2))\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+icx^2}\right)}{2c} + \frac{3b^3\,\mathrm{PolyLog}\left(3,1-\frac{2}{1+icx^2}\right)}{4c}$$

```
[Out] 1/2*I*(a+b*arctan(c*x^2))^3/c+1/2*x^2*(a+b*arctan(c*x^2))^3+3/2*b*(a+b*arct
an(c*x^2))^2*ln(2/(1+I*c*x^2))/c+3/2*I*b^2*(a+b*arctan(c*x^2))*polylog(2,1-
2/(1+I*c*x^2))/c+3/4*b^3*polylog(3,1-2/(1+I*c*x^2))/c
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4948, 4930, 5040, 4964, 5004, 5114, 6745}

$$\int x(a+b\arctan(cx^2))^3\,dx = \frac{3ib^2\,\mathrm{PolyLog}\left(2,1-\frac{2}{icx^2+1}\right)(a+b\arctan(cx^2))}{2c} + \frac{1}{2}x^2(a+b\arctan(cx^2))^3 + \frac{i(a+b\arctan(cx^2))^3}{2c} + \frac{3b\log\left(\frac{2}{1+icx^2}\right)(a+b\arctan(cx^2))^2}{2c} + \frac{3b^3\,\mathrm{PolyLog}\left(3,1-\frac{2}{icx^2+1}\right)}{4c}$$

```
[In] Int[x*(a + b*ArcTan[c*x^2])^3,x]
```

```
[Out] ((I/2)*(a + b*ArcTan[c*x^2])^3)/c + (x^2*(a + b*ArcTan[c*x^2])^3)/2 + (3*b*
(a + b*ArcTan[c*x^2])^2*Log[2/(1 + I*c*x^2)])/(2*c) + (((3*I)/2)*b^2*(a + b
*ArcTan[c*x^2])*PolyLog[2, 1 - 2/(1 + I*c*x^2)])/c + (3*b^3*PolyLog[3, 1 -
2/(1 + I*c*x^2)])/(4*c)
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplify
[(m + 1)/n]]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
```

```
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int (a+b\arctan(cx))^3\,dx, x, x^2\right)$$

$$= \frac{1}{2}x^2\left(a+b\arctan\left(cx^2\right)\right)^3 - \frac{1}{2}(3bc)\text{Subst}\left(\int \frac{x(a+b\arctan(cx))^2}{1+c^2x^2}\,dx, x, x^2\right)$$

$$= \frac{i\left(a+b\arctan\left(cx^2\right)\right)^3}{2c} + \frac{1}{2}x^2\left(a+b\arctan\left(cx^2\right)\right)^3 + \frac{1}{2}(3b)\text{Subst}\left(\int \frac{(a+b\arctan(cx))^2}{i-cx}\,dx, x, x^2\right)$$

$$= \frac{i\left(a+b\arctan\left(cx^2\right)\right)^3}{2c} + \frac{1}{2}x^2\left(a+b\arctan\left(cx^2\right)\right)^3 + \frac{3b\left(a+b\arctan\left(cx^2\right)\right)^2\log\left(\frac{2}{1+icx^2}\right)}{2c} - \left(3b^2\right)\text{Subst}\left(\int \frac{(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx, x, x^2\right)$$

$$= \frac{i\left(a+b\arctan\left(cx^2\right)\right)^3}{2c} + \frac{1}{2}x^2\left(a+b\arctan\left(cx^2\right)\right)^3 + \frac{3b\left(a+b\arctan\left(cx^2\right)\right)^2\log\left(\frac{2}{1+icx^2}\right)}{2c} + \frac{3ib^2\left(a+b\arctan\left(cx^2\right)\right)\text{PolyLog}\left(2,1-\frac{2}{1+icx^2}\right)}{2c} - \frac{1}{2}\left(3ib^3\right)\text{Subst}\left(\int \frac{\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx, x, x^2\right)$$

$$= \frac{i\left(a+b\arctan\left(cx^2\right)\right)^3}{2c} + \frac{1}{2}x^2\left(a+b\arctan\left(cx^2\right)\right)^3 + \frac{3b\left(a+b\arctan\left(cx^2\right)\right)^2\log\left(\frac{2}{1+icx^2}\right)}{2c} + \frac{3ib^2\left(a+b\arctan\left(cx^2\right)\right)\text{PolyLog}\left(2,1-\frac{2}{1+icx^2}\right)}{2c} + \frac{3b^3\,\text{PolyLog}\left(3,1-\frac{2}{1+icx^2}\right)}{4c}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.56

$$\int x\left(a+b\arctan\left(cx^2\right)\right)^3\,dx$$
$$=\frac{2a^3cx^2+6a^2bcx^2\arctan\left(cx^2\right)-6iab^2\arctan\left(cx^2\right)^2+6ab^2cx^2\arctan\left(cx^2\right)^2-2ib^3\arctan\left(cx^2\right)^3+2b^3cx^2}{}$$

```
[In] Integrate[x*(a + b*ArcTan[c*x^2])^3,x]
```

```
[Out] (2*a^3*c*x^2 + 6*a^2*b*c*x^2*ArcTan[c*x^2] - (6*I)*a*b^2*ArcTan[c*x^2]^2 +
6*a*b^2*c*x^2*ArcTan[c*x^2]^2 - (2*I)*b^3*ArcTan[c*x^2]^3 + 2*b^3*c*x^2*Arc
Tan[c*x^2]^3 + 12*a*b^2*ArcTan[c*x^2]*Log[1 + E^((2*I)*ArcTan[c*x^2])] + 6*
b^3*ArcTan[c*x^2]^2*Log[1 + E^((2*I)*ArcTan[c*x^2])] - 3*a^2*b*Log[1 + c^2*
x^4] - (6*I)*b^2*(a + b*ArcTan[c*x^2])*PolyLog[2, -E^((2*I)*ArcTan[c*x^2])]
 + 3*b^3*PolyLog[3, -E^((2*I)*ArcTan[c*x^2])])/(4*c)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 274 vs. 2(129) = 258.

Time = 9.39 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.91

method	result
derivativedivides	$\frac{a^3cx^2+b^3\left(\arctan(cx^2)^3(cx^2+i)-2i\arctan(cx^2)^3+3\arctan(cx^2)^2\ln\left(1+\frac{(icx^2+1)^2}{c^2x^4+1}\right)-3i\arctan(cx^2)\operatorname{polylog}\left(2,\right.\right.}{}$
default	$\frac{a^3cx^2+b^3\left(\arctan(cx^2)^3(cx^2+i)-2i\arctan(cx^2)^3+3\arctan(cx^2)^2\ln\left(1+\frac{(icx^2+1)^2}{c^2x^4+1}\right)-3i\arctan(cx^2)\operatorname{polylog}\left(2,\right.\right.}{}$
parts	$\frac{a^3x^2}{2}+\frac{b^3\left(\arctan(cx^2)^3(cx^2+i)-2i\arctan(cx^2)^3+3\arctan(cx^2)^2\ln\left(1+\frac{(icx^2+1)^2}{c^2x^4+1}\right)-3i\arctan(cx^2)\operatorname{polylog}\left(2\right.\right.}{2c}$

```
[In] int(x*(a+b*arctan(c*x^2))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/c*(a^3*c*x^2+b^3*(arctan(c*x^2)^3*(c*x^2+I)-2*I*arctan(c*x^2)^3+3*arcta
n(c*x^2)^2*ln(1+(1+I*c*x^2)^2/(c^2*x^4+1))-3*I*arctan(c*x^2)*polylog(2,-(1+
I*c*x^2)^2/(c^2*x^4+1))+3/2*polylog(3,-(1+I*c*x^2)^2/(c^2*x^4+1)))+3*a*b^2*
```

```
(arctan(c*x^2)^2*(c*x^2+I)+2*arctan(c*x^2)*ln(1+(1+I*c*x^2)^2/(c^2*x^4+1))-
2*I*arctan(c*x^2)^2-I*polylog(2,-(1+I*c*x^2)^2/(c^2*x^4+1)))+3*a^2*b*(c*x^2
*arctan(c*x^2)-1/2*ln(c^2*x^4+1)))
```

Fricas [F]

$$\int x(a+b\arctan(cx^2))^3\,dx = \int (b\arctan(cx^2)+a)^3x\,dx$$

```
[In] integrate(x*(a+b*arctan(c*x^2))^3,x, algorithm="fricas")
[Out] integral(b^3*x*arctan(c*x^2)^3 + 3*a*b^2*x*arctan(c*x^2)^2 + 3*a^2*b*x*arct
an(c*x^2) + a^3*x, x)
```

Sympy [F]

$$\int x(a+b\arctan(cx^2))^3\,dx = \int x(a+b\,\mathrm{atan}(cx^2))^3\,dx$$

```
[In] integrate(x*(a+b*atan(c*x**2))**3,x)
[Out] Integral(x*(a + b*atan(c*x**2))**3, x)
```

Maxima [F]

$$\int x(a+b\arctan(cx^2))^3\,dx = \int (b\arctan(cx^2)+a)^3x\,dx$$

```
[In] integrate(x*(a+b*arctan(c*x^2))^3,x, algorithm="maxima")
[Out] 1/16*b^3*x^2*arctan(c*x^2)^3 - 3/64*b^3*x^2*arctan(c*x^2)*log(c^2*x^4 + 1)^
2 + 7/64*b^3*arctan(c*x^2)^4/c + 28*b^3*c^2*integrate(1/32*x^5*arctan(c*x^2
)^3/(c^2*x^4 + 1), x) + 3*b^3*c^2*integrate(1/32*x^5*arctan(c*x^2)*log(c^2*
x^4 + 1)^2/(c^2*x^4 + 1), x) + 96*a*b^2*c^2*integrate(1/32*x^5*arctan(c*x^2
)^2/(c^2*x^4 + 1), x) + 12*b^3*c^2*integrate(1/32*x^5*arctan(c*x^2)*log(c^2
*x^4 + 1)/(c^2*x^4 + 1), x) + 1/2*a^3*x^2 + 1/2*a*b^2*arctan(c*x^2)^3/c - 1
2*b^3*c*integrate(1/32*x^3*arctan(c*x^2)^2/(c^2*x^4 + 1), x) + 3*b^3*c*inte
grate(1/32*x^3*log(c^2*x^4 + 1)^2/(c^2*x^4 + 1), x) + 3*b^3*integrate(1/32*
x*arctan(c*x^2)*log(c^2*x^4 + 1)^2/(c^2*x^4 + 1), x) + 3/4*(2*c*x^2*arctan(
c*x^2) - log(c^2*x^4 + 1))*a^2*b/c
```

Giac [F]

$$\int x\left(a+b\arctan\left(cx^2\right)\right)^3\,dx=\int\left(b\arctan\left(cx^2\right)+a\right)^3x\,dx$$

```
[In] integrate(x*(a+b*arctan(c*x^2))^3,x, algorithm="giac")
[Out] integrate((b*arctan(c*x^2) + a)^3*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a+b\arctan\left(cx^2\right)\right)^3\,dx=\int x\,\left(a+b\,\mathrm{atan}\left(c\,x^2\right)\right)^3\,dx$$

```
[In] int(x*(a + b*atan(c*x^2))^3,x)
[Out] int(x*(a + b*atan(c*x^2))^3, x)
```

3.88 $\int \frac{(a+b\arctan(cx^2))^3}{x}\,dx$

Optimal result . . . 526
Rubi [A] (verified) . . . 527
Mathematica [A] (verified) . . . 530
Maple [F] . . . 531
Fricas [F] . . . 531
Sympy [F] . . . 531
Maxima [F] . . . 532
Giac [F] . . . 532
Mupad [F(-1)] . . . 532

Optimal result

Integrand size = 16, antiderivative size = 229

$$\begin{aligned}\int \frac{(a+b\arctan(cx^2))^3}{x}\,dx =& (a+b\arctan(cx^2))^3\operatorname{arctanh}\left(1-\frac{2}{1+icx^2}\right)\\&-\frac{3}{4}ib(a+b\arctan(cx^2))^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx^2}\right)\\&+\frac{3}{4}ib(a+b\arctan(cx^2))^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx^2}\right)\\&-\frac{3}{4}b^2(a+b\arctan(cx^2))\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx^2}\right)\\&+\frac{3}{4}b^2(a+b\arctan(cx^2))\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx^2}\right)\\&+\frac{3}{8}ib^3\operatorname{PolyLog}\left(4,1-\frac{2}{1+icx^2}\right)\\&-\frac{3}{8}ib^3\operatorname{PolyLog}\left(4,-1+\frac{2}{1+icx^2}\right)\end{aligned}$$

```
[Out] -(a+b*arctan(c*x^2))^3*arctanh(-1+2/(1+I*c*x^2))-3/4*I*b*(a+b*arctan(c*x^2)
)^2*polylog(2,1-2/(1+I*c*x^2))+3/4*I*b*(a+b*arctan(c*x^2))^2*polylog(2,-1+2
/(1+I*c*x^2))-3/4*b^2*(a+b*arctan(c*x^2))*polylog(3,1-2/(1+I*c*x^2))+3/4*b^
2*(a+b*arctan(c*x^2))*polylog(3,-1+2/(1+I*c*x^2))+3/8*I*b^3*polylog(4,1-2/(
1+I*c*x^2))-3/8*I*b^3*polylog(4,-1+2/(1+I*c*x^2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {4944, 4942, 5108, 5004, 5114, 5118, 6745}

$$\begin{aligned}\int \frac{(a+b\arctan(cx^2))^3}{x}\,dx &= \operatorname{arctanh}\left(1-\frac{2}{1+icx^2}\right)(a+b\arctan(cx^2))^3 \\ &\quad -\frac{3}{4}b^2\operatorname{PolyLog}\left(3,1-\frac{2}{icx^2+1}\right)(a+b\arctan(cx^2)) \\ &\quad +\frac{3}{4}b^2\operatorname{PolyLog}\left(3,\frac{2}{icx^2+1}-1\right)(a+b\arctan(cx^2)) \\ &\quad -\frac{3}{4}ib\operatorname{PolyLog}\left(2,1-\frac{2}{icx^2+1}\right)(a+b\arctan(cx^2))^2 \\ &\quad +\frac{3}{4}ib\operatorname{PolyLog}\left(2,\frac{2}{icx^2+1}-1\right)(a+b\arctan(cx^2))^2 \\ &\quad +\frac{3}{8}ib^3\operatorname{PolyLog}\left(4,1-\frac{2}{icx^2+1}\right) \\ &\quad -\frac{3}{8}ib^3\operatorname{PolyLog}\left(4,\frac{2}{icx^2+1}-1\right)\end{aligned}$$

```
[In] Int[(a + b*ArcTan[c*x^2])^3/x,x]
```

```
[Out] (a + b*ArcTan[c*x^2])^3*ArcTanh[1 - 2/(1 + I*c*x^2)] - ((3*I)/4)*b*(a + b*A
rcTan[c*x^2])^2*PolyLog[2, 1 - 2/(1 + I*c*x^2)] + ((3*I)/4)*b*(a + b*ArcTan
[c*x^2])^2*PolyLog[2, -1 + 2/(1 + I*c*x^2)] - (3*b^2*(a + b*ArcTan[c*x^2])*
PolyLog[3, 1 - 2/(1 + I*c*x^2)])/4 + (3*b^2*(a + b*ArcTan[c*x^2])*PolyLog[3
, -1 + 2/(1 + I*c*x^2)])/4 + ((3*I)/8)*b^3*PolyLog[4, 1 - 2/(1 + I*c*x^2)]
- ((3*I)/8)*b^3*PolyLog[4, -1 + 2/(1 + I*c*x^2)]
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1
/n, Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[p, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5108

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e*
x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5118

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k + 1,
 u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+b\arctan(cx))^3}{x}\,dx, x, x^2\right) \\
&= \left(a+b\arctan\left(cx^2\right)\right)^3\operatorname{arctanh}\left(1-\frac{2}{1+icx^2}\right) \\
&\quad -(3bc)\text{Subst}\left(\int \frac{(a+b\arctan(cx))^2\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx, x, x^2\right)
\end{aligned}$$

$$
\begin{aligned}
&= \left(a+b\arctan\left(cx^2\right)\right)^3\operatorname{arctanh}\left(1-\frac{2}{1+icx^2}\right) \\
&\quad+\frac{1}{2}(3bc)\text{Subst}\left(\int\frac{(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx,x,x^2\right) \\
&\quad-\frac{1}{2}(3bc)\text{Subst}\left(\int\frac{(a+b\arctan(cx))^2\log\left(2-\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx,x,x^2\right) \\
&= \left(a+b\arctan\left(cx^2\right)\right)^3\operatorname{arctanh}\left(1-\frac{2}{1+icx^2}\right) \\
&\quad-\frac{3}{4}ib\left(a+b\arctan\left(cx^2\right)\right)^2\text{PolyLog}\left(2,1-\frac{2}{1+icx^2}\right) \\
&\quad+\frac{3}{4}ib\left(a+b\arctan\left(cx^2\right)\right)^2\text{PolyLog}\left(2,-1+\frac{2}{1+icx^2}\right) \\
&\quad+\frac{1}{2}\left(3ib^2c\right)\text{Subst}\left(\int\frac{(a+b\arctan(cx))\,\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx,x,x^2\right) \\
&\quad-\frac{1}{2}\left(3ib^2c\right)\text{Subst}\left(\int\frac{(a+b\arctan(cx))\,\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx,x,x^2\right) \\
&= \left(a+b\arctan\left(cx^2\right)\right)^3\operatorname{arctanh}\left(1-\frac{2}{1+icx^2}\right) \\
&\quad-\frac{3}{4}ib\left(a+b\arctan\left(cx^2\right)\right)^2\text{PolyLog}\left(2,1-\frac{2}{1+icx^2}\right) \\
&\quad+\frac{3}{4}ib\left(a+b\arctan\left(cx^2\right)\right)^2\text{PolyLog}\left(2,-1+\frac{2}{1+icx^2}\right) \\
&\quad-\frac{3}{4}b^2\left(a+b\arctan\left(cx^2\right)\right)\text{PolyLog}\left(3,1-\frac{2}{1+icx^2}\right) \\
&\quad+\frac{3}{4}b^2\left(a+b\arctan\left(cx^2\right)\right)\text{PolyLog}\left(3,-1+\frac{2}{1+icx^2}\right) \\
&\quad+\frac{1}{4}\left(3b^3c\right)\text{Subst}\left(\int\frac{\text{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx,x,x^2\right) \\
&\quad-\frac{1}{4}\left(3b^3c\right)\text{Subst}\left(\int\frac{\text{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx,x,x^2\right)
\end{aligned}
$$

$$= \left(a+b\arctan\left(cx^2\right)\right)^3\operatorname{arctanh}\left(1-\frac{2}{1+icx^2}\right) - \frac{3}{4}ib\left(a+b\arctan\left(cx^2\right)\right)^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx^2}\right) + \frac{3}{4}ib\left(a+b\arctan\left(cx^2\right)\right)^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx^2}\right) - \frac{3}{4}b^2\left(a+b\arctan\left(cx^2\right)\right)\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx^2}\right) + \frac{3}{4}b^2\left(a+b\arctan\left(cx^2\right)\right)\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx^2}\right) + \frac{3}{8}ib^3\operatorname{PolyLog}\left(4,1-\frac{2}{1+icx^2}\right) - \frac{3}{8}ib^3\operatorname{PolyLog}\left(4,-1+\frac{2}{1+icx^2}\right)$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 417, normalized size of antiderivative = 1.82

$$\int \frac{\left(a+b\arctan\left(cx^2\right)\right)^3}{x}\,dx = a^3\log(x) + \frac{3}{4}ia^2b\left(\operatorname{PolyLog}\left(2,-icx^2\right)-\operatorname{PolyLog}\left(2,icx^2\right)\right) + \frac{1}{16}ab^2\left(-i\pi^3+16i\arctan\left(cx^2\right)^3 + 24\arctan\left(cx^2\right)^2\log\left(1-e^{-2i\arctan(cx^2)}\right) - 24\arctan\left(cx^2\right)^2\log\left(1+e^{2i\arctan(cx^2)}\right) + 24i\arctan\left(cx^2\right)\operatorname{PolyLog}\left(2,e^{-2i\arctan(cx^2)}\right) + 24i\arctan\left(cx^2\right)\operatorname{PolyLog}\left(2,-e^{2i\arctan(cx^2)}\right) + 12\operatorname{PolyLog}\left(3,e^{-2i\arctan(cx^2)}\right) - 12\operatorname{PolyLog}\left(3,-e^{2i\arctan(cx^2)}\right)\right) - \frac{1}{128}ib^3\left(\pi^4 - 32\arctan\left(cx^2\right)^4 + 64i\arctan\left(cx^2\right)^3\log\left(1-e^{-2i\arctan(cx^2)}\right) - 64i\arctan\left(cx^2\right)^3\log\left(1+e^{2i\arctan(cx^2)}\right) - 96\arctan\left(cx^2\right)^2\operatorname{PolyLog}\left(2,e^{-2i\arctan(cx^2)}\right) - 96\arctan\left(cx^2\right)^2\operatorname{PolyLog}\left(2,-e^{2i\arctan(cx^2)}\right) + 96i\arctan\left(cx^2\right)\operatorname{PolyLog}\left(3,e^{-2i\arctan(cx^2)}\right) - 96i\arctan\left(cx^2\right)\operatorname{PolyLog}\left(3,-e^{2i\arctan(cx^2)}\right) + 48\operatorname{PolyLog}\left(4,e^{-2i\arctan(cx^2)}\right) + 48\operatorname{PolyLog}\left(4,-e^{2i\arctan(cx^2)}\right)\right)$$

```
[In] Integrate[(a + b*ArcTan[c*x^2])^3/x,x]
```

```
[Out] a^3*Log[x] + ((3*I)/4)*a^2*b*(PolyLog[2, (-I)*c*x^2] - PolyLog[2, I*c*x^2])
 + (a*b^2*((-I)*Pi^3 + (16*I)*ArcTan[c*x^2]^3 + 24*ArcTan[c*x^2]^2*Log[1 -
E^((-2*I)*ArcTan[c*x^2])] - 24*ArcTan[c*x^2]^2*Log[1 + E^((2*I)*ArcTan[c*x^
2])] + (24*I)*ArcTan[c*x^2]*PolyLog[2, E^((-2*I)*ArcTan[c*x^2])] + (24*I)*A
rcTan[c*x^2]*PolyLog[2, -E^((2*I)*ArcTan[c*x^2])] + 12*PolyLog[3, E^((-2*I)
*ArcTan[c*x^2])] - 12*PolyLog[3, -E^((2*I)*ArcTan[c*x^2])]))/16 - (I/128)*b
^3*(Pi^4 - 32*ArcTan[c*x^2]^4 + (64*I)*ArcTan[c*x^2]^3*Log[1 - E^((-2*I)*Ar
cTan[c*x^2])] - (64*I)*ArcTan[c*x^2]^3*Log[1 + E^((2*I)*ArcTan[c*x^2])] - 9
6*ArcTan[c*x^2]^2*PolyLog[2, E^((-2*I)*ArcTan[c*x^2])] - 96*ArcTan[c*x^2]^2
*PolyLog[2, -E^((2*I)*ArcTan[c*x^2])] + (96*I)*ArcTan[c*x^2]*PolyLog[3, E^(
(-2*I)*ArcTan[c*x^2])] - (96*I)*ArcTan[c*x^2]*PolyLog[3, -E^((2*I)*ArcTan[c
*x^2])] + 48*PolyLog[4, E^((-2*I)*ArcTan[c*x^2])] + 48*PolyLog[4, -E^((2*I)
*ArcTan[c*x^2])])
```

Maple [F]

$$\int \frac{(a+b\arctan(c\,x^2))^3}{x}dx$$

```
[In] int((a+b*arctan(c*x^2))^3/x,x)
```

```
[Out] int((a+b*arctan(c*x^2))^3/x,x)
```

Fricas [F]

$$\int \frac{(a+b\arctan(cx^2))^3}{x}\,dx = \int \frac{(b\arctan(cx^2)+a)^3}{x}\,dx$$

```
[In] integrate((a+b*arctan(c*x^2))^3/x,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(c*x^2)^3 + 3*a*b^2*arctan(c*x^2)^2 + 3*a^2*b*arctan(c*
x^2) + a^3)/x, x)
```

Sympy [F]

$$\int \frac{(a+b\arctan(cx^2))^3}{x}\,dx = \int \frac{(a+b\operatorname{atan}(cx^2))^3}{x}\,dx$$

```
[In] integrate((a+b*atan(c*x**2))**3/x,x)
```

```
[Out] Integral((a + b*atan(c*x**2))**3/x, x)
```

Maxima [F]

$$\int \frac{(a+b\arctan(cx^2))^3}{x}\,dx=\int \frac{(b\arctan(cx^2)+a)^3}{x}\,dx$$

```
[In] integrate((a+b*arctan(c*x^2))^3/x,x, algorithm="maxima")
```

```
[Out] a^3*log(x) + 1/32*integrate((28*b^3*arctan(c*x^2)^3 + 3*b^3*arctan(c*x^2)*l
og(c^2*x^4 + 1)^2 + 96*a*b^2*arctan(c*x^2)^2 + 96*a^2*b*arctan(c*x^2))/x, x
)
```

Giac [F]

$$\int \frac{(a+b\arctan(cx^2))^3}{x}\,dx=\int \frac{(b\arctan(cx^2)+a)^3}{x}\,dx$$

```
[In] integrate((a+b*arctan(c*x^2))^3/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^2) + a)^3/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx^2))^3}{x}\,dx=\int \frac{(a+b\,\mathrm{atan}(c\,x^2))^3}{x}\,dx$$

```
[In] int((a + b*atan(c*x^2))^3/x,x)
```

```
[Out] int((a + b*atan(c*x^2))^3/x, x)
```

3.89 $\int \frac{(a+b\arctan(cx^2))^3}{x^3} dx$

Optimal result . . . 533
Rubi [A] (verified) . . . 534
Mathematica [A] (verified) . . . 536
Maple [F] . . . 537
Fricas [F] . . . 537
Sympy [F] . . . 537
Maxima [F] . . . 537
Giac [F] . . . 538
Mupad [F(-1)] . . . 538

Optimal result

Integrand size = 16, antiderivative size = 138

$$\int \frac{(a+b\arctan(cx^2))^3}{x^3}\,dx = -\frac{1}{2}ic\big(a+b\arctan(cx^2)\big)^3 - \frac{(a+b\arctan(cx^2))^3}{2x^2} + \frac{3}{2}bc\big(a+b\arctan(cx^2)\big)^2\log\left(2-\frac{2}{1-icx^2}\right) - \frac{3}{2}ib^2c\big(a+b\arctan(cx^2)\big)\operatorname{PolyLog}\left(2,-1+\frac{2}{1-icx^2}\right) + \frac{3}{4}b^3c\operatorname{PolyLog}\left(3,-1+\frac{2}{1-icx^2}\right)$$

```
[Out] -1/2*I*c*(a+b*arctan(c*x^2))^3-1/2*(a+b*arctan(c*x^2))^3/x^2+3/2*b*c*(a+b*a
rctan(c*x^2))^2*ln(2-2/(1-I*c*x^2))-3/2*I*b^2*c*(a+b*arctan(c*x^2))*polylog
(2,-1+2/(1-I*c*x^2))+3/4*b^3*c*polylog(3,-1+2/(1-I*c*x^2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {4948, 4946, 5044, 4988, 5004, 5112, 6745}

$$\int \frac{(a+b\arctan(cx^2))^3}{x^3}\,dx = -\frac{3}{2}ib^2c\,\text{PolyLog}\left(2,\frac{2}{1-icx^2}-1\right)(a+b\arctan(cx^2)) - \frac{1}{2}ic(a+b\arctan(cx^2))^3 - \frac{(a+b\arctan(cx^2))^3}{2x^2} + \frac{3}{2}bc\log\left(2-\frac{2}{1-icx^2}\right)(a+b\arctan(cx^2))^2 + \frac{3}{4}b^3c\,\text{PolyLog}\left(3,\frac{2}{1-icx^2}-1\right)$$

```
[In] Int[(a + b*ArcTan[c*x^2])^3/x^3,x]
```

```
[Out] (-1/2*I)*c*(a + b*ArcTan[c*x^2])^3 - (a + b*ArcTan[c*x^2])^3/(2*x^2) + (3*b
*c*(a + b*ArcTan[c*x^2])^2*Log[2 - 2/(1 - I*c*x^2)])/2 - ((3*I)/2)*b^2*c*(a
 + b*ArcTan[c*x^2])*PolyLog[2, -1 + 2/(1 - I*c*x^2)] + (3*b^3*c*PolyLog[3,
-1 + 2/(1 - I*c*x^2)])/4
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplify
[(m + 1)/n]]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5112

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+b\arctan(cx))^3}{x^2}\,dx, x, x^2\right)\\
&= -\frac{\left(a+b\arctan\left(cx^2\right)\right)^3}{2x^2} + \frac{1}{2}(3bc)\text{Subst}\left(\int \frac{(a+b\arctan(cx))^2}{x\left(1+c^2x^2\right)}\,dx, x, x^2\right)\\
&= -\frac{1}{2}ic\left(a+b\arctan\left(cx^2\right)\right)^3 - \frac{\left(a+b\arctan\left(cx^2\right)\right)^3}{2x^2}\\
&\quad + \frac{1}{2}(3ibc)\text{Subst}\left(\int \frac{(a+b\arctan(cx))^2}{x(i+cx)}\,dx, x, x^2\right)\\
&= -\frac{1}{2}ic\left(a+b\arctan\left(cx^2\right)\right)^3 - \frac{\left(a+b\arctan\left(cx^2\right)\right)^3}{2x^2}\\
&\quad + \frac{3}{2}bc\left(a+b\arctan\left(cx^2\right)\right)^2\log\left(2-\frac{2}{1-icx^2}\right)\\
&\quad - \left(3b^2c^2\right)\text{Subst}\left(\int \frac{(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right)}{1+c^2x^2}\,dx, x, x^2\right)
\end{aligned}
$$

$$= -\frac{1}{2}ic\left(a+b\arctan\left(cx^2\right)\right)^3 - \frac{\left(a+b\arctan\left(cx^2\right)\right)^3}{2x^2} + \frac{3}{2}bc\left(a+b\arctan\left(cx^2\right)\right)^2\log\left(2-\frac{2}{1-icx^2}\right) - \frac{3}{2}ib^2c\left(a+b\arctan\left(cx^2\right)\right)\text{PolyLog}\left(2,-1+\frac{2}{1-icx^2}\right) + \frac{1}{2}\left(3ib^3c^2\right)\text{Subst}\left(\int\frac{\text{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)}{1+c^2x^2}dx,x,x^2\right)$$

$$= -\frac{1}{2}ic\left(a+b\arctan\left(cx^2\right)\right)^3 - \frac{\left(a+b\arctan\left(cx^2\right)\right)^3}{2x^2} + \frac{3}{2}bc\left(a+b\arctan\left(cx^2\right)\right)^2\log\left(2-\frac{2}{1-icx^2}\right) - \frac{3}{2}ib^2c\left(a+b\arctan\left(cx^2\right)\right)\text{PolyLog}\left(2,-1+\frac{2}{1-icx^2}\right) + \frac{3}{4}b^3c\,\text{PolyLog}\left(3,-1+\frac{2}{1-icx^2}\right)$$

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.73

$$\int\frac{\left(a+b\arctan\left(cx^2\right)\right)^3}{x^3}dx$$
$$= \frac{1}{4}\left(-\frac{2a^3}{x^2} - \frac{6a^2b\arctan\left(cx^2\right)}{x^2} + 12a^2bc\log(x) - 3a^2bc\log\left(1+c^2x^4\right) + 6ab^2c\left(\arctan\left(cx^2\right)\left(\left(-i-\frac{1}{cx^2}\right)\arctan\left(cx^2\right) + 2\log\left(1-e^{2i\arctan\left(cx^2\right)}\right)\right) - i\,\text{PolyLog}\left(2,e^{2i\arctan\left(cx^2\right)}\right)\right) + 2b^3c\left(-\frac{i\pi^3}{8} + i\arctan\left(cx^2\right)^3 - \frac{\arctan\left(cx^2\right)^3}{cx^2} + 3\arctan\left(cx^2\right)^2\log\left(1-e^{-2i\arctan\left(cx^2\right)}\right) + 3i\arctan\left(cx^2\right)\text{PolyLog}\left(2,e^{-2i\arctan\left(cx^2\right)}\right) + \frac{3}{2}\text{PolyLog}\left(3,e^{-2i\arctan\left(cx^2\right)}\right)\right)\right)$$

```
[In] Integrate[(a + b*ArcTan[c*x^2])^3/x^3,x]
```

```
[Out] ((-2*a^3)/x^2 - (6*a^2*b*ArcTan[c*x^2])/x^2 + 12*a^2*b*c*Log[x] - 3*a^2*b*c
*Log[1 + c^2*x^4] + 6*a*b^2*c*(ArcTan[c*x^2]*((-I - 1/(c*x^2))*ArcTan[c*x^2
] + 2*Log[1 - E^((2*I)*ArcTan[c*x^2])]) - I*PolyLog[2, E^((2*I)*ArcTan[c*x^
2])]) + 2*b^3*c*((-1/8*I)*Pi^3 + I*ArcTan[c*x^2]^3 - ArcTan[c*x^2]^3/(c*x^2
```

```
) + 3*ArcTan[c*x^2]^2*Log[1 - E^((-2*I)*ArcTan[c*x^2])] + (3*I)*ArcTan[c*x^
2]*PolyLog[2, E^((-2*I)*ArcTan[c*x^2])] + (3*PolyLog[3, E^((-2*I)*ArcTan[c*
x^2])])/2))/4
```

Maple [F]

$$\int \frac{(a+b\arctan(c\,x^2))^3}{x^3}dx$$

```
[In] int((a+b*arctan(c*x^2))^3/x^3,x)
[Out] int((a+b*arctan(c*x^2))^3/x^3,x)
```

Fricas [F]

$$\int \frac{(a+b\arctan(cx^2))^3}{x^3}\,dx = \int \frac{(b\arctan(cx^2)+a)^3}{x^3}\,dx$$

```
[In] integrate((a+b*arctan(c*x^2))^3/x^3,x, algorithm="fricas")
[Out] integral((b^3*arctan(c*x^2)^3 + 3*a*b^2*arctan(c*x^2)^2 + 3*a^2*b*arctan(c*
x^2) + a^3)/x^3, x)
```

Sympy [F]

$$\int \frac{(a+b\arctan(cx^2))^3}{x^3}\,dx = \int \frac{(a+b\,\mathrm{atan}(cx^2))^3}{x^3}\,dx$$

```
[In] integrate((a+b*atan(c*x**2))**3/x**3,x)
[Out] Integral((a + b*atan(c*x**2))**3/x**3, x)
```

Maxima [F]

$$\int \frac{(a+b\arctan(cx^2))^3}{x^3}\,dx = \int \frac{(b\arctan(cx^2)+a)^3}{x^3}\,dx$$

```
[In] integrate((a+b*arctan(c*x^2))^3/x^3,x, algorithm="maxima")
[Out] -3/4*(c*(log(c^2*x^4 + 1) - log(x^4)) + 2*arctan(c*x^2)/x^2)*a^2*b - 1/2*a^
3/x^2 - 1/64*(4*b^3*arctan(c*x^2)^3 - 3*b^3*arctan(c*x^2)*log(c^2*x^4 + 1)^
2 - 64*x^2*integrate(-1/32*(12*b^3*c^2*x^4*arctan(c*x^2)*log(c^2*x^4 + 1) -
 28*(b^3*c^2*x^4 + b^3)*arctan(c*x^2)^3 - 12*(8*a*b^2*c^2*x^4 + b^3*c*x^2 +
 8*a*b^2)*arctan(c*x^2)^2 + 3*(b^3*c*x^2 - (b^3*c^2*x^4 + b^3)*arctan(c*x^2
))*log(c^2*x^4 + 1)^2)/(c^2*x^7 + x^3), x))/x^2
```

Giac [F]

$$\int \frac{(a+b\arctan(cx^2))^3}{x^3}\,dx = \int \frac{(b\arctan(cx^2)+a)^3}{x^3}\,dx$$

```
[In] integrate((a+b*arctan(c*x^2))^3/x^3,x, algorithm="giac")
[Out] integrate((b*arctan(c*x^2) + a)^3/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx^2))^3}{x^3}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x^2))^3}{x^3}\,dx$$

```
[In] int((a + b*atan(c*x^2))^3/x^3,x)
[Out] int((a + b*atan(c*x^2))^3/x^3, x)
```

3.90 $\int \frac{(a+b\arctan(cx^2))^3}{x^5} dx$

Optimal result 539
Rubi [A] (verified) 539
Mathematica [A] (verified) 542
Maple [C] (warning: unable to verify) 542
Fricas [F] 543
Sympy [F] 543
Maxima [F] 543
Giac [F] 544
Mupad [F(-1)] 544

Optimal result

Integrand size = 16, antiderivative size = 149

$$\begin{aligned}\int \frac{(a+b\arctan(cx^2))^3}{x^5} dx = & -\frac{3}{4}ibc^2(a+b\arctan(cx^2))^2 - \frac{3bc(a+b\arctan(cx^2))^2}{4x^2} \\ & -\frac{1}{4}c^2(a+b\arctan(cx^2))^3 - \frac{(a+b\arctan(cx^2))^3}{4x^4} \\ & +\frac{3}{2}b^2c^2(a+b\arctan(cx^2))\log\left(2-\frac{2}{1-icx^2}\right) \\ & -\frac{3}{4}ib^3c^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1-icx^2}\right)\end{aligned}$$

```
[Out] -3/4*I*b*c^2*(a+b*arctan(c*x^2))^2-3/4*b*c*(a+b*arctan(c*x^2))^2/x^2-1/4*c^
2*(a+b*arctan(c*x^2))^3-1/4*(a+b*arctan(c*x^2))^3/x^4+3/2*b^2*c^2*(a+b*arct
an(c*x^2))*ln(2-2/(1-I*c*x^2))-3/4*I*b^3*c^2*polylog(2,-1+2/(1-I*c*x^2))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {4948, 4946, 5038, 5044, 4988, 2497, 5004}

$$\begin{aligned}\int \frac{(a+b\arctan(cx^2))^3}{x^5} dx = & \frac{3}{2}b^2c^2\log\left(2-\frac{2}{1-icx^2}\right)(a+b\arctan(cx^2)) \\ & -\frac{3}{4}ibc^2(a+b\arctan(cx^2))^2 - \frac{1}{4}c^2(a+b\arctan(cx^2))^3 \\ & -\frac{3bc(a+b\arctan(cx^2))^2}{4x^2} - \frac{(a+b\arctan(cx^2))^3}{4x^4} \\ & -\frac{3}{4}ib^3c^2\operatorname{PolyLog}\left(2,\frac{2}{1-icx^2}-1\right)\end{aligned}$$

```
[In] Int[(a + b*ArcTan[c*x^2])^3/x^5,x]
```

```
[Out] ((-3*I)/4)*b*c^2*(a + b*ArcTan[c*x^2])^2 - (3*b*c*(a + b*ArcTan[c*x^2])^2)/
(4*x^2) - (c^2*(a + b*ArcTan[c*x^2])^3)/4 - (a + b*ArcTan[c*x^2])^3/(4*x^4)
 + (3*b^2*c^2*(a + b*ArcTan[c*x^2])*Log[2 - 2/(1 - I*c*x^2)])/2 - ((3*I)/4)
*b^3*c^2*PolyLog[2, -1 + 2/(1 - I*c*x^2)]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplify
[(m + 1)/n]]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
```

```
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+b\arctan(cx))^3}{x^3}\,dx, x, x^2\right)\\
&= -\frac{\left(a+b\arctan\left(cx^2\right)\right)^3}{4x^4} + \frac{1}{4}(3bc)\text{Subst}\left(\int \frac{(a+b\arctan(cx))^2}{x^2\left(1+c^2x^2\right)}\,dx, x, x^2\right)\\
&= -\frac{\left(a+b\arctan\left(cx^2\right)\right)^3}{4x^4} + \frac{1}{4}(3bc)\text{Subst}\left(\int \frac{(a+b\arctan(cx))^2}{x^2}\,dx, x, x^2\right)\\
&\quad - \frac{1}{4}\left(3bc^3\right)\text{Subst}\left(\int \frac{(a+b\arctan(cx))^2}{1+c^2x^2}\,dx, x, x^2\right)\\
&= -\frac{3bc\left(a+b\arctan\left(cx^2\right)\right)^2}{4x^2} - \frac{1}{4}c^2\left(a+b\arctan\left(cx^2\right)\right)^3\\
&\quad - \frac{\left(a+b\arctan\left(cx^2\right)\right)^3}{4x^4} + \frac{1}{2}\left(3b^2c^2\right)\text{Subst}\left(\int \frac{a+b\arctan(cx)}{x\left(1+c^2x^2\right)}\,dx, x, x^2\right)\\
&= -\frac{3}{4}ibc^2\left(a+b\arctan\left(cx^2\right)\right)^2 - \frac{3bc\left(a+b\arctan\left(cx^2\right)\right)^2}{4x^2} - \frac{1}{4}c^2\left(a+b\arctan\left(cx^2\right)\right)^3\\
&\quad - \frac{\left(a+b\arctan\left(cx^2\right)\right)^3}{4x^4} + \frac{1}{2}\left(3ib^2c^2\right)\text{Subst}\left(\int \frac{a+b\arctan(cx)}{x(i+cx)}\,dx, x, x^2\right)\\
&= -\frac{3}{4}ibc^2\left(a+b\arctan\left(cx^2\right)\right)^2 - \frac{3bc\left(a+b\arctan\left(cx^2\right)\right)^2}{4x^2} - \frac{1}{4}c^2\left(a+b\arctan\left(cx^2\right)\right)^3\\
&\quad - \frac{\left(a+b\arctan\left(cx^2\right)\right)^3}{4x^4} + \frac{3}{2}b^2c^2\left(a+b\arctan\left(cx^2\right)\right)\log\left(2-\frac{2}{1-icx^2}\right)\\
&\quad - \frac{1}{2}\left(3b^3c^3\right)\text{Subst}\left(\int \frac{\log\left(2-\frac{2}{1-icx}\right)}{1+c^2x^2}\,dx, x, x^2\right)\\
&= -\frac{3}{4}ibc^2\left(a+b\arctan\left(cx^2\right)\right)^2 - \frac{3bc\left(a+b\arctan\left(cx^2\right)\right)^2}{4x^2} - \frac{1}{4}c^2\left(a+b\arctan\left(cx^2\right)\right)^3\\
&\quad - \frac{\left(a+b\arctan\left(cx^2\right)\right)^3}{4x^4} + \frac{3}{2}b^2c^2\left(a+b\arctan\left(cx^2\right)\right)\log\left(2-\frac{2}{1-icx^2}\right)\\
&\quad - \frac{3}{4}ib^3c^2\,\text{PolyLog}\left(2, -1+\frac{2}{1-icx^2}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.32

$$\int \frac{(a+b\arctan(cx^2))^3}{x^5}\,dx =$$

$$-\frac{3b^2(a+ac^2x^4+bcx^2(1+icx^2))\arctan(cx^2)^2+b^3(1+c^2x^4)\arctan(cx^2)^3+3b\arctan(cx^2)\Big(a(a+2bcx$$

[In] Integrate[(a + b*ArcTan[c*x^2])^3/x^5,x]

[Out] -1/4*(3*b^2*(a + a*c^2*x^4 + b*c*x^2*(1 + I*c*x^2))*ArcTan[c*x^2]^2 + b^3*(1 + c^2*x^4)*ArcTan[c*x^2]^3 + 3*b*ArcTan[c*x^2]*(a*(a + 2*b*c*x^2 + a*c^2*x^4) - 2*b^2*c^2*x^4*Log[1 - E^((2*I)*ArcTan[c*x^2])]) + a*(a*(a + 3*b*c*x^2) - 6*b^2*c^2*x^4*Log[(c*x^2)/Sqrt[1 + c^2*x^4]]) + (3*I)*b^3*c^2*x^4*PolyLog[2, E^((2*I)*ArcTan[c*x^2])])/x^4

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.42 (sec) , antiderivative size = 465, normalized size of antiderivative = 3.12

method	result
default	$-\frac{a^3}{4x^4}-\frac{b^3\arctan(c\,x^2)^3}{4x^4}-\frac{3b^3c\arctan(c\,x^2)^2}{4x^2}-\frac{b^3\arctan(c\,x^2)^3c^2}{4}+3b^3c^2\arctan(c\,x^2)\ln(x)-\frac{3b^3c^2\arctan(c\,x^2)}{4}$
parts	$-\frac{a^3}{4x^4}-\frac{b^3\arctan(c\,x^2)^3}{4x^4}-\frac{3b^3c\arctan(c\,x^2)^2}{4x^2}-\frac{b^3\arctan(c\,x^2)^3c^2}{4}+3b^3c^2\arctan(c\,x^2)\ln(x)-\frac{3b^3c^2\arctan(c\,x^2)}{4}$

[In] int((a+b*arctan(c*x^2))^3/x^5,x,method=_RETURNVERBOSE)

[Out] -1/4*a^3/x^4-1/4*b^3/x^4*arctan(c*x^2)^3-3/4*b^3*c*arctan(c*x^2)^2/x^2-1/4*b^3*arctan(c*x^2)^3*c^2+3*b^3*c^2*arctan(c*x^2)*ln(x)-3/4*b^3*c^2*arctan(c*x^2)*ln(c^2*x^4+1)+3/16*b^3*c*sum(1/_alpha^2*(2*ln(x-_alpha)*ln(c^2*x^4+1)-c*(1/c/_alpha^3*ln(x-_alpha)^2+2/_alpha*ln(x-_alpha)*(_alpha^2*ln(1/2*(x+_a

```
lpha)/_alpha)*c-ln((_alpha^3*c+x)/_alpha/(_alpha^2*c+1))+ln((_alpha^3*c-x)/
_alpha/(_alpha^2*c-1)))+2/_alpha*(_alpha^2*dilog(1/2*(x+_alpha)/_alpha)*c-d
ilog((_alpha^3*c+x)/_alpha/(_alpha^2*c+1))+dilog((_alpha^3*c-x)/_alpha/(_al
pha^2*c-1))))),_alpha=RootOf(_Z^4*c^2+1))-3/2*b^3*c*sum(1/_R1^2*(ln(x)*ln((
_R1-x)/_R1)+dilog((_R1-x)/_R1)),_R1=RootOf(_Z^4*c^2+1))-3/4*a*b^2/x^4*arcta
n(c*x^2)^2-3/2*a*b^2*c*arctan(c*x^2)/x^2-3/4*a*b^2*arctan(c*x^2)^2*c^2+3*a*
b^2*c^2*ln(x)-3/4*a*b^2*c^2*ln(c^2*x^4+1)-3/4*a^2*b/x^4*arctan(c*x^2)-3/4*a
^2*b*c/x^2-3/4*a^2*b*c^2*arctan(c*x^2)
```

Fricas [F]

$$\int \frac{(a+b\arctan(cx^2))^3}{x^5}\,dx=\int \frac{(b\arctan(cx^2)+a)^3}{x^5}\,dx$$

```
[In] integrate((a+b*arctan(c*x^2))^3/x^5,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(c*x^2)^3 + 3*a*b^2*arctan(c*x^2)^2 + 3*a^2*b*arctan(c*
x^2) + a^3)/x^5, x)
```

Sympy [F]

$$\int \frac{(a+b\arctan(cx^2))^3}{x^5}\,dx=\int \frac{(a+b\,\mathrm{atan}(cx^2))^3}{x^5}\,dx$$

```
[In] integrate((a+b*atan(c*x**2))**3/x**5,x)
```

```
[Out] Integral((a + b*atan(c*x**2))**3/x**5, x)
```

Maxima [F]

$$\int \frac{(a+b\arctan(cx^2))^3}{x^5}\,dx=\int \frac{(b\arctan(cx^2)+a)^3}{x^5}\,dx$$

```
[In] integrate((a+b*arctan(c*x^2))^3/x^5,x, algorithm="maxima")
```

```
[Out] -3/4*((c*arctan(c*x^2) + 1/x^2)*c + arctan(c*x^2)/x^4)*a^2*b + 3/4*((arctan
(c*x^2)^2 - log(c^2*x^4 + 1) + 4*log(x))*c^2 - 2*(c*arctan(c*x^2) + 1/x^2)*
c*arctan(c*x^2))*a*b^2 - 3/4*a*b^2*arctan(c*x^2)^2/x^4 + 1/128*(128*x^4*int
egrate(-1/64*(12*c^2*x^4*arctan(c*x^2)*log(c^2*x^4 + 1) - 12*c*x^2*arctan(c
*x^2)^2 - 56*(c^2*x^4 + 1)*arctan(c*x^2)^3 + 3*(c*x^2 - 2*(c^2*x^4 + 1)*arc
tan(c*x^2))*log(c^2*x^4 + 1)^2)/(c^2*x^9 + x^5), x) - 4*arctan(c*x^2)^3 + 3
*arctan(c*x^2)*log(c^2*x^4 + 1)^2)*b^3/x^4 - 1/4*a^3/x^4
```

Giac [F]

$$\int \frac{(a + b \arctan(cx^2))^3}{x^5} \, dx = \int \frac{(b \arctan(cx^2) + a)^3}{x^5} \, dx$$

```
[In] integrate((a+b*arctan(c*x^2))^3/x^5,x, algorithm="giac")
[Out] integrate((b*arctan(c*x^2) + a)^3/x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b \arctan(cx^2))^3}{x^5} \, dx = \int \frac{(a + b \operatorname{atan}(c x^2))^3}{x^5} \, dx$$

```
[In] int((a + b*atan(c*x^2))^3/x^5,x)
[Out] int((a + b*atan(c*x^2))^3/x^5, x)
```

3.91 $\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)^3 \, dx$

Optimal result 545
Rubi [N/A] 545
Mathematica [N/A] 546
Maple [N/A] (verified) 546
Fricas [N/A] 546
Sympy [F(-1)] 546
Maxima [N/A] 547
Giac [N/A] 547
Mupad [N/A] 547

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)^3 \, dx = \text{Int}\left((dx)^m \left(a+b\arctan\left(cx^2\right)\right)^3, x\right)$$

```
[Out] Unintegrable((d*x)^m*(a+b*arctan(c*x^2))^3,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)^3 \, dx = \int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)^3 \, dx$$

```
[In] Int[(d*x)^m*(a + b*ArcTan[c*x^2])^3,x]
[Out] Defer[Int][(d*x)^m*(a + b*ArcTan[c*x^2])^3, x]
```

Rubi steps

$$\text{integral} = \int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)^3 \, dx$$

Mathematica [N/A]

Not integrable

Time = 2.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)^3 \, dx=\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)^3 \, dx$$

```
[In] Integrate[(d*x)^m*(a + b*ArcTan[c*x^2])^3,x]
[Out] Integrate[(d*x)^m*(a + b*ArcTan[c*x^2])^3, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (dx)^m \left(a+b\arctan\left(c\,x^2\right)\right)^3 dx$$

```
[In] int((d*x)^m*(a+b*arctan(c*x^2))^3,x)
[Out] int((d*x)^m*(a+b*arctan(c*x^2))^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.78

$$\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)^3 \, dx=\int \left(b\arctan\left(cx^2\right)+a\right)^3 (dx)^m \, dx$$

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^2))^3,x, algorithm="fricas")
[Out] integral((b^3*arctan(c*x^2)^3 + 3*a*b^2*arctan(c*x^2)^2 + 3*a^2*b*arctan(c*
x^2) + a^3)*(d*x)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)^3 \, dx=\text{Timed out}$$

```
[In] integrate((d*x)**m*(a+b*atan(c*x**2))**3,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 3.97 (sec) , antiderivative size = 406, normalized size of antiderivative = 22.56

$$\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)^3 \, dx = \int \left(b\arctan\left(cx^2\right)+a\right)^3 (dx)^m \, dx$$

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^2))^3,x, algorithm="maxima")
```

```
[Out] (d*x)^(m + 1)*a^3/(d*(m + 1)) + 1/32*(4*b^3*d^m*x*x^m*arctan(c*x^2)^3 - 3*b
^3*d^m*x*x^m*arctan(c*x^2)*log(c^2*x^4 + 1)^2 + 32*(m + 1)*integrate(1/32*(
24*b^3*c^2*d^m*x^4*x^m*arctan(c*x^2)*log(c^2*x^4 + 1) + 28*(b^3*d^m*m + (b^
3*c^2*d^m*m + b^3*c^2*d^m)*x^4 + b^3*d^m)*x^m*arctan(c*x^2)^3 - 24*(b^3*c*d
^m*x^2 - 4*a*b^2*d^m*m - 4*(a*b^2*c^2*d^m*m + a*b^2*c^2*d^m)*x^4 - 4*a*b^2*
d^m)*x^m*arctan(c*x^2)^2 + 96*(a^2*b*d^m*m + (a^2*b*c^2*d^m*m + a^2*b*c^2*d
^m)*x^4 + a^2*b*d^m)*x^m*arctan(c*x^2) + 3*(2*b^3*c*d^m*x^2*x^m + (b^3*d^m*
m + (b^3*c^2*d^m*m + b^3*c^2*d^m)*x^4 + b^3*d^m)*x^m*arctan(c*x^2))*log(c^2
*x^4 + 1)^2)/((c^2*m + c^2)*x^4 + m + 1), x))/(m + 1)
```

Giac [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)^3 \, dx = \int \left(b\arctan\left(cx^2\right)+a\right)^3 (dx)^m \, dx$$

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^2))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^2) + a)^3*(d*x)^m, x)
```

Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)^3 \, dx = \int (d\,x)^m \left(a+b\,\mathrm{atan}\left(c\,x^2\right)\right)^3 dx$$

```
[In] int((d*x)^m*(a + b*atan(c*x^2))^3,x)
```

```
[Out] int((d*x)^m*(a + b*atan(c*x^2))^3, x)
```

3.92 $\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)^2 \, dx$

Optimal result . . . 548
Rubi [N/A] . . . 548
Mathematica [N/A] . . . 549
Maple [N/A] (verified) . . . 549
Fricas [N/A] . . . 549
Sympy [N/A] . . . 549
Maxima [N/A] . . . 550
Giac [N/A] . . . 550
Mupad [N/A] . . . 550

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)^2 \, dx = \text{Int}\left((dx)^m \left(a+b\arctan\left(cx^2\right)\right)^2, x\right)$$

```
[Out] Unintegrable((d*x)^m*(a+b*arctan(c*x^2))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)^2 \, dx = \int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)^2 \, dx$$

```
[In] Int[(d*x)^m*(a + b*ArcTan[c*x^2])^2,x]
```

```
[Out] Defer[Int][(d*x)^m*(a + b*ArcTan[c*x^2])^2, x]
```

Rubi steps

$$\text{integral} = \int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)^2 \, dx$$

Mathematica [N/A]

Not integrable

Time = 1.47 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)^2\,dx=\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)^2\,dx$$

```
[In] Integrate[(d*x)^m*(a + b*ArcTan[c*x^2])^2,x]
[Out] Integrate[(d*x)^m*(a + b*ArcTan[c*x^2])^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (dx)^m \left(a+b\arctan\left(c\,x^2\right)\right)^2 dx$$

```
[In] int((d*x)^m*(a+b*arctan(c*x^2))^2,x)
[Out] int((d*x)^m*(a+b*arctan(c*x^2))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.89

$$\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)^2\,dx=\int \left(b\arctan\left(cx^2\right)+a\right)^2(dx)^m\,dx$$

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^2))^2,x, algorithm="fricas")
[Out] integral((b^2*arctan(c*x^2)^2 + 2*a*b*arctan(c*x^2) + a^2)*(d*x)^m, x)
```

Sympy [N/A]

Not integrable

Time = 111.97 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)^2\,dx=\int (dx)^m \left(a+b\,\mathrm{atan}\left(cx^2\right)\right)^2\,dx$$

```
[In] integrate((d*x)**m*(a+b*atan(c*x**2))**2,x)
[Out] Integral((d*x)**m*(a + b*atan(c*x**2))**2, x)
```

Maxima [N/A]

Not integrable

Time = 2.35 (sec) , antiderivative size = 303, normalized size of antiderivative = 16.83

$$\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)^2 \, dx = \int \left(b\arctan\left(cx^2\right)+a\right)^2 (dx)^m \, dx$$

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^2))^2,x, algorithm="maxima")
```

```
[Out] (d*x)^(m + 1)*a^2/(d*(m + 1)) + 1/16*(4*b^2*d^m*x*x^m*arctan(c*x^2)^2 - b^2
*d^m*x*x^m*log(c^2*x^4 + 1)^2 + 16*(m + 1)*integrate(1/16*(8*b^2*c^2*d^m*x^
4*x^m*log(c^2*x^4 + 1) + 12*((b^2*c^2*d^m*m + b^2*c^2*d^m)*x^4 + b^2*d^m*m
+ b^2*d^m)*x^m*arctan(c*x^2)^2 + ((b^2*c^2*d^m*m + b^2*c^2*d^m)*x^4 + b^2*d
^m*m + b^2*d^m)*x^m*log(c^2*x^4 + 1)^2 - 16*(b^2*c*d^m*x^2 - 2*(a*b*c^2*d^m
*m + a*b*c^2*d^m)*x^4 - 2*a*b*d^m*m - 2*a*b*d^m)*x^m*arctan(c*x^2))/((c^2*m
 + c^2)*x^4 + m + 1), x))/(m + 1)
```

Giac [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)^2 \, dx = \int \left(b\arctan\left(cx^2\right)+a\right)^2 (dx)^m \, dx$$

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^2))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^2) + a)^2*(d*x)^m, x)
```

Mupad [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)^2 \, dx = \int {\left(d\,x\right)}^m\,{\left(a+b\,\mathrm{atan}\left(c\,x^2\right)\right)}^2 \,d x$$

```
[In] int((d*x)^m*(a + b*atan(c*x^2))^2,x)
```

```
[Out] int((d*x)^m*(a + b*atan(c*x^2))^2, x)
```

3.93 $\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)\,dx$

Optimal result 551
Rubi [A] (verified) 551
Mathematica [A] (verified) 552
Maple [F] 552
Fricas [F] 553
Sympy [F] 553
Maxima [F] 553
Giac [F] 553
Mupad [F(-1)] 554

Optimal result

Integrand size = 16, antiderivative size = 75

$$\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)\,dx=\frac{(dx)^{1+m}\left(a+b\arctan\left(cx^2\right)\right)}{d(1+m)}-\frac{2bc(dx)^{3+m}\,\text{Hypergeometric2F1}\left(1,\frac{3+m}{4},\frac{7+m}{4},-c^2x^4\right)}{d^3(1+m)(3+m)}$$

```
[Out] (d*x)^(1+m)*(a+b*arctan(c*x^2))/d/(1+m)-2*b*c*(d*x)^(3+m)*hypergeom([1, 3/4
+1/4*m],[7/4+1/4*m],-c^2*x^4)/d^3/(1+m)/(3+m)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4958, 371}

$$\int (dx)^m \left(a+b\arctan\left(cx^2\right)\right)\,dx=\frac{(dx)^{m+1}\left(a+b\arctan\left(cx^2\right)\right)}{d(m+1)}-\frac{2bc(dx)^{m+3}\,\text{Hypergeometric2F1}\left(1,\frac{m+3}{4},\frac{m+7}{4},-c^2x^4\right)}{d^3(m+1)(m+3)}$$

```
[In] Int[(d*x)^m*(a + b*ArcTan[c*x^2]),x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*ArcTan[c*x^2]))/(d*(1 + m)) - (2*b*c*(d*x)^(3 + m)*Hy
pergeometric2F1[1, (3 + m)/4, (7 + m)/4, -(c^2*x^4)])/(d^3*(1 + m)*(3 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4958

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))*((d_)*(x_))^(m_), x_Symbol] :>
 Simp[(d*x)^(m + 1)*((a + b*ArcTan[c*x^n])/(d*(m + 1))), x] - Dist[b*c*(n/(
d^n*(m + 1))), Int[(d*x)^(m + n)/(1 + c^2*x^(2*n)), x], x] /; FreeQ[{a, b,
c, d, m, n}, x] && IntegerQ[n] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(dx)^{1+m}(a+b\arctan(cx^2))}{d(1+m)} - \frac{(2bc)\int\frac{(dx)^{2+m}}{1+c^2x^4}\,dx}{d^2(1+m)}\\ &= \frac{(dx)^{1+m}(a+b\arctan(cx^2))}{d(1+m)} - \frac{2bc(dx)^{3+m}\,\text{Hypergeometric2F1}\left(1,\frac{3+m}{4},\frac{7+m}{4},-c^2x^4\right)}{d^3(1+m)(3+m)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.87

$$\int (dx)^m\left(a+b\arctan\left(cx^2\right)\right)\,dx = -\frac{x(dx)^m\left(-((3+m)(a+b\arctan(cx^2)))+2bcx^2\,\text{Hypergeometric2F1}\left(1,\frac{3+m}{4},\frac{7+m}{4},-c^2x^4\right)\right)}{(1+m)(3+m)}$$

```
[In] Integrate[(d*x)^m*(a + b*ArcTan[c*x^2]),x]
[Out] -((x*(d*x)^m*(-((3 + m)*(a + b*ArcTan[c*x^2])) + 2*b*c*x^2*Hypergeometric2F
1[1, (3 + m)/4, (7 + m)/4, -(c^2*x^4)]))/((1 + m)*(3 + m)))
```

Maple [F]

$$\int (dx)^m\left(a+b\arctan\left(c\,x^2\right)\right)dx$$

```
[In] int((d*x)^m*(a+b*arctan(c*x^2)),x)
[Out] int((d*x)^m*(a+b*arctan(c*x^2)),x)
```

Fricas [F]

$$\int (dx)^m \left(a + b\arctan\left(cx^2\right)\right) \, dx = \int \left(b\arctan\left(cx^2\right) + a\right)(dx)^m \, dx$$

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^2)),x, algorithm="fricas")
[Out] integral((b*arctan(c*x^2) + a)*(d*x)^m, x)
```

Sympy [F]

$$\int (dx)^m \left(a + b\arctan\left(cx^2\right)\right) \, dx = \int (dx)^m \left(a + b\operatorname{atan}\left(cx^2\right)\right) \, dx$$

```
[In] integrate((d*x)**m*(a+b*atan(c*x**2)),x)
[Out] Integral((d*x)**m*(a + b*atan(c*x**2)), x)
```

Maxima [F]

$$\int (dx)^m \left(a + b\arctan\left(cx^2\right)\right) \, dx = \int \left(b\arctan\left(cx^2\right) + a\right)(dx)^m \, dx$$

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^2)),x, algorithm="maxima")
[Out] (d^m*x*x^m*arctan(c*x^2) - 2*(c*d^m*m + c*d^m)*integrate(x^2*x^m/((c^2*m +
c^2)*x^4 + m + 1), x))*b/(m + 1) + (d*x)^(m + 1)*a/(d*(m + 1))
```

Giac [F]

$$\int (dx)^m \left(a + b\arctan\left(cx^2\right)\right) \, dx = \int \left(b\arctan\left(cx^2\right) + a\right)(dx)^m \, dx$$

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^2)),x, algorithm="giac")
[Out] integrate((b*arctan(c*x^2) + a)*(d*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m \left(a+b \arctan \left(cx^2\right)\right) \, dx = \int (d\,x)^m \, \left(a+b\,\mathrm{atan}\left(c\,x^2\right)\right) \, dx$$

```
[In] int((d*x)^m*(a + b*atan(c*x^2)),x)
[Out] int((d*x)^m*(a + b*atan(c*x^2)), x)
```

3.94 $\int \frac{(dx)^m}{a+b\arctan(cx^2)}\,dx$

Optimal result 555
Rubi [N/A] 555
Mathematica [N/A] 556
Maple [N/A] (verified) 556
Fricas [N/A] 556
Sympy [F(-1)] 556
Maxima [N/A] 557
Giac [N/A] 557
Mupad [N/A] 557

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{(dx)^m}{a+b\arctan(cx^2)}\,dx = \operatorname{Int}\left(\frac{(dx)^m}{a+b\arctan(cx^2)},x\right)$$

```
[Out] Unintegrable((d*x)^m/(a+b*arctan(c*x^2)),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(dx)^m}{a+b\arctan(cx^2)}\,dx = \int \frac{(dx)^m}{a+b\arctan(cx^2)}\,dx$$

```
[In] Int[(d*x)^m/(a + b*ArcTan[c*x^2]),x]
```

```
[Out] Defer[Int][(d*x)^m/(a + b*ArcTan[c*x^2]), x]
```

Rubi steps

$$\text{integral} = \int \frac{(dx)^m}{a+b\arctan(cx^2)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{a+b\arctan(cx^2)}\,dx=\int \frac{(dx)^m}{a+b\arctan(cx^2)}\,dx$$

```
[In] Integrate[(d*x)^m/(a + b*ArcTan[c*x^2]),x]
[Out] Integrate[(d*x)^m/(a + b*ArcTan[c*x^2]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^m}{a+b\arctan(c\,x^2)}dx$$

```
[In] int((d*x)^m/(a+b*arctan(c*x^2)),x)
[Out] int((d*x)^m/(a+b*arctan(c*x^2)),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{a+b\arctan(cx^2)}\,dx=\int \frac{(dx)^m}{b\arctan(cx^2)+a}\,dx$$

```
[In] integrate((d*x)^m/(a+b*arctan(c*x^2)),x, algorithm="fricas")
[Out] integral((d*x)^m/(b*arctan(c*x^2) + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{a+b\arctan(cx^2)}\,dx=\text{Timed out}$$

```
[In] integrate((d*x)**m/(a+b*atan(c*x**2)),x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{a + b\arctan(cx^2)}\,dx = \int \frac{(dx)^m}{b\arctan(cx^2) + a}\,dx$$

```
[In] integrate((d*x)^m/(a+b*arctan(c*x^2)),x, algorithm="maxima")
[Out] integrate((d*x)^m/(b*arctan(c*x^2) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{a + b\arctan(cx^2)}\,dx = \int \frac{(dx)^m}{b\arctan(cx^2) + a}\,dx$$

```
[In] integrate((d*x)^m/(a+b*arctan(c*x^2)),x, algorithm="giac")
[Out] integrate((d*x)^m/(b*arctan(c*x^2) + a), x)
```

Mupad [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{a + b\arctan(cx^2)}\,dx = \int \frac{(d\,x)^m}{a + b\,\mathrm{atan}(c\,x^2)}\,dx$$

```
[In] int((d*x)^m/(a + b*atan(c*x^2)),x)
[Out] int((d*x)^m/(a + b*atan(c*x^2)), x)
```

3.95 $\int \frac{(dx)^m}{(a+b\arctan(cx^2))^2}\,dx$

Optimal result 558
Rubi [N/A] 558
Mathematica [N/A] 559
Maple [N/A] (verified) 559
Fricas [N/A] 559
Sympy [F(-1)] 559
Maxima [N/A] 560
Giac [N/A] 560
Mupad [N/A] 560

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{(dx)^m}{(a+b\arctan(cx^2))^2}\,dx = \text{Int}\left(\frac{(dx)^m}{(a+b\arctan(cx^2))^2},x\right)$$

```
[Out] Unintegrable((d*x)^m/(a+b*arctan(c*x^2))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(dx)^m}{(a+b\arctan(cx^2))^2}\,dx = \int \frac{(dx)^m}{(a+b\arctan(cx^2))^2}\,dx$$

```
[In] Int[(d*x)^m/(a + b*ArcTan[c*x^2])^2,x]
[Out] Defer[Int][(d*x)^m/(a + b*ArcTan[c*x^2])^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{(dx)^m}{(a+b\arctan(cx^2))^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{(a+b\arctan(cx^2))^2}\,dx = \int \frac{(dx)^m}{(a+b\arctan(cx^2))^2}\,dx$$

```
[In] Integrate[(d*x)^m/(a + b*ArcTan[c*x^2])^2,x]
[Out] Integrate[(d*x)^m/(a + b*ArcTan[c*x^2])^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^m}{(a+b\arctan(c\,x^2))^2}dx$$

```
[In] int((d*x)^m/(a+b*arctan(c*x^2))^2,x)
[Out] int((d*x)^m/(a+b*arctan(c*x^2))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.00

$$\int \frac{(dx)^m}{(a+b\arctan(cx^2))^2}\,dx = \int \frac{(dx)^m}{(b\arctan(cx^2)+a)^2}\,dx$$

```
[In] integrate((d*x)^m/(a+b*arctan(c*x^2))^2,x, algorithm="fricas")
[Out] integral((d*x)^m/(b^2*arctan(c*x^2)^2 + 2*a*b*arctan(c*x^2) + a^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{(a+b\arctan(cx^2))^2}\,dx = \text{Timed out}$$

```
[In] integrate((d*x)**m/(a+b*atan(c*x**2))**2,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 124, normalized size of antiderivative = 6.89

$$\int \frac{(dx)^m}{(a+b\arctan(cx^2))^2}\,dx = \int \frac{(dx)^m}{(b\arctan(cx^2)+a)^2}\,dx$$

```
[In] integrate((d*x)^m/(a+b*arctan(c*x^2))^2,x, algorithm="maxima")
```

```
[Out] -1/2*((c^2*d^m*x^4 + d^m)*x^m - 2*(b^2*c*x*arctan(c*x^2) + a*b*c*x)*integra
te(1/2*((c^2*d^m*m + 3*c^2*d^m)*x^4 + d^m*m - d^m)*x^m/(b^2*c*x^2*arctan(c*
x^2) + a*b*c*x^2), x))/(b^2*c*x*arctan(c*x^2) + a*b*c*x)
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{(a+b\arctan(cx^2))^2}\,dx = \int \frac{(dx)^m}{(b\arctan(cx^2)+a)^2}\,dx$$

```
[In] integrate((d*x)^m/(a+b*arctan(c*x^2))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b*arctan(c*x^2) + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{(a+b\arctan(cx^2))^2}\,dx = \int \frac{(d\,x)^m}{(a+b\,\mathrm{atan}(c\,x^2))^2}\,dx$$

```
[In] int((d*x)^m/(a + b*atan(c*x^2))^2,x)
```

```
[Out] int((d*x)^m/(a + b*atan(c*x^2))^2, x)
```

3.96 $\int x^{11}(a+b\arctan(cx^3))\,dx$

Optimal result . . . 561
Rubi [A] (verified) . . . 561
Mathematica [A] (verified) . . . 562
Maple [A] (verified) . . . 563
Fricas [A] (verification not implemented) . . . 563
Sympy [A] (verification not implemented) . . . 563
Maxima [A] (verification not implemented) . . . 564
Giac [A] (verification not implemented) . . . 564
Mupad [B] (verification not implemented) . . . 564

Optimal result

Integrand size = 14, antiderivative size = 54

$$\int x^{11}(a+b\arctan(cx^3))\,dx=\frac{bx^3}{12c^3}-\frac{bx^9}{36c}-\frac{b\arctan(cx^3)}{12c^4}+\frac{1}{12}x^{12}(a+b\arctan(cx^3))$$

```
[Out] 1/12*b*x^3/c^3-1/36*b*x^9/c-1/12*b*arctan(c*x^3)/c^4+1/12*x^12*(a+b*arctan(
c*x^3))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4946, 281, 308, 209}

$$\int x^{11}(a+b\arctan(cx^3))\,dx=\frac{1}{12}x^{12}(a+b\arctan(cx^3))-\frac{b\arctan(cx^3)}{12c^4}+\frac{bx^3}{12c^3}-\frac{bx^9}{36c}$$

```
[In] Int[x^11*(a + b*ArcTan[c*x^3]),x]
```

```
[Out] (b*x^3)/(12*c^3) - (b*x^9)/(36*c) - (b*ArcTan[c*x^3])/(12*c^4) + (x^12*(a +
 b*ArcTan[c*x^3]))/12
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{12}x^{12}\left(a+b\arctan\left(cx^3\right)\right)-\frac{1}{4}(bc)\int\frac{x^{14}}{1+c^2x^6}\,dx\\
&= \frac{1}{12}x^{12}\left(a+b\arctan\left(cx^3\right)\right)-\frac{1}{12}(bc)\text{Subst}\left(\int\frac{x^4}{1+c^2x^2}\,dx,x,x^3\right)\\
&= \frac{1}{12}x^{12}\left(a+b\arctan\left(cx^3\right)\right)-\frac{1}{12}(bc)\text{Subst}\left(\int\left(-\frac{1}{c^4}+\frac{x^2}{c^2}+\frac{1}{c^4\left(1+c^2x^2\right)}\right)\,dx,x,x^3\right)\\
&= \frac{bx^3}{12c^3}-\frac{bx^9}{36c}+\frac{1}{12}x^{12}\left(a+b\arctan\left(cx^3\right)\right)-\frac{b\text{Subst}\left(\int\frac{1}{1+c^2x^2}\,dx,x,x^3\right)}{12c^3}\\
&= \frac{bx^3}{12c^3}-\frac{bx^9}{36c}-\frac{b\arctan\left(cx^3\right)}{12c^4}+\frac{1}{12}x^{12}\left(a+b\arctan\left(cx^3\right)\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.09

$$
\int x^{11}\left(a+b\arctan\left(cx^3\right)\right)\,dx=\frac{bx^3}{12c^3}-\frac{bx^9}{36c}+\frac{ax^{12}}{12}-\frac{b\arctan\left(cx^3\right)}{12c^4}+\frac{1}{12}bx^{12}\arctan\left(cx^3\right)
$$

```
[In] Integrate[x^11*(a + b*ArcTan[c*x^3]),x]
[Out] (b*x^3)/(12*c^3) - (b*x^9)/(36*c) + (a*x^12)/12 - (b*ArcTan[c*x^3])/(12*c^4
) + (b*x^12*ArcTan[c*x^3])/12
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{a\,x^{12}}{12}+\frac{b\,x^{12}\arctan\left(c\,x^{3}\right)}{12}-\frac{b\,x^{9}}{36c}+\frac{b\,x^{3}}{12c^{3}}-\frac{b\arctan\left(c\,x^{3}\right)}{12c^{4}}$	50
parts	$\frac{a\,x^{12}}{12}+\frac{b\,x^{12}\arctan\left(c\,x^{3}\right)}{12}-\frac{b\,x^{9}}{36c}+\frac{b\,x^{3}}{12c^{3}}-\frac{b\arctan\left(c\,x^{3}\right)}{12c^{4}}$	50
parallelrisch	$\frac{3b\arctan\left(c\,x^{3}\right)x^{12}c^{4}+3a\,c^{4}x^{12}-b\,c^{3}x^{9}+3bc\,x^{3}-3b\arctan\left(c\,x^{3}\right)}{36c^{4}}$	56
risch	$-\frac{i x^{12} b\ln\left(i c\,x^{3}+1\right)}{24}+\frac{i x^{12} b\ln\left(-i c\,x^{3}+1\right)}{24}+\frac{a\,x^{12}}{12}-\frac{b\,x^{9}}{36c}+\frac{b\,x^{3}}{12c^{3}}-\frac{b\arctan\left(c\,x^{3}\right)}{12c^{4}}$	72

```
[In] int(x^11*(a+b*arctan(c*x^3)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*a*x^12+1/12*b*x^12*arctan(c*x^3)-1/36*b*x^9/c+1/12*b*x^3/c^3-1/12*b*ar
ctan(c*x^3)/c^4
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

$$\int x^{11}(a+b\arctan(cx^3))\,dx=\frac{3\,ac^4x^{12}-bc^3x^9+3\,bcx^3+3\,(bc^4x^{12}-b)\arctan(cx^3)}{36\,c^4}$$

```
[In] integrate(x^11*(a+b*arctan(c*x^3)),x, algorithm="fricas")
```

```
[Out] 1/36*(3*a*c^4*x^12 - b*c^3*x^9 + 3*b*c*x^3 + 3*(b*c^4*x^12 - b)*arctan(c*x^
3))/c^4
```

Sympy [A] (verification not implemented)

Time = 129.73 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.07

$$\int x^{11}(a+b\arctan(cx^3))\,dx=\begin{cases}\frac{ax^{12}}{12}+\frac{bx^{12}\operatorname{atan}(cx^3)}{12}-\frac{bx^9}{36c}+\frac{bx^3}{12c^3}-\frac{b\operatorname{atan}(cx^3)}{12c^4} & \text{for}\ c\neq 0\\ \frac{ax^{12}}{12} & \text{otherwise}\end{cases}$$

```
[In] integrate(x**11*(a+b*atan(c*x**3)),x)
```

```
[Out] Piecewise((a*x**12/12 + b*x**12*atan(c*x**3)/12 - b*x**9/(36*c) + b*x**3/(1
2*c**3) - b*atan(c*x**3)/(12*c**4), Ne(c, 0)), (a*x**12/12, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int x^{11}(a+b\arctan(cx^3))\,dx = \frac{1}{12}ax^{12}+\frac{1}{36}\left(3x^{12}\arctan(cx^3)-c\left(\frac{c^2x^9-3x^3}{c^4}+\frac{3\arctan(cx^3)}{c^5}\right)\right)b$$

```
[In] integrate(x^11*(a+b*arctan(c*x^3)),x, algorithm="maxima")
[Out] 1/12*a*x^12 + 1/36*(3*x^12*arctan(c*x^3) - c*((c^2*x^9 - 3*x^3)/c^4 + 3*arc
tan(c*x^3)/c^5))*b
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.11

$$\int x^{11}(a+b\arctan(cx^3))\,dx = \frac{3\,acx^{12}+\left(3\,cx^{12}\arctan(cx^3)-\frac{3\arctan(cx^3)}{c^3}-\frac{c^9x^9-3\,c^7x^3}{c^9}\right)b}{36\,c}$$

```
[In] integrate(x^11*(a+b*arctan(c*x^3)),x, algorithm="giac")
[Out] 1/36*(3*a*c*x^12 + (3*c*x^12*arctan(c*x^3) - 3*arctan(c*x^3)/c^3 - (c^9*x^9
 - 3*c^7*x^3)/c^9)*b)/c
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

$$\int x^{11}(a+b\arctan(cx^3))\,dx = \frac{a\,x^{12}}{12}+\frac{b\,x^3}{12\,c^3}-\frac{b\,x^9}{36\,c}-\frac{b\,\mathrm{atan}(c\,x^3)}{12\,c^4}+\frac{b\,x^{12}\,\mathrm{atan}(c\,x^3)}{12}$$

```
[In] int(x^11*(a + b*atan(c*x^3)),x)
[Out] (a*x^12)/12 + (b*x^3)/(12*c^3) - (b*x^9)/(36*c) - (b*atan(c*x^3))/(12*c^4)
+ (b*x^12*atan(c*x^3))/12
```

3.97 $\int x^8(a+b\arctan(cx^3))\,dx$

Optimal result 565
Rubi [A] (verified) 565
Mathematica [A] (verified) 566
Maple [A] (verified) 566
Fricas [A] (verification not implemented) 567
Sympy [B] (verification not implemented) 567
Maxima [A] (verification not implemented) 568
Giac [A] (verification not implemented) 568
Mupad [B] (verification not implemented) 568

Optimal result

Integrand size = 14, antiderivative size = 47

$$\int x^8(a+b\arctan(cx^3))\,dx=-\frac{bx^6}{18c}+\frac{1}{9}x^9(a+b\arctan(cx^3))+\frac{b\log(1+c^2x^6)}{18c^3}$$

```
[Out] -1/18*b*x^6/c+1/9*x^9*(a+b*arctan(c*x^3))+1/18*b*ln(c^2*x^6+1)/c^3
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4946, 272, 45}

$$\int x^8(a+b\arctan(cx^3))\,dx=\frac{1}{9}x^9(a+b\arctan(cx^3))+\frac{b\log(c^2x^6+1)}{18c^3}-\frac{bx^6}{18c}$$

```
[In] Int[x^8*(a + b*ArcTan[c*x^3]),x]
```

```
[Out] -1/18*(b*x^6)/c + (x^9*(a + b*ArcTan[c*x^3]))/9 + (b*Log[1 + c^2*x^6])/(18*
c^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{9}x^9\left(a+b\arctan\left(cx^3\right)\right)-\frac{1}{3}(bc)\int\frac{x^{11}}{1+c^2x^6}\,dx\\
&= \frac{1}{9}x^9\left(a+b\arctan\left(cx^3\right)\right)-\frac{1}{18}(bc)\text{Subst}\left(\int\frac{x}{1+c^2x}\,dx,x,x^6\right)\\
&= \frac{1}{9}x^9\left(a+b\arctan\left(cx^3\right)\right)-\frac{1}{18}(bc)\text{Subst}\left(\int\left(\frac{1}{c^2}-\frac{1}{c^2\left(1+c^2x\right)}\right)\,dx,x,x^6\right)\\
&= -\frac{bx^6}{18c}+\frac{1}{9}x^9\left(a+b\arctan\left(cx^3\right)\right)+\frac{b\log\left(1+c^2x^6\right)}{18c^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.11

$$\int x^8\left(a+b\arctan\left(cx^3\right)\right)\,dx=-\frac{bx^6}{18c}+\frac{ax^9}{9}+\frac{1}{9}bx^9\arctan\left(cx^3\right)+\frac{b\log\left(1+c^2x^6\right)}{18c^3}$$

```
[In] Integrate[x^8*(a + b*ArcTan[c*x^3]),x]
[Out] -1/18*(b*x^6)/c + (a*x^9)/9 + (b*x^9*ArcTan[c*x^3])/9 + (b*Log[1 + c^2*x^6]
)/(18*c^3)
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96

method	result	size
default	$\frac{x^{9} a}{9}+\frac{b\,x^{9} \arctan\left(c\,x^{3}\right)}{9}-\frac{b\,x^{6}}{18 c}+\frac{b \ln\left(c^{2} x^{6}+1\right)}{18 c^{3}}$	45
parts	$\frac{x^{9} a}{9}+\frac{b\,x^{9} \arctan\left(c\,x^{3}\right)}{9}-\frac{b\,x^{6}}{18 c}+\frac{b \ln\left(c^{2} x^{6}+1\right)}{18 c^{3}}$	45
parallelrisch	$\frac{2 x^{9} \arctan\left(c\,x^{3}\right) b\,c^{3}+2 a\,c^{3} x^{9}-c^{2} b\,x^{6}+b \ln\left(c^{2} x^{6}+1\right)}{18 c^{3}}$	52
risch	$-\frac{i x^{9} b \ln\left(i c\,x^{3}+1\right)}{18}+\frac{i x^{9} b \ln\left(-i c\,x^{3}+1\right)}{18}+\frac{x^{9} a}{9}-\frac{b\,x^{6}}{18 c}+\frac{b \ln\left(-c^{2} x^{6}-1\right)}{18 c^{3}}$	68

```
[In] int(x^8*(a+b*arctan(c*x^3)),x,method=_RETURNVERBOSE)
[Out] 1/9*x^9*a+1/9*b*x^9*arctan(c*x^3)-1/18*b*x^6/c+1/18*b*ln(c^2*x^6+1)/c^3
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.09

$$\int x^8\left(a+b\arctan\left(cx^3\right)\right)\,dx=\frac{2\,bc^3x^9\arctan\left(cx^3\right)+2\,ac^3x^9-bc^2x^6+b\log\left(c^2x^6+1\right)}{18\,c^3}$$

```
[In] integrate(x^8*(a+b*arctan(c*x^3)),x, algorithm="fricas")
[Out] 1/18*(2*b*c^3*x^9*arctan(c*x^3) + 2*a*c^3*x^9 - b*c^2*x^6 + b*log(c^2*x^6 +
 1))/c^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(39) = 78.

Time = 73.04 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.49

$$\int x^8\left(a+b\arctan\left(cx^3\right)\right)\,dx$$
$$=\begin{cases}\frac{ax^9}{9}+\frac{bx^9\operatorname{atan}\left(cx^3\right)}{9}-\frac{bx^6}{18c}-\frac{b\sqrt{-\frac{1}{c^2}}\operatorname{atan}\left(cx^3\right)}{9c^2}+\frac{b\log\left(x-\sqrt[6]{-\frac{1}{c^2}}\right)}{9c^3}+\frac{b\log\left(4x^2+4x\sqrt[6]{-\frac{1}{c^2}}+4\sqrt[3]{-\frac{1}{c^2}}\right)}{9c^3} & \text{for } c\neq 0\\ \frac{ax^9}{9} & \text{otherwise}\end{cases}$$

```
[In] integrate(x**8*(a+b*atan(c*x**3)),x)
[Out] Piecewise((a*x**9/9 + b*x**9*atan(c*x**3)/9 - b*x**6/(18*c) - b*sqrt(-1/c**
2)*atan(c*x**3)/(9*c**2) + b*log(x - (-1/c**2)**(1/6))/(9*c**3) + b*log(4*x
**2 + 4*x*(-1/c**2)**(1/6) + 4*(-1/c**2)**(1/3))/(9*c**3), Ne(c, 0)), (a*x*
*9/9, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

$$\int x^8(a+b\arctan(cx^3))\,dx=\frac{1}{9}ax^9+\frac{1}{18}\left(2x^9\arctan(cx^3)-\left(\frac{x^6}{c^2}-\frac{\log(c^2x^6+1)}{c^4}\right)c\right)b$$

```
[In] integrate(x^8*(a+b*arctan(c*x^3)),x, algorithm="maxima")
[Out] 1/9*a*x^9 + 1/18*(2*x^9*arctan(c*x^3) - (x^6/c^2 - log(c^2*x^6 + 1)/c^4)*c)
*b
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int x^8(a+b\arctan(cx^3))\,dx=\frac{2acx^9+\left(2cx^9\arctan(cx^3)-x^6+\frac{\log(c^2x^6+1)}{c^2}\right)b}{18c}$$

```
[In] integrate(x^8*(a+b*arctan(c*x^3)),x, algorithm="giac")
[Out] 1/18*(2*a*c*x^9 + (2*c*x^9*arctan(c*x^3) - x^6 + log(c^2*x^6 + 1)/c^2)*b)/c
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

$$\int x^8(a+b\arctan(cx^3))\,dx=\frac{a\,x^9}{9}+\frac{b\,\ln(c^2\,x^6+1)}{18\,c^3}-\frac{b\,x^6}{18\,c}+\frac{b\,x^9\,\mathrm{atan}(c\,x^3)}{9}$$

```
[In] int(x^8*(a + b*atan(c*x^3)),x)
[Out] (a*x^9)/9 + (b*log(c^2*x^6 + 1))/(18*c^3) - (b*x^6)/(18*c) + (b*x^9*atan(c*
x^3))/9
```

3.98 $\int x^5(a+b\arctan(cx^3))\,dx$

Optimal result 569
Rubi [A] (verified) 569
Mathematica [A] (verified) 570
Maple [A] (verified) 571
Fricas [A] (verification not implemented) 571
Sympy [A] (verification not implemented) 571
Maxima [A] (verification not implemented) 572
Giac [A] (verification not implemented) 572
Mupad [B] (verification not implemented) 572

Optimal result

Integrand size = 14, antiderivative size = 43

$$\int x^5(a+b\arctan(cx^3))\,dx=-\frac{bx^3}{6c}+\frac{b\arctan(cx^3)}{6c^2}+\frac{1}{6}x^6(a+b\arctan(cx^3))$$

```
[Out] -1/6*b*x^3/c+1/6*b*arctan(c*x^3)/c^2+1/6*x^6*(a+b*arctan(c*x^3))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4946, 281, 327, 209}

$$\int x^5(a+b\arctan(cx^3))\,dx=\frac{1}{6}x^6(a+b\arctan(cx^3))+\frac{b\arctan(cx^3)}{6c^2}-\frac{bx^3}{6c}$$

```
[In] Int[x^5*(a + b*ArcTan[c*x^3]),x]
```

```
[Out] -1/6*(b*x^3)/c + (b*ArcTan[c*x^3])/(6*c^2) + (x^6*(a + b*ArcTan[c*x^3]))/6
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{6}x^6\left(a+b\arctan\left(cx^3\right)\right)-\frac{1}{2}(bc)\int\frac{x^8}{1+c^2x^6}\,dx\\
&= \frac{1}{6}x^6\left(a+b\arctan\left(cx^3\right)\right)-\frac{1}{6}(bc)\text{Subst}\left(\int\frac{x^2}{1+c^2x^2}\,dx,x,x^3\right)\\
&= -\frac{bx^3}{6c}+\frac{1}{6}x^6\left(a+b\arctan\left(cx^3\right)\right)+\frac{b\text{Subst}\left(\int\frac{1}{1+c^2x^2}\,dx,x,x^3\right)}{6c}\\
&= -\frac{bx^3}{6c}+\frac{b\arctan\left(cx^3\right)}{6c^2}+\frac{1}{6}x^6\left(a+b\arctan\left(cx^3\right)\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.12

$$\int x^5\left(a+b\arctan\left(cx^3\right)\right)\,dx=-\frac{bx^3}{6c}+\frac{ax^6}{6}+\frac{b\arctan\left(cx^3\right)}{6c^2}+\frac{1}{6}bx^6\arctan\left(cx^3\right)$$

```
[In] Integrate[x^5*(a + b*ArcTan[c*x^3]),x]
```

```
[Out] -1/6*(b*x^3)/c + (a*x^6)/6 + (b*ArcTan[c*x^3])/(6*c^2) + (b*x^6*ArcTan[c*x^
3])/6
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{a\,x^{6}}{6}+\frac{b\,x^{6}\arctan\left(c\,x^{3}\right)}{6}-\frac{b\,x^{3}}{6c}+\frac{b\arctan\left(c\,x^{3}\right)}{6c^{2}}$	41
parts	$\frac{a\,x^{6}}{6}+\frac{b\,x^{6}\arctan\left(c\,x^{3}\right)}{6}-\frac{b\,x^{3}}{6c}+\frac{b\arctan\left(c\,x^{3}\right)}{6c^{2}}$	41
parallelrisch	$\frac{\arctan\left(c\,x^{3}\right)b\,c^{2}x^{6}+a\,c^{2}x^{6}-bc\,x^{3}+b\arctan\left(c\,x^{3}\right)}{6c^{2}}$	44
risch	$-\frac{i x^{6} b\ln\left(i c\,x^{3}+1\right)}{12}+\frac{i x^{6} b\ln\left(-i c\,x^{3}+1\right)}{12}+\frac{a\,x^{6}}{6}-\frac{b\,x^{3}}{6c}+\frac{b\arctan\left(c\,x^{3}\right)}{6c^{2}}+\frac{b^{2}}{24a\,c^{2}}$	74

```
[In] int(x^5*(a+b*arctan(c*x^3)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*a*x^6+1/6*b*x^6*arctan(c*x^3)-1/6*b*x^3/c+1/6*b*arctan(c*x^3)/c^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

$$\int x^5(a+b\arctan(cx^3))\,dx=\frac{ac^2x^6-bcx^3+(bc^2x^6+b)\arctan(cx^3)}{6\,c^2}$$

```
[In] integrate(x^5*(a+b*arctan(c*x^3)),x, algorithm="fricas")
```

```
[Out] 1/6*(a*c^2*x^6 - b*c*x^3 + (b*c^2*x^6 + b)*arctan(c*x^3))/c^2
```

Sympy [A] (verification not implemented)

Time = 36.16 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.12

$$\int x^5(a+b\arctan(cx^3))\,dx=\begin{cases}\frac{ax^6}{6}+\frac{bx^6\operatorname{atan}(cx^3)}{6}-\frac{bx^3}{6c}+\frac{b\operatorname{atan}(cx^3)}{6c^2} & \text{for } c\neq 0\\ \frac{ax^6}{6} & \text{otherwise}\end{cases}$$

```
[In] integrate(x**5*(a+b*atan(c*x**3)),x)
```

```
[Out] Piecewise((a*x**6/6 + b*x**6*atan(c*x**3)/6 - b*x**3/(6*c) + b*atan(c*x**3)
/(6*c**2), Ne(c, 0)), (a*x**6/6, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int x^5(a+b\arctan(cx^3))\,dx=\frac{1}{6}ax^6+\frac{1}{6}\left(x^6\arctan(cx^3)-c\left(\frac{x^3}{c^2}-\frac{\arctan(cx^3)}{c^3}\right)\right)b$$

```
[In] integrate(x^5*(a+b*arctan(c*x^3)),x, algorithm="maxima")
[Out] 1/6*a*x^6 + 1/6*(x^6*arctan(c*x^3) - c*(x^3/c^2 - arctan(c*x^3)/c^3))*b
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int x^5(a+b\arctan(cx^3))\,dx=\frac{acx^6+\frac{(c^2x^6\arctan(cx^3)-cx^3+\arctan(cx^3))b}{c}}{6c}$$

```
[In] integrate(x^5*(a+b*arctan(c*x^3)),x, algorithm="giac")
[Out] 1/6*(a*c*x^6 + (c^2*x^6*arctan(c*x^3) - c*x^3 + arctan(c*x^3))*b/c)/c
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int x^5(a+b\arctan(cx^3))\,dx=\frac{a\,x^6}{6}-\frac{b\,x^3}{6\,c}+\frac{b\,\mathrm{atan}(c\,x^3)}{6\,c^2}+\frac{b\,x^6\,\mathrm{atan}(c\,x^3)}{6}$$

```
[In] int(x^5*(a + b*atan(c*x^3)),x)
[Out] (a*x^6)/6 - (b*x^3)/(6*c) + (b*atan(c*x^3))/(6*c^2) + (b*x^6*atan(c*x^3))/6
```

3.99 $\int x^2(a+b\arctan(cx^3))\,dx$

Optimal result 573
Rubi [A] (verified) 573
Mathematica [A] (verified) 574
Maple [A] (verified) 574
Fricas [A] (verification not implemented) 575
Sympy [B] (verification not implemented) 575
Maxima [A] (verification not implemented) 575
Giac [A] (verification not implemented) 576
Mupad [B] (verification not implemented) 576

Optimal result

Integrand size = 14, antiderivative size = 36

$$\int x^2\left(a+b\arctan\left(cx^3\right)\right)\,dx=\frac{1}{3}x^3\left(a+b\arctan\left(cx^3\right)\right)-\frac{b\log\left(1+c^2x^6\right)}{6c}$$

```
[Out] 1/3*x^3*(a+b*arctan(c*x^3))-1/6*b*ln(c^2*x^6+1)/c
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4946, 266}

$$\int x^2\left(a+b\arctan\left(cx^3\right)\right)\,dx=\frac{1}{3}x^3\left(a+b\arctan\left(cx^3\right)\right)-\frac{b\log\left(c^2x^6+1\right)}{6c}$$

```
[In] Int[x^2*(a + b*ArcTan[c*x^3]),x]
```

```
[Out] (x^3*(a + b*ArcTan[c*x^3]))/3 - (b*Log[1 + c^2*x^6])/(6*c)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
```

```
IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{1}{3}x^3\left(a + b\arctan\left(cx^3\right)\right) - (bc)\int \frac{x^5}{1 + c^2x^6}\,dx$$
$$= \frac{1}{3}x^3\left(a + b\arctan\left(cx^3\right)\right) - \frac{b\log\left(1 + c^2x^6\right)}{6c}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.14

$$\int x^2\left(a + b\arctan\left(cx^3\right)\right)\,dx = \frac{ax^3}{3} + \frac{1}{3}bx^3\arctan\left(cx^3\right) - \frac{b\log\left(1 + c^2x^6\right)}{6c}$$

```
[In] Integrate[x^2*(a + b*ArcTan[c*x^3]),x]
[Out] (a*x^3)/3 + (b*x^3*ArcTan[c*x^3])/3 - (b*Log[1 + c^2*x^6])/(6*c)
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

method	result	size
parts	$\frac{x^3 a}{3} + \frac{b\arctan\left(c\,x^3\right)x^3}{3} - \frac{b\ln\left(c^2x^6+1\right)}{6c}$	36
derivativedivides	$\frac{ac\,x^3+b\left(c\,x^3\arctan\left(c\,x^3\right)-\frac{\ln\left(c^2x^6+1\right)}{2}\right)}{3c}$	39
default	$\frac{ac\,x^3+b\left(c\,x^3\arctan\left(c\,x^3\right)-\frac{\ln\left(c^2x^6+1\right)}{2}\right)}{3c}$	39
parallelrisch	$-\frac{-2x^3\arctan\left(c\,x^3\right)bc-2ac\,x^3+b\ln\left(c^2x^6+1\right)}{6c}$	39
risch	$-\frac{ix^3b\ln\left(ic\,x^3+1\right)}{6} + \frac{ib\,x^3\ln\left(-ic\,x^3+1\right)}{6} + \frac{x^3a}{3} - \frac{b\ln\left(-c^2x^6-1\right)}{6c}$	59

```
[In] int(x^2*(a+b*arctan(c*x^3)),x,method=_RETURNVERBOSE)
[Out] 1/3*x^3*a+1/3*b*arctan(c*x^3)*x^3-1/6*b*ln(c^2*x^6+1)/c
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

$$\int x^2(a+b\arctan(cx^3))\,dx = \frac{2\,bcx^3\arctan(cx^3)+2\,acx^3-b\log(c^2x^6+1)}{6\,c}$$

```
[In] integrate(x^2*(a+b*arctan(c*x^3)),x, algorithm="fricas")
[Out] 1/6*(2*b*c*x^3*arctan(c*x^3) + 2*a*c*x^3 - b*log(c^2*x^6 + 1))/c
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. 2(29) = 58.

Time = 20.58 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.83

$$\int x^2(a+b\arctan(cx^3))\,dx$$
$$=\begin{cases}\frac{ax^3}{3}+\frac{bx^3\operatorname{atan}(cx^3)}{3}+\frac{b\sqrt{-\frac{1}{c^2}}\operatorname{atan}(cx^3)}{3}-\frac{b\log\left(x-\sqrt[6]{-\frac{1}{c^2}}\right)}{3c}-\frac{b\log\left(4x^2+4x\sqrt[6]{-\frac{1}{c^2}}+4\sqrt[3]{-\frac{1}{c^2}}\right)}{3c} & \text{for } c\neq 0\\ \frac{ax^3}{3} & \text{otherwise}\end{cases}$$

```
[In] integrate(x**2*(a+b*atan(c*x**3)),x)
[Out] Piecewise((a*x**3/3 + b*x**3*atan(c*x**3)/3 + b*sqrt(-1/c**2)*atan(c*x**3)/
3 - b*log(x - (-1/c**2)**(1/6))/(3*c) - b*log(4*x**2 + 4*x*(-1/c**2)**(1/6)
 + 4*(-1/c**2)**(1/3))/(3*c), Ne(c, 0)), (a*x**3/3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int x^2(a+b\arctan(cx^3))\,dx = \frac{1}{3}\,ax^3+\frac{(2\,cx^3\arctan(cx^3)-\log(c^2x^6+1))b}{6\,c}$$

```
[In] integrate(x^2*(a+b*arctan(c*x^3)),x, algorithm="maxima")
[Out] 1/3*a*x^3 + 1/6*(2*c*x^3*arctan(c*x^3) - log(c^2*x^6 + 1))*b/c
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11

$$\int x^2(a+b\arctan(cx^3))\,dx = \frac{2\,acx^3 + (2\,cx^3\arctan(cx^3) - \log(c^2x^6+1))b}{6\,c}$$

```
[In] integrate(x^2*(a+b*arctan(c*x^3)),x, algorithm="giac")
[Out] 1/6*(2*a*c*x^3 + (2*c*x^3*arctan(c*x^3) - log(c^2*x^6 + 1))*b)/c
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

$$\int x^2(a+b\arctan(cx^3))\,dx = \frac{a\,x^3}{3} - \frac{b\,\ln(c^2\,x^6+1)}{6\,c} + \frac{b\,x^3\,\mathrm{atan}(c\,x^3)}{3}$$

```
[In] int(x^2*(a + b*atan(c*x^3)),x)
[Out] (a*x^3)/3 - (b*log(c^2*x^6 + 1))/(6*c) + (b*x^3*atan(c*x^3))/3
```

3.100 $\int \frac{a+b\arctan(cx^3)}{x}\,dx$

Optimal result 577
Rubi [A] (verified) 577
Mathematica [A] (verified) 578
Maple [C] (verified) 578
Fricas [F] 579
Sympy [F] 579
Maxima [F] 579
Giac [F] 579
Mupad [B] (verification not implemented) 580

Optimal result

Integrand size = 14, antiderivative size = 39

$$\int \frac{a+b\arctan(cx^3)}{x}\,dx = a\log(x) + \frac{1}{6}ib\,\text{PolyLog}\left(2,-icx^3\right) - \frac{1}{6}ib\,\text{PolyLog}\left(2,icx^3\right)$$

```
[Out] a*ln(x)+1/6*I*b*polylog(2,-I*c*x^3)-1/6*I*b*polylog(2,I*c*x^3)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4944, 4940, 2438}

$$\int \frac{a+b\arctan(cx^3)}{x}\,dx = a\log(x) + \frac{1}{6}ib\,\text{PolyLog}\left(2,-icx^3\right) - \frac{1}{6}ib\,\text{PolyLog}\left(2,icx^3\right)$$

```
[In] Int[(a + b*ArcTan[c*x^3])/x,x]
```

```
[Out] a*Log[x] + (I/6)*b*PolyLog[2, (-I)*c*x^3] - (I/6)*b*PolyLog[2, I*c*x^3]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1
/n, Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{a+b\arctan(cx)}{x}\,dx, x, x^3\right)\\ &= a\log(x)+\frac{1}{6}(ib)\text{Subst}\left(\int \frac{\log(1-icx)}{x}\,dx, x, x^3\right)-\frac{1}{6}(ib)\text{Subst}\left(\int \frac{\log(1+icx)}{x}\,dx, x, x^3\right)\\ &= a\log(x)+\frac{1}{6}ib\,\text{PolyLog}\left(2,-icx^3\right)-\frac{1}{6}ib\,\text{PolyLog}\left(2,icx^3\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{a+b\arctan\left(cx^3\right)}{x}\,dx = a\log(x)+\frac{1}{6}ib\,\text{PolyLog}\left(2,-icx^3\right)-\frac{1}{6}ib\,\text{PolyLog}\left(2,icx^3\right)$$

```
[In] Integrate[(a + b*ArcTan[c*x^3])/x,x]
[Out] a*Log[x] + (I/6)*b*PolyLog[2, (-I)*c*x^3] - (I/6)*b*PolyLog[2, I*c*x^3]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.52 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.62

method	result
default	$a\ln(x)+b\ln(x)\arctan\left(c\,x^3\right)-\frac{b\left(\sum_{\_R1=\text{RootOf}\left(c^2\_Z^6+1\right)}\frac{\ln(x)\ln\left(\frac{\_R1-x}{\_R1}\right)+\text{dilog}\left(\frac{\_R1-x}{\_R1}\right)}{\_R1^3}\right)}{2c}$
parts	$a\ln(x)+b\ln(x)\arctan\left(c\,x^3\right)-\frac{b\left(\sum_{\_R1=\text{RootOf}\left(c^2\_Z^6+1\right)}\frac{\ln(x)\ln\left(\frac{\_R1-x}{\_R1}\right)+\text{dilog}\left(\frac{\_R1-x}{\_R1}\right)}{\_R1^3}\right)}{2c}$
risch	$-\frac{i\left(\sum_{\_R1=\text{RootOf}\left(c\_Z^3+\text{RootOf}\left(\_Z^2+1,\text{index}=1\right)\right)}\left(\ln(x)\ln\left(\frac{\_R1-x}{\_R1}\right)+\text{dilog}\left(\frac{\_R1-x}{\_R1}\right)\right)\right)b}{2}+\frac{i\ln(x)\ln\left(-ic\,x^3+1\right)b}{2}+a\ln$

```
[In] int((a+b*arctan(c*x^3))/x,x,method=_RETURNVERBOSE)
[Out] a*ln(x)+b*ln(x)*arctan(c*x^3)-1/2*b/c*sum(1/_R1^3*(ln(x)*ln((_R1-x)/_R1)+di
log((_R1-x)/_R1)),_R1=RootOf(_Z^6*c^2+1))
```

Fricas [F]

$$\int \frac{a + b \arctan(cx^3)}{x} \, dx = \int \frac{b \arctan(cx^3) + a}{x} \, dx$$

```
[In] integrate((a+b*arctan(c*x^3))/x,x, algorithm="fricas")
[Out] integral((b*arctan(c*x^3) + a)/x, x)
```

Sympy [F]

$$\int \frac{a + b \arctan(cx^3)}{x} \, dx = \int \frac{a + b \operatorname{atan}(cx^3)}{x} \, dx$$

```
[In] integrate((a+b*atan(c*x**3))/x,x)
[Out] Integral((a + b*atan(c*x**3))/x, x)
```

Maxima [F]

$$\int \frac{a + b \arctan(cx^3)}{x} \, dx = \int \frac{b \arctan(cx^3) + a}{x} \, dx$$

```
[In] integrate((a+b*arctan(c*x^3))/x,x, algorithm="maxima")
[Out] b*integrate(arctan(c*x^3)/x, x) + a*log(x)
```

Giac [F]

$$\int \frac{a + b \arctan(cx^3)}{x} \, dx = \int \frac{b \arctan(cx^3) + a}{x} \, dx$$

```
[In] integrate((a+b*arctan(c*x^3))/x,x, algorithm="giac")
[Out] integrate((b*arctan(c*x^3) + a)/x, x)
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{a+b\arctan\left(cx^3\right)}{x}\,dx = a\,\ln\left(x\right)-\frac{b\,\left(\mathrm{Li}_2(1-c\,x^3\,1\mathrm{i})-\mathrm{Li}_2(1\mathrm{i}\,c\,x^3+1)\right)\,1\mathrm{i}}{6}$$

```
[In] int((a + b*atan(c*x^3))/x,x)
```

```
[Out] a*log(x) - (b*(dilog(1 - c*x^3*1i) - dilog(c*x^3*1i + 1))*1i)/6
```

3.101 $\int \frac{a+b\arctan(cx^3)}{x^4}\,dx$

Optimal result 581
Rubi [A] (verified) 581
Mathematica [A] (verified) 582
Maple [A] (verified) 583
Fricas [A] (verification not implemented) 583
Sympy [B] (verification not implemented) 583
Maxima [A] (verification not implemented) 584
Giac [A] (verification not implemented) 584
Mupad [B] (verification not implemented) 584

Optimal result

Integrand size = 14, antiderivative size = 39

$$\int \frac{a+b\arctan(cx^3)}{x^4}\,dx=-\frac{a+b\arctan(cx^3)}{3x^3}+bc\log(x)-\frac{1}{6}bc\log\left(1+c^2x^6\right)$$

```
[Out] 1/3*(-a-b*arctan(c*x^3))/x^3+b*c*ln(x)-1/6*b*c*ln(c^2*x^6+1)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {4946, 272, 36, 29, 31}

$$\int \frac{a+b\arctan(cx^3)}{x^4}\,dx=-\frac{a+b\arctan(cx^3)}{3x^3}-\frac{1}{6}bc\log\left(c^2x^6+1\right)+bc\log(x)$$

```
[In] Int[(a + b*ArcTan[c*x^3])/x^4,x]
[Out] -1/3*(a + b*ArcTan[c*x^3])/x^3 + b*c*Log[x] - (b*c*Log[1 + c^2*x^6])/6
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a+b\arctan(cx^3)}{3x^3}+(bc)\int\frac{1}{x(1+c^2x^6)}\,dx\\
&= -\frac{a+b\arctan(cx^3)}{3x^3}+\frac{1}{6}(bc)\text{Subst}\left(\int\frac{1}{x(1+c^2x)}\,dx,x,x^6\right)\\
&= -\frac{a+b\arctan(cx^3)}{3x^3}+\frac{1}{6}(bc)\text{Subst}\left(\int\frac{1}{x}\,dx,x,x^6\right)-\frac{1}{6}(bc^3)\,\text{Subst}\left(\int\frac{1}{1+c^2x}\,dx,x,x^6\right)\\
&= -\frac{a+b\arctan(cx^3)}{3x^3}+bc\log(x)-\frac{1}{6}bc\log\left(1+c^2x^6\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.13

$$\int\frac{a+b\arctan(cx^3)}{x^4}\,dx=-\frac{a}{3x^3}-\frac{b\arctan(cx^3)}{3x^3}+bc\log(x)-\frac{1}{6}bc\log\left(1+c^2x^6\right)$$

```
[In] Integrate[(a + b*ArcTan[c*x^3])/x^4,x]
[Out] -1/3*a/x^3 - (b*ArcTan[c*x^3])/(3*x^3) + b*c*Log[x] - (b*c*Log[1 + c^2*x^6]
)/6
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

method	result	size
default	$-\frac{a}{3x^3}+b\left(-\frac{\arctan(c\,x^3)}{3x^3}+c\left(\ln(x)-\frac{\ln(c^2x^6+1)}{6}\right)\right)$	39
parts	$-\frac{a}{3x^3}+b\left(-\frac{\arctan(c\,x^3)}{3x^3}+c\left(\ln(x)-\frac{\ln(c^2x^6+1)}{6}\right)\right)$	39
parallelrisch	$\frac{6bc\ln(x)x^3-bc\ln(c^2x^6+1)x^3-2b\arctan(c\,x^3)-2a}{6x^3}$	45
risch	$\frac{ib\ln(ic\,x^3+1)}{6x^3}-\frac{-6bc\ln(x)x^3+bc\ln(-c^2x^6-1)x^3+ib\ln(-ic\,x^3+1)+2a}{6x^3}$	68

```
[In] int((a+b*arctan(c*x^3))/x^4,x,method=_RETURNVERBOSE)
[Out] -1/3*a/x^3+b*(-1/3/x^3*arctan(c*x^3)+c*(ln(x)-1/6*ln(c^2*x^6+1)))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10

$$\int \frac{a+b\arctan(cx^3)}{x^4}\,dx = -\frac{bcx^3\log(c^2x^6+1)-6\,bcx^3\log(x)+2\,b\arctan(cx^3)+2\,a}{6\,x^3}$$

```
[In] integrate((a+b*arctan(c*x^3))/x^4,x, algorithm="fricas")
[Out] -1/6*(b*c*x^3*log(c^2*x^6 + 1) - 6*b*c*x^3*log(x) + 2*b*arctan(c*x^3) + 2*a
)/x^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. 2(39) = 78.

Time = 39.53 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.82

$$\int \frac{a+b\arctan(cx^3)}{x^4}\,dx = \begin{cases} -\frac{a}{3x^3}+bc\log(x)-\frac{bc\log\left(x-\sqrt[6]{-\frac{1}{c^2}}\right)}{3}-\frac{bc\log\left(4x^2+4x\sqrt[6]{-\frac{1}{c^2}}+4\sqrt[3]{-\frac{1}{c^2}}\right)}{3}-\frac{b\,\mathrm{atan}(cx^3)}{3\sqrt{-\frac{1}{c^2}}}-\frac{b\,\mathrm{atan}(cx^3)}{3x^3} & \text{for } c\neq 0 \\ -\frac{a}{3x^3} & \text{otherwis} \end{cases}$$

```
[In] integrate((a+b*atan(c*x**3))/x**4,x)
```

```
[Out] Piecewise((-a/(3*x**3) + b*c*log(x) - b*c*log(x - (-1/c**2)**(1/6))/3 - b*c
*log(4*x**2 + 4*x*(-1/c**2)**(1/6) + 4*(-1/c**2)**(1/3))/3 - b*atan(c*x**3)
/(3*sqrt(-1/c**2)) - b*atan(c*x**3)/(3*x**3), Ne(c, 0)), (-a/(3*x**3), True
))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{a+b\arctan(cx^3)}{x^4}\,dx = -\frac{1}{6}\left(c\left(\log\left(c^2x^6+1\right)-\log\left(x^6\right)\right)+\frac{2\arctan(cx^3)}{x^3}\right)b-\frac{a}{3\,x^3}$$

```
[In] integrate((a+b*arctan(c*x^3))/x^4,x, algorithm="maxima")
[Out] -1/6*(c*(log(c^2*x^6 + 1) - log(x^6)) + 2*arctan(c*x^3)/x^3)*b - 1/3*a/x^3
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.54

$$\int \frac{a+b\arctan(cx^3)}{x^4}\,dx = -\frac{bc^3x^3\log\left(c^2x^6+1\right)-2\,bc^3x^3\log\left(cx^3\right)+2\,bc^2\arctan\left(cx^3\right)+2\,ac^2}{6\,c^2x^3}$$

```
[In] integrate((a+b*arctan(c*x^3))/x^4,x, algorithm="giac")
[Out] -1/6*(b*c^3*x^3*log(c^2*x^6 + 1) - 2*b*c^3*x^3*log(c*x^3) + 2*b*c^2*arctan(
c*x^3) + 2*a*c^2)/(c^2*x^3)
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.97

$$\int \frac{a+b\arctan(cx^3)}{x^4}\,dx = b\,c\,\ln(x)-\frac{a}{3\,x^3}-\frac{b\,\mathrm{atan}(c\,x^3)}{3\,x^3}-\frac{b\,c\,\ln\left(c^2\,x^6+1\right)}{6}$$

```
[In] int((a + b*atan(c*x^3))/x^4,x)
[Out] b*c*log(x) - a/(3*x^3) - (b*atan(c*x^3))/(3*x^3) - (b*c*log(c^2*x^6 + 1))/6
```

3.102 $\int \frac{a+b\arctan(cx^3)}{x^7}\,dx$

Optimal result 585
Rubi [A] (verified) 585
Mathematica [C] (verified) 586
Maple [A] (verified) 587
Fricas [A] (verification not implemented) 587
Sympy [A] (verification not implemented) 587
Maxima [A] (verification not implemented) 588
Giac [C] (verification not implemented) 588
Mupad [B] (verification not implemented) 588

Optimal result

Integrand size = 14, antiderivative size = 41

$$\int \frac{a+b\arctan(cx^3)}{x^7}\,dx = -\frac{bc}{6x^3}-\frac{1}{6}bc^2\arctan(cx^3)-\frac{a+b\arctan(cx^3)}{6x^6}$$

```
[Out] -1/6*b*c/x^3-1/6*b*c^2*arctan(c*x^3)+1/6*(-a-b*arctan(c*x^3))/x^6
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4946, 281, 331, 209}

$$\int \frac{a+b\arctan(cx^3)}{x^7}\,dx = -\frac{a+b\arctan(cx^3)}{6x^6}-\frac{1}{6}bc^2\arctan(cx^3)-\frac{bc}{6x^3}$$

```
[In] Int[(a + b*ArcTan[c*x^3])/x^7,x]
```

```
[Out] -1/6*(b*c)/x^3 - (b*c^2*ArcTan[c*x^3])/6 - (a + b*ArcTan[c*x^3])/(6*x^6)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a+b\arctan(cx^3)}{6x^6}+\frac{1}{2}(bc)\int\frac{1}{x^4(1+c^2x^6)}\,dx\\
&= -\frac{a+b\arctan(cx^3)}{6x^6}+\frac{1}{6}(bc)\text{Subst}\left(\int\frac{1}{x^2(1+c^2x^2)}\,dx,x,x^3\right)\\
&= -\frac{bc}{6x^3}-\frac{a+b\arctan(cx^3)}{6x^6}-\frac{1}{6}\left(bc^3\right)\text{Subst}\left(\int\frac{1}{1+c^2x^2}\,dx,x,x^3\right)\\
&= -\frac{bc}{6x^3}-\frac{1}{6}bc^2\arctan\left(cx^3\right)-\frac{a+b\arctan(cx^3)}{6x^6}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.17

$$\int\frac{a+b\arctan(cx^3)}{x^7}\,dx=-\frac{a}{6x^6}-\frac{b\arctan(cx^3)}{6x^6}-\frac{bc\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-c^2x^6\right)}{6x^3}$$

```
[In] Integrate[(a + b*ArcTan[c*x^3])/x^7,x]
[Out] -1/6*a/x^6 - (b*ArcTan[c*x^3])/(6*x^6) - (b*c*Hypergeometric2F1[-1/2, 1, 1/
2, -(c^2*x^6)])/(6*x^3)
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

method	result	size
default	$-\frac{a}{6x^{6}}-\frac{b\arctan\left(c\,x^{3}\right)}{6x^{6}}-\frac{b\,c^{2}\arctan\left(c\,x^{3}\right)}{6}-\frac{bc}{6x^{3}}$	39
parts	$-\frac{a}{6x^{6}}-\frac{b\arctan\left(c\,x^{3}\right)}{6x^{6}}-\frac{b\,c^{2}\arctan\left(c\,x^{3}\right)}{6}-\frac{bc}{6x^{3}}$	39
parallelrisch	$-\frac{\arctan\left(c\,x^{3}\right)b\,c^{2}x^{6}-a\,c^{2}x^{6}+bc\,x^{3}+b\arctan\left(c\,x^{3}\right)+a}{6x^{6}}$	45
risch	$\frac{ib\ln\left(ic\,x^{3}+1\right)}{12x^{6}}-\frac{ib\,c^{2}\ln\left(c\,x^{3}+i\right)x^{6}-ib\,c^{2}\ln\left(c\,x^{3}-i\right)x^{6}+2bc\,x^{3}+ib\ln\left(-ic\,x^{3}+1\right)+2a}{12x^{6}}$	87

```
[In] int((a+b*arctan(c*x^3))/x^7,x,method=_RETURNVERBOSE)
[Out] -1/6*a/x^6-1/6*b/x^6*arctan(c*x^3)-1/6*b*c^2*arctan(c*x^3)-1/6*b*c/x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.73

$$\int \frac{a+b\arctan\left(cx^3\right)}{x^7}\,dx=-\frac{bcx^3+\left(bc^2x^6+b\right)\arctan\left(cx^3\right)+a}{6\,x^6}$$

```
[In] integrate((a+b*arctan(c*x^3))/x^7,x, algorithm="fricas")
[Out] -1/6*(b*c*x^3 + (b*c^2*x^6 + b)*arctan(c*x^3) + a)/x^6
```

Sympy [A] (verification not implemented)

Time = 37.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

$$\int \frac{a+b\arctan\left(cx^3\right)}{x^7}\,dx=-\frac{a}{6x^6}-\frac{bc^2\operatorname{atan}\left(cx^3\right)}{6}-\frac{bc}{6x^3}-\frac{b\operatorname{atan}\left(cx^3\right)}{6x^6}$$

```
[In] integrate((a+b*atan(c*x**3))/x**7,x)
[Out] -a/(6*x**6) - b*c**2*atan(c*x**3)/6 - b*c/(6*x**3) - b*atan(c*x**3)/(6*x**6
)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \frac{a+b\arctan(cx^3)}{x^7}\,dx = -\frac{1}{6}\left(\left(c\arctan(cx^3)+\frac{1}{x^3}\right)c+\frac{\arctan(cx^3)}{x^6}\right)b-\frac{a}{6\,x^6}$$

```
[In] integrate((a+b*arctan(c*x^3))/x^7,x, algorithm="maxima")
[Out] -1/6*((c*arctan(c*x^3) + 1/x^3)*c + arctan(c*x^3)/x^6)*b - 1/6*a/x^6
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.76

$$\int \frac{a+b\arctan(cx^3)}{x^7}\,dx = \frac{i\,bc^5x^6\log(i\,cx^3+1)-i\,bc^5x^6\log(-i\,cx^3+1)-2\,bc^4x^3-2\,bc^3\arctan(cx^3)-2\,ac^3}{12\,c^3x^6}$$

```
[In] integrate((a+b*arctan(c*x^3))/x^7,x, algorithm="giac")
[Out] 1/12*(I*b*c^5*x^6*log(I*c*x^3 + 1) - I*b*c^5*x^6*log(-I*c*x^3 + 1) - 2*b*c^
4*x^3 - 2*b*c^3*arctan(c*x^3) - 2*a*c^3)/(c^3*x^6)
```

Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{a+b\arctan(cx^3)}{x^7}\,dx = -\frac{\frac{b\,c\,x^3}{3}+\frac{a}{3}}{2\,x^6}-\frac{b\,c^2\,\mathrm{atan}(c\,x^3)}{6}-\frac{b\,\mathrm{atan}(c\,x^3)}{6\,x^6}$$

```
[In] int((a + b*atan(c*x^3))/x^7,x)
[Out] - (a/3 + (b*c*x^3)/3)/(2*x^6) - (b*c^2*atan(c*x^3))/6 - (b*atan(c*x^3))/(6*
x^6)
```

3.103 $\int \frac{a+b\arctan(cx^3)}{x^{10}}\,dx$

Optimal result . . . 589
Rubi [A] (verified) . . . 589
Mathematica [A] (verified) . . . 590
Maple [A] (verified) . . . 591
Fricas [A] (verification not implemented) . . . 591
Sympy [B] (verification not implemented) . . . 592
Maxima [A] (verification not implemented) . . . 592
Giac [A] (verification not implemented) . . . 592
Mupad [B] (verification not implemented) . . . 593

Optimal result

Integrand size = 14, antiderivative size = 55

$$\int \frac{a+b\arctan(cx^3)}{x^{10}}\,dx = -\frac{bc}{18x^6}-\frac{a+b\arctan(cx^3)}{9x^9}-\frac{1}{3}bc^3\log(x)+\frac{1}{18}bc^3\log\left(1+c^2x^6\right)$$

```
[Out] -1/18*b*c/x^6+1/9*(-a-b*arctan(c*x^3))/x^9-1/3*b*c^3*ln(x)+1/18*b*c^3*ln(c^
2*x^6+1)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4946, 272, 46}

$$\int \frac{a+b\arctan(cx^3)}{x^{10}}\,dx = -\frac{a+b\arctan(cx^3)}{9x^9}-\frac{1}{3}bc^3\log(x)+\frac{1}{18}bc^3\log\left(c^2x^6+1\right)-\frac{bc}{18x^6}$$

```
[In] Int[(a + b*ArcTan[c*x^3])/x^10,x]
```

```
[Out] -1/18*(b*c)/x^6 - (a + b*ArcTan[c*x^3])/(9*x^9) - (b*c^3*Log[x])/3 + (b*c^3
*Log[1 + c^2*x^6])/18
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a+b\arctan\left(cx^3\right)}{9x^9}+\frac{1}{3}(bc)\int\frac{1}{x^7\left(1+c^2x^6\right)}\,dx\\
&= -\frac{a+b\arctan\left(cx^3\right)}{9x^9}+\frac{1}{18}(bc)\text{Subst}\left(\int\frac{1}{x^2\left(1+c^2x\right)}\,dx,x,x^6\right)\\
&= -\frac{a+b\arctan\left(cx^3\right)}{9x^9}+\frac{1}{18}(bc)\text{Subst}\left(\int\left(\frac{1}{x^2}-\frac{c^2}{x}+\frac{c^4}{1+c^2x}\right)\,dx,x,x^6\right)\\
&= -\frac{bc}{18x^6}-\frac{a+b\arctan\left(cx^3\right)}{9x^9}-\frac{1}{3}bc^3\log(x)+\frac{1}{18}bc^3\log\left(1+c^2x^6\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.09

$$\int\frac{a+b\arctan\left(cx^3\right)}{x^{10}}\,dx=-\frac{a}{9x^9}-\frac{bc}{18x^6}-\frac{b\arctan\left(cx^3\right)}{9x^9}-\frac{1}{3}bc^3\log(x)+\frac{1}{18}bc^3\log\left(1+c^2x^6\right)$$

```
[In] Integrate[(a + b*ArcTan[c*x^3])/x^10,x]
[Out] -1/9*a/x^9 - (b*c)/(18*x^6) - (b*ArcTan[c*x^3])/(9*x^9) - (b*c^3*Log[x])/3
+ (b*c^3*Log[1 + c^2*x^6])/18
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

method	result	size
default	$-\frac{a}{9x^{9}}+b\left(-\frac{\arctan\left(c\,x^{3}\right)}{9x^{9}}+\frac{c\left(-\frac{1}{6x^{6}}-c^{2}\ln\left(x\right)+\frac{c^{2}\ln\left(c^{2}x^{6}+1\right)}{6}\right)}{3}\right)$	53
parts	$-\frac{a}{9x^{9}}+b\left(-\frac{\arctan\left(c\,x^{3}\right)}{9x^{9}}+\frac{c\left(-\frac{1}{6x^{6}}-c^{2}\ln\left(x\right)+\frac{c^{2}\ln\left(c^{2}x^{6}+1\right)}{6}\right)}{3}\right)$	53
parallelrisch	$-\frac{6b\,c^{3}\ln\left(x\right)x^{9}-b\,c^{3}\ln\left(c^{2}x^{6}+1\right)x^{9}-b\,c^{3}x^{9}+bc\,x^{3}+2b\arctan\left(c\,x^{3}\right)+2a}{18x^{9}}$	64
risch	$\frac{ib\ln\left(ic\,x^{3}+1\right)}{18x^{9}}-\frac{6b\,c^{3}\ln\left(x\right)x^{9}-b\,c^{3}\ln\left(c^{2}x^{6}+1\right)x^{9}+bc\,x^{3}+ib\ln\left(-ic\,x^{3}+1\right)+2a}{18x^{9}}$	78

```
[In] int((a+b*arctan(c*x^3))/x^10,x,method=_RETURNVERBOSE)
[Out] -1/9*a/x^9+b*(-1/9/x^9*arctan(c*x^3)+1/3*c*(-1/6/x^6-c^2*ln(x)+1/6*c^2*ln(c
^2*x^6+1)))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.98

$$\int \frac{a+b\arctan\left(cx^{3}\right)}{x^{10}}\,dx = \frac{bc^{3}x^{9}\log\left(c^{2}x^{6}+1\right)-6\,bc^{3}x^{9}\log\left(x\right)-bcx^{3}-2\,b\arctan\left(cx^{3}\right)-2\,a}{18\,x^{9}}$$

```
[In] integrate((a+b*arctan(c*x^3))/x^10,x, algorithm="fricas")
[Out] 1/18*(b*c^3*x^9*log(c^2*x^6 + 1) - 6*b*c^3*x^9*log(x) - b*c*x^3 - 2*b*arcta
n(c*x^3) - 2*a)/x^9
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(53) = 106$.

Time = 135.23 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.35

$$\int \frac{a + b\arctan(cx^3)}{x^{10}}\,dx = \begin{cases} -\frac{a}{9x^9} - \frac{bc^4\sqrt{-\frac{1}{c^2}}\operatorname{atan}(cx^3)}{9} - \frac{bc^3\log(x)}{3} + \frac{bc^3\log\left(x-\sqrt[6]{-\frac{1}{c^2}}\right)}{9} + \frac{bc^3\log\left(4x^2+4x\sqrt[6]{-\frac{1}{c^2}}+4\sqrt[3]{-\frac{1}{c^2}}\right)}{9} - \frac{bc}{18x^6} - \frac{b\operatorname{atan}(cx^3)}{9x^9} \\ -\frac{a}{9x^9} \end{cases}$$

```
[In] integrate((a+b*atan(c*x**3))/x**10,x)
```

```
[Out] Piecewise((-a/(9*x**9) - b*c**4*sqrt(-1/c**2)*atan(c*x**3)/9 - b*c**3*log(x
)/3 + b*c**3*log(x - (-1/c**2)**(1/6))/9 + b*c**3*log(4*x**2 + 4*x*(-1/c**2
)**(1/6) + 4*(-1/c**2)**(1/3))/9 - b*c/(18*x**6) - b*atan(c*x**3)/(9*x**9),
 Ne(c, 0)), (-a/(9*x**9), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int \frac{a + b\arctan(cx^3)}{x^{10}}\,dx = \frac{1}{18}\left(\left(c^2\log(c^2x^6+1) - c^2\log(x^6) - \frac{1}{x^6}\right)c - \frac{2\arctan(cx^3)}{x^9}\right)b - \frac{a}{9x^9}$$

```
[In] integrate((a+b*arctan(c*x^3))/x^10,x, algorithm="maxima")
```

```
[Out] 1/18*((c^2*log(c^2*x^6 + 1) - c^2*log(x^6) - 1/x^6)*c - 2*arctan(c*x^3)/x^9
)*b - 1/9*a/x^9
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.25

$$\int \frac{a + b\arctan(cx^3)}{x^{10}}\,dx = \frac{bc^7x^9\log(c^2x^6+1) - 2\,bc^7x^9\log(cx^3) - bc^5x^3 - 2\,bc^4\arctan(cx^3) - 2\,ac^4}{18\,c^4x^9}$$

```
[In] integrate((a+b*arctan(c*x^3))/x^10,x, algorithm="giac")
```

```
[Out] 1/18*(b*c^7*x^9*log(c^2*x^6 + 1) - 2*b*c^7*x^9*log(c*x^3) - b*c^5*x^3 - 2*b
*c^4*arctan(c*x^3) - 2*a*c^4)/(c^4*x^9)
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

$$\int \frac{a+b\arctan(cx^3)}{x^{10}}\,dx=\frac{b\,c^3\,\ln\left(c^2\,x^6+1\right)}{18}-\frac{a}{9\,x^9}-\frac{b\,c^3\,\ln\left(x\right)}{3}-\frac{b\,\mathrm{atan}(c\,x^3)}{9\,x^9}-\frac{b\,c}{18\,x^6}$$

```
[In] int((a + b*atan(c*x^3))/x^10,x)
```

```
[Out] (b*c^3*log(c^2*x^6 + 1))/18 - a/(9*x^9) - (b*c^3*log(x))/3 - (b*atan(c*x^3)
)/(9*x^9) - (b*c)/(18*x^6)
```

3.104 $\int x^3(a+b\arctan(cx^3))\,dx$

Optimal result . . . 594
Rubi [A] (verified) . . . 594
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Optimal result

Integrand size = 14, antiderivative size = 174

$$\int x^3\left(a+b\arctan\left(cx^3\right)\right)\,dx=-\frac{3bx}{4c}+\frac{b\arctan\left(\sqrt[3]{c}x\right)}{4c^{4/3}}+\frac{1}{4}x^4\left(a+b\arctan\left(cx^3\right)\right)-\frac{b\arctan\left(\sqrt{3}-2\sqrt[3]{c}x\right)}{8c^{4/3}}+\frac{b\arctan\left(\sqrt{3}+2\sqrt[3]{c}x\right)}{8c^{4/3}}-\frac{\sqrt{3}b\log\left(1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)}{16c^{4/3}}+\frac{\sqrt{3}b\log\left(1+\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)}{16c^{4/3}}$$

```
[Out] -3/4*b*x/c+1/4*b*arctan(c^(1/3)*x)/c^(4/3)+1/4*x^4*(a+b*arctan(c*x^3))+1/8*
b*arctan(2*c^(1/3)*x-3^(1/2))/c^(4/3)+1/8*b*arctan(2*c^(1/3)*x+3^(1/2))/c^(
4/3)-1/16*b*ln(1+c^(2/3)*x^2-c^(1/3)*x*3^(1/2))*3^(1/2)/c^(4/3)+1/16*b*ln(1
+c^(2/3)*x^2+c^(1/3)*x*3^(1/2))*3^(1/2)/c^(4/3)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {4946, 327, 215, 648, 632, 210, 642, 209}

$$\int x^3\left(a+b\arctan\left(cx^3\right)\right)\,dx=\frac{1}{4}x^4\left(a+b\arctan\left(cx^3\right)\right)+\frac{b\arctan\left(\sqrt[3]{c}x\right)}{4c^{4/3}}-\frac{b\arctan\left(\sqrt{3}-2\sqrt[3]{c}x\right)}{8c^{4/3}}+\frac{b\arctan\left(2\sqrt[3]{c}x+\sqrt{3}\right)}{8c^{4/3}}-\frac{\sqrt{3}b\log\left(c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1\right)}{16c^{4/3}}+\frac{\sqrt{3}b\log\left(c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1\right)}{16c^{4/3}}-\frac{3bx}{4c}$$

```
[In] Int[x^3*(a + b*ArcTan[c*x^3]),x]
```

```
[Out] (-3*b*x)/(4*c) + (b*ArcTan[c^(1/3)*x])/(4*c^(4/3)) + (x^4*(a + b*ArcTan[c*x
^3]))/4 - (b*ArcTan[Sqrt[3] - 2*c^(1/3)*x])/(8*c^(4/3)) + (b*ArcTan[Sqrt[3]
 + 2*c^(1/3)*x])/(8*c^(4/3)) - (Sqrt[3]*b*Log[1 - Sqrt[3]*c^(1/3)*x + c^(2/
3)*x^2])/(16*c^(4/3)) + (Sqrt[3]*b*Log[1 + Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2]
)/(16*c^(4/3))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{4}x^4\left(a+b\arctan\left(cx^3\right)\right)-\frac{1}{4}(3bc)\int\frac{x^6}{1+c^2x^6}\,dx\\
&= -\frac{3bx}{4c}+\frac{1}{4}x^4\left(a+b\arctan\left(cx^3\right)\right)+\frac{(3b)\int\frac{1}{1+c^2x^6}\,dx}{4c}\\
&= -\frac{3bx}{4c}+\frac{1}{4}x^4\left(a+b\arctan\left(cx^3\right)\right)+\frac{b\int\frac{1}{1+c^{2/3}x^2}\,dx}{4c}\\
&\quad+\frac{b\int\frac{1-\frac{1}{2}\sqrt{3}\sqrt[3]{c}x}{1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2}\,dx}{4c}+\frac{b\int\frac{1+\frac{1}{2}\sqrt{3}\sqrt[3]{c}x}{1+\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2}\,dx}{4c}\\
&= -\frac{3bx}{4c}+\frac{b\arctan\left(\sqrt[3]{c}x\right)}{4c^{4/3}}+\frac{1}{4}x^4\left(a+b\arctan\left(cx^3\right)\right)-\frac{\left(\sqrt{3}b\right)\int\frac{-\sqrt{3}\sqrt[3]{c}+2c^{2/3}x}{1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2}\,dx}{16c^{4/3}}\\
&\quad+\frac{\left(\sqrt{3}b\right)\int\frac{\sqrt{3}\sqrt[3]{c}+2c^{2/3}x}{1+\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2}\,dx}{16c^{4/3}}+\frac{b\int\frac{1}{1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2}\,dx}{16c}+\frac{b\int\frac{1}{1+\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2}\,dx}{16c}\\
&= -\frac{3bx}{4c}+\frac{b\arctan\left(\sqrt[3]{c}x\right)}{4c^{4/3}}+\frac{1}{4}x^4\left(a+b\arctan\left(cx^3\right)\right)\\
&\quad-\frac{\sqrt{3}b\log\left(1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)}{16c^{4/3}}+\frac{\sqrt{3}b\log\left(1+\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)}{16c^{4/3}}\\
&\quad+\frac{b\text{Subst}\left(\int\frac{1}{-\frac{1}{3}-x^2}\,dx,x,1-\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)}{8\sqrt{3}c^{4/3}}-\frac{b\text{Subst}\left(\int\frac{1}{-\frac{1}{3}-x^2}\,dx,x,1+\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)}{8\sqrt{3}c^{4/3}}
\end{aligned}
$$

$$= -\frac{3bx}{4c} + \frac{b \arctan\left(\sqrt[3]{c}x\right)}{4c^{4/3}} + \frac{1}{4}x^4\left(a + b\arctan\left(cx^3\right)\right) - \frac{b\arctan\left(\sqrt{3} - 2\sqrt[3]{c}x\right)}{8c^{4/3}} + \frac{b\arctan\left(\sqrt{3} + 2\sqrt[3]{c}x\right)}{8c^{4/3}} - \frac{\sqrt{3}b\log\left(1 - \sqrt{3}\sqrt[3]{c}x + c^{2/3}x^2\right)}{16c^{4/3}} + \frac{\sqrt{3}b\log\left(1 + \sqrt{3}\sqrt[3]{c}x + c^{2/3}x^2\right)}{16c^{4/3}}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.03

$$\int x^3\left(a + b\arctan\left(cx^3\right)\right)\,dx = -\frac{3bx}{4c} + \frac{ax^4}{4} + \frac{b\arctan\left(\sqrt[3]{c}x\right)}{4c^{4/3}} + \frac{1}{4}bx^4\arctan\left(cx^3\right) - \frac{b\arctan\left(\sqrt{3} - 2\sqrt[3]{c}x\right)}{8c^{4/3}} + \frac{b\arctan\left(\sqrt{3} + 2\sqrt[3]{c}x\right)}{8c^{4/3}} - \frac{\sqrt{3}b\log\left(1 - \sqrt{3}\sqrt[3]{c}x + c^{2/3}x^2\right)}{16c^{4/3}} + \frac{\sqrt{3}b\log\left(1 + \sqrt{3}\sqrt[3]{c}x + c^{2/3}x^2\right)}{16c^{4/3}}$$

```
[In] Integrate[x^3*(a + b*ArcTan[c*x^3]),x]
```

```
[Out] (-3*b*x)/(4*c) + (a*x^4)/4 + (b*ArcTan[c^(1/3)*x])/(4*c^(4/3)) + (b*x^4*Arc
Tan[c*x^3])/4 - (b*ArcTan[Sqrt[3] - 2*c^(1/3)*x])/(8*c^(4/3)) + (b*ArcTan[S
qrt[3] + 2*c^(1/3)*x])/(8*c^(4/3)) - (Sqrt[3]*b*Log[1 - Sqrt[3]*c^(1/3)*x +
 c^(2/3)*x^2])/(16*c^(4/3)) + (Sqrt[3]*b*Log[1 + Sqrt[3]*c^(1/3)*x + c^(2/3
)*x^2])/(16*c^(4/3))
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.89

method	result
default	$\frac{a\,x^{4}}{4}+b\left(\frac{x^{4}\arctan\left(c\,x^{3}\right)}{4}-\frac{3c\left(\frac{x}{c^{2}}-\frac{\frac{\sqrt{3}\,\left(\frac{1}{c^{2}}\right)^{\frac{1}{6}}\ln\left(x^{2}+\sqrt{3}\,\left(\frac{1}{c^{2}}\right)^{\frac{1}{6}}x+\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}\right)}{12}+\frac{\left(\frac{1}{c^{2}}\right)^{\frac{1}{6}}\arctan\left(\frac{2x}{\left(\frac{1}{c^{2}}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{6}-\frac{\sqrt{3}\,\left(\frac{1}{c^{2}}\right)^{\frac{1}{6}}\ln\left(x^{2}-\cdots\right.}{\cdots}}{c^{2}}\right.}{4}\cdots\right.$
parts	$\frac{a\,x^{4}}{4}+b\left(\frac{x^{4}\arctan\left(c\,x^{3}\right)}{4}-\frac{3c\left(\frac{x}{c^{2}}-\frac{\frac{\sqrt{3}\,\left(\frac{1}{c^{2}}\right)^{\frac{1}{6}}\ln\left(x^{2}+\sqrt{3}\,\left(\frac{1}{c^{2}}\right)^{\frac{1}{6}}x+\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}\right)}{12}+\frac{\left(\frac{1}{c^{2}}\right)^{\frac{1}{6}}\arctan\left(\frac{2x}{\left(\frac{1}{c^{2}}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{6}-\frac{\sqrt{3}\,\left(\frac{1}{c^{2}}\right)^{\frac{1}{6}}\ln\left(x^{2}-\cdots\right.}{\cdots}}{c^{2}}\right.}{4}\cdots\right.$

```
[In] int(x^3*(a+b*arctan(c*x^3)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a*x^4+b*(1/4*x^4*arctan(c*x^3)-3/4*c*(1/c^2*x-(1/12*3^(1/2)*(1/c^2)^(1/
6)*ln(x^2+3^(1/2)*(1/c^2)^(1/6)*x+(1/c^2)^(1/3))+1/6*(1/c^2)^(1/6)*arctan(2
*x/(1/c^2)^(1/6)+3^(1/2))-1/12*3^(1/2)*(1/c^2)^(1/6)*ln(x^2-3^(1/2)*(1/c^2)
^(1/6)*x+(1/c^2)^(1/3))+1/6*(1/c^2)^(1/6)*arctan(2*x/(1/c^2)^(1/6)-3^(1/2))
+1/3*(1/c^2)^(1/6)*arctan(x/(1/c^2)^(1/6)))/c^2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(126) = 252$.

Time = 0.25 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.55

$$\int x^3\left(a+b\arctan\left(cx^3\right)\right)\,dx$$
$$= \frac{4\,bcx^4\arctan\left(cx^3\right)+4\,acx^4+\left(\sqrt{-3}c+c\right)\left(-\frac{b^6}{c^8}\right)^{\frac{1}{6}}\log\left(bx+\frac{1}{2}\left(\sqrt{-3}c+c\right)\left(-\frac{b^6}{c^8}\right)^{\frac{1}{6}}\right)-\left(\sqrt{-3}c+c\right)\left(-\frac{b}{c}\right.}{}$$

```
[In] integrate(x^3*(a+b*arctan(c*x^3)),x, algorithm="fricas")
```

```
[Out] 1/16*(4*b*c*x^4*arctan(c*x^3) + 4*a*c*x^4 + (sqrt(-3)*c + c)*(-b^6/c^8)^(1/
6)*log(b*x + 1/2*(sqrt(-3)*c + c)*(-b^6/c^8)^(1/6)) - (sqrt(-3)*c + c)*(-b^
6/c^8)^(1/6)*log(b*x - 1/2*(sqrt(-3)*c + c)*(-b^6/c^8)^(1/6)) + (sqrt(-3)*c
 - c)*(-b^6/c^8)^(1/6)*log(b*x + 1/2*(sqrt(-3)*c - c)*(-b^6/c^8)^(1/6)) - (
sqrt(-3)*c - c)*(-b^6/c^8)^(1/6)*log(b*x - 1/2*(sqrt(-3)*c - c)*(-b^6/c^8)^
(1/6)) + 2*c*(-b^6/c^8)^(1/6)*log(b*x + c*(-b^6/c^8)^(1/6)) - 2*c*(-b^6/c^8
)^(1/6)*log(b*x - c*(-b^6/c^8)^(1/6)) - 12*b*x)/c
```

Sympy [A] (verification not implemented)

Time = 24.30 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.47

$$\int x^3\left(a+b\arctan\left(cx^3\right)\right)\,dx$$
$$= \begin{cases} \frac{ax^4}{4}+\frac{bx^4\operatorname{atan}\left(cx^3\right)}{4}-\frac{3bx}{4c}-\frac{3b\sqrt[6]{-\frac{1}{c^2}}\log\left(4x^2-4x\sqrt[6]{-\frac{1}{c^2}}+4\sqrt[3]{-\frac{1}{c^2}}\right)}{16c}+\frac{3b\sqrt[6]{-\frac{1}{c^2}}\log\left(4x^2+4x\sqrt[6]{-\frac{1}{c^2}}+4\sqrt[3]{-\frac{1}{c^2}}\right)}{16c}+ \\ \frac{ax^4}{4} \end{cases}$$

```
[In] integrate(x**3*(a+b*atan(c*x**3)),x)
```

```
[Out] Piecewise((a*x**4/4 + b*x**4*atan(c*x**3)/4 - 3*b*x/(4*c) - 3*b*(-1/c**2)**
(1/6)*log(4*x**2 - 4*x*(-1/c**2)**(1/6) + 4*(-1/c**2)**(1/3))/(16*c) + 3*b*
(-1/c**2)**(1/6)*log(4*x**2 + 4*x*(-1/c**2)**(1/6) + 4*(-1/c**2)**(1/3))/(1
6*c) + sqrt(3)*b*(-1/c**2)**(1/6)*atan(2*sqrt(3)*x/(3*(-1/c**2)**(1/6)) - s
qrt(3)/3)/(8*c) + sqrt(3)*b*(-1/c**2)**(1/6)*atan(2*sqrt(3)*x/(3*(-1/c**2)*
*(1/6)) + sqrt(3)/3)/(8*c) + b*atan(c*x**3)/(4*c**2*(-1/c**2)**(1/3)), Ne(c
, 0)), (a*x**4/4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.85

$$\int x^3\left(a+b\arctan\left(cx^3\right)\right)\,dx=\frac{1}{4}\,ax^4+\frac{1}{16}\left(4\,x^4\arctan\left(cx^3\right)+c\left(\frac{\frac{\sqrt{3}\log\left(c^{\frac{2}{3}}x^2+\sqrt{3}c^{\frac{1}{3}}x+1\right)}{c^{\frac{1}{3}}}-\frac{\sqrt{3}\log\left(c^{\frac{2}{3}}x^2-\sqrt{3}c^{\frac{1}{3}}x+1\right)}{c^{\frac{1}{3}}}+\frac{4\arctan\left(c^{\frac{1}{3}}x\right)}{c^{\frac{1}{3}}}+\frac{2\arctan\left(\frac{2c^{\frac{2}{3}}x+\sqrt{3}c^{\frac{1}{3}}}{c^{\frac{1}{3}}}\right)}{c^{\frac{1}{3}}}$$

[In] integrate(x^3*(a+b*arctan(c*x^3)),x, algorithm="maxima")

[Out] 1/4*a*x^4 + 1/16*(4*x^4*arctan(c*x^3) + c*((sqrt(3)*log(c^(2/3)*x^2 + sqrt(3)*c^(1/3)*x + 1)/c^(1/3) - sqrt(3)*log(c^(2/3)*x^2 - sqrt(3)*c^(1/3)*x + 1)/c^(1/3) + 4*arctan(c^(1/3)*x)/c^(1/3) + 2*arctan((2*c^(2/3)*x + sqrt(3)*c^(1/3))/c^(1/3))/c^(1/3) + 2*arctan((2*c^(2/3)*x - sqrt(3)*c^(1/3))/c^(1/3))/c^(1/3))/c^2 - 12*x/c^2))*b

Giac [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.96

$$\int x^3\left(a+b\arctan\left(cx^3\right)\right)\,dx=\frac{1}{16}\,bc^7\left(\frac{\sqrt{3}\log\left(x^2+\frac{\sqrt{3}x}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{c^8|c|^{\frac{1}{3}}}-\frac{\sqrt{3}\log\left(x^2-\frac{\sqrt{3}x}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{c^8|c|^{\frac{1}{3}}}+\frac{2\arctan\left(\left(2x+\frac{\sqrt{3}}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{c^8|c|^{\frac{1}{3}}}+\frac{2\arcta}{}\right.$$

$$+\frac{bcx^4\arctan\left(cx^3\right)+acx^4-3\,bx}{4\,c}$$

[In] integrate(x^3*(a+b*arctan(c*x^3)),x, algorithm="giac")

[Out] 1/16*b*c^7*(sqrt(3)*log(x^2 + sqrt(3)*x/abs(c)^(1/3) + 1/abs(c)^(2/3))/(c^8*abs(c)^(1/3)) - sqrt(3)*log(x^2 - sqrt(3)*x/abs(c)^(1/3) + 1/abs(c)^(2/3))/(c^8*abs(c)^(1/3)) + 2*arctan((2*x + sqrt(3)/abs(c)^(1/3))*abs(c)^(1/3))/(c^8*abs(c)^(1/3)) + 2*arctan((2*x - sqrt(3)/abs(c)^(1/3))*abs(c)^(1/3))/(c^8*abs(c)^(1/3)) + 4*arctan(x*abs(c)^(1/3))/(c^8*abs(c)^(1/3))) + 1/4*(b*c*x^4*arctan(c*x^3) + a*c*x^4 - 3*b*x)/c

Mupad [B] (verification not implemented)

Time = 1.26 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.66

$$\int x^3(a+b\arctan(cx^3))\,dx$$
$$=\frac{a\,x^4}{4}-\frac{b\left(\mathrm{atan}\left((-1)^{2/3}\,c^{1/3}\,x\right)-\mathrm{atan}\left(\frac{c^{1/3}\,x\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)+2\,\mathrm{atan}\left(\frac{(-1)^{2/3}\,c^{1/3}\,x\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\right)}{8\,c^{4/3}}+\frac{b\,x^4\,\mathrm{atan}(c\,x^3)}{4}-\frac{3\,b\,x}{4\,c}-\frac{\sqrt{3}\,b\,\left(\mathrm{atan}\left(\frac{c^{1/3}\,x\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)+\mathrm{atan}\left((-1)^{2/3}\,c^{1/3}\,x\right)\right)\,1\mathrm{i}}{8\,c^{4/3}}$$

```
[In] int(x^3*(a + b*atan(c*x^3)),x)
```

```
[Out] (a*x^4)/4 - (b*(atan((-1)^(2/3)*c^(1/3)*x) - atan((c^(1/3)*x*(3^(1/2)*1i +
1))/2) + 2*atan(((-1)^(2/3)*c^(1/3)*x*(3^(1/2)*1i + 1))/2)))/(8*c^(4/3)) +
(b*x^4*atan(c*x^3))/4 - (3*b*x)/(4*c) - (3^(1/2)*b*(atan((c^(1/3)*x*(3^(1/2
)*1i + 1))/2) + atan((-1)^(2/3)*c^(1/3)*x))*1i)/(8*c^(4/3))
```

3.105 $\int (a+b\arctan(cx^3))\,dx$

Optimal result 602
Rubi [A] (verified) 602
Mathematica [A] (verified) 604
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Optimal result

Integrand size = 10, antiderivative size = 101

$$\int (a+b\arctan(cx^3))\,dx = ax+bx\arctan(cx^3)+\frac{\sqrt{3}b\arctan\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right)}{2\sqrt[3]{c}}+\frac{b\log\left(1+c^{2/3}x^2\right)}{2\sqrt[3]{c}}-\frac{b\log\left(1-c^{2/3}x^2+c^{4/3}x^4\right)}{4\sqrt[3]{c}}$$

```
[Out] a*x+b*x*arctan(c*x^3)+1/2*b*ln(1+c^(2/3)*x^2)/c^(1/3)-1/4*b*ln(1-c^(2/3)*x^
2+c^(4/3)*x^4)/c^(1/3)+1/2*b*arctan(1/3*(1-2*c^(2/3)*x^2)*3^(1/2))*3^(1/2)/
c^(1/3)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {4930, 281, 298, 31, 648, 631, 210, 642}

$$\int (a+b\arctan(cx^3))\,dx = ax+\frac{\sqrt{3}b\arctan\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right)}{2\sqrt[3]{c}}+bx\arctan(cx^3)+\frac{b\log\left(c^{2/3}x^2+1\right)}{2\sqrt[3]{c}}-\frac{b\log\left(c^{4/3}x^4-c^{2/3}x^2+1\right)}{4\sqrt[3]{c}}$$

```
[In] Int[a + b*ArcTan[c*x^3],x]
```

```
[Out] a*x + b*x*ArcTan[c*x^3] + (Sqrt[3]*b*ArcTan[(1 - 2*c^(2/3)*x^2)/Sqrt[3]])/(
2*c^(1/3)) + (b*Log[1 + c^(2/3)*x^2])/(2*c^(1/3)) - (b*Log[1 - c^(2/3)*x^2
+ c^(4/3)*x^4])/(4*c^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
```

```
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= ax + b\int \arctan\left(cx^3\right)\,dx\\
&= ax + bx\arctan\left(cx^3\right) - (3bc)\int \frac{x^3}{1+c^2x^6}\,dx\\
&= ax + bx\arctan\left(cx^3\right) - \frac{1}{2}(3bc)\text{Subst}\left(\int \frac{x}{1+c^2x^3}\,dx, x, x^2\right)\\
&= ax + bx\arctan\left(cx^3\right) + \frac{1}{2}\left(b\sqrt[3]{c}\right)\text{Subst}\left(\int \frac{1}{1+c^{2/3}x}\,dx, x, x^2\right)\\
&\quad - \frac{1}{2}\left(b\sqrt[3]{c}\right)\text{Subst}\left(\int \frac{1+c^{2/3}x}{1-c^{2/3}x+c^{4/3}x^2}\,dx, x, x^2\right)\\
&= ax + bx\arctan\left(cx^3\right) + \frac{b\log\left(1+c^{2/3}x^2\right)}{2\sqrt[3]{c}} - \frac{b\text{Subst}\left(\int \frac{-c^{2/3}+2c^{4/3}x}{1-c^{2/3}x+c^{4/3}x^2}\,dx, x, x^2\right)}{4\sqrt[3]{c}}\\
&\quad - \frac{1}{4}\left(3b\sqrt[3]{c}\right)\text{Subst}\left(\int \frac{1}{1-c^{2/3}x+c^{4/3}x^2}\,dx, x, x^2\right)\\
&= ax + bx\arctan\left(cx^3\right) + \frac{b\log\left(1+c^{2/3}x^2\right)}{2\sqrt[3]{c}}\\
&\quad - \frac{b\log\left(1-c^{2/3}x^2+c^{4/3}x^4\right)}{4\sqrt[3]{c}} - \frac{(3b)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-2c^{2/3}x^2\right)}{2\sqrt[3]{c}}\\
&= ax + bx\arctan\left(cx^3\right) + \frac{\sqrt{3}b\arctan\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right)}{2\sqrt[3]{c}}\\
&\quad + \frac{b\log\left(1+c^{2/3}x^2\right)}{2\sqrt[3]{c}} - \frac{b\log\left(1-c^{2/3}x^2+c^{4/3}x^4\right)}{4\sqrt[3]{c}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.30

$$\int \left(a+b\arctan\left(cx^3\right)\right)\,dx = ax + bx\arctan\left(cx^3\right) - \frac{b\left(-2\sqrt{3}\arctan\left(\sqrt{3}-2\sqrt[3]{c}x\right) - 2\sqrt{3}\arctan\left(\sqrt{3}+2\sqrt[3]{c}x\right) - 2\log\left(1+c^{2/3}x^2\right) + \log\left(1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}\right.\right.}{4\sqrt[3]{c}}$$

```
[In] Integrate[a + b*ArcTan[c*x^3],x]
```

```
[Out] a*x + b*x*ArcTan[c*x^3] - (b*(-2*Sqrt[3]*ArcTan[Sqrt[3] - 2*c^(1/3)*x] - 2*
Sqrt[3]*ArcTan[Sqrt[3] + 2*c^(1/3)*x] - 2*Log[1 + c^(2/3)*x^2] + Log[1 - Sq
rt[3]*c^(1/3)*x + c^(2/3)*x^2] + Log[1 + Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2]))
/(4*c^(1/3))
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.97

method	result	siz
default	$ax+bx\arctan\left(c\,x^{3}\right)+\frac{b\ln\left(x^{2}+\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}\right)}{2c\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}}-\frac{b\ln\left(x^{4}-\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}x^{2}+\left(\frac{1}{c^{2}}\right)^{\frac{2}{3}}\right)}{4c\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}}-\frac{b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{2}}{\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{2c\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}}$	98
parts	$ax+bx\arctan\left(c\,x^{3}\right)+\frac{b\ln\left(x^{2}+\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}\right)}{2c\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}}-\frac{b\ln\left(x^{4}-\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}x^{2}+\left(\frac{1}{c^{2}}\right)^{\frac{2}{3}}\right)}{4c\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}}-\frac{b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{2}}{\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{2c\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}}$	98

```
[In] int(a+b*arctan(c*x^3),x,method=_RETURNVERBOSE)
```

```
[Out] a*x+b*x*arctan(c*x^3)+1/2*b/c/(1/c^2)^(1/3)*ln(x^2+(1/c^2)^(1/3))-1/4*b/c/(
1/c^2)^(1/3)*ln(x^4-(1/c^2)^(1/3)*x^2+(1/c^2)^(2/3))-1/2*b*3^(1/2)/c/(1/c^2
)^(1/3)*arctan(1/3*3^(1/2)*(2*x^2/(1/c^2)^(1/3)-1))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.32

$$\int \left(a+b\arctan\left(cx^{3}\right)\right)\,dx$$
$$=\left[\frac{4\,bcx\arctan\left(cx^{3}\right)+\sqrt{3}bc\sqrt{-\frac{1}{c^{\frac{2}{3}}}}\log\left(\frac{2\,c^{2}x^{6}-3\,c^{\frac{2}{3}}x^{2}-\sqrt{3}\left(2\,c^{\frac{5}{3}}x^{4}+cx^{2}-c^{\frac{1}{3}}\right)\sqrt{-\frac{1}{c^{\frac{2}{3}}}}-1}{c^{2}x^{6}+1}\right)+4\,acx-bc^{\frac{2}{3}}\log\left(c^{2}x^{4}-\right.}{4\,c}\right.$$

```
[In] integrate(a+b*arctan(c*x^3),x, algorithm="fricas")
```

```
[Out] [1/4*(4*b*c*x*arctan(c*x^3) + sqrt(3)*b*c*sqrt(-1/c^(2/3))*log((2*c^2*x^6 -
 3*c^(2/3)*x^2 - sqrt(3)*(2*c^(5/3)*x^4 + c*x^2 - c^(1/3))*sqrt(-1/c^(2/3))
 - 1)/(c^2*x^6 + 1)) + 4*a*c*x - b*c^(2/3)*log(c^2*x^4 - c^(4/3)*x^2 + c^(2
/3)) + 2*b*c^(2/3)*log(c*x^2 + c^(1/3)))/c, 1/4*(4*b*c*x*arctan(c*x^3) + 2*
sqrt(3)*b*c^(2/3)*arctan(-1/3*sqrt(3)*(2*c*x^2 - c^(1/3))/c^(1/3)) + 4*a*c*
x - b*c^(2/3)*log(c^2*x^4 - c^(4/3)*x^2 + c^(2/3)) + 2*b*c^(2/3)*log(c*x^2
+ c^(1/3)))/c]
```

Sympy [A] (verification not implemented)

Time = 13.07 (sec) , antiderivative size = 755, normalized size of antiderivative = 7.48

$$\int \left(a+b\arctan\left(cx^3\right)\right)\,dx = ax + b\left(\begin{cases}0\\ -\infty ix\\ \infty ix\\ -\frac{4c^4x^6\left(-\frac{1}{c^2}\right)^{\frac{5}{3}}\log\left(x-\sqrt[6]{-\frac{1}{c^2}}\right)}{4cx^6+\frac{4}{c}}+\frac{3c^4x^6\left(-\frac{1}{c^2}\right)^{\frac{5}{3}}\log\left(4x^2-4x\sqrt[6]{-\frac{1}{c^2}}+4\sqrt[3]{-\frac{1}{c^2}}\right)}{4cx^6+\frac{4}{c}}-\frac{c^4x^6\left(-\frac{1}{c^2}\right)^{\frac{5}{3}}\log\left(4x^2+4x\sqrt[6]{-\frac{1}{c^2}}+\cdots\right.}{4cx^6+\frac{4}{c}}\end{cases}\right.$$

```
[In] integrate(a+b*atan(c*x**3),x)
```

```
[Out] a*x + b*Piecewise((0, Eq(c, 0)), (-oo*I*x, Eq(c, -I/x**3)), (oo*I*x, Eq(c,
I/x**3)), (-4*c**4*x**6*(-1/c**2)**(5/3)*log(x - (-1/c**2)**(1/6))/(4*c*x**
6 + 4/c) + 3*c**4*x**6*(-1/c**2)**(5/3)*log(4*x**2 - 4*x*(-1/c**2)**(1/6) +
 4*(-1/c**2)**(1/3))/(4*c*x**6 + 4/c) - c**4*x**6*(-1/c**2)**(5/3)*log(4*x*
*2 + 4*x*(-1/c**2)**(1/6) + 4*(-1/c**2)**(1/3))/(4*c*x**6 + 4/c) - 2*sqrt(3
)*c**4*x**6*(-1/c**2)**(5/3)*atan(2*sqrt(3)*x/(3*(-1/c**2)**(1/6)) - sqrt(3
)/3)/(4*c*x**6 + 4/c) + 2*sqrt(3)*c**4*x**6*(-1/c**2)**(5/3)*atan(2*sqrt(3)
*x/(3*(-1/c**2)**(1/6)) + sqrt(3)/3)/(4*c*x**6 + 4/c) - 4*c**4*x**6*(-1/c**
2)**(5/3)*log(2)/(4*c*x**6 + 4/c) - 4*c**3*x**6*(-1/c**2)**(7/6)*atan(c*x**
3)/(4*c*x**6 + 4/c) - 4*c**2*(-1/c**2)**(5/3)*log(x - (-1/c**2)**(1/6))/(4*
c*x**6 + 4/c) + 3*c**2*(-1/c**2)**(5/3)*log(4*x**2 - 4*x*(-1/c**2)**(1/6) +
 4*(-1/c**2)**(1/3))/(4*c*x**6 + 4/c) - c**2*(-1/c**2)**(5/3)*log(4*x**2 +
4*x*(-1/c**2)**(1/6) + 4*(-1/c**2)**(1/3))/(4*c*x**6 + 4/c) - 2*sqrt(3)*c**
2*(-1/c**2)**(5/3)*atan(2*sqrt(3)*x/(3*(-1/c**2)**(1/6)) - sqrt(3)/3)/(4*c*
x**6 + 4/c) + 2*sqrt(3)*c**2*(-1/c**2)**(5/3)*atan(2*sqrt(3)*x/(3*(-1/c**2)
**(1/6)) + sqrt(3)/3)/(4*c*x**6 + 4/c) - 4*c**2*(-1/c**2)**(5/3)*log(2)/(4*
c*x**6 + 4/c) + 4*c*x**7*atan(c*x**3)/(4*c*x**6 + 4/c) - 4*c*(-1/c**2)**(7/
6)*atan(c*x**3)/(4*c*x**6 + 4/c) + 4*x*atan(c*x**3)/(4*c**2*x**6 + 4), True
))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.91

$$\int \left(a+b\arctan\left(cx^3\right)\right)\,dx = -\frac{1}{4}\left(c\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2c^{\frac{4}{3}}x^2-c^{\frac{2}{3}}\right)}{3c^{\frac{2}{3}}}\right)}{c^{\frac{4}{3}}}+\frac{\log\left(c^{\frac{4}{3}}x^4-c^{\frac{2}{3}}x^2+1\right)}{c^{\frac{4}{3}}}-\frac{2\log\left(\frac{c^{\frac{2}{3}}x^2+1}{c^{\frac{2}{3}}}\right)}{c^{\frac{4}{3}}}\right)-4x\arctan\left(cx^3\right)\right) + ax$$

```
[In] integrate(a+b*arctan(c*x^3),x, algorithm="maxima")
```

```
[Out] -1/4*(c*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*c^(4/3)*x^2 - c^(2/3))/c^(2/3))/c^
(4/3) + log(c^(4/3)*x^4 - c^(2/3)*x^2 + 1)/c^(4/3) - 2*log((c^(2/3)*x^2 + 1
)/c^(2/3))/c^(4/3)) - 4*x*arctan(c*x^3))*b + a*x
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.94

$$\int \left(a+b\arctan\left(cx^3\right)\right)\,dx = -\frac{1}{4}\left(c\left(\frac{2\sqrt{3}|c|^{\frac{2}{3}}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-\frac{1}{|c|^{\frac{2}{3}}}\right)|c|^{\frac{2}{3}}\right)}{c^2}+\frac{|c|^{\frac{2}{3}}\log\left(x^4-\frac{x^2}{|c|^{\frac{2}{3}}}+\frac{1}{|c|^{\frac{4}{3}}}\right)}{c^2}-\frac{2\log\left(x^2+\frac{1}{|c|^{\frac{2}{3}}}\right)}{|c|^{\frac{4}{3}}}\right)\right) + ax$$

```
[In] integrate(a+b*arctan(c*x^3),x, algorithm="giac")
```

```
[Out] -1/4*(c*(2*sqrt(3)*abs(c)^(2/3)*arctan(1/3*sqrt(3)*(2*x^2 - 1/abs(c)^(2/3))
*abs(c)^(2/3))/c^2 + abs(c)^(2/3)*log(x^4 - x^2/abs(c)^(2/3) + 1/abs(c)^(4/
3))/c^2 - 2*log(x^2 + 1/abs(c)^(2/3))/abs(c)^(4/3)) - 4*x*arctan(c*x^3))*b
+ a*x
```

Mupad [B] (verification not implemented)

Time = 2.43 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.90

$$\int (a + b\arctan(cx^3)) \, dx = a\,x + b\,x\,\mathrm{atan}\left(c\,x^3\right) + \frac{b\,\ln\left(c^{2/3}\,x^2 + 1\right)}{2\,c^{1/3}} - \frac{\ln\left(2 - 4\,c^{2/3}\,x^2 + \sqrt{3}\,2\mathrm{i}\right)\,\left(b - \sqrt{3}\,b\,1\mathrm{i}\right)}{4\,c^{1/3}} - \frac{\ln\left(4\,c^{2/3}\,x^2 - 2 + \sqrt{3}\,2\mathrm{i}\right)\,\left(b + \sqrt{3}\,b\,1\mathrm{i}\right)}{4\,c^{1/3}}$$

```
[In] int(a + b*atan(c*x^3),x)
```

```
[Out] a*x + b*x*atan(c*x^3) + (b*log(c^(2/3)*x^2 + 1))/(2*c^(1/3)) - (log(3^(1/2)
*2i - 4*c^(2/3)*x^2 + 2)*(b - 3^(1/2)*b*1i))/(4*c^(1/3)) - (log(3^(1/2)*2i
+ 4*c^(2/3)*x^2 - 2)*(b + 3^(1/2)*b*1i))/(4*c^(1/3))
```

3.106 $\int \frac{a+b\arctan(cx^3)}{x^3}\,dx$

Optimal result . . . 609
Rubi [A] (verified) . . . 609
Mathematica [A] (verified) . . . 611
Maple [A] (verified) . . . 612
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Optimal result

Integrand size = 14, antiderivative size = 165

$$\int \frac{a+b\arctan(cx^3)}{x^3}\,dx = \frac{1}{2}bc^{2/3}\arctan\left(\sqrt[3]{c}x\right)-\frac{a+b\arctan(cx^3)}{2x^2}-\frac{1}{4}bc^{2/3}\arctan\left(\sqrt{3}-2\sqrt[3]{c}x\right)+\frac{1}{4}bc^{2/3}\arctan\left(\sqrt{3}+2\sqrt[3]{c}x\right)-\frac{1}{8}\sqrt{3}bc^{2/3}\log\left(1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)+\frac{1}{8}\sqrt{3}bc^{2/3}\log\left(1+\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)$$

```
[Out] 1/2*b*c^(2/3)*arctan(c^(1/3)*x)+1/2*(-a-b*arctan(c*x^3))/x^2+1/4*b*c^(2/3)*
arctan(2*c^(1/3)*x-3^(1/2))+1/4*b*c^(2/3)*arctan(2*c^(1/3)*x+3^(1/2))-1/8*b
*c^(2/3)*ln(1+c^(2/3)*x^2-c^(1/3)*x*3^(1/2))*3^(1/2)+1/8*b*c^(2/3)*ln(1+c^(
2/3)*x^2+c^(1/3)*x*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4946, 215, 648, 632, 210, 642, 209}

$$\int \frac{a+b\arctan(cx^3)}{x^3}\,dx = -\frac{a+b\arctan(cx^3)}{2x^2}+\frac{1}{2}bc^{2/3}\arctan\left(\sqrt[3]{c}x\right)-\frac{1}{4}bc^{2/3}\arctan\left(\sqrt{3}-2\sqrt[3]{c}x\right)+\frac{1}{4}bc^{2/3}\arctan\left(2\sqrt[3]{c}x+\sqrt{3}\right)-\frac{1}{8}\sqrt{3}bc^{2/3}\log\left(c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1\right)+\frac{1}{8}\sqrt{3}bc^{2/3}\log\left(c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1\right)$$

```
[In] Int[(a + b*ArcTan[c*x^3])/x^3,x]
```

```
[Out] (b*c^(2/3)*ArcTan[c^(1/3)*x])/2 - (a + b*ArcTan[c*x^3])/(2*x^2) - (b*c^(2/3
)*ArcTan[Sqrt[3] - 2*c^(1/3)*x])/4 + (b*c^(2/3)*ArcTan[Sqrt[3] + 2*c^(1/3)*
x])/4 - (Sqrt[3]*b*c^(2/3)*Log[1 - Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/8 + (S
qrt[3]*b*c^(2/3)*Log[1 + Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/8
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{a+b\arctan(cx^3)}{2x^2}+\frac{1}{2}(3bc)\int\frac{1}{1+c^2x^6}\,dx$$
$$= -\frac{a+b\arctan(cx^3)}{2x^2}+\frac{1}{2}(bc)\int\frac{1}{1+c^{2/3}x^2}\,dx + \frac{1}{2}(bc)\int\frac{1-\frac{1}{2}\sqrt{3}\sqrt[3]{c}x}{1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2}\,dx+\frac{1}{2}(bc)\int\frac{1+\frac{1}{2}\sqrt{3}\sqrt[3]{c}x}{1+\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2}\,dx$$
$$= \frac{1}{2}bc^{2/3}\arctan\left(\sqrt[3]{c}x\right)-\frac{a+b\arctan(cx^3)}{2x^2}-\frac{1}{8}\left(\sqrt{3}bc^{2/3}\right)\int\frac{-\sqrt{3}\sqrt[3]{c}+2c^{2/3}x}{1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2}\,dx + \frac{1}{8}\left(\sqrt{3}bc^{2/3}\right)\int\frac{\sqrt{3}\sqrt[3]{c}+2c^{2/3}x}{1+\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2}\,dx+\frac{1}{8}(bc)\int\frac{1}{1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2}\,dx+\frac{1}{8}(bc)\int\frac{1}{1+\sqrt{3}\sqrt[3]{}}$$
$$= \frac{1}{2}bc^{2/3}\arctan\left(\sqrt[3]{c}x\right)-\frac{a+b\arctan(cx^3)}{2x^2} -\frac{1}{8}\sqrt{3}bc^{2/3}\log\left(1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)+\frac{1}{8}\sqrt{3}bc^{2/3}\log\left(1+\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)+\frac{\left(bc^{2/3}\right)\text{Subst}\left(\int\frac{1}{-\frac{1}{3}}\right.}{4}$$
$$= \frac{1}{2}bc^{2/3}\arctan\left(\sqrt[3]{c}x\right)-\frac{a+b\arctan(cx^3)}{2x^2}-\frac{1}{4}bc^{2/3}\arctan\left(\sqrt{3}-2\sqrt[3]{c}x\right) +\frac{1}{4}bc^{2/3}\arctan\left(\sqrt{3}+2\sqrt[3]{c}x\right)-\frac{1}{8}\sqrt{3}bc^{2/3}\log\left(1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)+\frac{1}{8}\sqrt{3}bc^{2/3}\log\left(1+\sqrt{3}\sqrt[3]{c}x+c\right.$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.03

$$\int\frac{a+b\arctan(cx^3)}{x^3}\,dx$$
$$= -\frac{a}{2x^2}+\frac{1}{2}bc^{2/3}\arctan\left(\sqrt[3]{c}x\right)-\frac{b\arctan(cx^3)}{2x^2}-\frac{1}{4}bc^{2/3}\arctan\left(\sqrt{3}-2\sqrt[3]{c}x\right) +\frac{1}{4}bc^{2/3}\arctan\left(\sqrt{3}+2\sqrt[3]{c}x\right)-\frac{1}{8}\sqrt{3}bc^{2/3}\log\left(1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)+\frac{1}{8}\sqrt{3}bc^{2/3}\log\left(1+\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)$$

```
[In] Integrate[(a + b*ArcTan[c*x^3])/x^3,x]
```

[Out] -1/2*a/x^2 + (b*c^(2/3)*ArcTan[c^(1/3)*x])/2 - (b*ArcTan[c*x^3])/(2*x^2) - (b*c^(2/3)*ArcTan[Sqrt[3] - 2*c^(1/3)*x])/4 + (b*c^(2/3)*ArcTan[Sqrt[3] + 2*c^(1/3)*x])/4 - (Sqrt[3]*b*c^(2/3)*Log[1 - Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/8 + (Sqrt[3]*b*c^(2/3)*Log[1 + Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/8

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.87

method	result
default	$-\frac{a}{2x^2}+b\left(-\frac{\arctan(c\,x^3)}{2x^2}+\frac{3c\left(\frac{\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{1}{6}}\ln\left(x^2+\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{1}{6}}x+\left(\frac{1}{c^2}\right)^{\frac{1}{3}}\right)}{12}+\frac{\left(\frac{1}{c^2}\right)^{\frac{1}{6}}\arctan\left(\frac{2x}{\left(\frac{1}{c^2}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{6}-\frac{\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{1}{6}}\ln\left(x^2-\sqrt{3}\cdots\right.}{1\cdots}\right.}{2}\right.$
parts	$-\frac{a}{2x^2}+b\left(-\frac{\arctan(c\,x^3)}{2x^2}+\frac{3c\left(\frac{\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{1}{6}}\ln\left(x^2+\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{1}{6}}x+\left(\frac{1}{c^2}\right)^{\frac{1}{3}}\right)}{12}+\frac{\left(\frac{1}{c^2}\right)^{\frac{1}{6}}\arctan\left(\frac{2x}{\left(\frac{1}{c^2}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{6}-\frac{\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{1}{6}}\ln\left(x^2-\sqrt{3}\cdots\right.}{1\cdots}\right.}{2}\right.$

[In] int((a+b*arctan(c*x^3))/x^3,x,method=_RETURNVERBOSE)

[Out] -1/2*a/x^2+b*(-1/2/x^2*arctan(c*x^3)+3/2*c*(1/12*3^(1/2)*(1/c^2)^(1/6)*ln(x^2+3^(1/2)*(1/c^2)^(1/6)*x+(1/c^2)^(1/3))+1/6*(1/c^2)^(1/6)*arctan(2*x/(1/c^2)^(1/6)+3^(1/2))-1/12*3^(1/2)*(1/c^2)^(1/6)*ln(x^2-3^(1/2)*(1/c^2)^(1/6)*x+(1/c^2)^(1/3))+1/6*(1/c^2)^(1/6)*arctan(2*x/(1/c^2)^(1/6)-3^(1/2))+1/3*(1/c^2)^(1/6)*arctan(x/(1/c^2)^(1/6))))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(119) = 238$.

Time = 0.25 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.64

$$\int \frac{a + b \arctan(cx^3)}{x^3} dx = \frac{2 (-b^6c^4)^{\frac{1}{6}} x^2 \log\left(bcx + (-b^6c^4)^{\frac{1}{6}}\right) - 2 (-b^6c^4)^{\frac{1}{6}} x^2 \log\left(bcx - (-b^6c^4)^{\frac{1}{6}}\right) + (-b^6c^4)^{\frac{1}{6}} \left(\sqrt{-3}x^2 + x^2\right) \log\left(\right.}{}$$

```
[In] integrate((a+b*arctan(c*x^3))/x^3,x, algorithm="fricas")
```

```
[Out] 1/8*(2*(-b^6*c^4)^(1/6)*x^2*log(b*c*x + (-b^6*c^4)^(1/6)) - 2*(-b^6*c^4)^(1
/6)*x^2*log(b*c*x - (-b^6*c^4)^(1/6)) + (-b^6*c^4)^(1/6)*(sqrt(-3)*x^2 + x^
2)*log(2*b*c*x + (-b^6*c^4)^(1/6)*(sqrt(-3) + 1)) - (-b^6*c^4)^(1/6)*(sqrt(
-3)*x^2 + x^2)*log(2*b*c*x - (-b^6*c^4)^(1/6)*(sqrt(-3) + 1)) + (-b^6*c^4)^
(1/6)*(sqrt(-3)*x^2 - x^2)*log(2*b*c*x + (-b^6*c^4)^(1/6)*(sqrt(-3) - 1)) -
 (-b^6*c^4)^(1/6)*(sqrt(-3)*x^2 - x^2)*log(2*b*c*x - (-b^6*c^4)^(1/6)*(sqrt
(-3) - 1)) - 4*b*arctan(c*x^3) - 4*a)/x^2
```

Sympy [A] (verification not implemented)

Time = 29.93 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.48

$$\int \frac{a + b \arctan(cx^3)}{x^3} dx = \begin{cases} -\frac{a}{2x^2} + \frac{b \operatorname{atan}(cx^3)}{2 \sqrt[3]{-\frac{1}{c^2}}} - \frac{b \operatorname{atan}(cx^3)}{2x^2} + \frac{3b \log\left(4x^2 - 4x \sqrt[6]{-\frac{1}{c^2}} + 4 \sqrt[3]{-\frac{1}{c^2}}\right)}{8c\left(-\frac{1}{c^2}\right)^{\frac{5}{6}}} - \frac{3b \log\left(4x^2 + 4x \sqrt[6]{-\frac{1}{c^2}} + 4 \sqrt[3]{-\frac{1}{c^2}}\right)}{8c\left(-\frac{1}{c^2}\right)^{\frac{5}{6}}} - \frac{\sqrt{3} b \operatorname{atan}}{4} \\ -\frac{a}{2x^2} \end{cases}$$

```
[In] integrate((a+b*atan(c*x**3))/x**3,x)
```

```
[Out] Piecewise((-a/(2*x**2) + b*atan(c*x**3)/(2*(-1/c**2)**(1/3)) - b*atan(c*x**
3)/(2*x**2) + 3*b*log(4*x**2 - 4*x*(-1/c**2)**(1/6) + 4*(-1/c**2)**(1/3))/(
8*c*(-1/c**2)**(5/6)) - 3*b*log(4*x**2 + 4*x*(-1/c**2)**(1/6) + 4*(-1/c**2)
**(1/3))/(8*c*(-1/c**2)**(5/6)) - sqrt(3)*b*atan(2*sqrt(3)*x/(3*(-1/c**2)**
(1/6)) - sqrt(3)/3)/(4*c*(-1/c**2)**(5/6)) - sqrt(3)*b*atan(2*sqrt(3)*x/(3*
(-1/c**2)**(1/6)) + sqrt(3)/3)/(4*c*(-1/c**2)**(5/6)), Ne(c, 0)), (-a/(2*x*
*2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.83

$$\int \frac{a+b\arctan(cx^3)}{x^3}\,dx$$
$$=\frac{1}{8}\left(\left(\left(\frac{\sqrt{3}\log\left(c^{\frac{2}{3}}x^2+\sqrt{3}c^{\frac{1}{3}}x+1\right)}{c^{\frac{1}{3}}}-\frac{\sqrt{3}\log\left(c^{\frac{2}{3}}x^2-\sqrt{3}c^{\frac{1}{3}}x+1\right)}{c^{\frac{1}{3}}}+\frac{4\arctan\left(c^{\frac{1}{3}}x\right)}{c^{\frac{1}{3}}}+\frac{2\arctan\left(\frac{2c^{\frac{2}{3}}x+\sqrt{}}{c^{\frac{1}{3}}}\right.}{c^{\frac{1}{3}}}\right.\right.\right.$$
$$-\frac{a}{2x^2}$$

[In] integrate((a+b*arctan(c*x^3))/x^3,x, algorithm="maxima")

[Out] 1/8*((sqrt(3)*log(c^(2/3)*x^2 + sqrt(3)*c^(1/3)*x + 1)/c^(1/3) - sqrt(3)*log(c^(2/3)*x^2 - sqrt(3)*c^(1/3)*x + 1)/c^(1/3) + 4*arctan(c^(1/3)*x)/c^(1/3) + 2*arctan((2*c^(2/3)*x + sqrt(3)*c^(1/3))/c^(1/3))/c^(1/3) + 2*arctan((2*c^(2/3)*x - sqrt(3)*c^(1/3))/c^(1/3))/c^(1/3))*c - 4*arctan(c*x^3)/x^2)*b - 1/2*a/x^2

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.83

$$\int \frac{a+b\arctan(cx^3)}{x^3}\,dx$$
$$=\frac{1}{8}\left(\frac{\sqrt{3}\log\left(x^2+\frac{\sqrt{3}x}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{|c|^{\frac{1}{3}}}-\frac{\sqrt{3}\log\left(x^2-\frac{\sqrt{3}x}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{|c|^{\frac{1}{3}}}+\frac{2\arctan\left(\left(2x+\frac{\sqrt{3}}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{|c|^{\frac{1}{3}}}+\frac{2\arctan\left(\right.}{}\right.$$
$$-\frac{b\arctan(cx^3)+a}{2x^2}$$

[In] integrate((a+b*arctan(c*x^3))/x^3,x, algorithm="giac")

[Out] 1/8*(sqrt(3)*log(x^2 + sqrt(3)*x/abs(c)^(1/3) + 1/abs(c)^(2/3))/abs(c)^(1/3) - sqrt(3)*log(x^2 - sqrt(3)*x/abs(c)^(1/3) + 1/abs(c)^(2/3))/abs(c)^(1/3) + 2*arctan((2*x + sqrt(3)/abs(c)^(1/3))*abs(c)^(1/3))/abs(c)^(1/3) + 2*arctan((2*x - sqrt(3)/abs(c)^(1/3))*abs(c)^(1/3))/abs(c)^(1/3) + 4*arctan(x*abs(c)^(1/3))/abs(c)^(1/3))*b*c - 1/2*(b*arctan(c*x^3) + a)/x^2

Mupad [B] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.65

$$\int \frac{a+b\arctan(cx^3)}{x^3}\,dx$$
$$= -\frac{a}{2\,x^2}-\frac{b\,c^{2/3}\left(\frac{\mathrm{atan}\left({\left(-1\right)}^{2/3}\,c^{1/3}\,x\right)}{2}-\frac{\mathrm{atan}\left(\frac{c^{1/3}\,x\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}\right)}{2}+\mathrm{atan}\left(\frac{{\left(-1\right)}^{2/3}\,c^{1/3}\,x\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}\right)\right)}{2}-\frac{b\,\mathrm{atan}\left(c\,x^3\right)}{2\,x^2}-\frac{\sqrt{3}\,b\,c^{2/3}\,\left(\mathrm{atan}\left(\frac{c^{1/3}\,x\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}\right)+\mathrm{atan}\left({\left(-1\right)}^{2/3}\,c^{1/3}\,x\right)\right)\,1{}\mathrm{i}}{4}$$

```
[In] int((a + b*atan(c*x^3))/x^3,x)
```

```
[Out] - a/(2*x^2) - (b*c^(2/3)*(atan((-1)^(2/3)*c^(1/3)*x)/2 - atan((c^(1/3)*x*(3
^(1/2)*1i + 1))/2)/2 + atan(((-1)^(2/3)*c^(1/3)*x*(3^(1/2)*1i + 1))/2)))/2
- (b*atan(c*x^3))/(2*x^2) - (3^(1/2)*b*c^(2/3)*(atan((c^(1/3)*x*(3^(1/2)*1i
 + 1))/2) + atan((-1)^(2/3)*c^(1/3)*x))*1i)/4
```

3.107 $\int \frac{a+b\arctan(cx^3)}{x^6}\,dx$

Optimal result 616
Rubi [A] (verified) 616
Mathematica [A] (verified) 619
Maple [A] (verified) 619
Fricas [A] (verification not implemented) 620
Sympy [B] (verification not implemented) 620
Maxima [A] (verification not implemented) 621
Giac [A] (verification not implemented) 621
Mupad [B] (verification not implemented) 622

Optimal result

Integrand size = 14, antiderivative size = 115

$$\int \frac{a+b\arctan(cx^3)}{x^6}\,dx = -\frac{3bc}{10x^2} - \frac{a+b\arctan(cx^3)}{5x^5} + \frac{1}{10}\sqrt{3}bc^{5/3}\arctan\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right) + \frac{1}{10}bc^{5/3}\log\left(1+c^{2/3}x^2\right) - \frac{1}{20}bc^{5/3}\log\left(1-c^{2/3}x^2+c^{4/3}x^4\right)$$

```
[Out] -3/10*b*c/x^2+1/5*(-a-b*arctan(c*x^3))/x^5+1/10*b*c^(5/3)*ln(1+c^(2/3)*x^2)
-1/20*b*c^(5/3)*ln(1-c^(2/3)*x^2+c^(4/3)*x^4)+1/10*b*c^(5/3)*arctan(1/3*(1-
2*c^(2/3)*x^2)*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {4946, 281, 331, 298, 31, 648, 631, 210, 642}

$$\int \frac{a+b\arctan(cx^3)}{x^6}\,dx = -\frac{a+b\arctan(cx^3)}{5x^5} + \frac{1}{10}\sqrt{3}bc^{5/3}\arctan\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right) + \frac{1}{10}bc^{5/3}\log\left(c^{2/3}x^2+1\right) - \frac{1}{20}bc^{5/3}\log\left(c^{4/3}x^4-c^{2/3}x^2+1\right) - \frac{3bc}{10x^2}$$

```
[In] Int[(a + b*ArcTan[c*x^3])/x^6,x]
```

```
[Out] (-3*b*c)/(10*x^2) - (a + b*ArcTan[c*x^3])/(5*x^5) + (Sqrt[3]*b*c^(5/3)*ArcT
an[(1 - 2*c^(2/3)*x^2)/Sqrt[3]])/10 + (b*c^(5/3)*Log[1 + c^(2/3)*x^2])/10 -
 (b*c^(5/3)*Log[1 - c^(2/3)*x^2 + c^(4/3)*x^4])/20
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{a+b\arctan\left(cx^3\right)}{5x^5}+\frac{1}{5}(3bc)\int\frac{1}{x^3\left(1+c^2x^6\right)}\,dx$$

$$= -\frac{a+b\arctan\left(cx^3\right)}{5x^5}+\frac{1}{10}(3bc)\text{Subst}\left(\int\frac{1}{x^2\left(1+c^2x^3\right)}\,dx,x,x^2\right)$$

$$= -\frac{3bc}{10x^2}-\frac{a+b\arctan\left(cx^3\right)}{5x^5}-\frac{1}{10}\left(3bc^3\right)\text{Subst}\left(\int\frac{x}{1+c^2x^3}\,dx,x,x^2\right)$$

$$= -\frac{3bc}{10x^2}-\frac{a+b\arctan\left(cx^3\right)}{5x^5}+\frac{1}{10}\left(bc^{7/3}\right)\text{Subst}\left(\int\frac{1}{1+c^{2/3}x}\,dx,x,x^2\right) - \frac{1}{10}\left(bc^{7/3}\right)\text{Subst}\left(\int\frac{1+c^{2/3}x}{1-c^{2/3}x+c^{4/3}x^2}\,dx,x,x^2\right)$$

$$= -\frac{3bc}{10x^2}-\frac{a+b\arctan\left(cx^3\right)}{5x^5}+\frac{1}{10}bc^{5/3}\log\left(1+c^{2/3}x^2\right) - \frac{1}{20}\left(bc^{5/3}\right)\text{Subst}\left(\int\frac{-c^{2/3}+2c^{4/3}x}{1-c^{2/3}x+c^{4/3}x^2}\,dx,x,x^2\right)-\frac{1}{20}\left(3bc^{7/3}\right)\text{Subst}\left(\int\frac{1}{1-c^{2/3}x+c^{4/3}x^2}\,dx,x\right.$$

$$= -\frac{3bc}{10x^2}-\frac{a+b\arctan\left(cx^3\right)}{5x^5}+\frac{1}{10}bc^{5/3}\log\left(1+c^{2/3}x^2\right) - \frac{1}{20}bc^{5/3}\log\left(1-c^{2/3}x^2+c^{4/3}x^4\right)-\frac{1}{10}\left(3bc^{5/3}\right)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-2c^{2/3}x^2\right)$$

$$= -\frac{3bc}{10x^2}-\frac{a+b\arctan\left(cx^3\right)}{5x^5}+\frac{1}{10}\sqrt{3}bc^{5/3}\arctan\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right) + \frac{1}{10}bc^{5/3}\log\left(1+c^{2/3}x^2\right)-\frac{1}{20}bc^{5/3}\log\left(1-c^{2/3}x^2+c^{4/3}x^4\right)$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.59

$$\int \frac{a+b\arctan(cx^3)}{x^6}\,dx = -\frac{a}{5x^5}-\frac{3bc}{10x^2}-\frac{b\arctan(cx^3)}{5x^5} +\frac{1}{10}\sqrt{3}bc^{5/3}\arctan\left(\sqrt{3}-2\sqrt[3]{c}x\right)+\frac{1}{10}\sqrt{3}bc^{5/3}\arctan\left(\sqrt{3}+2\sqrt[3]{c}x\right) +\frac{1}{10}bc^{5/3}\log\left(1+c^{2/3}x^2\right)-\frac{1}{20}bc^{5/3}\log\left(1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)-\frac{1}{20}bc^{5/3}\log\left(1+\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)$$

```
[In] Integrate[(a + b*ArcTan[c*x^3])/x^6,x]
```

```
[Out] -1/5*a/x^5 - (3*b*c)/(10*x^2) - (b*ArcTan[c*x^3])/(5*x^5) + (Sqrt[3]*b*c^(5
/3)*ArcTan[Sqrt[3] - 2*c^(1/3)*x])/10 + (Sqrt[3]*b*c^(5/3)*ArcTan[Sqrt[3] +
 2*c^(1/3)*x])/10 + (b*c^(5/3)*Log[1 + c^(2/3)*x^2])/10 - (b*c^(5/3)*Log[1
- Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/20 - (b*c^(5/3)*Log[1 + Sqrt[3]*c^(1/3)
*x + c^(2/3)*x^2])/20
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.91

method	result
default	$-\frac{a}{5x^5}-\frac{b\arctan(c\,x^3)}{5x^5}-\frac{3bc}{10x^2}+\frac{bc\ln\left(x^2+\left(\frac{1}{c^2}\right)^{\frac{1}{3}}\right)}{10\left(\frac{1}{c^2}\right)^{\frac{1}{3}}}-\frac{bc\ln\left(x^4-\left(\frac{1}{c^2}\right)^{\frac{1}{3}}x^2+\left(\frac{1}{c^2}\right)^{\frac{2}{3}}\right)}{20\left(\frac{1}{c^2}\right)^{\frac{1}{3}}}-\frac{bc\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x^2}{\left(\frac{1}{c^2}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{10\left(\frac{1}{c^2}\right)^{\frac{1}{3}}}$
parts	$-\frac{a}{5x^5}-\frac{b\arctan(c\,x^3)}{5x^5}-\frac{3bc}{10x^2}+\frac{bc\ln\left(x^2+\left(\frac{1}{c^2}\right)^{\frac{1}{3}}\right)}{10\left(\frac{1}{c^2}\right)^{\frac{1}{3}}}-\frac{bc\ln\left(x^4-\left(\frac{1}{c^2}\right)^{\frac{1}{3}}x^2+\left(\frac{1}{c^2}\right)^{\frac{2}{3}}\right)}{20\left(\frac{1}{c^2}\right)^{\frac{1}{3}}}-\frac{bc\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x^2}{\left(\frac{1}{c^2}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{10\left(\frac{1}{c^2}\right)^{\frac{1}{3}}}$
risch	$\frac{ib\ln(ic\,x^3+1)}{10x^5}-\frac{ib\ln(-ic\,x^3+1)}{10x^5}-\frac{3bc}{10x^2}-\frac{bc\ln\left(x+\left(\frac{i}{c}\right)^{\frac{1}{3}}\right)}{10\left(\frac{i}{c}\right)^{\frac{2}{3}}}+\frac{bc\ln\left(x^2-\left(\frac{i}{c}\right)^{\frac{1}{3}}x+\left(\frac{i}{c}\right)^{\frac{2}{3}}\right)}{20\left(\frac{i}{c}\right)^{\frac{2}{3}}}-\frac{bc\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{i}{c}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{10\left(\frac{i}{c}\right)^{\frac{2}{3}}}$

```
[In] int((a+b*arctan(c*x^3))/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*a/x^5-1/5*b/x^5*arctan(c*x^3)-3/10*b*c/x^2+1/10*b*c/(1/c^2)^(1/3)*ln(x
^2+(1/c^2)^(1/3))-1/20*b*c/(1/c^2)^(1/3)*ln(x^4-(1/c^2)^(1/3)*x^2+(1/c^2)^(
2/3))-1/10*b*c*3^(1/2)/(1/c^2)^(1/3)*arctan(1/3*3^(1/2)*(2*x^2/(1/c^2)^(1/3
)-1))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.05

$$\int \frac{a + b \arctan(cx^3)}{x^6}\, dx = -\frac{2\sqrt{3}b(c^2)^{\frac{1}{3}}cx^5 \arctan\left(\frac{2}{3}\sqrt{3}(c^2)^{\frac{1}{3}}x^2 - \frac{1}{3}\sqrt{3}\right) + b(c^2)^{\frac{1}{3}}cx^5 \log\left(c^2x^4 - (c^2)^{\frac{2}{3}}x^2 + (c^2)^{\frac{1}{3}}\right) - 2\, b(c^2)^{\frac{1}{3}}cx^5 \log}{20\, x^5}$$

```
[In] integrate((a+b*arctan(c*x^3))/x^6,x, algorithm="fricas")
```

```
[Out] -1/20*(2*sqrt(3)*b*(c^2)^(1/3)*c*x^5*arctan(2/3*sqrt(3)*(c^2)^(1/3)*x^2 - 1
/3*sqrt(3)) + b*(c^2)^(1/3)*c*x^5*log(c^2*x^4 - (c^2)^(2/3)*x^2 + (c^2)^(1/
3)) - 2*b*(c^2)^(1/3)*c*x^5*log(c^2*x^2 + (c^2)^(2/3)) + 6*b*c*x^3 + 4*b*ar
ctan(c*x^3) + 4*a)/x^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(112) = 224$.

Time = 61.14 (sec) , antiderivative size = 286, normalized size of antiderivative = 2.49

$$\int \frac{a + b \arctan(cx^3)}{x^6}\, dx = \begin{cases} -\frac{a}{5x^5} + \frac{bc^2\sqrt[6]{-\frac{1}{c^2}}\operatorname{atan}(cx^3)}{5} - \frac{bc\log\left(x - \sqrt[6]{-\frac{1}{c^2}}\right)}{5\sqrt[3]{-\frac{1}{c^2}}} + \frac{3bc\log\left(4x^2 - 4x\sqrt[6]{-\frac{1}{c^2}} + 4\sqrt[3]{-\frac{1}{c^2}}\right)}{20\sqrt[3]{-\frac{1}{c^2}}} - \frac{bc\log\left(4x^2 + 4x\sqrt[6]{-\frac{1}{c^2}} + 4\sqrt[3]{-\right.}}{20\sqrt[3]{-\frac{1}{c^2}}} \\ -\frac{a}{5x^5} \end{cases}$$

```
[In] integrate((a+b*atan(c*x**3))/x**6,x)
```

```
[Out] Piecewise((-a/(5*x**5) + b*c**2*(-1/c**2)**(1/6)*atan(c*x**3)/5 - b*c*log(x
 - (-1/c**2)**(1/6))/(5*(-1/c**2)**(1/3)) + 3*b*c*log(4*x**2 - 4*x*(-1/c**2
)**(1/6) + 4*(-1/c**2)**(1/3))/(20*(-1/c**2)**(1/3)) - b*c*log(4*x**2 + 4*x
*(-1/c**2)**(1/6) + 4*(-1/c**2)**(1/3))/(20*(-1/c**2)**(1/3)) - sqrt(3)*b*c
*atan(2*sqrt(3)*x/(3*(-1/c**2)**(1/6)) - sqrt(3)/3)/(10*(-1/c**2)**(1/3)) +
```

```
 sqrt(3)*b*c*atan(2*sqrt(3)*x/(3*(-1/c**2)**(1/6)) + sqrt(3)/3)/(10*(-1/c**
2)**(1/3)) - 3*b*c/(10*x**2) - b*atan(c*x**3)/(5*x**5), Ne(c, 0)), (-a/(5*x
**5), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.89

$$\int \frac{a+b\arctan(cx^3)}{x^6}\,dx = -\frac{1}{20}\left(\left(2\sqrt{3}c^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2c^{\frac{4}{3}}x^2-c^{\frac{2}{3}}\right)}{3c^{\frac{2}{3}}}\right)+c^{\frac{2}{3}}\log\left(c^{\frac{4}{3}}x^4-c^{\frac{2}{3}}x^2+1\right)-2c^{\frac{2}{3}}\log\left(\frac{c^{\frac{2}{3}}x^2+1}{c^{\frac{2}{3}}}\right)+\frac{6}{x^2}\right.\right. - \frac{a}{5x^5}$$

```
[In] integrate((a+b*arctan(c*x^3))/x^6,x, algorithm="maxima")
```

```
[Out] -1/20*((2*sqrt(3)*c^(2/3)*arctan(1/3*sqrt(3)*(2*c^(4/3)*x^2 - c^(2/3))/c^(2
/3)) + c^(2/3)*log(c^(4/3)*x^4 - c^(2/3)*x^2 + 1) - 2*c^(2/3)*log((c^(2/3)*
x^2 + 1)/c^(2/3)) + 6/x^2)*c + 4*arctan(c*x^3)/x^5)*b - 1/5*a/x^5
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.94

$$\int \frac{a+b\arctan(cx^3)}{x^6}\,dx = -\frac{1}{20}bc^3\left(\frac{2\sqrt{3}|c|^{\frac{2}{3}}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-\frac{1}{|c|^{\frac{2}{3}}}\right)|c|^{\frac{2}{3}}\right)}{c^2}+\frac{|c|^{\frac{2}{3}}\log\left(x^4-\frac{x^2}{|c|^{\frac{2}{3}}}+\frac{1}{|c|^{\frac{4}{3}}}\right)}{c^2}-\frac{2\log\left(x^2+\frac{1}{|c|^{\frac{2}{3}}}\right)}{|c|^{\frac{4}{3}}}\right) - \frac{3bcx^3+2b\arctan(cx^3)+2a}{10x^5}$$

```
[In] integrate((a+b*arctan(c*x^3))/x^6,x, algorithm="giac")
```

```
[Out] -1/20*b*c^3*(2*sqrt(3)*abs(c)^(2/3)*arctan(1/3*sqrt(3)*(2*x^2 - 1/abs(c)^(2
/3))*abs(c)^(2/3))/c^2 + abs(c)^(2/3)*log(x^4 - x^2/abs(c)^(2/3) + 1/abs(c)
^(4/3))/c^2 - 2*log(x^2 + 1/abs(c)^(2/3))/abs(c)^(4/3)) - 1/10*(3*b*c*x^3 +
 2*b*arctan(c*x^3) + 2*a)/x^5
```

Mupad [B] (verification not implemented)

Time = 2.71 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.03

$$\int \frac{a+b\arctan(cx^3)}{x^6}\,dx = \frac{b\,c^{5/3}\,\ln\left(c^{2/3}\,x^2+1\right)}{10} - \frac{\frac{3\,b\,c\,x^3}{2}+a}{5\,x^5} - \frac{b\,\mathrm{atan}(c\,x^3)}{5\,x^5} - \frac{b\,c^{5/3}\,\ln\left(\sqrt{3}\,c^{2/3}\,x^2 - c^{2/3}\,x^2\,1\mathrm{i} + 2\mathrm{i}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{20} + \frac{b\,c^{5/3}\,\ln\left(-c^{2/3}\,x^2\,1\mathrm{i} - \sqrt{3}\,c^{2/3}\,x^2 + 2\mathrm{i}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{20}$$

```
[In] int((a + b*atan(c*x^3))/x^6,x)
```

```
[Out] (b*c^(5/3)*log(c^(2/3)*x^2 + 1))/10 - (a + (3*b*c*x^3)/2)/(5*x^5) - (b*atan
(c*x^3))/(5*x^5) - (b*c^(5/3)*log(3^(1/2)*c^(2/3)*x^2 - c^(2/3)*x^2*1i + 2i
)*(3^(1/2)*1i + 1))/20 + (b*c^(5/3)*log(2i - 3^(1/2)*c^(2/3)*x^2 - c^(2/3)*
x^2*1i)*(3^(1/2)*1i - 1))/20
```

3.108 $\int x^7(a+b\arctan(cx^3))\,dx$

Optimal result 623
Rubi [A] (verified) 623
Mathematica [A] (verified) 626
Maple [A] (verified) 627
Fricas [B] (verification not implemented) 628
Sympy [A] (verification not implemented) 628
Maxima [A] (verification not implemented) 629
Giac [A] (verification not implemented) 629
Mupad [B] (verification not implemented) 630

Optimal result

Integrand size = 14, antiderivative size = 176

$$\int x^7\left(a+b\arctan\left(cx^3\right)\right)\,dx=-\frac{3bx^5}{40c}+\frac{b\arctan\left(\sqrt[3]{c}x\right)}{8c^{8/3}}+\frac{1}{8}x^8\left(a+b\arctan\left(cx^3\right)\right)-\frac{b\arctan\left(\sqrt{3}-2\sqrt[3]{c}x\right)}{16c^{8/3}}+\frac{b\arctan\left(\sqrt{3}+2\sqrt[3]{c}x\right)}{16c^{8/3}}+\frac{\sqrt{3}b\log\left(1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)}{32c^{8/3}}-\frac{\sqrt{3}b\log\left(1+\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)}{32c^{8/3}}$$

```
[Out] -3/40*b*x^5/c+1/8*b*arctan(c^(1/3)*x)/c^(8/3)+1/8*x^8*(a+b*arctan(c*x^3))+1
/16*b*arctan(2*c^(1/3)*x-3^(1/2))/c^(8/3)+1/16*b*arctan(2*c^(1/3)*x+3^(1/2)
)/c^(8/3)+1/32*b*ln(1+c^(2/3)*x^2-c^(1/3)*x*3^(1/2))*3^(1/2)/c^(8/3)-1/32*b
*ln(1+c^(2/3)*x^2+c^(1/3)*x*3^(1/2))*3^(1/2)/c^(8/3)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {4946, 327, 301, 648, 632, 210, 642, 209}

$$\int x^7\left(a+b\arctan\left(cx^3\right)\right)\,dx=\frac{1}{8}x^8\left(a+b\arctan\left(cx^3\right)\right)+\frac{b\arctan\left(\sqrt[3]{c}x\right)}{8c^{8/3}}-\frac{b\arctan\left(\sqrt{3}-2\sqrt[3]{c}x\right)}{16c^{8/3}}+\frac{b\arctan\left(2\sqrt[3]{c}x+\sqrt{3}\right)}{16c^{8/3}}+\frac{\sqrt{3}b\log\left(c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1\right)}{32c^{8/3}}-\frac{\sqrt{3}b\log\left(c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1\right)}{32c^{8/3}}-\frac{3bx^5}{40c}$$

```
[In] Int[x^7*(a + b*ArcTan[c*x^3]),x]
```

```
[Out] (-3*b*x^5)/(40*c) + (b*ArcTan[c^(1/3)*x])/(8*c^(8/3)) + (x^8*(a + b*ArcTan[
c*x^3]))/8 - (b*ArcTan[Sqrt[3] - 2*c^(1/3)*x])/(16*c^(8/3)) + (b*ArcTan[Sqr
t[3] + 2*c^(1/3)*x])/(16*c^(8/3)) + (Sqrt[3]*b*Log[1 - Sqrt[3]*c^(1/3)*x +
c^(2/3)*x^2])/(32*c^(8/3)) - (Sqrt[3]*b*Log[1 + Sqrt[3]*c^(1/3)*x + c^(2/3)
*x^2])/(32*c^(8/3))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 301

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r^
(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{8}x^8\left(a+b\arctan\left(cx^3\right)\right)-\frac{1}{8}(3bc)\int\frac{x^{10}}{1+c^2x^6}\,dx\\
&= -\frac{3bx^5}{40c}+\frac{1}{8}x^8\left(a+b\arctan\left(cx^3\right)\right)+\frac{(3b)\int\frac{x^4}{1+c^2x^6}\,dx}{8c}\\
&= -\frac{3bx^5}{40c}+\frac{1}{8}x^8\left(a+b\arctan\left(cx^3\right)\right)+\frac{b\int\frac{1}{1+c^{2/3}x^2}\,dx}{8c^{7/3}}\\
&\quad+\frac{b\int\frac{-\frac{1}{2}+\frac{1}{2}\sqrt{3}\sqrt[3]{c}x}{1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2}\,dx}{8c^{7/3}}+\frac{b\int\frac{-\frac{1}{2}-\frac{1}{2}\sqrt{3}\sqrt[3]{c}x}{1+\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2}\,dx}{8c^{7/3}}\\
&= -\frac{3bx^5}{40c}+\frac{b\arctan\left(\sqrt[3]{c}x\right)}{8c^{8/3}}+\frac{1}{8}x^8\left(a+b\arctan\left(cx^3\right)\right)+\frac{\left(\sqrt{3}b\right)\int\frac{-\sqrt{3}\sqrt[3]{c}+2c^{2/3}x}{1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2}\,dx}{32c^{8/3}}\\
&\quad-\frac{\left(\sqrt{3}b\right)\int\frac{\sqrt{3}\sqrt[3]{c}+2c^{2/3}x}{1+\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2}\,dx}{32c^{8/3}}+\frac{b\int\frac{1}{1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2}\,dx}{32c^{7/3}}+\frac{b\int\frac{1}{1+\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2}\,dx}{32c^{7/3}}\\
&= -\frac{3bx^5}{40c}+\frac{b\arctan\left(\sqrt[3]{c}x\right)}{8c^{8/3}}+\frac{1}{8}x^8\left(a+b\arctan\left(cx^3\right)\right)\\
&\quad+\frac{\sqrt{3}b\log\left(1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)}{32c^{8/3}}-\frac{\sqrt{3}b\log\left(1+\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)}{32c^{8/3}}\\
&\quad+\frac{b\text{Subst}\left(\int\frac{1}{-\frac{1}{3}-x^2}\,dx,x,1-\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)}{16\sqrt{3}c^{8/3}}-\frac{b\text{Subst}\left(\int\frac{1}{-\frac{1}{3}-x^2}\,dx,x,1+\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)}{16\sqrt{3}c^{8/3}}
\end{aligned}$$

$$= -\frac{3bx^5}{40c} + \frac{b\arctan\left(\sqrt[3]{c}x\right)}{8c^{8/3}} + \frac{1}{8}x^8\left(a + b\arctan\left(cx^3\right)\right) - \frac{b\arctan\left(\sqrt{3} - 2\sqrt[3]{c}x\right)}{16c^{8/3}} + \frac{b\arctan\left(\sqrt{3} + 2\sqrt[3]{c}x\right)}{16c^{8/3}} + \frac{\sqrt{3}b\log\left(1 - \sqrt{3}\sqrt[3]{c}x + c^{2/3}x^2\right)}{32c^{8/3}} - \frac{\sqrt{3}b\log\left(1 + \sqrt{3}\sqrt[3]{c}x + c^{2/3}x^2\right)}{32c^{8/3}}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.03

$$\int x^7\left(a + b\arctan\left(cx^3\right)\right)\,dx = -\frac{3bx^5}{40c} + \frac{ax^8}{8} + \frac{b\arctan\left(\sqrt[3]{c}x\right)}{8c^{8/3}} + \frac{1}{8}bx^8\arctan\left(cx^3\right) - \frac{b\arctan\left(\sqrt{3} - 2\sqrt[3]{c}x\right)}{16c^{8/3}} + \frac{b\arctan\left(\sqrt{3} + 2\sqrt[3]{c}x\right)}{16c^{8/3}} + \frac{\sqrt{3}b\log\left(1 - \sqrt{3}\sqrt[3]{c}x + c^{2/3}x^2\right)}{32c^{8/3}} - \frac{\sqrt{3}b\log\left(1 + \sqrt{3}\sqrt[3]{c}x + c^{2/3}x^2\right)}{32c^{8/3}}$$

```
[In] Integrate[x^7*(a + b*ArcTan[c*x^3]),x]
```

```
[Out] (-3*b*x^5)/(40*c) + (a*x^8)/8 + (b*ArcTan[c^(1/3)*x])/(8*c^(8/3)) + (b*x^8*
ArcTan[c*x^3])/8 - (b*ArcTan[Sqrt[3] - 2*c^(1/3)*x])/(16*c^(8/3)) + (b*ArcT
an[Sqrt[3] + 2*c^(1/3)*x])/(16*c^(8/3)) + (Sqrt[3]*b*Log[1 - Sqrt[3]*c^(1/3
)*x + c^(2/3)*x^2])/(32*c^(8/3)) - (Sqrt[3]*b*Log[1 + Sqrt[3]*c^(1/3)*x + c
^(2/3)*x^2])/(32*c^(8/3))
```

Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.95

method	result
default	$\frac{x^8 a}{8}+b\left(\frac{x^8\arctan\left(c\,x^3\right)}{8}-\frac{3c\left(\frac{x^5}{5c^2}-\frac{-\frac{\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{5}{6}}\ln\left(x^2+\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{1}{6}}x+\left(\frac{1}{c^2}\right)^{\frac{1}{3}}\right)}{12}+\frac{\arctan\left(\frac{2x}{\left(\frac{1}{c^2}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{6c^2\left(\frac{1}{c^2}\right)^{\frac{1}{6}}}+\frac{\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{5}{6}}\ln\left(x^2-\sqrt{3}\cdots\right.}{12}\cdots}{c^2}\right.}{8}\cdots\right.$
parts	$\frac{x^8 a}{8}+b\left(\frac{x^8\arctan\left(c\,x^3\right)}{8}-\frac{3c\left(\frac{x^5}{5c^2}-\frac{-\frac{\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{5}{6}}\ln\left(x^2+\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{1}{6}}x+\left(\frac{1}{c^2}\right)^{\frac{1}{3}}\right)}{12}+\frac{\arctan\left(\frac{2x}{\left(\frac{1}{c^2}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{6c^2\left(\frac{1}{c^2}\right)^{\frac{1}{6}}}+\frac{\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{5}{6}}\ln\left(x^2-\sqrt{3}\cdots\right.}{12}\cdots}{c^2}\right.}{8}\cdots\right.$

```
[In] int(x^7*(a+b*arctan(c*x^3)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*x^8*a+b*(1/8*x^8*arctan(c*x^3)-3/8*c*(1/5/c^2*x^5-(-1/12*3^(1/2)*(1/c^2
)^(5/6)*ln(x^2+3^(1/2)*(1/c^2)^(1/6)*x+(1/c^2)^(1/3))+1/6/c^2/(1/c^2)^(1/6)
*arctan(2*x/(1/c^2)^(1/6)+3^(1/2))+1/12*3^(1/2)*(1/c^2)^(5/6)*ln(x^2-3^(1/2
)*(1/c^2)^(1/6)*x+(1/c^2)^(1/3))+1/6/c^2/(1/c^2)^(1/6)*arctan(2*x/(1/c^2)^(
1/6)-3^(1/2))+1/3/c^2/(1/c^2)^(1/6)*arctan(x/(1/c^2)^(1/6)))/c^2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 305 vs. 2(128) = 256.

Time = 0.26 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.73

$$\int x^7\left(a+b\arctan\left(cx^3\right)\right)\,dx$$

$$=\frac{20\,bcx^8\arctan\left(cx^3\right)+20\,acx^8-12\,bx^5+10\,c\left(-\frac{b^6}{c^{16}}\right)^{\frac{1}{6}}\log\left(c^{13}\left(-\frac{b^6}{c^{16}}\right)^{\frac{5}{6}}+b^5x\right)-10\,c\left(-\frac{b^6}{c^{16}}\right)^{\frac{1}{6}}\log\left(-c^{13}\left(\right.\right.}{}$$

```
[In] integrate(x^7*(a+b*arctan(c*x^3)),x, algorithm="fricas")
```

```
[Out] 1/160*(20*b*c*x^8*arctan(c*x^3) + 20*a*c*x^8 - 12*b*x^5 + 10*c*(-b^6/c^16)^
(1/6)*log(c^13*(-b^6/c^16)^(5/6) + b^5*x) - 10*c*(-b^6/c^16)^(1/6)*log(-c^1
3*(-b^6/c^16)^(5/6) + b^5*x) - 5*(sqrt(-3)*c - c)*(-b^6/c^16)^(1/6)*log(b^5
*x + 1/2*(sqrt(-3)*c^13 + c^13)*(-b^6/c^16)^(5/6)) + 5*(sqrt(-3)*c - c)*(-b
^6/c^16)^(1/6)*log(b^5*x - 1/2*(sqrt(-3)*c^13 + c^13)*(-b^6/c^16)^(5/6)) -
5*(sqrt(-3)*c + c)*(-b^6/c^16)^(1/6)*log(b^5*x + 1/2*(sqrt(-3)*c^13 - c^13)
*(-b^6/c^16)^(5/6)) + 5*(sqrt(-3)*c + c)*(-b^6/c^16)^(1/6)*log(b^5*x - 1/2*
(sqrt(-3)*c^13 - c^13)*(-b^6/c^16)^(5/6)))/c
```

Sympy [A] (verification not implemented)

Time = 63.81 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.50

$$\int x^7\left(a+b\arctan\left(cx^3\right)\right)\,dx$$

$$=\begin{cases}\frac{ax^8}{8}+\frac{bx^8\operatorname{atan}\left(cx^3\right)}{8}-\frac{3bx^5}{40c}+\frac{3b\log\left(4x^2-4x\sqrt[6]{-\frac{1}{c^2}}+4\sqrt[3]{-\frac{1}{c^2}}\right)}{32c^3\sqrt[6]{-\frac{1}{c^2}}}-\frac{3b\log\left(4x^2+4x\sqrt[6]{-\frac{1}{c^2}}+4\sqrt[3]{-\frac{1}{c^2}}\right)}{32c^3\sqrt[6]{-\frac{1}{c^2}}}+\frac{\sqrt{3}b\operatorname{atan}\left(\frac{2\sqrt{3}x}{3\sqrt[6]{-\ldots}}\right.}{16c^3\sqrt[6]{-\ldots}} \\ \frac{ax^8}{8}\end{cases}$$

```
[In] integrate(x**7*(a+b*atan(c*x**3)),x)
```

```
[Out] Piecewise((a*x**8/8 + b*x**8*atan(c*x**3)/8 - 3*b*x**5/(40*c) + 3*b*log(4*x
**2 - 4*x*(-1/c**2)**(1/6) + 4*(-1/c**2)**(1/3))/(32*c**3*(-1/c**2)**(1/6))
 - 3*b*log(4*x**2 + 4*x*(-1/c**2)**(1/6) + 4*(-1/c**2)**(1/3))/(32*c**3*(-1
/c**2)**(1/6)) + sqrt(3)*b*atan(2*sqrt(3)*x/(3*(-1/c**2)**(1/6)) - sqrt(3)/
3)/(16*c**3*(-1/c**2)**(1/6)) + sqrt(3)*b*atan(2*sqrt(3)*x/(3*(-1/c**2)**(1
/6)) + sqrt(3)/3)/(16*c**3*(-1/c**2)**(1/6)) - b*atan(c*x**3)/(8*c**4*(-1/c
**2)**(2/3)), Ne(c, 0)), (a*x**8/8, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.86

$$\int x^7 \left(a + b\arctan\left(cx^3\right)\right)\,dx = \frac{1}{8}\,ax^8 + \frac{1}{160}\left(20\,x^8\arctan\left(cx^3\right) - \left(\frac{12\,x^5}{c^2} + \frac{5\left(\frac{\sqrt{3}\log\left(c^{\frac{2}{3}}x^2 + \sqrt{3}c^{\frac{1}{3}}x + 1\right)}{c^{\frac{5}{3}}} - \frac{\sqrt{3}\log\left(c^{\frac{2}{3}}x^2 - \sqrt{3}c^{\frac{1}{3}}x + 1\right)}{c^{\frac{5}{3}}} - \frac{4\arctan\left(c^{\frac{1}{3}}x\right)}{c^{\frac{5}{3}}} - \cdots\right.}{c^2}\right.\right.$$

[In] integrate(x^7*(a+b*arctan(c*x^3)),x, algorithm="maxima")

[Out] 1/8*a*x^8 + 1/160*(20*x^8*arctan(c*x^3) - (12*x^5/c^2 + 5*(sqrt(3)*log(c^(2/3)*x^2 + sqrt(3)*c^(1/3)*x + 1)/c^(5/3) - sqrt(3)*log(c^(2/3)*x^2 - sqrt(3)*c^(1/3)*x + 1)/c^(5/3) - 4*arctan(c^(1/3)*x)/c^(5/3) - 2*arctan((2*c^(2/3)*x + sqrt(3)*c^(1/3))/c^(1/3))/c^(5/3) - 2*arctan((2*c^(2/3)*x - sqrt(3)*c^(1/3))/c^(1/3))/c^(5/3))/c^2)*c)*b

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.97

$$\int x^7 \left(a + b\arctan\left(cx^3\right)\right)\,dx = -\frac{1}{32}\,bc^{15}\left(\frac{\sqrt{3}|c|^{\frac{1}{3}}\log\left(x^2 + \frac{\sqrt{3}x}{|c|^{\frac{1}{3}}} + \frac{1}{|c|^{\frac{2}{3}}}\right)}{c^{18}} - \frac{\sqrt{3}\log\left(x^2 - \frac{\sqrt{3}x}{|c|^{\frac{1}{3}}} + \frac{1}{|c|^{\frac{2}{3}}}\right)}{c^{16}|c|^{\frac{5}{3}}} - \frac{2\arctan\left(\left(2\,x + \frac{\sqrt{3}}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{c^{16}|c|^{\frac{5}{3}}} - \cdots\right. + \frac{5\,bcx^8\arctan\left(cx^3\right) + 5\,acx^8 - 3\,bx^5}{40\,c}$$

[In] integrate(x^7*(a+b*arctan(c*x^3)),x, algorithm="giac")

[Out] -1/32*b*c^15*(sqrt(3)*abs(c)^(1/3)*log(x^2 + sqrt(3)*x/abs(c)^(1/3) + 1/abs(c)^(2/3))/c^18 - sqrt(3)*log(x^2 - sqrt(3)*x/abs(c)^(1/3) + 1/abs(c)^(2/3))/(c^16*abs(c)^(5/3)) - 2*arctan((2*x + sqrt(3)/abs(c)^(1/3))*abs(c)^(1/3))/(c^16*abs(c)^(5/3)) - 2*arctan((2*x - sqrt(3)/abs(c)^(1/3))*abs(c)^(1/3))/(c^16*abs(c)^(5/3)) - 4*arctan(x*abs(c)^(1/3))/(c^16*abs(c)^(5/3))) + 1/40*(5*b*c*x^8*arctan(c*x^3) + 5*a*c*x^8 - 3*b*x^5)/c

Mupad [B] (verification not implemented)

Time = 1.06 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.69

$$\int x^7(a+b\arctan(cx^3))\,dx=\frac{a\,x^8}{8}-\frac{3\,b\,x^5}{40\,c}-\frac{b\left(\mathrm{atan}\left((-1)^{2/3}\,c^{1/3}\,x\right)+\mathrm{atan}\left(\frac{(-1)^{2/3}\,c^{1/3}\,x\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)+2\,\mathrm{atan}\left(\frac{(-1)^{2/3}\,c^{1/3}\,x\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\right)}{16\,c^{8/3}}+\frac{b\,x^8\,\mathrm{atan}(c\,x^3)}{8}+\frac{\sqrt{3}\,b\left(\mathrm{atan}\left((-1)^{2/3}\,c^{1/3}\,x\right)-\mathrm{atan}\left(\frac{(-1)^{2/3}\,c^{1/3}\,x\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\right)\,1\mathrm{i}}{16\,c^{8/3}}$$

```
[In] int(x^7*(a + b*atan(c*x^3)),x)
```

```
[Out] (a*x^8)/8 - (3*b*x^5)/(40*c) - (b*(atan((-1)^(2/3)*c^(1/3)*x) + atan(((-1)^
(2/3)*c^(1/3)*x*(3^(1/2)*1i - 1))/2) + 2*atan(((-1)^(2/3)*c^(1/3)*x*(3^(1/2
)*1i + 1))/2)))/(16*c^(8/3)) + (b*x^8*atan(c*x^3))/8 + (3^(1/2)*b*(atan((-1
)^(2/3)*c^(1/3)*x) - atan(((-1)^(2/3)*c^(1/3)*x*(3^(1/2)*1i - 1))/2))*1i)/(
16*c^(8/3))
```

3.109 $\int x^4(a+b\arctan(cx^3))\,dx$

Optimal result . . . 631
Rubi [A] (verified) . . . 631
Mathematica [A] (verified) . . . 634
Maple [A] (verified) . . . 634
Fricas [A] (verification not implemented) . . . 635
Sympy [B] (verification not implemented) . . . 635
Maxima [A] (verification not implemented) . . . 636
Giac [A] (verification not implemented) . . . 636
Mupad [B] (verification not implemented) . . . 637

Optimal result

Integrand size = 14, antiderivative size = 117

$$\int x^4(a+b\arctan(cx^3))\,dx=-\frac{3bx^2}{10c}+\frac{1}{5}x^5(a+b\arctan(cx^3))-\frac{\sqrt{3}b\arctan\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right)}{10c^{5/3}}+\frac{b\log\left(1+c^{2/3}x^2\right)}{10c^{5/3}}-\frac{b\log\left(1-c^{2/3}x^2+c^{4/3}x^4\right)}{20c^{5/3}}$$

```
[Out] -3/10*b*x^2/c+1/5*x^5*(a+b*arctan(c*x^3))+1/10*b*ln(1+c^(2/3)*x^2)/c^(5/3)-
1/20*b*ln(1-c^(2/3)*x^2+c^(4/3)*x^4)/c^(5/3)-1/10*b*arctan(1/3*(1-2*c^(2/3)
*x^2)*3^(1/2))*3^(1/2)/c^(5/3)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {4946, 281, 327, 206, 31, 648, 631, 210, 642}

$$\int x^4(a+b\arctan(cx^3))\,dx=\frac{1}{5}x^5(a+b\arctan(cx^3))-\frac{\sqrt{3}b\arctan\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right)}{10c^{5/3}}+\frac{b\log\left(c^{2/3}x^2+1\right)}{10c^{5/3}}-\frac{b\log\left(c^{4/3}x^4-c^{2/3}x^2+1\right)}{20c^{5/3}}-\frac{3bx^2}{10c}$$

```
[In] Int[x^4*(a + b*ArcTan[c*x^3]),x]
```

```
[Out] (-3*b*x^2)/(10*c) + (x^5*(a + b*ArcTan[c*x^3]))/5 - (Sqrt[3]*b*ArcTan[(1 -
2*c^(2/3)*x^2)/Sqrt[3]])/(10*c^(5/3)) + (b*Log[1 + c^(2/3)*x^2])/(10*c^(5/3
)) - (b*Log[1 - c^(2/3)*x^2 + c^(4/3)*x^4])/(20*c^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{5}x^5\left(a+b\arctan\left(cx^3\right)\right)-\frac{1}{5}(3bc)\int\frac{x^7}{1+c^2x^6}\,dx\\
&= \frac{1}{5}x^5\left(a+b\arctan\left(cx^3\right)\right)-\frac{1}{10}(3bc)\text{Subst}\left(\int\frac{x^3}{1+c^2x^3}\,dx,x,x^2\right)\\
&= -\frac{3bx^2}{10c}+\frac{1}{5}x^5\left(a+b\arctan\left(cx^3\right)\right)+\frac{(3b)\text{Subst}\left(\int\frac{1}{1+c^2x^3}\,dx,x,x^2\right)}{10c}\\
&= -\frac{3bx^2}{10c}+\frac{1}{5}x^5\left(a+b\arctan\left(cx^3\right)\right)+\frac{b\text{Subst}\left(\int\frac{1}{1+c^{2/3}x}\,dx,x,x^2\right)}{10c}\\
&\quad+\frac{b\text{Subst}\left(\int\frac{2-c^{2/3}x}{1-c^{2/3}x+c^{4/3}x^2}\,dx,x,x^2\right)}{10c}\\
&= -\frac{3bx^2}{10c}+\frac{1}{5}x^5\left(a+b\arctan\left(cx^3\right)\right)+\frac{b\log\left(1+c^{2/3}x^2\right)}{10c^{5/3}}\\
&\quad-\frac{b\text{Subst}\left(\int\frac{-c^{2/3}+2c^{4/3}x}{1-c^{2/3}x+c^{4/3}x^2}\,dx,x,x^2\right)}{20c^{5/3}}+\frac{(3b)\text{Subst}\left(\int\frac{1}{1-c^{2/3}x+c^{4/3}x^2}\,dx,x,x^2\right)}{20c}\\
&= -\frac{3bx^2}{10c}+\frac{1}{5}x^5\left(a+b\arctan\left(cx^3\right)\right)+\frac{b\log\left(1+c^{2/3}x^2\right)}{10c^{5/3}}\\
&\quad-\frac{b\log\left(1-c^{2/3}x^2+c^{4/3}x^4\right)}{20c^{5/3}}+\frac{(3b)\text{Subst}\left(\int\frac{1}{-3-x^2}\,dx,x,1-2c^{2/3}x^2\right)}{10c^{5/3}}\\
&= -\frac{3bx^2}{10c}+\frac{1}{5}x^5\left(a+b\arctan\left(cx^3\right)\right)-\frac{\sqrt{3}b\arctan\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right)}{10c^{5/3}}\\
&\quad+\frac{b\log\left(1+c^{2/3}x^2\right)}{10c^{5/3}}-\frac{b\log\left(1-c^{2/3}x^2+c^{4/3}x^4\right)}{20c^{5/3}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.58

$$\int x^4\left(a+b\arctan\left(cx^3\right)\right)\,dx=-\frac{3bx^2}{10c}+\frac{ax^5}{5}+\frac{1}{5}bx^5\arctan\left(cx^3\right)-\frac{\sqrt{3}b\arctan\left(\sqrt{3}-2\sqrt[3]{c}x\right)}{10c^{5/3}}-\frac{\sqrt{3}b\arctan\left(\sqrt{3}+2\sqrt[3]{c}x\right)}{10c^{5/3}}+\frac{b\log\left(1+c^{2/3}x^2\right)}{10c^{5/3}}-\frac{b\log\left(1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)}{20c^{5/3}}-\frac{b\log\left(1+\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)}{20c^{5/3}}$$

```
[In] Integrate[x^4*(a + b*ArcTan[c*x^3]),x]
```

```
[Out] (-3*b*x^2)/(10*c) + (a*x^5)/5 + (b*x^5*ArcTan[c*x^3])/5 - (Sqrt[3]*b*ArcTan
[Sqrt[3] - 2*c^(1/3)*x])/(10*c^(5/3)) - (Sqrt[3]*b*ArcTan[Sqrt[3] + 2*c^(1/
3)*x])/(10*c^(5/3)) + (b*Log[1 + c^(2/3)*x^2])/(10*c^(5/3)) - (b*Log[1 - Sq
rt[3]*c^(1/3)*x + c^(2/3)*x^2])/(20*c^(5/3)) - (b*Log[1 + Sqrt[3]*c^(1/3)*x
 + c^(2/3)*x^2])/(20*c^(5/3))
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.97

method	result
default	$\frac{a x^5}{5}+\frac{b x^5 \arctan(c x^3)}{5}-\frac{3b x^2}{10c}+\frac{b \ln\left(x^2+\left(\frac{1}{c^2}\right)^{\frac13}\right)}{10c^3\left(\frac{1}{c^2}\right)^{\frac23}}-\frac{b\ln\left(x^4-\left(\frac{1}{c^2}\right)^{\frac13}x^2+\left(\frac{1}{c^2}\right)^{\frac23}\right)}{20c^3\left(\frac{1}{c^2}\right)^{\frac23}}+\frac{b\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x^2}{\left(\frac{1}{c^2}\right)^{\frac13}}-1\right)}{3}\right)}{10c^3\left(\frac{1}{c^2}\right)^{\frac23}}$
parts	$\frac{a x^5}{5}+\frac{b x^5 \arctan(c x^3)}{5}-\frac{3b x^2}{10c}+\frac{b \ln\left(x^2+\left(\frac{1}{c^2}\right)^{\frac13}\right)}{10c^3\left(\frac{1}{c^2}\right)^{\frac23}}-\frac{b\ln\left(x^4-\left(\frac{1}{c^2}\right)^{\frac13}x^2+\left(\frac{1}{c^2}\right)^{\frac23}\right)}{20c^3\left(\frac{1}{c^2}\right)^{\frac23}}+\frac{b\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x^2}{\left(\frac{1}{c^2}\right)^{\frac13}}-1\right)}{3}\right)}{10c^3\left(\frac{1}{c^2}\right)^{\frac23}}$
risch	$-\frac{i x^5 b\ln(icx^3+1)}{10}+\frac{ibx^5\ln(-icx^3+1)}{10}-\frac{3bx^2}{10c}-\frac{ib\ln\left(x+\left(\frac{i}{c}\right)^{\frac13}\right)}{10c^2\left(\frac{i}{c}\right)^{\frac13}}+\frac{ib\ln\left(x^2-\left(\frac{i}{c}\right)^{\frac13}x+\left(\frac{i}{c}\right)^{\frac23}\right)}{20c^2\left(\frac{i}{c}\right)^{\frac13}}+\frac{ib\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{i}{c}\right)^{\frac13}}\right.}{3}\right.}{10c^2\left(\frac{i}{c}\right)^{\frac13}}$

```
[In] int(x^4*(a+b*arctan(c*x^3)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*a*x^5+1/5*b*x^5*arctan(c*x^3)-3/10*b*x^2/c+1/10*b/c^3/(1/c^2)^(2/3)*ln(
x^2+(1/c^2)^(1/3))-1/20*b/c^3/(1/c^2)^(2/3)*ln(x^4-(1/c^2)^(1/3)*x^2+(1/c^2
)^(2/3))+1/10*b/c^3/(1/c^2)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2*x^2/(1/c^2)
^(1/3)-1))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.17

$$\int x^4\left(a+b\arctan\left(cx^3\right)\right)\,dx$$

$$=\frac{4\,bc^3x^5\arctan\left(cx^3\right)+4\,ac^3x^5-6\,bc^2x^2+2\,\sqrt{3}b(c^2)^{\frac{1}{6}}c\arctan\left(\frac{\sqrt{3}\left(2\,(c^2)^{\frac{2}{3}}x^2-(c^2)^{\frac{1}{3}}\right)(c^2)^{\frac{1}{6}}}{3\,c}\right)-b(c^2)^{\frac{2}{3}}\log\left(c^2x\right.}{20\,c^3}$$

```
[In] integrate(x^4*(a+b*arctan(c*x^3)),x, algorithm="fricas")
```

```
[Out] 1/20*(4*b*c^3*x^5*arctan(c*x^3) + 4*a*c^3*x^5 - 6*b*c^2*x^2 + 2*sqrt(3)*b*(
c^2)^(1/6)*c*arctan(1/3*sqrt(3)*(2*(c^2)^(2/3)*x^2 - (c^2)^(1/3))*(c^2)^(1/
6)/c) - b*(c^2)^(2/3)*log(c^2*x^4 - (c^2)^(2/3)*x^2 + (c^2)^(1/3)) + 2*b*(c
^2)^(2/3)*log(c^2*x^2 + (c^2)^(2/3)))/c^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 292 vs. 2(109) = 218.

Time = 31.58 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.50

$$\int x^4\left(a+b\arctan\left(cx^3\right)\right)\,dx$$

$$=\begin{cases}\frac{ax^5}{5}-\frac{bc^3\left(-\frac{1}{c^2}\right)^{\frac{7}{3}}\log\left(x-\sqrt[6]{-\frac{1}{c^2}}\right)}{5}+\frac{3bc^3\left(-\frac{1}{c^2}\right)^{\frac{7}{3}}\log\left(4x^2-4x\sqrt[6]{-\frac{1}{c^2}}+4\sqrt[3]{-\frac{1}{c^2}}\right)}{20}-\frac{bc^3\left(-\frac{1}{c^2}\right)^{\frac{7}{3}}\log\left(4x^2+4x\sqrt[6]{-\frac{1}{c^2}}+4\sqrt[3]{\right.}}{20}\\ \frac{ax^5}{5}\end{cases}$$

```
[In] integrate(x**4*(a+b*atan(c*x**3)),x)
```

```
[Out] Piecewise((a*x**5/5 - b*c**3*(-1/c**2)**(7/3)*log(x - (-1/c**2)**(1/6))/5 +
 3*b*c**3*(-1/c**2)**(7/3)*log(4*x**2 - 4*x*(-1/c**2)**(1/6) + 4*(-1/c**2)*
*(1/3))/20 - b*c**3*(-1/c**2)**(7/3)*log(4*x**2 + 4*x*(-1/c**2)**(1/6) + 4*
```

```
(-1/c**2)**(1/3))/20 + sqrt(3)*b*c**3*(-1/c**2)**(7/3)*atan(2*sqrt(3)*x/(3*
(-1/c**2)**(1/6)) - sqrt(3)/3)/10 - sqrt(3)*b*c**3*(-1/c**2)**(7/3)*atan(2*
sqrt(3)*x/(3*(-1/c**2)**(1/6)) + sqrt(3)/3)/10 - b*c**2*(-1/c**2)**(11/6)*a
tan(c*x**3)/5 + b*x**5*atan(c*x**3)/5 - 3*b*x**2/(10*c), Ne(c, 0)), (a*x**5
/5, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.91

$$\int x^4\left(a+b\arctan\left(cx^3\right)\right)\,dx=\frac{1}{5}\,ax^5+\frac{1}{20}\left(4\,x^5\arctan\left(cx^3\right)-c\left(\frac{6\,x^2}{c^2}-\frac{2\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,c^{\frac{4}{3}}x^2-c^{\frac{2}{3}}\right)}{3\,c^{\frac{2}{3}}}\right)}{c^{\frac{8}{3}}}+\frac{\log\left(c^{\frac{4}{3}}x^4-c^{\frac{2}{3}}x^2+1\right)}{c^{\frac{8}{3}}}-\frac{2\,\log\left(\frac{c^{\frac{2}{3}}x^2+1}{c^{\frac{2}{3}}}\right)}{c^{\frac{8}{3}}}\right)\right)b$$

```
[In] integrate(x^4*(a+b*arctan(c*x^3)),x, algorithm="maxima")
```

```
[Out] 1/5*a*x^5 + 1/20*(4*x^5*arctan(c*x^3) - c*(6*x^2/c^2 - 2*sqrt(3)*arctan(1/3
*sqrt(3)*(2*c^(4/3)*x^2 - c^(2/3))/c^(2/3))/c^(8/3) + log(c^(4/3)*x^4 - c^(
2/3)*x^2 + 1)/c^(8/3) - 2*log((c^(2/3)*x^2 + 1)/c^(2/3))/c^(8/3)))*b
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.02

$$\int x^4\left(a+b\arctan\left(cx^3\right)\right)\,dx=\frac{1}{20}\,bc^9\left(\frac{2\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^2-\frac{1}{|c|^{\frac{2}{3}}}\right)|c|^{\frac{2}{3}}\right)}{c^{10}|c|^{\frac{2}{3}}}-\frac{\log\left(x^4-\frac{x^2}{|c|^{\frac{2}{3}}}+\frac{1}{|c|^{\frac{4}{3}}}\right)}{c^{10}|c|^{\frac{2}{3}}}+\frac{2\,\log\left(x^2+\frac{1}{|c|^{\frac{2}{3}}}\right)}{c^{10}|c|^{\frac{2}{3}}}\right)+\frac{2\,bcx^5\arctan\left(cx^3\right)+2\,acx^5-3\,bx^2}{10\,c}$$

```
[In] integrate(x^4*(a+b*arctan(c*x^3)),x, algorithm="giac")
```

```
[Out] 1/20*b*c^9*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1/abs(c)^(2/3))*abs(c)^(2
/3))/(c^10*abs(c)^(2/3)) - log(x^4 - x^2/abs(c)^(2/3) + 1/abs(c)^(4/3))/(c^
10*abs(c)^(2/3)) + 2*log(x^2 + 1/abs(c)^(2/3))/(c^10*abs(c)^(2/3))) + 1/10*
(2*b*c*x^5*arctan(c*x^3) + 2*a*c*x^5 - 3*b*x^2)/c
```

Mupad [B] (verification not implemented)

Time = 2.07 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.91

$$\int x^4\left(a+b\arctan\left(cx^3\right)\right)\,dx=\frac{a\,x^5}{5}+\frac{b\,\ln\left(c^{2/3}\,x^2+1\right)}{10\,c^{5/3}}-\frac{3\,b\,x^2}{10\,c}-\frac{\ln\left(1-2\,c^{2/3}\,x^2+\sqrt{3}\,1\mathrm{i}\right)\,\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)}{20\,c^{5/3}}-\frac{\ln\left(2\,c^{2/3}\,x^2-1+\sqrt{3}\,1\mathrm{i}\right)\,\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)}{20\,c^{5/3}}+\frac{b\,x^5\,\mathrm{atan}(c\,x^3)}{5}$$

```
[In] int(x^4*(a + b*atan(c*x^3)),x)
```

```
[Out] (a*x^5)/5 + (b*log(c^(2/3)*x^2 + 1))/(10*c^(5/3)) - (3*b*x^2)/(10*c) - (log
(3^(1/2)*1i - 2*c^(2/3)*x^2 + 1)*(b + 3^(1/2)*b*1i))/(20*c^(5/3)) - (log(3^
(1/2)*1i + 2*c^(2/3)*x^2 - 1)*(b - 3^(1/2)*b*1i))/(20*c^(5/3)) + (b*x^5*ata
n(c*x^3))/5
```

3.110 $\int x(a+b\arctan(cx^3))\,dx$

Optimal result 638
Rubi [A] (verified) 638
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Optimal result

Integrand size = 12, antiderivative size = 165

$$\int x(a+b\arctan(cx^3))\,dx=-\frac{b\arctan\left(\sqrt[3]{c}x\right)}{2c^{2/3}}+\frac{1}{2}x^2(a+b\arctan(cx^3))+\frac{b\arctan\left(\sqrt{3}-2\sqrt[3]{c}x\right)}{4c^{2/3}}-\frac{b\arctan\left(\sqrt{3}+2\sqrt[3]{c}x\right)}{4c^{2/3}}-\frac{\sqrt{3}b\log\left(1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)}{8c^{2/3}}+\frac{\sqrt{3}b\log\left(1+\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)}{8c^{2/3}}$$

```
[Out] -1/2*b*arctan(c^(1/3)*x)/c^(2/3)+1/2*x^2*(a+b*arctan(c*x^3))-1/4*b*arctan(2
*c^(1/3)*x-3^(1/2))/c^(2/3)-1/4*b*arctan(2*c^(1/3)*x+3^(1/2))/c^(2/3)-1/8*b
*ln(1+c^(2/3)*x^2-c^(1/3)*x*3^(1/2))*3^(1/2)/c^(2/3)+1/8*b*ln(1+c^(2/3)*x^2
+c^(1/3)*x*3^(1/2))*3^(1/2)/c^(2/3)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {4946, 301, 648, 632, 210, 642, 209}

$$\int x(a+b\arctan(cx^3))\,dx=\frac{1}{2}x^2(a+b\arctan(cx^3))-\frac{b\arctan\left(\sqrt[3]{c}x\right)}{2c^{2/3}}+\frac{b\arctan\left(\sqrt{3}-2\sqrt[3]{c}x\right)}{4c^{2/3}}-\frac{b\arctan\left(2\sqrt[3]{c}x+\sqrt{3}\right)}{4c^{2/3}}-\frac{\sqrt{3}b\log\left(c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1\right)}{8c^{2/3}}+\frac{\sqrt{3}b\log\left(c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1\right)}{8c^{2/3}}$$

```
[In] Int[x*(a + b*ArcTan[c*x^3]),x]
```

```
[Out] -1/2*(b*ArcTan[c^(1/3)*x])/c^(2/3) + (x^2*(a + b*ArcTan[c*x^3]))/2 + (b*Arc
Tan[Sqrt[3] - 2*c^(1/3)*x])/(4*c^(2/3)) - (b*ArcTan[Sqrt[3] + 2*c^(1/3)*x])
/(4*c^(2/3)) - (Sqrt[3]*b*Log[1 - Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/(8*c^(2
/3)) + (Sqrt[3]*b*Log[1 + Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/(8*c^(2/3))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 301

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r^
(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{1}{2}x^2\left(a+b\arctan\left(cx^3\right)\right)-\frac{1}{2}(3bc)\int\frac{x^4}{1+c^2x^6}\,dx$$

$$=\frac{1}{2}x^2\left(a+b\arctan\left(cx^3\right)\right)-\frac{b\int\frac{1}{1+c^{2/3}x^2}\,dx}{2\sqrt[3]{c}}-\frac{b\int\frac{-\frac{1}{2}+\frac{1}{2}\sqrt{3}\sqrt[3]{c}x}{1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2}\,dx}{2\sqrt[3]{c}}-\frac{b\int\frac{-\frac{1}{2}-\frac{1}{2}\sqrt{3}\sqrt[3]{c}x}{1+\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2}\,dx}{2\sqrt[3]{c}}$$

$$=-\frac{b\arctan\left(\sqrt[3]{c}x\right)}{2c^{2/3}}+\frac{1}{2}x^2\left(a+b\arctan\left(cx^3\right)\right)-\frac{\left(\sqrt{3}b\right)\int\frac{-\sqrt{3}\sqrt[3]{c}+2c^{2/3}x}{1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2}\,dx}{8c^{2/3}}+\frac{\left(\sqrt{3}b\right)\int\frac{\sqrt{3}\sqrt[3]{c}+2c^{2/3}x}{1+\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2}\,dx}{8c^{2/3}}-\frac{b\int\frac{1}{1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2}\,dx}{8\sqrt[3]{c}}-\frac{b\int\frac{1}{1+\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2}\,dx}{8\sqrt[3]{c}}$$

$$=-\frac{b\arctan\left(\sqrt[3]{c}x\right)}{2c^{2/3}}+\frac{1}{2}x^2\left(a+b\arctan\left(cx^3\right)\right)-\frac{\sqrt{3}b\log\left(1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)}{8c^{2/3}}+\frac{\sqrt{3}b\log\left(1+\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)}{8c^{2/3}}-\frac{b\text{Subst}\left(\int\frac{1}{-\frac{1}{3}-x^2}\,dx,x,1-\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)}{4\sqrt{3}c^{2/3}}+\frac{b\text{Subst}\left(\int\frac{1}{-\frac{1}{3}-x^2}\,dx,x,1+\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)}{4\sqrt{3}c^{2/3}}$$

$$=-\frac{b\arctan\left(\sqrt[3]{c}x\right)}{2c^{2/3}}+\frac{1}{2}x^2\left(a+b\arctan\left(cx^3\right)\right)+\frac{b\arctan\left(\sqrt{3}-2\sqrt[3]{c}x\right)}{4c^{2/3}}-\frac{b\arctan\left(\sqrt{3}+2\sqrt[3]{c}x\right)}{4c^{2/3}}-\frac{\sqrt{3}b\log\left(1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)}{8c^{2/3}}+\frac{\sqrt{3}b\log\left(1+\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)}{8c^{2/3}}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.03

$$\int x\left(a+b\arctan\left(cx^3\right)\right)\,dx=\frac{ax^2}{2}-\frac{b\arctan\left(\sqrt[3]{c}x\right)}{2c^{2/3}}+\frac{1}{2}bx^2\arctan\left(cx^3\right)+\frac{b\arctan\left(\sqrt{3}-2\sqrt[3]{c}x\right)}{4c^{2/3}}-\frac{b\arctan\left(\sqrt{3}+2\sqrt[3]{c}x\right)}{4c^{2/3}}-\frac{\sqrt{3}b\log\left(1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)}{8c^{2/3}}+\frac{\sqrt{3}b\log\left(1+\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)}{8c^{2/3}}$$

```
[In] Integrate[x*(a + b*ArcTan[c*x^3]),x]
```

```
[Out] (a*x^2)/2 - (b*ArcTan[c^(1/3)*x])/(2*c^(2/3)) + (b*x^2*ArcTan[c*x^3])/2 + (
b*ArcTan[Sqrt[3] - 2*c^(1/3)*x])/(4*c^(2/3)) - (b*ArcTan[Sqrt[3] + 2*c^(1/3
)*x])/(4*c^(2/3)) - (Sqrt[3]*b*Log[1 - Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/(8
*c^(2/3)) + (Sqrt[3]*b*Log[1 + Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/(8*c^(2/3)
)
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.93

method	result
default	$\frac{a\,x^2}{2}+b\left(\frac{x^2\arctan\left(c\,x^3\right)}{2}-\frac{3c\left(-\frac{\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{5}{6}}\ln\left(x^2+\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{1}{6}}x+\left(\frac{1}{c^2}\right)^{\frac{1}{3}}\right)}{12}+\frac{\arctan\left(\frac{2x}{\left(\frac{1}{c^2}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{6c^2\left(\frac{1}{c^2}\right)^{\frac{1}{6}}}+\frac{\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{5}{6}}\ln\left(x^2-\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{1}{6}}\cdots\right.}{12}\cdots\right.}{2}\cdots\right.$
parts	$\frac{a\,x^2}{2}+b\left(\frac{x^2\arctan\left(c\,x^3\right)}{2}-\frac{3c\left(-\frac{\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{5}{6}}\ln\left(x^2+\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{1}{6}}x+\left(\frac{1}{c^2}\right)^{\frac{1}{3}}\right)}{12}+\frac{\arctan\left(\frac{2x}{\left(\frac{1}{c^2}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{6c^2\left(\frac{1}{c^2}\right)^{\frac{1}{6}}}+\frac{\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{5}{6}}\ln\left(x^2-\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{1}{6}}\cdots\right.}{12}\cdots\right.}{2}\cdots\right.$

```
[In] int(x*(a+b*arctan(c*x^3)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a*x^2+b*(1/2*x^2*arctan(c*x^3)-3/2*c*(-1/12*3^(1/2)*(1/c^2)^(5/6)*ln(x^
2+3^(1/2)*(1/c^2)^(1/6)*x+(1/c^2)^(1/3))+1/6/c^2/(1/c^2)^(1/6)*arctan(2*x/(
1/c^2)^(1/6)+3^(1/2))+1/12*3^(1/2)*(1/c^2)^(5/6)*ln(x^2-3^(1/2)*(1/c^2)^(1/
6)*x+(1/c^2)^(1/3))+1/6/c^2/(1/c^2)^(1/6)*arctan(2*x/(1/c^2)^(1/6)-3^(1/2))
+1/3/c^2/(1/c^2)^(1/6)*arctan(x/(1/c^2)^(1/6))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 278 vs. $2(119) = 238$.

Time = 0.25 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.68

$$\int x\left(a+b\arctan\left(cx^{3}\right)\right)\,dx=\frac{1}{2}\,bx^{2}\arctan\left(cx^{3}\right)+\frac{1}{2}\,ax^{2}+\frac{1}{8}\left(-\frac{b^{6}}{c^{4}}\right)^{\frac{1}{6}}\left(\sqrt{-3}-1\right)\log\left(b^{5}x+\frac{1}{2}\left(\sqrt{-3}c^{3}+c^{3}\right)\left(-\frac{b^{6}}{c^{4}}\right)^{\frac{5}{6}}\right)-\frac{1}{8}\left(-\frac{b^{6}}{c^{4}}\right)^{\frac{1}{6}}\left(\sqrt{-3}-1\right)\log\left(b^{5}x-\frac{1}{2}\left(\sqrt{-3}c^{3}+c^{3}\right)\left(-\frac{b^{6}}{c^{4}}\right)^{\frac{5}{6}}\right)+\frac{1}{8}\left(-\frac{b^{6}}{c^{4}}\right)^{\frac{1}{6}}\left(\sqrt{-3}+1\right)\log\left(b^{5}x+\frac{1}{2}\left(\sqrt{-3}c^{3}-c^{3}\right)\left(-\frac{b^{6}}{c^{4}}\right)^{\frac{5}{6}}\right)-\frac{1}{8}\left(-\frac{b^{6}}{c^{4}}\right)^{\frac{1}{6}}\left(\sqrt{-3}+1\right)\log\left(b^{5}x-\frac{1}{2}\left(\sqrt{-3}c^{3}-c^{3}\right)\left(-\frac{b^{6}}{c^{4}}\right)^{\frac{5}{6}}\right)-\frac{1}{4}\left(-\frac{b^{6}}{c^{4}}\right)^{\frac{1}{6}}\log\left(b^{5}x+\left(-\frac{b^{6}}{c^{4}}\right)^{\frac{5}{6}}c^{3}\right)+\frac{1}{4}\left(-\frac{b^{6}}{c^{4}}\right)^{\frac{1}{6}}\log\left(b^{5}x-\left(-\frac{b^{6}}{c^{4}}\right)^{\frac{5}{6}}c^{3}\right)$$

```
[In] integrate(x*(a+b*arctan(c*x^3)),x, algorithm="fricas")
```

```
[Out] 1/2*b*x^2*arctan(c*x^3) + 1/2*a*x^2 + 1/8*(-b^6/c^4)^(1/6)*(sqrt(-3) - 1)*l
og(b^5*x + 1/2*(sqrt(-3)*c^3 + c^3)*(-b^6/c^4)^(5/6)) - 1/8*(-b^6/c^4)^(1/6
)*(sqrt(-3) - 1)*log(b^5*x - 1/2*(sqrt(-3)*c^3 + c^3)*(-b^6/c^4)^(5/6)) + 1
/8*(-b^6/c^4)^(1/6)*(sqrt(-3) + 1)*log(b^5*x + 1/2*(sqrt(-3)*c^3 - c^3)*(-b
^6/c^4)^(5/6)) - 1/8*(-b^6/c^4)^(1/6)*(sqrt(-3) + 1)*log(b^5*x - 1/2*(sqrt(
-3)*c^3 - c^3)*(-b^6/c^4)^(5/6)) - 1/4*(-b^6/c^4)^(1/6)*log(b^5*x + (-b^6/c
^4)^(5/6)*c^3) + 1/4*(-b^6/c^4)^(1/6)*log(b^5*x - (-b^6/c^4)^(5/6)*c^3)
```

Sympy [A] (verification not implemented)

Time = 18.14 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.49

$$\int x\left(a+b\arctan\left(cx^3\right)\right)\,dx$$
$$=\begin{cases}\frac{ax^2}{2}+\frac{bx^2\operatorname{atan}\left(cx^3\right)}{2}-\frac{3b\log\left(4x^2-4x\sqrt[6]{-\frac{1}{c^2}}+4\sqrt[3]{-\frac{1}{c^2}}\right)}{8c\sqrt[6]{-\frac{1}{c^2}}}+\frac{3b\log\left(4x^2+4x\sqrt[6]{-\frac{1}{c^2}}+4\sqrt[3]{-\frac{1}{c^2}}\right)}{8c\sqrt[6]{-\frac{1}{c^2}}}-\frac{\sqrt{3}b\operatorname{atan}\left(\frac{2\sqrt{3}x}{3\sqrt[6]{-\frac{1}{c^2}}}-\frac{\sqrt{3}}{3}\right)}{4c\sqrt[6]{-\frac{1}{c^2}}} \\ \frac{ax^2}{2}\end{cases}$$

[In] integrate(x*(a+b*atan(c*x**3)),x)

[Out] Piecewise((a*x**2/2 + b*x**2*atan(c*x**3)/2 - 3*b*log(4*x**2 - 4*x*(-1/c**2)**(1/6) + 4*(-1/c**2)**(1/3))/(8*c*(-1/c**2)**(1/6)) + 3*b*log(4*x**2 + 4* x*(-1/c**2)**(1/6) + 4*(-1/c**2)**(1/3))/(8*c*(-1/c**2)**(1/6)) - sqrt(3)*b *atan(2*sqrt(3)*x/(3*(-1/c**2)**(1/6)) - sqrt(3)/3)/(4*c*(-1/c**2)**(1/6)) - sqrt(3)*b*atan(2*sqrt(3)*x/(3*(-1/c**2)**(1/6)) + sqrt(3)/3)/(4*c*(-1/c** 2)**(1/6)) + b*atan(c*x**3)/(2*c**2*(-1/c**2)**(2/3)), Ne(c, 0)), (a*x**2/2 , True))

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.83

$$\int x\left(a+b\arctan\left(cx^3\right)\right)\,dx=\frac{1}{2}\,ax^2$$
$$+\frac{1}{8}\left(4\,x^2\arctan\left(cx^3\right)+c\left(\frac{\sqrt{3}\log\left(c^{\frac{2}{3}}x^2+\sqrt{3}c^{\frac{1}{3}}x+1\right)}{c^{\frac{5}{3}}}-\frac{\sqrt{3}\log\left(c^{\frac{2}{3}}x^2-\sqrt{3}c^{\frac{1}{3}}x+1\right)}{c^{\frac{5}{3}}}-\frac{4\,\arctan\left(c^{\frac{1}{3}}x\right)}{c^{\frac{5}{3}}}\right.\right.$$

[In] integrate(x*(a+b*arctan(c*x^3)),x, algorithm="maxima")

[Out] 1/2*a*x^2 + 1/8*(4*x^2*arctan(c*x^3) + c*(sqrt(3)*log(c^(2/3)*x^2 + sqrt(3) *c^(1/3)*x + 1)/c^(5/3) - sqrt(3)*log(c^(2/3)*x^2 - sqrt(3)*c^(1/3)*x + 1)/ c^(5/3) - 4*arctan(c^(1/3)*x)/c^(5/3) - 2*arctan((2*c^(2/3)*x + sqrt(3)*c^(1/3))/c^(1/3))/c^(5/3) - 2*arctan((2*c^(2/3)*x - sqrt(3)*c^(1/3))/c^(1/3))/ c^(5/3)))*b

Giac [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.95

$$\int x\left(a+b\arctan\left(cx^3\right)\right)\,dx$$
$$=\frac{1}{8}\,bc^5\left(\frac{\sqrt{3}|c|^{\frac{1}{3}}\log\left(x^2+\frac{\sqrt{3}x}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{c^6}-\frac{\sqrt{3}|c|^{\frac{1}{3}}\log\left(x^2-\frac{\sqrt{3}x}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{c^6}-\frac{2\arctan\left(\left(2\,x+\frac{\sqrt{3}}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{c^4|c|^{\frac{5}{3}}}-\right.$$
$$+\frac{1}{2}\,bx^2\arctan\left(cx^3\right)+\frac{1}{2}\,ax^2$$

```
[In] integrate(x*(a+b*arctan(c*x^3)),x, algorithm="giac")
```

```
[Out] 1/8*b*c^5*(sqrt(3)*abs(c)^(1/3)*log(x^2 + sqrt(3)*x/abs(c)^(1/3) + 1/abs(c)
^(2/3))/c^6 - sqrt(3)*abs(c)^(1/3)*log(x^2 - sqrt(3)*x/abs(c)^(1/3) + 1/abs
(c)^(2/3))/c^6 - 2*arctan((2*x + sqrt(3)/abs(c)^(1/3))*abs(c)^(1/3))/(c^4*a
bs(c)^(5/3)) - 2*arctan((2*x - sqrt(3)/abs(c)^(1/3))*abs(c)^(1/3))/(c^4*abs
(c)^(5/3)) - 4*arctan(x*abs(c)^(1/3))/(c^4*abs(c)^(5/3))) + 1/2*b*x^2*arcta
n(c*x^3) + 1/2*a*x^2
```

Mupad [B] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.68

$$\int x\left(a+b\arctan\left(cx^3\right)\right)\,dx=\frac{a\,x^2}{2}$$
$$+\frac{b\left(\mathrm{atan}\left((-1)^{2/3}\,c^{1/3}\,x\right)+\mathrm{atan}\left(\frac{(-1)^{2/3}\,c^{1/3}\,x\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)+2\,\mathrm{atan}\left(\frac{(-1)^{2/3}\,c^{1/3}\,x\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\right)}{4\,c^{2/3}}$$
$$+\frac{b\,x^2\,\mathrm{atan}(c\,x^3)}{2}-\frac{\sqrt{3}\,b\left(\mathrm{atan}\left((-1)^{2/3}\,c^{1/3}\,x\right)-\mathrm{atan}\left(\frac{(-1)^{2/3}\,c^{1/3}\,x\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\right)\,1\mathrm{i}}{4\,c^{2/3}}$$

```
[In] int(x*(a + b*atan(c*x^3)),x)
```

```
[Out] (a*x^2)/2 + (b*(atan((-1)^(2/3)*c^(1/3)*x) + atan(((-1)^(2/3)*c^(1/3)*x*(3^
(1/2)*1i - 1))/2) + 2*atan(((-1)^(2/3)*c^(1/3)*x*(3^(1/2)*1i + 1))/2)))/(4*
c^(2/3)) + (b*x^2*atan(c*x^3))/2 - (3^(1/2)*b*(atan((-1)^(2/3)*c^(1/3)*x) -
 atan(((-1)^(2/3)*c^(1/3)*x*(3^(1/2)*1i - 1))/2))*1i)/(4*c^(2/3))
```

3.111 $\int \frac{a+b\arctan(cx^3)}{x^2}\,dx$

Optimal result 646
Rubi [A] (verified) 646
Mathematica [A] (verified) 648
Maple [A] (verified) 649
Fricas [A] (verification not implemented) 650
Sympy [B] (verification not implemented) 650
Maxima [A] (verification not implemented) 651
Giac [A] (verification not implemented) 651
Mupad [B] (verification not implemented) 652

Optimal result

Integrand size = 14, antiderivative size = 104

$$\int \frac{a+b\arctan(cx^3)}{x^2}\,dx = -\frac{a+b\arctan(cx^3)}{x} - \frac{1}{2}\sqrt{3}b\sqrt[3]{c}\arctan\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right) + \frac{1}{2}b\sqrt[3]{c}\log\left(1+c^{2/3}x^2\right) - \frac{1}{4}b\sqrt[3]{c}\log\left(1-c^{2/3}x^2+c^{4/3}x^4\right)$$

```
[Out] (-a-b*arctan(c*x^3))/x+1/2*b*c^(1/3)*ln(1+c^(2/3)*x^2)-1/4*b*c^(1/3)*ln(1-c
^(2/3)*x^2+c^(4/3)*x^4)-1/2*b*c^(1/3)*arctan(1/3*(1-2*c^(2/3)*x^2)*3^(1/2))
*3^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {4946, 281, 206, 31, 648, 631, 210, 642}

$$\int \frac{a+b\arctan(cx^3)}{x^2}\,dx = -\frac{a+b\arctan(cx^3)}{x} - \frac{1}{2}\sqrt{3}b\sqrt[3]{c}\arctan\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right) + \frac{1}{2}b\sqrt[3]{c}\log\left(c^{2/3}x^2+1\right) - \frac{1}{4}b\sqrt[3]{c}\log\left(c^{4/3}x^4-c^{2/3}x^2+1\right)$$

```
[In] Int[(a + b*ArcTan[c*x^3])/x^2,x]
```

```
[Out] -((a + b*ArcTan[c*x^3])/x) - (Sqrt[3]*b*c^(1/3)*ArcTan[(1 - 2*c^(2/3)*x^2)/
Sqrt[3]])/2 + (b*c^(1/3)*Log[1 + c^(2/3)*x^2])/2 - (b*c^(1/3)*Log[1 - c^(2/
3)*x^2 + c^(4/3)*x^4])/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
```

```
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{a+b\arctan(cx^3)}{x} + (3bc)\int \frac{x}{1+c^2x^6}\,dx \\
&= -\frac{a+b\arctan(cx^3)}{x} + \frac{1}{2}(3bc)\text{Subst}\left(\int \frac{1}{1+c^2x^3}\,dx, x, x^2\right) \\
&= -\frac{a+b\arctan(cx^3)}{x} + \frac{1}{2}(bc)\text{Subst}\left(\int \frac{1}{1+c^{2/3}x}\,dx, x, x^2\right) \\
&\quad + \frac{1}{2}(bc)\text{Subst}\left(\int \frac{2-c^{2/3}x}{1-c^{2/3}x+c^{4/3}x^2}\,dx, x, x^2\right) \\
&= -\frac{a+b\arctan(cx^3)}{x} + \frac{1}{2}b\sqrt[3]{c}\log\left(1+c^{2/3}x^2\right) \\
&\quad -\frac{1}{4}(b\sqrt[3]{c})\,\text{Subst}\left(\int \frac{-c^{2/3}+2c^{4/3}x}{1-c^{2/3}x+c^{4/3}x^2}\,dx, x, x^2\right) + \frac{1}{4}(3bc)\text{Subst}\left(\int \frac{1}{1-c^{2/3}x+c^{4/3}x^2}\,dx, x, x^2\right) \\
&= -\frac{a+b\arctan(cx^3)}{x} + \frac{1}{2}b\sqrt[3]{c}\log\left(1+c^{2/3}x^2\right) \\
&\quad -\frac{1}{4}b\sqrt[3]{c}\log\left(1-c^{2/3}x^2+c^{4/3}x^4\right) + \frac{1}{2}(3b\sqrt[3]{c})\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-2c^{2/3}x^2\right) \\
&= -\frac{a+b\arctan(cx^3)}{x} - \frac{1}{2}\sqrt{3}b\sqrt[3]{c}\arctan\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right) \\
&\quad + \frac{1}{2}b\sqrt[3]{c}\log\left(1+c^{2/3}x^2\right) - \frac{1}{4}b\sqrt[3]{c}\log\left(1-c^{2/3}x^2+c^{4/3}x^4\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.63

$$
\begin{aligned}
\int \frac{a+b\arctan(cx^3)}{x^2}\,dx = &-\frac{a}{x} - \frac{b\arctan(cx^3)}{x} - \frac{1}{2}\sqrt{3}b\sqrt[3]{c}\arctan\left(\sqrt{3}-2\sqrt[3]{c}x\right) \\
&- \frac{1}{2}\sqrt{3}b\sqrt[3]{c}\arctan\left(\sqrt{3}+2\sqrt[3]{c}x\right) \\
&+ \frac{1}{2}b\sqrt[3]{c}\log\left(1+c^{2/3}x^2\right) - \frac{1}{4}b\sqrt[3]{c}\log\left(1-\sqrt{3}\sqrt[3]{c}x\right. \\
&\left.+c^{2/3}x^2\right) - \frac{1}{4}b\sqrt[3]{c}\log\left(1+\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)
\end{aligned}
$$

```
[In] Integrate[(a + b*ArcTan[c*x^3])/x^2,x]
```

[Out] -(a/x) - (b*ArcTan[c*x^3])/x - (Sqrt[3]*b*c^(1/3)*ArcTan[Sqrt[3] - 2*c^(1/3)*x])/2 - (Sqrt[3]*b*c^(1/3)*ArcTan[Sqrt[3] + 2*c^(1/3)*x])/2 + (b*c^(1/3)*Log[1 + c^(2/3)*x^2])/2 - (b*c^(1/3)*Log[1 - Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/4 - (b*c^(1/3)*Log[1 + Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/4

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00

method	result
default	$-\frac{a}{x}-\frac{b\arctan(c x^3)}{x}+\frac{b\ln\left(x^2+\left(\frac{1}{c^2}\right)^{\frac{1}{3}}\right)}{2c\left(\frac{1}{c^2}\right)^{\frac{2}{3}}}-\frac{b\ln\left(x^4-\left(\frac{1}{c^2}\right)^{\frac{1}{3}}x^2+\left(\frac{1}{c^2}\right)^{\frac{2}{3}}\right)}{4c\left(\frac{1}{c^2}\right)^{\frac{2}{3}}}+\frac{b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^2}{\left(\frac{1}{c^2}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{2c\left(\frac{1}{c^2}\right)^{\frac{2}{3}}}$
parts	$-\frac{a}{x}-\frac{b\arctan(c x^3)}{x}+\frac{b\ln\left(x^2+\left(\frac{1}{c^2}\right)^{\frac{1}{3}}\right)}{2c\left(\frac{1}{c^2}\right)^{\frac{2}{3}}}-\frac{b\ln\left(x^4-\left(\frac{1}{c^2}\right)^{\frac{1}{3}}x^2+\left(\frac{1}{c^2}\right)^{\frac{2}{3}}\right)}{4c\left(\frac{1}{c^2}\right)^{\frac{2}{3}}}+\frac{b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^2}{\left(\frac{1}{c^2}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{2c\left(\frac{1}{c^2}\right)^{\frac{2}{3}}}$
risch	$\frac{ib\ln(icx^3+1)}{2x}-\frac{ib\ln(-icx^3+1)}{2x}-\frac{ib\ln\left(x+\left(\frac{i}{c}\right)^{\frac{1}{3}}\right)}{2\left(\frac{i}{c}\right)^{\frac{1}{3}}}+\frac{ib\ln\left(x^2-\left(\frac{i}{c}\right)^{\frac{1}{3}}x+\left(\frac{i}{c}\right)^{\frac{2}{3}}\right)}{4\left(\frac{i}{c}\right)^{\frac{1}{3}}}+\frac{ib\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{i}{c}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{2\left(\frac{i}{c}\right)^{\frac{1}{3}}}-\frac{a}{x}-$

[In] int((a+b*arctan(c*x^3))/x^2,x,method=_RETURNVERBOSE)

[Out] -a/x-b/x*arctan(c*x^3)+1/2*b/c/(1/c^2)^(2/3)*ln(x^2+(1/c^2)^(1/3))-1/4*b/c/(1/c^2)^(2/3)*ln(x^4-(1/c^2)^(1/3)*x^2+(1/c^2)^(2/3))+1/2*b/c/(1/c^2)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2*x^2/(1/c^2)^(1/3)-1))

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.87

$$\int \frac{a + b\arctan(cx^3)}{x^2}\,dx = \frac{2\sqrt{3}bc^{\frac{1}{3}}x\arctan\left(\frac{2}{3}\sqrt{3}c^{\frac{2}{3}}x^2 - \frac{1}{3}\sqrt{3}\right) - bc^{\frac{1}{3}}x\log\left(c^2x^4 - c^{\frac{4}{3}}x^2 + c^{\frac{2}{3}}\right) + 2\,bc^{\frac{1}{3}}x\log\left(cx^2 + c^{\frac{1}{3}}\right) - 4\,b\arctan(}{4\,x}$$

```
[In] integrate((a+b*arctan(c*x^3))/x^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*sqrt(3)*b*c^(1/3)*x*arctan(2/3*sqrt(3)*c^(2/3)*x^2 - 1/3*sqrt(3)) -
b*c^(1/3)*x*log(c^2*x^4 - c^(4/3)*x^2 + c^(2/3)) + 2*b*c^(1/3)*x*log(c*x^2
+ c^(1/3)) - 4*b*arctan(c*x^3) - 4*a)/x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. 2(97) = 194.

Time = 25.23 (sec) , antiderivative size = 262, normalized size of antiderivative = 2.52

$$\int \frac{a + b\arctan(cx^3)}{x^2}\,dx = \begin{cases} -\frac{a}{x} + bc^2\left(-\frac{1}{c^2}\right)^{\frac{5}{6}}\operatorname{atan}(cx^3) - bc\sqrt[3]{-\frac{1}{c^2}}\log\left(x - \sqrt[6]{-\frac{1}{c^2}}\right) + \frac{3bc\sqrt[3]{-\frac{1}{c^2}}\log\left(4x^2 - 4x\sqrt[6]{-\frac{1}{c^2}} + 4\sqrt[3]{-\frac{1}{c^2}}\right)}{4} - \frac{bc\sqrt[3]{-} \\ -\frac{a}{x} \end{cases}$$

```
[In] integrate((a+b*atan(c*x**3))/x**2,x)
```

```
[Out] Piecewise((-a/x + b*c**2*(-1/c**2)**(5/6)*atan(c*x**3) - b*c*(-1/c**2)**(1/
3)*log(x - (-1/c**2)**(1/6)) + 3*b*c*(-1/c**2)**(1/3)*log(4*x**2 - 4*x*(-1/
c**2)**(1/6) + 4*(-1/c**2)**(1/3))/4 - b*c*(-1/c**2)**(1/3)*log(4*x**2 + 4*
x*(-1/c**2)**(1/6) + 4*(-1/c**2)**(1/3))/4 + sqrt(3)*b*c*(-1/c**2)**(1/3)*a
tan(2*sqrt(3)*x/(3*(-1/c**2)**(1/6)) - sqrt(3)/3)/2 - sqrt(3)*b*c*(-1/c**2)
**(1/3)*atan(2*sqrt(3)*x/(3*(-1/c**2)**(1/6)) + sqrt(3)/3)/2 - b*atan(c*x**
3)/x, Ne(c, 0)), (-a/x, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.94

$$\int \frac{a+b\arctan(cx^3)}{x^2}\,dx = \frac{1}{4}\left(c\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2c^{\frac{4}{3}}x^2-c^{\frac{2}{3}}\right)}{3c^{\frac{2}{3}}}\right)}{c^{\frac{2}{3}}}-\frac{\log\left(c^{\frac{4}{3}}x^4-c^{\frac{2}{3}}x^2+1\right)}{c^{\frac{2}{3}}}+\frac{2\log\left(\frac{c^{\frac{2}{3}}x^2+1}{c^{\frac{2}{3}}}\right)}{c^{\frac{2}{3}}}\right)-\frac{4\arctan(cx^3)}{x}\right)b - \frac{a}{x}$$

```
[In] integrate((a+b*arctan(c*x^3))/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*(c*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*c^(4/3)*x^2 - c^(2/3))/c^(2/3))/c^(
2/3) - log(c^(4/3)*x^4 - c^(2/3)*x^2 + 1)/c^(2/3) + 2*log((c^(2/3)*x^2 + 1)
/c^(2/3))/c^(2/3)) - 4*arctan(c*x^3)/x)*b - a/x
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.88

$$\int \frac{a+b\arctan(cx^3)}{x^2}\,dx = \frac{1}{4}bc\left(\frac{2\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-\frac{1}{|c|^{\frac{2}{3}}}\right)|c|^{\frac{2}{3}}\right)}{|c|^{\frac{2}{3}}}-\frac{\log\left(x^4-\frac{x^2}{|c|^{\frac{2}{3}}}+\frac{1}{|c|^{\frac{4}{3}}}\right)}{|c|^{\frac{2}{3}}}+\frac{2\log\left(x^2+\frac{1}{|c|^{\frac{2}{3}}}\right)}{|c|^{\frac{2}{3}}}\right) - \frac{b\arctan(cx^3)+a}{x}$$

```
[In] integrate((a+b*arctan(c*x^3))/x^2,x, algorithm="giac")
```

```
[Out] 1/4*b*c*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1/abs(c)^(2/3))*abs(c)^(2/3)
)/abs(c)^(2/3) - log(x^4 - x^2/abs(c)^(2/3) + 1/abs(c)^(4/3))/abs(c)^(2/3)
+ 2*log(x^2 + 1/abs(c)^(2/3))/abs(c)^(2/3)) - (b*arctan(c*x^3) + a)/x
```

Mupad [B] (verification not implemented)

Time = 2.02 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.95

$$\int \frac{a+b\arctan(cx^3)}{x^2}\,dx = \frac{b\,c^{1/3}\,\ln\left(c^{2/3}\,x^2+1\right)}{2}-\frac{a}{x}-\frac{b\,\mathrm{atan}(c\,x^3)}{x}-\frac{b\,c^{1/3}\,\ln\left(-\sqrt{3}-c^{2/3}\,x^2\,2\mathrm{i}+1\mathrm{i}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{4}+\frac{b\,c^{1/3}\,\ln\left(-\sqrt{3}+c^{2/3}\,x^2\,2\mathrm{i}-\mathrm{i}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{4}$$

```
[In] int((a + b*atan(c*x^3))/x^2,x)
```

```
[Out] (b*c^(1/3)*log(c^(2/3)*x^2 + 1))/2 - a/x - (b*atan(c*x^3))/x - (b*c^(1/3)*l
og(1i - c^(2/3)*x^2*2i - 3^(1/2))*(3^(1/2)*1i + 1))/4 + (b*c^(1/3)*log(c^(2
/3)*x^2*2i - 3^(1/2) - 1i)*(3^(1/2)*1i - 1))/4
```

3.112 $\int \frac{a+b\arctan(cx^3)}{x^5}\,dx$

Optimal result 653
Rubi [A] (verified) 653
Mathematica [A] (verified) 656
Maple [A] (verified) 656
Fricas [B] (verification not implemented) 657
Sympy [A] (verification not implemented) 657
Maxima [A] (verification not implemented) 658
Giac [A] (verification not implemented) 658
Mupad [B] (verification not implemented) 659

Optimal result

Integrand size = 14, antiderivative size = 174

$$\int \frac{a+b\arctan(cx^3)}{x^5}\,dx = -\frac{3bc}{4x}-\frac{1}{4}bc^{4/3}\arctan\left(\sqrt[3]{c}x\right)-\frac{a+b\arctan(cx^3)}{4x^4}+\frac{1}{8}bc^{4/3}\arctan\left(\sqrt{3}-2\sqrt[3]{c}x\right) -\frac{1}{8}bc^{4/3}\arctan\left(\sqrt{3}+2\sqrt[3]{c}x\right)-\frac{1}{16}\sqrt{3}bc^{4/3}\log\left(1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)+\frac{1}{16}\sqrt{3}bc^{4/3}\log\left(1+\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)$$

```
[Out] -3/4*b*c/x-1/4*b*c^(4/3)*arctan(c^(1/3)*x)+1/4*(-a-b*arctan(c*x^3))/x^4-1/8
*b*c^(4/3)*arctan(2*c^(1/3)*x-3^(1/2))-1/8*b*c^(4/3)*arctan(2*c^(1/3)*x+3^(
1/2))-1/16*b*c^(4/3)*ln(1+c^(2/3)*x^2-c^(1/3)*x*3^(1/2))*3^(1/2)+1/16*b*c^(
4/3)*ln(1+c^(2/3)*x^2+c^(1/3)*x*3^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {4946, 331, 301, 648, 632, 210, 642, 209}

$$\int \frac{a+b\arctan(cx^3)}{x^5}\,dx = -\frac{a+b\arctan(cx^3)}{4x^4}-\frac{1}{4}bc^{4/3}\arctan\left(\sqrt[3]{c}x\right)+\frac{1}{8}bc^{4/3}\arctan\left(\sqrt{3}-2\sqrt[3]{c}x\right) -\frac{1}{8}bc^{4/3}\arctan\left(2\sqrt[3]{c}x+\sqrt{3}\right)-\frac{1}{16}\sqrt{3}bc^{4/3}\log\left(c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}x+1\right)+\frac{1}{16}\sqrt{3}bc^{4/3}\log\left(c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}x+1\right)$$

```
[In] Int[(a + b*ArcTan[c*x^3])/x^5,x]
```

```
[Out] (-3*b*c)/(4*x) - (b*c^(4/3)*ArcTan[c^(1/3)*x])/4 - (a + b*ArcTan[c*x^3])/(4
*x^4) + (b*c^(4/3)*ArcTan[Sqrt[3] - 2*c^(1/3)*x])/8 - (b*c^(4/3)*ArcTan[Sqr
t[3] + 2*c^(1/3)*x])/8 - (Sqrt[3]*b*c^(4/3)*Log[1 - Sqrt[3]*c^(1/3)*x + c^(
2/3)*x^2])/16 + (Sqrt[3]*b*c^(4/3)*Log[1 + Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2]
)/16
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 301

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r^
(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{a+b\arctan\left(cx^3\right)}{4x^4}+\frac{1}{4}(3bc)\int\frac{1}{x^2\left(1+c^2x^6\right)}\,dx$$

$$= -\frac{3bc}{4x}-\frac{a+b\arctan\left(cx^3\right)}{4x^4}-\frac{1}{4}\left(3bc^3\right)\int\frac{x^4}{1+c^2x^6}\,dx$$

$$= -\frac{3bc}{4x}-\frac{a+b\arctan\left(cx^3\right)}{4x^4}-\frac{1}{4}\left(bc^{5/3}\right)\int\frac{1}{1+c^{2/3}x^2}\,dx$$
$$-\frac{1}{4}\left(bc^{5/3}\right)\int\frac{-\frac{1}{2}+\frac{1}{2}\sqrt{3}\sqrt[3]{c}x}{1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2}\,dx-\frac{1}{4}\left(bc^{5/3}\right)\int\frac{-\frac{1}{2}-\frac{1}{2}\sqrt{3}\sqrt[3]{c}x}{1+\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2}\,dx$$

$$= -\frac{3bc}{4x}-\frac{1}{4}bc^{4/3}\arctan\left(\sqrt[3]{c}x\right)-\frac{a+b\arctan\left(cx^3\right)}{4x^4}-\frac{1}{16}\left(\sqrt{3}bc^{4/3}\right)\int\frac{-\sqrt{3}\sqrt[3]{c}+2c^{2/3}x}{1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2}\,dx$$
$$+\frac{1}{16}\left(\sqrt{3}bc^{4/3}\right)\int\frac{\sqrt{3}\sqrt[3]{c}+2c^{2/3}x}{1+\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2}\,dx-\frac{1}{16}\left(bc^{5/3}\right)\int\frac{1}{1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2}\,dx-\frac{1}{16}\left(bc^{5/3}\right)\int$$

$$= -\frac{3bc}{4x}-\frac{1}{4}bc^{4/3}\arctan\left(\sqrt[3]{c}x\right)-\frac{a+b\arctan\left(cx^3\right)}{4x^4}$$
$$-\frac{1}{16}\sqrt{3}bc^{4/3}\log\left(1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)+\frac{1}{16}\sqrt{3}bc^{4/3}\log\left(1+\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)-\frac{\left(bc^{4/3}\right)\text{Subst}\left(\int\right.}{}$$

$$= -\frac{3bc}{4x}-\frac{1}{4}bc^{4/3}\arctan\left(\sqrt[3]{c}x\right)-\frac{a+b\arctan\left(cx^3\right)}{4x^4}+\frac{1}{8}bc^{4/3}\arctan\left(\sqrt{3}-2\sqrt[3]{c}x\right)$$
$$-\frac{1}{8}bc^{4/3}\arctan\left(\sqrt{3}+2\sqrt[3]{c}x\right)-\frac{1}{16}\sqrt{3}bc^{4/3}\log\left(1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)+\frac{1}{16}\sqrt{3}bc^{4/3}\log\left(1+\sqrt{3}\sqrt[3]{c}x\right.$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.03

$$\int \frac{a+b\arctan(cx^3)}{x^5}\,dx$$
$$= -\frac{a}{4x^4} - \frac{3bc}{4x} - \frac{1}{4}bc^{4/3}\arctan\left(\sqrt[3]{c}x\right) - \frac{b\arctan(cx^3)}{4x^4} + \frac{1}{8}bc^{4/3}\arctan\left(\sqrt{3}-2\sqrt[3]{c}x\right)$$
$$-\frac{1}{8}bc^{4/3}\arctan\left(\sqrt{3}+2\sqrt[3]{c}x\right) - \frac{1}{16}\sqrt{3}bc^{4/3}\log\left(1-\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right) + \frac{1}{16}\sqrt{3}bc^{4/3}\log\left(1+\sqrt{3}\sqrt[3]{c}x+c^{2/3}x^2\right)$$

```
[In] Integrate[(a + b*ArcTan[c*x^3])/x^5,x]
```

```
[Out] -1/4*a/x^4 - (3*b*c)/(4*x) - (b*c^(4/3)*ArcTan[c^(1/3)*x])/4 - (b*ArcTan[c*
x^3])/(4*x^4) + (b*c^(4/3)*ArcTan[Sqrt[3] - 2*c^(1/3)*x])/8 - (b*c^(4/3)*Ar
cTan[Sqrt[3] + 2*c^(1/3)*x])/8 - (Sqrt[3]*b*c^(4/3)*Log[1 - Sqrt[3]*c^(1/3)
*x + c^(2/3)*x^2])/16 + (Sqrt[3]*b*c^(4/3)*Log[1 + Sqrt[3]*c^(1/3)*x + c^(2
/3)*x^2])/16
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.94

method	result
default	$-\frac{a}{4x^4}+b\left(-\frac{\arctan(c\,x^3)}{4x^4}+\frac{3c\left(-\frac{1}{x}-\left(-\frac{\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{5}{6}}\ln\left(x^2+\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{1}{6}}x+\left(\frac{1}{c^2}\right)^{\frac{1}{3}}\right)}{12}+\frac{\arctan\left(\frac{2x}{\left(\frac{1}{c^2}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{6c^2\left(\frac{1}{c^2}\right)^{\frac{1}{6}}}+\frac{\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{5}{6}}\ln\left(x^2-\cdots\right.}{\cdots}\right.\right.}{4}\right.$
parts	$-\frac{a}{4x^4}+b\left(-\frac{\arctan(c\,x^3)}{4x^4}+\frac{3c\left(-\frac{1}{x}-\left(-\frac{\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{5}{6}}\ln\left(x^2+\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{1}{6}}x+\left(\frac{1}{c^2}\right)^{\frac{1}{3}}\right)}{12}+\frac{\arctan\left(\frac{2x}{\left(\frac{1}{c^2}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{6c^2\left(\frac{1}{c^2}\right)^{\frac{1}{6}}}+\frac{\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{5}{6}}\ln\left(x^2-\cdots\right.}{\cdots}\right.\right.}{4}\right.$

```
[In] int((a+b*arctan(c*x^3))/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*a/x^4+b*(-1/4/x^4*arctan(c*x^3)+3/4*c*(-1/x-(-1/12*3^(1/2)*(1/c^2)^(5/
6)*ln(x^2+3^(1/2)*(1/c^2)^(1/6)*x+(1/c^2)^(1/3))+1/6/c^2/(1/c^2)^(1/6)*arct
an(2*x/(1/c^2)^(1/6)+3^(1/2))+1/12*3^(1/2)*(1/c^2)^(5/6)*ln(x^2-3^(1/2)*(1/
c^2)^(1/6)*x+(1/c^2)^(1/3))+1/6/c^2/(1/c^2)^(1/6)*arctan(2*x/(1/c^2)^(1/6)-
3^(1/2))+1/3/c^2/(1/c^2)^(1/6)*arctan(x/(1/c^2)^(1/6)))*c^2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(126) = 252$.

Time = 0.28 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.73

$$\int \frac{a+b\arctan(cx^3)}{x^5}\,dx = -\frac{2\left(-b^6c^8\right)^{\frac{1}{6}}x^4\log\left(b^5c^7x+\left(-b^6c^8\right)^{\frac{5}{6}}\right)-2\left(-b^6c^8\right)^{\frac{1}{6}}x^4\log\left(b^5c^7x-\left(-b^6c^8\right)^{\frac{5}{6}}\right)+12\,bcx^3-\left(-b^6c^8\right)^{\frac{1}{6}}\left(\sqrt{\right.}$$

```
[In] integrate((a+b*arctan(c*x^3))/x^5,x, algorithm="fricas")
```

```
[Out] -1/16*(2*(-b^6*c^8)^(1/6)*x^4*log(b^5*c^7*x + (-b^6*c^8)^(5/6)) - 2*(-b^6*c
^8)^(1/6)*x^4*log(b^5*c^7*x - (-b^6*c^8)^(5/6)) + 12*b*c*x^3 - (-b^6*c^8)^(
1/6)*(sqrt(-3)*x^4 - x^4)*log(2*b^5*c^7*x + (-b^6*c^8)^(5/6)*(sqrt(-3) + 1)
) + (-b^6*c^8)^(1/6)*(sqrt(-3)*x^4 - x^4)*log(2*b^5*c^7*x - (-b^6*c^8)^(5/6
)*(sqrt(-3) + 1)) - (-b^6*c^8)^(1/6)*(sqrt(-3)*x^4 + x^4)*log(2*b^5*c^7*x +
 (-b^6*c^8)^(5/6)*(sqrt(-3) - 1)) + (-b^6*c^8)^(1/6)*(sqrt(-3)*x^4 + x^4)*l
og(2*b^5*c^7*x - (-b^6*c^8)^(5/6)*(sqrt(-3) - 1)) + 4*b*arctan(c*x^3) + 4*a
)/x^4
```

Sympy [A] (verification not implemented)

Time = 49.05 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.52

$$\int \frac{a+b\arctan(cx^3)}{x^5}\,dx = \begin{cases} -\frac{a}{4x^4}+\frac{3bc^3\left(-\frac{1}{c^2}\right)^{\frac{5}{6}}\log\left(4x^2-4x\sqrt[6]{-\frac{1}{c^2}}+4\sqrt[3]{-\frac{1}{c^2}}\right)}{16}-\frac{3bc^3\left(-\frac{1}{c^2}\right)^{\frac{5}{6}}\log\left(4x^2+4x\sqrt[6]{-\frac{1}{c^2}}+4\sqrt[3]{-\frac{1}{c^2}}\right)}{16}+\frac{\sqrt{3}bc^3\left(-\frac{1}{c^2}\right)^{\frac{5}{6}}\operatorname{atan}}{8} \\ -\frac{a}{4x^4} \end{cases}$$

```
[In] integrate((a+b*atan(c*x**3))/x**5,x)
```

```
[Out] Piecewise((-a/(4*x**4) + 3*b*c**3*(-1/c**2)**(5/6)*log(4*x**2 - 4*x*(-1/c**
2)**(1/6) + 4*(-1/c**2)**(1/3))/16 - 3*b*c**3*(-1/c**2)**(5/6)*log(4*x**2 +
```

```
 4*x*(-1/c**2)**(1/6) + 4*(-1/c**2)**(1/3))/16 + sqrt(3)*b*c**3*(-1/c**2)**
(5/6)*atan(2*sqrt(3)*x/(3*(-1/c**2)**(1/6)) - sqrt(3)/3)/8 + sqrt(3)*b*c**3
*(-1/c**2)**(5/6)*atan(2*sqrt(3)*x/(3*(-1/c**2)**(1/6)) + sqrt(3)/3)/8 - b*
c**2*(-1/c**2)**(1/3)*atan(c*x**3)/4 - 3*b*c/(4*x) - b*atan(c*x**3)/(4*x**4
), Ne(c, 0)), (-a/(4*x**4), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.84

$$\int \frac{a+b\arctan(cx^3)}{x^5}\,dx$$
$$=\frac{1}{16}\left(\left(\left(c^2\left(\frac{\sqrt{3}\log\left(c^{\frac{2}{3}}x^2+\sqrt{3}c^{\frac{1}{3}}x+1\right)}{c^{\frac{5}{3}}}-\frac{\sqrt{3}\log\left(c^{\frac{2}{3}}x^2-\sqrt{3}c^{\frac{1}{3}}x+1\right)}{c^{\frac{5}{3}}}-\frac{4\arctan\left(c^{\frac{1}{3}}x\right)}{c^{\frac{5}{3}}}-\frac{2\arctan\left(\frac{2c}{}\right.}{c^{\frac{5}{3}}}\right.\right.\right.\right.$$
$$-\frac{a}{4x^4}$$

```
[In] integrate((a+b*arctan(c*x^3))/x^5,x, algorithm="maxima")
```

```
[Out] 1/16*((c^2*(sqrt(3)*log(c^(2/3)*x^2 + sqrt(3)*c^(1/3)*x + 1)/c^(5/3) - sqrt
(3)*log(c^(2/3)*x^2 - sqrt(3)*c^(1/3)*x + 1)/c^(5/3) - 4*arctan(c^(1/3)*x)/
c^(5/3) - 2*arctan((2*c^(2/3)*x + sqrt(3)*c^(1/3))/c^(1/3))/c^(5/3) - 2*arc
tan((2*c^(2/3)*x - sqrt(3)*c^(1/3))/c^(1/3))/c^(5/3)) - 12/x)*c - 4*arctan(
c*x^3)/x^4)*b - 1/4*a/x^4
```

Giac [A] (verification not implemented)

none

Time = 0.41 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.93

$$\int \frac{a+b\arctan(cx^3)}{x^5}\,dx$$
$$=\frac{1}{16}bc^3\left(\frac{\sqrt{3}|c|^{\frac{1}{3}}\log\left(x^2+\frac{\sqrt{3}x}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{c^2}-\frac{\sqrt{3}|c|^{\frac{1}{3}}\log\left(x^2-\frac{\sqrt{3}x}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{c^2}-\frac{2|c|^{\frac{1}{3}}\arctan\left(\left(2x+\frac{\sqrt{3}}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right.}{c^2}\right.$$
$$-\frac{3bcx^3+b\arctan(cx^3)+a}{4x^4}$$

```
[In] integrate((a+b*arctan(c*x^3))/x^5,x, algorithm="giac")
```

```
[Out] 1/16*b*c^3*(sqrt(3)*abs(c)^(1/3)*log(x^2 + sqrt(3)*x/abs(c)^(1/3) + 1/abs(c
)^(2/3))/c^2 - sqrt(3)*abs(c)^(1/3)*log(x^2 - sqrt(3)*x/abs(c)^(1/3) + 1/ab
s(c)^(2/3))/c^2 - 2*abs(c)^(1/3)*arctan((2*x + sqrt(3)/abs(c)^(1/3))*abs(c)
^(1/3))/c^2 - 2*abs(c)^(1/3)*arctan((2*x - sqrt(3)/abs(c)^(1/3))*abs(c)^(1/
3))/c^2 - 4*abs(c)^(1/3)*arctan(x*abs(c)^(1/3))/c^2) - 1/4*(3*b*c*x^3 + b*a
rctan(c*x^3) + a)/x^4
```

Mupad [B] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.69

$$\int \frac{a+b\arctan(cx^3)}{x^5}\,dx = -\frac{a}{4\,x^4} + \frac{b\,c^{4/3}\left(\mathrm{atan}\left((-1)^{2/3}\,c^{1/3}\,x\right)+\mathrm{atan}\left(\frac{(-1)^{2/3}\,c^{1/3}\,x\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)+2\,\mathrm{atan}\left(\frac{(-1)^{2/3}\,c^{1/3}\,x\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\right)}{8} - \frac{b\,\mathrm{atan}(c\,x^3)}{4\,x^4} - \frac{3\,b\,c}{4\,x} - \frac{\sqrt{3}\,b\,c^{4/3}\left(\mathrm{atan}\left((-1)^{2/3}\,c^{1/3}\,x\right)-\mathrm{atan}\left(\frac{(-1)^{2/3}\,c^{1/3}\,x\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\right)\,1\mathrm{i}}{8}$$

```
[In] int((a + b*atan(c*x^3))/x^5,x)
```

```
[Out] (b*c^(4/3)*(atan((-1)^(2/3)*c^(1/3)*x) + atan(((-1)^(2/3)*c^(1/3)*x*(3^(1/2
)*1i - 1))/2) + 2*atan(((-1)^(2/3)*c^(1/3)*x*(3^(1/2)*1i + 1))/2)))/8 - a/(
4*x^4) - (b*atan(c*x^3))/(4*x^4) - (3*b*c)/(4*x) - (3^(1/2)*b*c^(4/3)*(atan
((-1)^(2/3)*c^(1/3)*x) - atan(((-1)^(2/3)*c^(1/3)*x*(3^(1/2)*1i - 1))/2))*1
i)/8
```

3.113 $\int x^{11}(a+b\arctan(cx^3))^2\,dx$

Optimal result 660
Rubi [A] (verified) 660
Mathematica [A] (verified) 663
Maple [A] (verified) 663
Fricas [A] (verification not implemented) 663
Sympy [B] (verification not implemented) 664
Maxima [A] (verification not implemented) 664
Giac [A] (verification not implemented) 665
Mupad [B] (verification not implemented) 665

Optimal result

Integrand size = 16, antiderivative size = 124

$$\int x^{11}\left(a+b\arctan\left(cx^3\right)\right)^2\,dx=\frac{abx^3}{6c^3}+\frac{b^2x^6}{36c^2}+\frac{b^2x^3\arctan(cx^3)}{6c^3}-\frac{bx^9(a+b\arctan(cx^3))}{18c}-\frac{(a+b\arctan(cx^3))^2}{12c^4}+\frac{1}{12}x^{12}\left(a+b\arctan\left(cx^3\right)\right)^2-\frac{b^2\log(1+c^2x^6)}{9c^4}$$

```
[Out] 1/6*a*b*x^3/c^3+1/36*b^2*x^6/c^2+1/6*b^2*x^3*arctan(c*x^3)/c^3-1/18*b*x^9*(
a+b*arctan(c*x^3))/c-1/12*(a+b*arctan(c*x^3))^2/c^4+1/12*x^12*(a+b*arctan(c
*x^3))^2-1/9*b^2*ln(c^2*x^6+1)/c^4
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4948, 4946, 5036, 272, 45, 4930, 266, 5004}

$$\int x^{11}\left(a+b\arctan\left(cx^3\right)\right)^2\,dx=-\frac{(a+b\arctan(cx^3))^2}{12c^4}+\frac{1}{12}x^{12}\left(a+b\arctan\left(cx^3\right)\right)^2-\frac{bx^9(a+b\arctan(cx^3))}{18c}+\frac{abx^3}{6c^3}+\frac{b^2x^3\arctan(cx^3)}{6c^3}+\frac{b^2x^6}{36c^2}-\frac{b^2\log(c^2x^6+1)}{9c^4}$$

```
[In] Int[x^11*(a + b*ArcTan[c*x^3])^2,x]
```

```
[Out] (a*b*x^3)/(6*c^3) + (b^2*x^6)/(36*c^2) + (b^2*x^3*ArcTan[c*x^3])/(6*c^3) -
(b*x^9*(a + b*ArcTan[c*x^3]))/(18*c) - (a + b*ArcTan[c*x^3])^2/(12*c^4) + (
x^12*(a + b*ArcTan[c*x^3])^2)/12 - (b^2*Log[1 + c^2*x^6])/(9*c^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplify
[(m + 1)/n]]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
```

```
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int x^3(a+b\arctan(cx))^2\,dx,x,x^3\right)\\
&= \frac{1}{12}x^{12}\left(a+b\arctan\left(cx^3\right)\right)^2-\frac{1}{6}(bc)\text{Subst}\left(\int \frac{x^4(a+b\arctan(cx))}{1+c^2x^2}\,dx,x,x^3\right)\\
&= \frac{1}{12}x^{12}\left(a+b\arctan\left(cx^3\right)\right)^2-\frac{b\text{Subst}\left(\int x^2(a+b\arctan(cx))\,dx,x,x^3\right)}{6c}\\
&\quad+\frac{b\text{Subst}\left(\int \frac{x^2(a+b\arctan(cx))}{1+c^2x^2}\,dx,x,x^3\right)}{6c}\\
&= -\frac{bx^9\left(a+b\arctan\left(cx^3\right)\right)}{18c}+\frac{1}{12}x^{12}\left(a+b\arctan\left(cx^3\right)\right)^2+\frac{1}{18}b^2\text{Subst}\left(\int \frac{x^3}{1+c^2x^2}\,dx,x,x^3\right)\\
&\quad+\frac{b\text{Subst}\left(\int(a+b\arctan(cx))\,dx,x,x^3\right)}{6c^3}-\frac{b\text{Subst}\left(\int \frac{a+b\arctan(cx)}{1+c^2x^2}\,dx,x,x^3\right)}{6c^3}\\
&= \frac{abx^3}{6c^3}-\frac{bx^9\left(a+b\arctan\left(cx^3\right)\right)}{18c}-\frac{\left(a+b\arctan\left(cx^3\right)\right)^2}{12c^4}+\frac{1}{12}x^{12}\left(a+b\arctan\left(cx^3\right)\right)^2\\
&\quad+\frac{1}{36}b^2\text{Subst}\left(\int \frac{x}{1+c^2x}\,dx,x,x^6\right)+\frac{b^2\text{Subst}\left(\int \arctan(cx)\,dx,x,x^3\right)}{6c^3}\\
&= \frac{abx^3}{6c^3}+\frac{b^2x^3\arctan\left(cx^3\right)}{6c^3}-\frac{bx^9\left(a+b\arctan\left(cx^3\right)\right)}{18c}\\
&\quad-\frac{\left(a+b\arctan\left(cx^3\right)\right)^2}{12c^4}+\frac{1}{12}x^{12}\left(a+b\arctan\left(cx^3\right)\right)^2\\
&\quad+\frac{1}{36}b^2\text{Subst}\left(\int\left(\frac{1}{c^2}-\frac{1}{c^2\left(1+c^2x\right)}\right)dx,x,x^6\right)-\frac{b^2\text{Subst}\left(\int \frac{x}{1+c^2x^2}\,dx,x,x^3\right)}{6c^2}\\
&= \frac{abx^3}{6c^3}+\frac{b^2x^6}{36c^2}+\frac{b^2x^3\arctan\left(cx^3\right)}{6c^3}-\frac{bx^9\left(a+b\arctan\left(cx^3\right)\right)}{18c}\\
&\quad-\frac{\left(a+b\arctan\left(cx^3\right)\right)^2}{12c^4}+\frac{1}{12}x^{12}\left(a+b\arctan\left(cx^3\right)\right)^2-\frac{b^2\log\left(1+c^2x^6\right)}{9c^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.98

$$\int x^{11}(a+b\arctan(cx^3))^2\,dx$$
$$=\frac{cx^3(6ab+b^2cx^3-2abc^2x^6+3a^2c^3x^9)-2b(bcx^3(-3+c^2x^6)+a(3-3c^4x^{12}))\arctan(cx^3)+3b^2(-1+c^4}{36c^4}$$

[In] Integrate[x^11*(a + b*ArcTan[c*x^3])^2,x]

[Out] (c*x^3*(6*a*b + b^2*c*x^3 - 2*a*b*c^2*x^6 + 3*a^2*c^3*x^9) - 2*b*(b*c*x^3*(-3 + c^2*x^6) + a*(3 - 3*c^4*x^12))*ArcTan[c*x^3] + 3*b^2*(-1 + c^4*x^12)*ArcTan[c*x^3]^2 - 4*b^2*Log[1 + c^2*x^6])/(36*c^4)

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.22

method	result
default	$\frac{a^2x^{12}}{12}+\frac{b^2x^{12}\arctan(cx^3)^2}{12}-\frac{b^2\arctan(cx^3)x^9}{18c}+\frac{b^2x^3\arctan(cx^3)}{6c^3}-\frac{b^2\arctan(cx^3)^2}{12c^4}+\frac{b^2x^6}{36c^2}-\frac{b^2\ln(c^2x^6+1)}{9c^4}$
parts	$\frac{a^2x^{12}}{12}+\frac{b^2x^{12}\arctan(cx^3)^2}{12}-\frac{b^2\arctan(cx^3)x^9}{18c}+\frac{b^2x^3\arctan(cx^3)}{6c^3}-\frac{b^2\arctan(cx^3)^2}{12c^4}+\frac{b^2x^6}{36c^2}-\frac{b^2\ln(c^2x^6+1)}{9c^4}$
parallelrisch	$-\frac{-3b^2\arctan(cx^3)^2x^{12}c^4-6ab\arctan(cx^3)x^{12}c^4-3c^4a^2x^{12}+2b^2\arctan(cx^3)x^9c^3+2abc^3x^9-x^6b^2c^2-6b^2\arctan(cx^3)x^3c}{36c^4}$
risch	$-\frac{b^2(c^4x^{12}-1)\ln(icx^3+1)^2}{48c^4}-\frac{ib(6ac^4x^{12}+3ibc^4x^{12}\ln(-icx^3+1)-2bc^3x^9+6bcx^3-3ib\ln(-icx^3+1))\ln(icx^3+1)}{72c^4}+\frac{ia}{}$

[In] int(x^11*(a+b*arctan(c*x^3))^2,x,method=_RETURNVERBOSE)

[Out] 1/12*a^2*x^12+1/12*b^2*x^12*arctan(c*x^3)^2-1/18*b^2*arctan(c*x^3)/c*x^9+1/6*b^2*x^3*arctan(c*x^3)/c^3-1/12*b^2/c^4*arctan(c*x^3)^2+1/36*b^2*x^6/c^2-1/9*b^2*ln(c^2*x^6+1)/c^4+1/6*a*b*x^12*arctan(c*x^3)-1/18*a*b/c*x^9+1/6*a*b*x^3/c^3-1/6*a*b/c^4*arctan(c*x^3)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.04

$$\int x^{11}(a+b\arctan(cx^3))^2\,dx$$
$$=\frac{3\,a^2c^4x^{12}-2\,abc^3x^9+b^2c^2x^6+6\,abcx^3+3\,(b^2c^4x^{12}-b^2)\arctan(cx^3)^2-4\,b^2\log(c^2x^6+1)+2\,(3\,abc^4x^{1}}{36\,c^4}$$

```
[In] integrate(x^11*(a+b*arctan(c*x^3))^2,x, algorithm="fricas")
```

```
[Out] 1/36*(3*a^2*c^4*x^12 - 2*a*b*c^3*x^9 + b^2*c^2*x^6 + 6*a*b*c*x^3 + 3*(b^2*c
^4*x^12 - b^2)*arctan(c*x^3)^2 - 4*b^2*log(c^2*x^6 + 1) + 2*(3*a*b*c^4*x^12
 - b^2*c^3*x^9 + 3*b^2*c*x^3 - 3*a*b)*arctan(c*x^3))/c^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. 2(112) = 224.

Time = 147.67 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.96

$$\int x^{11}\left(a+b\arctan\left(cx^3\right)\right)^2\,dx$$
$$=\begin{cases}\frac{a^2x^{12}}{12}+\frac{abx^{12}\operatorname{atan}\left(cx^3\right)}{6}-\frac{abx^9}{18c}+\frac{abx^3}{6c^3}-\frac{ab\operatorname{atan}\left(cx^3\right)}{6c^4}+\frac{b^2x^{12}\operatorname{atan}^2\left(cx^3\right)}{12}-\frac{b^2x^9\operatorname{atan}\left(cx^3\right)}{18c}+\frac{b^2x^6}{36c^2}+\frac{b^2x^3\operatorname{atan}\left(cx^3\right)}{6c^3}+\frac{2b \\ \frac{a^2x^{12}}{12}\end{cases}$$

```
[In] integrate(x**11*(a+b*atan(c*x**3))**2,x)
```

```
[Out] Piecewise((a**2*x**12/12 + a*b*x**12*atan(c*x**3)/6 - a*b*x**9/(18*c) + a*b
*x**3/(6*c**3) - a*b*atan(c*x**3)/(6*c**4) + b**2*x**12*atan(c*x**3)**2/12
- b**2*x**9*atan(c*x**3)/(18*c) + b**2*x**6/(36*c**2) + b**2*x**3*atan(c*x*
*3)/(6*c**3) + 2*b**2*sqrt(-1/c**2)*atan(c*x**3)/(9*c**3) - 2*b**2*log(x -
(-1/c**2)**(1/6))/(9*c**4) - 2*b**2*log(4*x**2 + 4*x*(-1/c**2)**(1/6) + 4*(
-1/c**2)**(1/3))/(9*c**4) - b**2*atan(c*x**3)**2/(12*c**4), Ne(c, 0)), (a**
2*x**12/12, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.36

$$\int x^{11}\left(a+b\arctan\left(cx^3\right)\right)^2\,dx=\frac{1}{12}\,b^2x^{12}\arctan\left(cx^3\right)^2+\frac{1}{12}\,a^2x^{12}$$
$$+\frac{1}{18}\left(3\,x^{12}\arctan\left(cx^3\right)-c\left(\frac{c^2x^9-3\,x^3}{c^4}+\frac{3\,\arctan\left(cx^3\right)}{c^5}\right)\right)ab$$
$$-\frac{1}{36}\left(2\,c\left(\frac{c^2x^9-3\,x^3}{c^4}+\frac{3\,\arctan\left(cx^3\right)}{c^5}\right)\arctan\left(cx^3\right)-\frac{c^2x^6+3\,\arctan\left(cx^3\right)^2-3\,\log\left(18\,c^7x^6+18\,c^5\right)}{c^4}\right.$$

```
[In] integrate(x^11*(a+b*arctan(c*x^3))^2,x, algorithm="maxima")
```

```
[Out] 1/12*b^2*x^12*arctan(c*x^3)^2 + 1/12*a^2*x^12 + 1/18*(3*x^12*arctan(c*x^3)
- c*((c^2*x^9 - 3*x^3)/c^4 + 3*arctan(c*x^3)/c^5))*a*b - 1/36*(2*c*((c^2*x^
9 - 3*x^3)/c^4 + 3*arctan(c*x^3)/c^5)*arctan(c*x^3) - (c^2*x^6 + 3*arctan(c
*x^3)^2 - 3*log(18*c^7*x^6 + 18*c^5) - log(c^2*x^6 + 1))/c^4)*b^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.17

$$\int x^{11}\left(a+b\arctan\left(cx^3\right)\right)^2\,dx$$
$$=\frac{3\,a^2cx^{12}+2\left(3\,cx^{12}\arctan\left(cx^3\right)-\frac{3\,\arctan\left(cx^3\right)}{c^3}-\frac{c^9x^9-3\,c^7x^3}{c^9}\right)ab+\left(3\,cx^{12}\arctan\left(cx^3\right)^2-\frac{2\,c^3x^9\arctan\left(cx^3\right)-c^2\cdots}{\cdots}\right.}{36\,c}$$

```
[In] integrate(x^11*(a+b*arctan(c*x^3))^2,x, algorithm="giac")
```

```
[Out] 1/36*(3*a^2*c*x^12 + 2*(3*c*x^12*arctan(c*x^3) - 3*arctan(c*x^3)/c^3 - (c^9
*x^9 - 3*c^7*x^3)/c^9)*a*b + (3*c*x^12*arctan(c*x^3)^2 - (2*c^3*x^9*arctan(
c*x^3) - c^2*x^6 - 6*c*x^3*arctan(c*x^3) + 3*arctan(c*x^3)^2 + 4*log(c^2*x^
6 + 1))/c^3)*b^2)/c
```

Mupad [B] (verification not implemented)

Time = 1.17 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.21

$$\int x^{11}\left(a+b\arctan\left(cx^3\right)\right)^2\,dx=\frac{a^2\,x^{12}}{12}-\frac{b^2\,\mathrm{atan}\left(c\,x^3\right)^2}{12\,c^4}+\frac{b^2\,x^{12}\,\mathrm{atan}\left(c\,x^3\right)^2}{12}-\frac{b^2\,\ln\left(c^2\,x^6+1\right)}{9\,c^4}+\frac{b^2\,x^6}{36\,c^2}+\frac{b^2\,x^3\,\mathrm{atan}\left(c\,x^3\right)}{6\,c^3}-\frac{b^2\,x^9\,\mathrm{atan}\left(c\,x^3\right)}{18\,c}+\frac{a\,b\,x^3}{6\,c^3}-\frac{a\,b\,x^9}{18\,c}-\frac{a\,b\,\mathrm{atan}\left(c\,x^3\right)}{6\,c^4}+\frac{a\,b\,x^{12}\,\mathrm{atan}\left(c\,x^3\right)}{6}$$

```
[In] int(x^11*(a + b*atan(c*x^3))^2,x)
```

```
[Out] (a^2*x^12)/12 - (b^2*atan(c*x^3)^2)/(12*c^4) + (b^2*x^12*atan(c*x^3)^2)/12
- (b^2*log(c^2*x^6 + 1))/(9*c^4) + (b^2*x^6)/(36*c^2) + (b^2*x^3*atan(c*x^3
))/(6*c^3) - (b^2*x^9*atan(c*x^3))/(18*c) + (a*b*x^3)/(6*c^3) - (a*b*x^9)/(
18*c) - (a*b*atan(c*x^3))/(6*c^4) + (a*b*x^12*atan(c*x^3))/6
```

3.114 $\int x^8(a+b\arctan(cx^3))^2\,dx$

Optimal result 666
Rubi [A] (verified) 666
Mathematica [A] (verified) 669
Maple [C] (warning: unable to verify) 669
Fricas [F] 670
Sympy [F] 670
Maxima [F] 670
Giac [F] 670
Mupad [F(-1)] 671

Optimal result

Integrand size = 16, antiderivative size = 154

$$\int x^8\left(a+b\arctan\left(cx^3\right)\right)^2\,dx=\frac{b^2x^3}{9c^2}-\frac{b^2\arctan\left(cx^3\right)}{9c^3}-\frac{bx^6(a+b\arctan\left(cx^3\right))}{9c}-\frac{i(a+b\arctan\left(cx^3\right))^2}{9c^3}+\frac{1}{9}x^9\left(a+b\arctan\left(cx^3\right)\right)^2-\frac{2b(a+b\arctan\left(cx^3\right))\log\left(\frac{2}{1+icx^3}\right)}{9c^3}-\frac{ib^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx^3}\right)}{9c^3}$$

```
[Out] 1/9*b^2*x^3/c^2-1/9*b^2*arctan(c*x^3)/c^3-1/9*b*x^6*(a+b*arctan(c*x^3))/c-1
/9*I*(a+b*arctan(c*x^3))^2/c^3+1/9*x^9*(a+b*arctan(c*x^3))^2-2/9*b*(a+b*arc
tan(c*x^3))*ln(2/(1+I*c*x^3))/c^3-1/9*I*b^2*polylog(2,1-2/(1+I*c*x^3))/c^3
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {4948, 4946, 5036, 327, 209, 5040, 4964, 2449, 2352}

$$\int x^8\left(a+b\arctan\left(cx^3\right)\right)^2\,dx=-\frac{i(a+b\arctan\left(cx^3\right))^2}{9c^3}-\frac{2b\log\left(\frac{2}{1+icx^3}\right)(a+b\arctan\left(cx^3\right))}{9c^3}+\frac{1}{9}x^9\left(a+b\arctan\left(cx^3\right)\right)^2-\frac{bx^6(a+b\arctan\left(cx^3\right))}{9c}-\frac{b^2\arctan\left(cx^3\right)}{9c^3}-\frac{ib^2\operatorname{PolyLog}\left(2,1-\frac{2}{icx^3+1}\right)}{9c^3}+\frac{b^2x^3}{9c^2}$$

```
[In] Int[x^8*(a + b*ArcTan[c*x^3])^2,x]
```

```
[Out] (b^2*x^3)/(9*c^2) - (b^2*ArcTan[c*x^3])/(9*c^3) - (b*x^6*(a + b*ArcTan[c*x^
3]))/(9*c) - ((I/9)*(a + b*ArcTan[c*x^3])^2)/c^3 + (x^9*(a + b*ArcTan[c*x^3
])^2)/9 - (2*b*(a + b*ArcTan[c*x^3])*Log[2/(1 + I*c*x^3)])/(9*c^3) - ((I/9)
*b^2*PolyLog[2, 1 - 2/(1 + I*c*x^3)])/c^3
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplify
[(m + 1)/n]]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int x^2(a+b\arctan(cx))^2\,dx,x,x^3\right)\\
&= \frac{1}{9}x^9\left(a+b\arctan\left(cx^3\right)\right)^2-\frac{1}{9}(2bc)\text{Subst}\left(\int\frac{x^3(a+b\arctan(cx))}{1+c^2x^2}\,dx,x,x^3\right)\\
&= \frac{1}{9}x^9\left(a+b\arctan\left(cx^3\right)\right)^2-\frac{(2b)\text{Subst}\left(\int x(a+b\arctan(cx))\,dx,x,x^3\right)}{9c}\\
&\quad+\frac{(2b)\text{Subst}\left(\int\frac{x(a+b\arctan(cx))}{1+c^2x^2}\,dx,x,x^3\right)}{9c}\\
&= -\frac{bx^6\left(a+b\arctan\left(cx^3\right)\right)}{9c}-\frac{i\left(a+b\arctan\left(cx^3\right)\right)^2}{9c^3}+\frac{1}{9}x^9\left(a+b\arctan\left(cx^3\right)\right)^2\\
&\quad+\frac{1}{9}b^2\text{Subst}\left(\int\frac{x^2}{1+c^2x^2}\,dx,x,x^3\right)-\frac{(2b)\text{Subst}\left(\int\frac{a+b\arctan(cx)}{i-cx}\,dx,x,x^3\right)}{9c^2}\\
&= \frac{b^2x^3}{9c^2}-\frac{bx^6\left(a+b\arctan\left(cx^3\right)\right)}{9c}-\frac{i\left(a+b\arctan\left(cx^3\right)\right)^2}{9c^3}\\
&\quad+\frac{1}{9}x^9\left(a+b\arctan\left(cx^3\right)\right)^2-\frac{2b\left(a+b\arctan\left(cx^3\right)\right)\log\left(\frac{2}{1+icx^3}\right)}{9c^3}\\
&\quad-\frac{b^2\text{Subst}\left(\int\frac{1}{1+c^2x^2}\,dx,x,x^3\right)}{9c^2}+\frac{\left(2b^2\right)\text{Subst}\left(\int\frac{\log\left(\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx,x,x^3\right)}{9c^2}
\end{aligned}$$

$$= \frac{b^2x^3}{9c^2} - \frac{b^2\arctan(cx^3)}{9c^3} - \frac{bx^6(a+b\arctan(cx^3))}{9c} - \frac{i(a+b\arctan(cx^3))^2}{9c^3} + \frac{1}{9}x^9(a+b\arctan(cx^3))^2 - \frac{2b(a+b\arctan(cx^3))\log\left(\frac{2}{1+icx^3}\right)}{9c^3} - \frac{(2ib^2)\operatorname{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1+icx^3}\right)}{9c^3}$$

$$= \frac{b^2x^3}{9c^2} - \frac{b^2\arctan(cx^3)}{9c^3} - \frac{bx^6(a+b\arctan(cx^3))}{9c} - \frac{i(a+b\arctan(cx^3))^2}{9c^3} + \frac{1}{9}x^9(a+b\arctan(cx^3))^2 - \frac{2b(a+b\arctan(cx^3))\log\left(\frac{2}{1+icx^3}\right)}{9c^3} - \frac{ib^2\operatorname{PolyLog}\left(2, 1-\frac{2}{1+icx^3}\right)}{9c^3}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.92

$$\int x^8(a+b\arctan(cx^3))^2\,dx = \frac{b^2cx^3 - abc^2x^6 + a^2c^3x^9 + b^2(i+c^3x^9)\arctan(cx^3)^2 - b\arctan(cx^3)\left(b+bc^2x^6-2ac^3x^9+2b\log\left(1+e\right.\right.}{9c^3}$$

```
[In] Integrate[x^8*(a + b*ArcTan[c*x^3])^2,x]
```

```
[Out] (b^2*c*x^3 - a*b*c^2*x^6 + a^2*c^3*x^9 + b^2*(I + c^3*x^9)*ArcTan[c*x^3]^2
- b*ArcTan[c*x^3]*(b + b*c^2*x^6 - 2*a*c^3*x^9 + 2*b*Log[1 + E^((2*I)*ArcTa
n[c*x^3])]) + a*b*Log[1 + c^2*x^6] + I*b^2*PolyLog[2, -E^((2*I)*ArcTan[c*x^
3])])/(9*c^3)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.69 (sec) , antiderivative size = 11449, normalized size of antiderivative = 74.34

method	result	size
default	Expression too large to display	11449
parts	Expression too large to display	11449

```
[In] int(x^8*(a+b*arctan(c*x^3))^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [F]

$$\int x^8 \left(a + b \arctan\left(c x^3\right)\right)^2 \, dx = \int \left(b \arctan\left(c x^3\right) + a\right)^2 x^8 \, dx$$

```
[In] integrate(x^8*(a+b*arctan(c*x^3))^2,x, algorithm="fricas")
[Out] integral(b^2*x^8*arctan(c*x^3)^2 + 2*a*b*x^8*arctan(c*x^3) + a^2*x^8, x)
```

Sympy [F]

$$\int x^8 \left(a + b \arctan\left(c x^3\right)\right)^2 \, dx = \int x^8 \left(a + b \operatorname{atan}\left(c x^3\right)\right)^2 \, dx$$

```
[In] integrate(x**8*(a+b*atan(c*x**3))**2,x)
[Out] Integral(x**8*(a + b*atan(c*x**3))**2, x)
```

Maxima [F]

$$\int x^8 \left(a + b \arctan\left(c x^3\right)\right)^2 \, dx = \int \left(b \arctan\left(c x^3\right) + a\right)^2 x^8 \, dx$$

```
[In] integrate(x^8*(a+b*arctan(c*x^3))^2,x, algorithm="maxima")
[Out] 1/9*a^2*x^9 + 1/9*(2*x^9*arctan(c*x^3) - (x^6/c^2 - log(c^2*x^6 + 1)/c^4)*c
)*a*b + 1/144*(4*x^9*arctan(c*x^3)^2 - x^9*log(c^2*x^6 + 1)^2 + 144*integra
te(1/48*(4*c^2*x^14*log(c^2*x^6 + 1) - 8*c*x^11*arctan(c*x^3) + 36*(c^2*x^1
4 + x^8)*arctan(c*x^3)^2 + 3*(c^2*x^14 + x^8)*log(c^2*x^6 + 1)^2)/(c^2*x^6
+ 1), x))*b^2
```

Giac [F]

$$\int x^8 \left(a + b \arctan\left(c x^3\right)\right)^2 \, dx = \int \left(b \arctan\left(c x^3\right) + a\right)^2 x^8 \, dx$$

```
[In] integrate(x^8*(a+b*arctan(c*x^3))^2,x, algorithm="giac")
[Out] integrate((b*arctan(c*x^3) + a)^2*x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^8(a+b\arctan(cx^3))^2\,dx = \int x^8\,\left(a+b\,\mathrm{atan}\left(c\,x^3\right)\right)^2\,dx$$

```
[In] int(x^8*(a + b*atan(c*x^3))^2,x)
[Out] int(x^8*(a + b*atan(c*x^3))^2, x)
```

3.115 $\int x^5(a+b\arctan(cx^3))^2 \, dx$

Optimal result 672
Rubi [A] (verified) 672
Mathematica [A] (verified) 674
Maple [A] (verified) 674
Fricas [A] (verification not implemented) 675
Sympy [B] (verification not implemented) 675
Maxima [A] (verification not implemented) 675
Giac [A] (verification not implemented) 676
Mupad [B] (verification not implemented) 676

Optimal result

Integrand size = 16, antiderivative size = 90

$$\int x^5(a+b\arctan(cx^3))^2 \, dx = -\frac{abx^3}{3c} - \frac{b^2x^3\arctan(cx^3)}{3c} + \frac{(a+b\arctan(cx^3))^2}{6c^2} + \frac{1}{6}x^6(a+b\arctan(cx^3))^2 + \frac{b^2\log(1+c^2x^6)}{6c^2}$$

```
[Out] -1/3*a*b*x^3/c-1/3*b^2*x^3*arctan(c*x^3)/c+1/6*(a+b*arctan(c*x^3))^2/c^2+1/
6*x^6*(a+b*arctan(c*x^3))^2+1/6*b^2*ln(c^2*x^6+1)/c^2
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4948, 4946, 5036, 4930, 266, 5004}

$$\int x^5(a+b\arctan(cx^3))^2 \, dx = \frac{(a+b\arctan(cx^3))^2}{6c^2} + \frac{1}{6}x^6(a+b\arctan(cx^3))^2 - \frac{abx^3}{3c} - \frac{b^2x^3\arctan(cx^3)}{3c} + \frac{b^2\log(c^2x^6+1)}{6c^2}$$

```
[In] Int[x^5*(a + b*ArcTan[c*x^3])^2,x]
```

```
[Out] -1/3*(a*b*x^3)/c - (b^2*x^3*ArcTan[c*x^3])/(3*c) + (a + b*ArcTan[c*x^3])^2/
(6*c^2) + (x^6*(a + b*ArcTan[c*x^3])^2)/6 + (b^2*Log[1 + c^2*x^6])/(6*c^2)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplify
[(m + 1)/n]]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int x(a+b\arctan(cx))^2\,dx,x,x^3\right)\\
&= \frac{1}{6}x^6\left(a+b\arctan\left(cx^3\right)\right)^2-\frac{1}{3}(bc)\text{Subst}\left(\int\frac{x^2(a+b\arctan(cx))}{1+c^2x^2}\,dx,x,x^3\right)\\
&= \frac{1}{6}x^6\left(a+b\arctan\left(cx^3\right)\right)^2-\frac{b\text{Subst}\left(\int(a+b\arctan(cx))\,dx,x,x^3\right)}{3c}\\
&\quad+\frac{b\text{Subst}\left(\int\frac{a+b\arctan(cx)}{1+c^2x^2}\,dx,x,x^3\right)}{3c}
\end{aligned}
$$

$$= -\frac{abx^3}{3c} + \frac{(a+b\arctan(cx^3))^2}{6c^2} + \frac{1}{6}x^6(a+b\arctan(cx^3))^2 - \frac{b^2\text{Subst}\left(\int \arctan(cx)\,dx, x, x^3\right)}{3c}$$

$$= -\frac{abx^3}{3c} - \frac{b^2x^3\arctan(cx^3)}{3c} + \frac{(a+b\arctan(cx^3))^2}{6c^2} + \frac{1}{6}x^6(a+b\arctan(cx^3))^2 + \frac{1}{3}b^2\text{Subst}\left(\int \frac{x}{1+c^2x^2}\,dx, x, x^3\right)$$

$$= -\frac{abx^3}{3c} - \frac{b^2x^3\arctan(cx^3)}{3c} + \frac{(a+b\arctan(cx^3))^2}{6c^2} + \frac{1}{6}x^6(a+b\arctan(cx^3))^2 + \frac{b^2\log(1+c^2x^6)}{6c^2}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.94

$$\int x^5(a+b\arctan(cx^3))^2\,dx = \frac{acx^3(-2b+acx^3)+2b(a-bcx^3+ac^2x^6)\arctan(cx^3)+b^2(1+c^2x^6)\arctan(cx^3)^2+b^2\log(1+c^2x^6)}{6c^2}$$

```
[In] Integrate[x^5*(a + b*ArcTan[c*x^3])^2,x]
```

```
[Out] (a*c*x^3*(-2*b + a*c*x^3) + 2*b*(a - b*c*x^3 + a*c^2*x^6)*ArcTan[c*x^3] + b
^2*(1 + c^2*x^6)*ArcTan[c*x^3]^2 + b^2*Log[1 + c^2*x^6])/(6*c^2)
```

Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.22

method	result
parallelrisch	$\frac{b^2\arctan(cx^3)^2x^6c^2+2ab\arctan(cx^3)x^6c^2+a^2c^2x^6-2b^2\arctan(cx^3)x^3c-2abcx^3+b^2\arctan(cx^3)^2+b^2\ln(c^2x^6+1)+2ab\,\text{arct}}{6c^2}$
default	$\frac{x^6a^2}{6}+\frac{b^2x^6\arctan(cx^3)^2}{6}-\frac{b^2x^3\arctan(cx^3)}{3c}+\frac{b^2\arctan(cx^3)^2}{6c^2}+\frac{b^2\ln(c^2x^6+1)}{6c^2}+\frac{abx^6\arctan(cx^3)}{3}-\frac{abx^3}{3c}+$
parts	$\frac{x^6a^2}{6}+\frac{b^2x^6\arctan(cx^3)^2}{6}-\frac{b^2x^3\arctan(cx^3)}{3c}+\frac{b^2\arctan(cx^3)^2}{6c^2}+\frac{b^2\ln(c^2x^6+1)}{6c^2}+\frac{abx^6\arctan(cx^3)}{3}-\frac{abx^3}{3c}+$
risch	$-\frac{b^2(c^2x^6+1)\ln(icx^3+1)^2}{24c^2}-\frac{ib(4a^2c^2x^6+2ix^6b\ln(-icx^3+1)ac^2-4abcx^3+b^2+2ib\ln(-icx^3+1)a)\ln(icx^3+1)}{24ac^2}+\frac{iabx^6\ln}{}$

```
[In] int(x^5*(a+b*arctan(c*x^3))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(b^2*arctan(c*x^3)^2*x^6*c^2+2*a*b*arctan(c*x^3)*x^6*c^2+a^2*c^2*x^6-2*
b^2*arctan(c*x^3)*x^3*c-2*a*b*c*x^3+b^2*arctan(c*x^3)^2+b^2*ln(c^2*x^6+1)+2
*a*b*arctan(c*x^3))/c^2
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.01

$$\int x^5 \left(a + b \arctan\left(cx^3\right)\right)^2 \, dx$$
$$= \frac{a^2c^2x^6 - 2\,abcx^3 + (b^2c^2x^6 + b^2) \arctan\left(cx^3\right)^2 + b^2 \log\left(c^2x^6 + 1\right) + 2\left(abc^2x^6 - b^2cx^3 + ab\right) \arctan\left(cx^3\right)}{6\,c^2}$$

```
[In] integrate(x^5*(a+b*arctan(c*x^3))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(a^2*c^2*x^6 - 2*a*b*c*x^3 + (b^2*c^2*x^6 + b^2)*arctan(c*x^3)^2 + b^2*
log(c^2*x^6 + 1) + 2*(a*b*c^2*x^6 - b^2*c*x^3 + a*b)*arctan(c*x^3))/c^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(78) = 156$.

Time = 50.12 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.16

$$\int x^5 \left(a + b \arctan\left(cx^3\right)\right)^2 \, dx$$
$$= \begin{cases} \frac{a^2x^6}{6} + \frac{abx^6 \operatorname{atan}\left(cx^3\right)}{3} - \frac{abx^3}{3c} + \frac{ab \operatorname{atan}\left(cx^3\right)}{3c^2} + \frac{b^2x^6 \operatorname{atan}^2\left(cx^3\right)}{6} - \frac{b^2x^3 \operatorname{atan}\left(cx^3\right)}{3c} - \frac{b^2\sqrt{-\frac{1}{c^2}} \operatorname{atan}\left(cx^3\right)}{3c} + \frac{b^2 \log\left(x - \sqrt[6]{-\right.}}{3c^2} \\ \frac{a^2x^6}{6} \end{cases}$$

```
[In] integrate(x**5*(a+b*atan(c*x**3))**2,x)
```

```
[Out] Piecewise((a**2*x**6/6 + a*b*x**6*atan(c*x**3)/3 - a*b*x**3/(3*c) + a*b*ata
n(c*x**3)/(3*c**2) + b**2*x**6*atan(c*x**3)**2/6 - b**2*x**3*atan(c*x**3)/(
3*c) - b**2*sqrt(-1/c**2)*atan(c*x**3)/(3*c) + b**2*log(x - (-1/c**2)**(1/6
))/(3*c**2) + b**2*log(4*x**2 + 4*x*(-1/c**2)**(1/6) + 4*(-1/c**2)**(1/3))/
(3*c**2) + b**2*atan(c*x**3)**2/(6*c**2), Ne(c, 0)), (a**2*x**6/6, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.40

$$\int x^5 \left(a + b \arctan\left(cx^3\right)\right)^2 \, dx$$
$$= \frac{1}{6}\,b^2x^6 \arctan\left(cx^3\right)^2 + \frac{1}{6}\,a^2x^6 + \frac{1}{3}\left(x^6 \arctan\left(cx^3\right) - c\left(\frac{x^3}{c^2} - \frac{\arctan\left(cx^3\right)}{c^3}\right)\right)ab$$
$$- \frac{1}{6}\left(2\,c\left(\frac{x^3}{c^2} - \frac{\arctan\left(cx^3\right)}{c^3}\right) \arctan\left(cx^3\right) + \frac{\arctan\left(cx^3\right)^2 - \log\left(6\,c^5x^6 + 6\,c^3\right)}{c^2}\right)b^2$$

```
[In] integrate(x^5*(a+b*arctan(c*x^3))^2,x, algorithm="maxima")
```

```
[Out] 1/6*b^2*x^6*arctan(c*x^3)^2 + 1/6*a^2*x^6 + 1/3*(x^6*arctan(c*x^3) - c*(x^3
/c^2 - arctan(c*x^3)/c^3))*a*b - 1/6*(2*c*(x^3/c^2 - arctan(c*x^3)/c^3)*arc
tan(c*x^3) + (arctan(c*x^3)^2 - log(6*c^5*x^6 + 6*c^3))/c^2)*b^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.11

$$\int x^5\left(a+b\arctan\left(cx^3\right)\right)^2\,dx = \frac{a^2cx^6 + \frac{2\left(c^2x^6\arctan\left(cx^3\right)-cx^3+\arctan\left(cx^3\right)\right)ab}{c} + \frac{\left(c^2x^6\arctan\left(cx^3\right)^2-2\,cx^3\arctan\left(cx^3\right)+\arctan\left(cx^3\right)^2+\log\left(c^2x^6+1\right)\right)b^2}{c}}{6\,c}$$

```
[In] integrate(x^5*(a+b*arctan(c*x^3))^2,x, algorithm="giac")
```

```
[Out] 1/6*(a^2*c*x^6 + 2*(c^2*x^6*arctan(c*x^3) - c*x^3 + arctan(c*x^3))*a*b/c +
(c^2*x^6*arctan(c*x^3)^2 - 2*c*x^3*arctan(c*x^3) + arctan(c*x^3)^2 + log(c^
2*x^6 + 1))*b^2/c)/c
```

Mupad [B] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.24

$$\int x^5\left(a+b\arctan\left(cx^3\right)\right)^2\,dx = \frac{a^2\,x^6}{6}+\frac{b^2\,\mathrm{atan}(c\,x^3)^2}{6\,c^2}+\frac{b^2\,x^6\,\mathrm{atan}(c\,x^3)^2}{6}+\frac{b^2\,\ln\left(c^2\,x^6+1\right)}{6\,c^2} -\frac{b^2\,x^3\,\mathrm{atan}(c\,x^3)}{3\,c}-\frac{a\,b\,x^3}{3\,c}+\frac{a\,b\,\mathrm{atan}(c\,x^3)}{3\,c^2}+\frac{a\,b\,x^6\,\mathrm{atan}(c\,x^3)}{3}$$

```
[In] int(x^5*(a + b*atan(c*x^3))^2,x)
```

```
[Out] (a^2*x^6)/6 + (b^2*atan(c*x^3)^2)/(6*c^2) + (b^2*x^6*atan(c*x^3)^2)/6 + (b^
2*log(c^2*x^6 + 1))/(6*c^2) - (b^2*x^3*atan(c*x^3))/(3*c) - (a*b*x^3)/(3*c)
 + (a*b*atan(c*x^3))/(3*c^2) + (a*b*x^6*atan(c*x^3))/3
```

3.116 $\int x^2(a+b\arctan(cx^3))^2\,dx$

Optimal result . . . 677
Rubi [A] (verified) . . . 677
Mathematica [A] (verified) . . . 679
Maple [A] (verified) . . . 679
Fricas [F] . . . 680
Sympy [F] . . . 680
Maxima [F] . . . 681
Giac [F] . . . 681
Mupad [F(-1)] . . . 681

Optimal result

Integrand size = 16, antiderivative size = 104

$$\int x^2\left(a+b\arctan\left(cx^3\right)\right)^2\,dx=\frac{i(a+b\arctan(cx^3))^2}{3c}+\frac{1}{3}x^3\left(a+b\arctan\left(cx^3\right)\right)^2+\frac{2b(a+b\arctan(cx^3))\log\left(\frac{2}{1+icx^3}\right)}{3c}+\frac{ib^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx^3}\right)}{3c}$$

```
[Out] 1/3*I*(a+b*arctan(c*x^3))^2/c+1/3*x^3*(a+b*arctan(c*x^3))^2+2/3*b*(a+b*arct
an(c*x^3))*ln(2/(1+I*c*x^3))/c+1/3*I*b^2*polylog(2,1-2/(1+I*c*x^3))/c
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4948, 4930, 5040, 4964, 2449, 2352}

$$\int x^2\left(a+b\arctan\left(cx^3\right)\right)^2\,dx=\frac{1}{3}x^3\left(a+b\arctan\left(cx^3\right)\right)^2+\frac{i(a+b\arctan(cx^3))^2}{3c}+\frac{2b\log\left(\frac{2}{1+icx^3}\right)\left(a+b\arctan\left(cx^3\right)\right)}{3c}+\frac{ib^2\operatorname{PolyLog}\left(2,1-\frac{2}{icx^3+1}\right)}{3c}$$

```
[In] Int[x^2*(a + b*ArcTan[c*x^3])^2,x]
```

```
[Out] ((I/3)*(a + b*ArcTan[c*x^3])^2)/c + (x^3*(a + b*ArcTan[c*x^3])^2)/3 + (2*b*
(a + b*ArcTan[c*x^3])*Log[2/(1 + I*c*x^3)])/(3*c) + ((I/3)*b^2*PolyLog[2, 1
 - 2/(1 + I*c*x^3)])/c
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplify
[(m + 1)/n]]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int (a+b\arctan(cx))^2\,dx, x, x^3\right)\\ &= \frac{1}{3}x^3\left(a+b\arctan\left(cx^3\right)\right)^2-\frac{1}{3}(2bc)\text{Subst}\left(\int \frac{x(a+b\arctan(cx))}{1+c^2x^2}\,dx, x, x^3\right)\end{aligned}$$

$$
\begin{aligned}
&= \frac{i\left(a+b\arctan\left(cx^3\right)\right)^2}{3c} + \frac{1}{3}x^3\left(a+b\arctan\left(cx^3\right)\right)^2 \\
&\quad + \frac{1}{3}(2b)\mathrm{Subst}\left(\int \frac{a+b\arctan(cx)}{i-cx}\,dx, x, x^3\right) \\
&= \frac{i\left(a+b\arctan\left(cx^3\right)\right)^2}{3c} + \frac{1}{3}x^3\left(a+b\arctan\left(cx^3\right)\right)^2 \\
&\quad + \frac{2b\left(a+b\arctan\left(cx^3\right)\right)\log\left(\frac{2}{1+icx^3}\right)}{3c} - \frac{1}{3}\left(2b^2\right)\mathrm{Subst}\left(\int \frac{\log\left(\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx, x, x^3\right) \\
&= \frac{i\left(a+b\arctan\left(cx^3\right)\right)^2}{3c} + \frac{1}{3}x^3\left(a+b\arctan\left(cx^3\right)\right)^2 \\
&\quad + \frac{2b\left(a+b\arctan\left(cx^3\right)\right)\log\left(\frac{2}{1+icx^3}\right)}{3c} + \frac{\left(2ib^2\right)\mathrm{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1+icx^3}\right)}{3c} \\
&= \frac{i\left(a+b\arctan\left(cx^3\right)\right)^2}{3c} + \frac{1}{3}x^3\left(a+b\arctan\left(cx^3\right)\right)^2 \\
&\quad + \frac{2b\left(a+b\arctan\left(cx^3\right)\right)\log\left(\frac{2}{1+icx^3}\right)}{3c} + \frac{ib^2\,\mathrm{PolyLog}\left(2, 1-\frac{2}{1+icx^3}\right)}{3c}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.03

$$
\int x^2\left(a+b\arctan\left(cx^3\right)\right)^2\,dx
= \frac{b^2\left(-i+cx^3\right)\arctan\left(cx^3\right)^2 + 2b\arctan\left(cx^3\right)\left(acx^3+b\log\left(1+e^{2i\arctan\left(cx^3\right)}\right)\right) + a\left(acx^3-b\log\left(1+c^2x^6\right.\right.}{3c}
$$

```
[In] Integrate[x^2*(a + b*ArcTan[c*x^3])^2,x]
```

```
[Out] (b^2*(-I + c*x^3)*ArcTan[c*x^3]^2 + 2*b*ArcTan[c*x^3]*(a*c*x^3 + b*Log[1 +
E^((2*I)*ArcTan[c*x^3])]) + a*(a*c*x^3 - b*Log[1 + c^2*x^6]) - I*b^2*PolyLo
g[2, -E^((2*I)*ArcTan[c*x^3])])/(3*c)
```

Maple [A] (verified)

Time = 5.02 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.35

method	result
parts	$\frac{a^2x^3}{3}+\frac{b^2\left(\arctan(cx^3)^2(cx^3+i)+2\arctan(cx^3)\ln\left(1+\frac{(icx^3+1)^2}{c^2x^6+1}\right)-2i\arctan(cx^3)^2-i\,\mathrm{polylog}\left(2,-\frac{(icx^3+1)^2}{c^2x^6+1}\right)\right)}{3c}$
derivativedivides	$\frac{cx^3a^2-i\arctan(cx^3)^2b^2+\arctan(cx^3)^2b^2cx^3-i\,\mathrm{polylog}\left(2,-\frac{(icx^3+1)^2}{c^2x^6+1}\right)b^2+2\arctan(cx^3)\ln\left(1+\frac{(icx^3+1)^2}{c^2x^6+1}\right)b^2+2ab}{3c}$
default	$\frac{cx^3a^2-i\arctan(cx^3)^2b^2+\arctan(cx^3)^2b^2cx^3-i\,\mathrm{polylog}\left(2,-\frac{(icx^3+1)^2}{c^2x^6+1}\right)b^2+2\arctan(cx^3)\ln\left(1+\frac{(icx^3+1)^2}{c^2x^6+1}\right)b^2+2ab}{3c}$
risch	$\frac{b^2\ln(icx^3+1)\ln(-icx^3+1)x^3}{6}-\frac{ba\ln(icx^3+1)}{3c}-\frac{i\ln(-icx^3+1)^2b^2}{12c}+\frac{i\ln(-icx^3+1)abx^3}{3}+\frac{ib^2\ln(c^2x^6+1)}{6c}-$

[In] int(x^2*(a+b*arctan(c*x^3))^2,x,method=_RETURNVERBOSE)

[Out] 1/3*a^2*x^3+1/3*b^2/c*(arctan(c*x^3)^2*(c*x^3+I)+2*arctan(c*x^3)*ln(1+(1+I*c*x^3)^2/(c^2*x^6+1))-2*I*arctan(c*x^3)^2-I*polylog(2,-(1+I*c*x^3)^2/(c^2*x^6+1)))+2/3*a*b*arctan(c*x^3)*x^3-1/3/c*a*b*ln(c^2*x^6+1)

Fricas [F]

$$\int x^2\left(a+b\arctan\left(cx^3\right)\right)^2\,dx=\int\left(b\arctan\left(cx^3\right)+a\right)^2x^2\,dx$$

[In] integrate(x^2*(a+b*arctan(c*x^3))^2,x, algorithm="fricas")

[Out] integral(b^2*x^2*arctan(c*x^3)^2 + 2*a*b*x^2*arctan(c*x^3) + a^2*x^2, x)

Sympy [F]

$$\int x^2\left(a+b\arctan\left(cx^3\right)\right)^2\,dx=\int x^2\left(a+b\,\mathrm{atan}\left(cx^3\right)\right)^2\,dx$$

[In] integrate(x**2*(a+b*atan(c*x**3))**2,x)

[Out] Integral(x**2*(a + b*atan(c*x**3))**2, x)

Maxima [F]

$$\int x^2 \left(a+b\arctan\left(cx^3\right)\right)^2\,dx = \int \left(b\arctan\left(cx^3\right)+a\right)^2 x^2\,dx$$

```
[In] integrate(x^2*(a+b*arctan(c*x^3))^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*x^3 + 1/48*(4*x^3*arctan(c*x^3)^2 - x^3*log(c^2*x^6 + 1)^2 + 576*c^
2*integrate(1/16*x^8*arctan(c*x^3)^2/(c^2*x^6 + 1), x) + 48*c^2*integrate(1
/16*x^8*log(c^2*x^6 + 1)^2/(c^2*x^6 + 1), x) + 192*c^2*integrate(1/16*x^8*l
og(c^2*x^6 + 1)/(c^2*x^6 + 1), x) + 4*arctan(c*x^3)^3/c - 384*c*integrate(1
/16*x^5*arctan(c*x^3)/(c^2*x^6 + 1), x) + 48*integrate(1/16*x^2*log(c^2*x^6
 + 1)^2/(c^2*x^6 + 1), x))*b^2 + 1/3*(2*c*x^3*arctan(c*x^3) - log(c^2*x^6 +
 1))*a*b/c
```

Giac [F]

$$\int x^2 \left(a+b\arctan\left(cx^3\right)\right)^2\,dx = \int \left(b\arctan\left(cx^3\right)+a\right)^2 x^2\,dx$$

```
[In] integrate(x^2*(a+b*arctan(c*x^3))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^3) + a)^2*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \left(a+b\arctan\left(cx^3\right)\right)^2\,dx = \int x^2\,\left(a+b\,\mathrm{atan}\left(c\,x^3\right)\right)^2\,dx$$

```
[In] int(x^2*(a + b*atan(c*x^3))^2,x)
```

```
[Out] int(x^2*(a + b*atan(c*x^3))^2, x)
```

3.117 $\int \frac{(a+b\arctan(cx^3))^2}{x}\,dx$

Optimal result 682
Rubi [A] (verified) 682
Mathematica [A] (verified) 685
Maple [F] 686
Fricas [F] 686
Sympy [F] 686
Maxima [F] 686
Giac [F] 687
Mupad [F(-1)] 687

Optimal result

Integrand size = 16, antiderivative size = 154

$$\int \frac{(a+b\arctan(cx^3))^2}{x}\,dx = \frac{2}{3}(a+b\arctan(cx^3))^2\operatorname{arctanh}\left(1-\frac{2}{1+icx^3}\right) - \frac{1}{3}ib(a+b\arctan(cx^3))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx^3}\right) + \frac{1}{3}ib(a+b\arctan(cx^3))\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx^3}\right) - \frac{1}{6}b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx^3}\right) + \frac{1}{6}b^2\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx^3}\right)$$

```
[Out] -2/3*(a+b*arctan(c*x^3))^2*arctanh(-1+2/(1+I*c*x^3))-1/3*I*b*(a+b*arctan(c*
x^3))*polylog(2,1-2/(1+I*c*x^3))+1/3*I*b*(a+b*arctan(c*x^3))*polylog(2,-1+2
/(1+I*c*x^3))-1/6*b^2*polylog(3,1-2/(1+I*c*x^3))+1/6*b^2*polylog(3,-1+2/(1+
I*c*x^3))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used

= {4944, 4942, 5108, 5004, 5114, 6745}

$$\int \frac{(a+b\arctan(cx^3))^2}{x}\,dx = \frac{2}{3}\text{arctanh}\left(1-\frac{2}{1+icx^3}\right)\left(a+b\arctan\left(cx^3\right)\right)^2 - \frac{1}{3}ib\,\text{PolyLog}\left(2,1-\frac{2}{icx^3+1}\right)\left(a+b\arctan\left(cx^3\right)\right) + \frac{1}{3}ib\,\text{PolyLog}\left(2,\frac{2}{icx^3+1}-1\right)\left(a+b\arctan\left(cx^3\right)\right) - \frac{1}{6}b^2\,\text{PolyLog}\left(3,1-\frac{2}{icx^3+1}\right) + \frac{1}{6}b^2\,\text{PolyLog}\left(3,\frac{2}{icx^3+1}-1\right)$$

```
[In] Int[(a + b*ArcTan[c*x^3])^2/x,x]
```

```
[Out] (2*(a + b*ArcTan[c*x^3])^2*ArcTanh[1 - 2/(1 + I*c*x^3)])/3 - (I/3)*b*(a + b
*ArcTan[c*x^3])*PolyLog[2, 1 - 2/(1 + I*c*x^3)] + (I/3)*b*(a + b*ArcTan[c*x
^3])*PolyLog[2, -1 + 2/(1 + I*c*x^3)] - (b^2*PolyLog[3, 1 - 2/(1 + I*c*x^3)
])/6 + (b^2*PolyLog[3, -1 + 2/(1 + I*c*x^3)])/6
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1
/n, Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[p, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5108

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e*
x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{(a+b\arctan(cx))^2}{x}\,dx, x, x^3\right) \\
&= \frac{2}{3}\left(a+b\arctan\left(cx^3\right)\right)^2\text{arctanh}\left(1-\frac{2}{1+icx^3}\right) \\
&\quad -\frac{1}{3}(4bc)\text{Subst}\left(\int \frac{(a+b\arctan(cx))\text{arctanh}\left(1-\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx, x, x^3\right) \\
&= \frac{2}{3}\left(a+b\arctan\left(cx^3\right)\right)^2\text{arctanh}\left(1-\frac{2}{1+icx^3}\right) \\
&\quad +\frac{1}{3}(2bc)\text{Subst}\left(\int \frac{(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx, x, x^3\right) \\
&\quad -\frac{1}{3}(2bc)\text{Subst}\left(\int \frac{(a+b\arctan(cx))\log\left(2-\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx, x, x^3\right) \\
&= \frac{2}{3}\left(a+b\arctan\left(cx^3\right)\right)^2\text{arctanh}\left(1-\frac{2}{1+icx^3}\right) \\
&\quad -\frac{1}{3}ib\left(a+b\arctan\left(cx^3\right)\right)\text{PolyLog}\left(2,1-\frac{2}{1+icx^3}\right) \\
&\quad +\frac{1}{3}ib\left(a+b\arctan\left(cx^3\right)\right)\text{PolyLog}\left(2,-1+\frac{2}{1+icx^3}\right) \\
&\quad +\frac{1}{3}\left(ib^2c\right)\text{Subst}\left(\int \frac{\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx, x, x^3\right) \\
&\quad -\frac{1}{3}\left(ib^2c\right)\text{Subst}\left(\int \frac{\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx, x, x^3\right)
\end{aligned}
$$

$$= \frac{2}{3}\left(a+b\arctan\left(cx^3\right)\right)^2\operatorname{arctanh}\left(1-\frac{2}{1+icx^3}\right)$$
$$-\frac{1}{3}ib\left(a+b\arctan\left(cx^3\right)\right)\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx^3}\right)$$
$$+\frac{1}{3}ib\left(a+b\arctan\left(cx^3\right)\right)\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx^3}\right)$$
$$-\frac{1}{6}b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx^3}\right)+\frac{1}{6}b^2\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx^3}\right)$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.31

$$\int \frac{\left(a+b\arctan\left(cx^3\right)\right)^2}{x}\,dx = a^2\log(x)+\frac{1}{3}iab\left(\operatorname{PolyLog}\left(2,-icx^3\right)-\operatorname{PolyLog}\left(2,icx^3\right)\right)$$
$$+\frac{1}{72}b^2\Big(-i\pi^3+16i\arctan\left(cx^3\right)^3$$
$$+24\arctan\left(cx^3\right)^2\log\left(1-e^{-2i\arctan\left(cx^3\right)}\right)$$
$$-24\arctan\left(cx^3\right)^2\log\left(1+e^{2i\arctan\left(cx^3\right)}\right)$$
$$+24i\arctan\left(cx^3\right)\operatorname{PolyLog}\left(2,e^{-2i\arctan\left(cx^3\right)}\right)$$
$$+24i\arctan\left(cx^3\right)\operatorname{PolyLog}\left(2,-e^{2i\arctan\left(cx^3\right)}\right)$$
$$+12\operatorname{PolyLog}\left(3,e^{-2i\arctan\left(cx^3\right)}\right)$$
$$-12\operatorname{PolyLog}\left(3,-e^{2i\arctan\left(cx^3\right)}\right)\Big)$$

```
[In] Integrate[(a + b*ArcTan[c*x^3])^2/x,x]
```

```
[Out] a^2*Log[x] + (I/3)*a*b*(PolyLog[2, (-I)*c*x^3] - PolyLog[2, I*c*x^3]) + (b^
2*((-I)*Pi^3 + (16*I)*ArcTan[c*x^3]^3 + 24*ArcTan[c*x^3]^2*Log[1 - E^((-2*I
)*ArcTan[c*x^3])] - 24*ArcTan[c*x^3]^2*Log[1 + E^((2*I)*ArcTan[c*x^3])] + (
24*I)*ArcTan[c*x^3]*PolyLog[2, E^((-2*I)*ArcTan[c*x^3])] + (24*I)*ArcTan[c*
x^3]*PolyLog[2, -E^((2*I)*ArcTan[c*x^3])] + 12*PolyLog[3, E^((-2*I)*ArcTan[
c*x^3])] - 12*PolyLog[3, -E^((2*I)*ArcTan[c*x^3])]))/72
```

Maple [F]

$$\int \frac{(a + b \arctan (c\, x^3))^2}{x} dx$$

```
[In] int((a+b*arctan(c*x^3))^2/x,x)
[Out] int((a+b*arctan(c*x^3))^2/x,x)
```

Fricas [F]

$$\int \frac{(a + b \arctan (cx^3))^2}{x} dx = \int \frac{(b \arctan (cx^3) + a)^2}{x} dx$$

```
[In] integrate((a+b*arctan(c*x^3))^2/x,x, algorithm="fricas")
[Out] integral((b^2*arctan(c*x^3)^2 + 2*a*b*arctan(c*x^3) + a^2)/x, x)
```

Sympy [F]

$$\int \frac{(a + b \arctan (cx^3))^2}{x} dx = \int \frac{(a + b \operatorname{atan} (cx^3))^2}{x} dx$$

```
[In] integrate((a+b*atan(c*x**3))**2/x,x)
[Out] Integral((a + b*atan(c*x**3))**2/x, x)
```

Maxima [F]

$$\int \frac{(a + b \arctan (cx^3))^2}{x} dx = \int \frac{(b \arctan (cx^3) + a)^2}{x} dx$$

```
[In] integrate((a+b*arctan(c*x^3))^2/x,x, algorithm="maxima")
[Out] a^2*log(x) + 1/16*integrate((12*b^2*arctan(c*x^3)^2 + b^2*log(c^2*x^6 + 1)^
2 + 32*a*b*arctan(c*x^3))/x, x)
```

Giac [F]

$$\int \frac{(a + b\arctan(cx^3))^2}{x} \, dx = \int \frac{(b\arctan(cx^3) + a)^2}{x} \, dx$$

```
[In] integrate((a+b*arctan(c*x^3))^2/x,x, algorithm="giac")
[Out] integrate((b*arctan(c*x^3) + a)^2/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\arctan(cx^3))^2}{x} \, dx = \int \frac{(a + b\,\mathrm{atan}(c\,x^3))^2}{x} \, dx$$

```
[In] int((a + b*atan(c*x^3))^2/x,x)
[Out] int((a + b*atan(c*x^3))^2/x, x)
```

3.118 $\int \frac{(a+b\arctan(cx^3))^2}{x^4} dx$

Optimal result . . . 688
Rubi [A] (verified) . . . 688
Mathematica [A] (verified) . . . 690
Maple [C] (warning: unable to verify) . . . 690
Fricas [F] . . . 691
Sympy [F] . . . 691
Maxima [F] . . . 691
Giac [F] . . . 691
Mupad [F(-1)] . . . 692

Optimal result

Integrand size = 16, antiderivative size = 100

$$\int \frac{(a+b\arctan(cx^3))^2}{x^4} dx = -\frac{1}{3}ic(a+b\arctan(cx^3))^2 - \frac{(a+b\arctan(cx^3))^2}{3x^3} + \frac{2}{3}bc(a+b\arctan(cx^3))\log\left(2-\frac{2}{1-icx^3}\right) - \frac{1}{3}ib^2c\,\text{PolyLog}\left(2,-1+\frac{2}{1-icx^3}\right)$$

```
[Out] -1/3*I*c*(a+b*arctan(c*x^3))^2-1/3*(a+b*arctan(c*x^3))^2/x^3+2/3*b*c*(a+b*a
rctan(c*x^3))*ln(2-2/(1-I*c*x^3))-1/3*I*b^2*c*polylog(2,-1+2/(1-I*c*x^3))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {4948, 4946, 5044, 4988, 2497}

$$\int \frac{(a+b\arctan(cx^3))^2}{x^4} dx = -\frac{1}{3}ic(a+b\arctan(cx^3))^2 - \frac{(a+b\arctan(cx^3))^2}{3x^3} + \frac{2}{3}bc\log\left(2-\frac{2}{1-icx^3}\right)(a+b\arctan(cx^3)) - \frac{1}{3}ib^2c\,\text{PolyLog}\left(2,\frac{2}{1-icx^3}-1\right)$$

```
[In] Int[(a + b*ArcTan[c*x^3])^2/x^4,x]
```

```
[Out] (-1/3*I)*c*(a + b*ArcTan[c*x^3])^2 - (a + b*ArcTan[c*x^3])^2/(3*x^3) + (2*b
*c*(a + b*ArcTan[c*x^3])*Log[2 - 2/(1 - I*c*x^3)])/3 - (I/3)*b^2*c*PolyLog[
2, -1 + 2/(1 - I*c*x^3)]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplify
[(m + 1)/n]]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{3}\text{Subst}\left(\int \frac{(a+b\arctan(cx))^2}{x^2}\,dx, x, x^3\right)$$

$$= -\frac{\left(a+b\arctan\left(cx^3\right)\right)^2}{3x^3} + \frac{1}{3}(2bc)\text{Subst}\left(\int \frac{a+b\arctan(cx)}{x\left(1+c^2x^2\right)}\,dx, x, x^3\right)$$

$$
\begin{aligned}
&= -\frac{1}{3} i c \left(a+b \arctan \left(c x^3\right)\right)^2-\frac{\left(a+b \arctan \left(c x^3\right)\right)^2}{3 x^3} \\
&\quad +\frac{1}{3}(2 i b c) \operatorname{Subst}\left(\int \frac{a+b \arctan (c x)}{x(i+c x)} d x, x, x^3\right) \\
&= -\frac{1}{3} i c \left(a+b \arctan \left(c x^3\right)\right)^2-\frac{\left(a+b \arctan \left(c x^3\right)\right)^2}{3 x^3} \\
&\quad +\frac{2}{3} b c \left(a+b \arctan \left(c x^3\right)\right) \log \left(2-\frac{2}{1-i c x^3}\right) \\
&\quad -\frac{1}{3}\left(2 b^2 c^2\right) \operatorname{Subst}\left(\int \frac{\log \left(2-\frac{2}{1-i c x}\right)}{1+c^2 x^2} d x, x, x^3\right) \\
&= -\frac{1}{3} i c \left(a+b \arctan \left(c x^3\right)\right)^2-\frac{\left(a+b \arctan \left(c x^3\right)\right)^2}{3 x^3} \\
&\quad +\frac{2}{3} b c \left(a+b \arctan \left(c x^3\right)\right) \log \left(2-\frac{2}{1-i c x^3}\right)-\frac{1}{3} i b^2 c \operatorname{PolyLog}\left(2,-1+\frac{2}{1-i c x^3}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.25

$$
\int \frac{\left(a+b \arctan \left(c x^3\right)\right)^2}{x^4} d x
= \frac{b^2\left(-1-i c x^3\right) \arctan \left(c x^3\right)^2+2 b \arctan \left(c x^3\right)\left(-a+b c x^3 \log \left(1-e^{2 i \arctan \left(c x^3\right)}\right)\right)-a\left(a-2 b c x^3 \log \left(c x^3\right)+\right.}{3 x^3}
$$

```
[In] Integrate[(a + b*ArcTan[c*x^3])^2/x^4,x]
```

```
[Out] (b^2*(-1 - I*c*x^3)*ArcTan[c*x^3]^2 + 2*b*ArcTan[c*x^3]*(-a + b*c*x^3*Log[1
 - E^((2*I)*ArcTan[c*x^3])]) - a*(a - 2*b*c*x^3*Log[c*x^3] + b*c*x^3*Log[1
+ c^2*x^6]) - I*b^2*c*x^3*PolyLog[2, E^((2*I)*ArcTan[c*x^3])])/(3*x^3)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.78 (sec) , antiderivative size = 11455, normalized size of antiderivative = 114.55

method	result	size
default	Expression too large to display	11455
parts	Expression too large to display	11455

```
[In] int((a+b*arctan(c*x^3))^2/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [F]

$$\int \frac{(a+b\arctan(cx^3))^2}{x^4}\,dx=\int \frac{(b\arctan(cx^3)+a)^2}{x^4}\,dx$$

```
[In] integrate((a+b*arctan(c*x^3))^2/x^4,x, algorithm="fricas")
[Out] integral((b^2*arctan(c*x^3)^2 + 2*a*b*arctan(c*x^3) + a^2)/x^4, x)
```

Sympy [F]

$$\int \frac{(a+b\arctan(cx^3))^2}{x^4}\,dx=\int \frac{(a+b\operatorname{atan}(cx^3))^2}{x^4}\,dx$$

```
[In] integrate((a+b*atan(c*x**3))**2/x**4,x)
[Out] Integral((a + b*atan(c*x**3))**2/x**4, x)
```

Maxima [F]

$$\int \frac{(a+b\arctan(cx^3))^2}{x^4}\,dx=\int \frac{(b\arctan(cx^3)+a)^2}{x^4}\,dx$$

```
[In] integrate((a+b*arctan(c*x^3))^2/x^4,x, algorithm="maxima")
[Out] -1/3*(c*(log(c^2*x^6 + 1) - log(x^6)) + 2*arctan(c*x^3)/x^3)*a*b + 1/48*(48
*x^3*integrate(-1/16*(4*c^2*x^6*log(c^2*x^6 + 1) - 8*c*x^3*arctan(c*x^3) -
12*(c^2*x^6 + 1)*arctan(c*x^3)^2 - (c^2*x^6 + 1)*log(c^2*x^6 + 1)^2)/(c^2*x
^10 + x^4), x) - 4*arctan(c*x^3)^2 + log(c^2*x^6 + 1)^2)*b^2/x^3 - 1/3*a^2/
x^3
```

Giac [F]

$$\int \frac{(a+b\arctan(cx^3))^2}{x^4}\,dx=\int \frac{(b\arctan(cx^3)+a)^2}{x^4}\,dx$$

```
[In] integrate((a+b*arctan(c*x^3))^2/x^4,x, algorithm="giac")
[Out] integrate((b*arctan(c*x^3) + a)^2/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b \arctan(cx^3))^2}{x^4} \, dx = \int \frac{(a + b \operatorname{atan}(c x^3))^2}{x^4} \, dx$$

```
[In] int((a + b*atan(c*x^3))^2/x^4,x)
[Out] int((a + b*atan(c*x^3))^2/x^4, x)
```

3.119 $\int \frac{(a+b\arctan(cx^3))^2}{x^7}\,dx$

Optimal result . . . 693
Rubi [A] (verified) . . . 693
Mathematica [A] (verified) . . . 695
Maple [A] (verified) . . . 696
Fricas [A] (verification not implemented) . . . 696
Sympy [B] (verification not implemented) . . . 696
Maxima [A] (verification not implemented) . . . 697
Giac [F] . . . 697
Mupad [B] (verification not implemented) . . . 698

Optimal result

Integrand size = 16, antiderivative size = 87

$$\int \frac{(a+b\arctan(cx^3))^2}{x^7}\,dx = -\frac{bc(a+b\arctan(cx^3))}{3x^3} - \frac{1}{6}c^2(a+b\arctan(cx^3))^2 - \frac{(a+b\arctan(cx^3))^2}{6x^6} + b^2c^2\log(x) - \frac{1}{6}b^2c^2\log(1+c^2x^6)$$

```
[Out] -1/3*b*c*(a+b*arctan(c*x^3))/x^3-1/6*c^2*(a+b*arctan(c*x^3))^2-1/6*(a+b*arc
tan(c*x^3))^2/x^6+b^2*c^2*ln(x)-1/6*b^2*c^2*ln(c^2*x^6+1)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4948, 4946, 5038, 272, 36, 29, 31, 5004}

$$\int \frac{(a+b\arctan(cx^3))^2}{x^7}\,dx = -\frac{1}{6}c^2(a+b\arctan(cx^3))^2 - \frac{bc(a+b\arctan(cx^3))}{3x^3} - \frac{(a+b\arctan(cx^3))^2}{6x^6} - \frac{1}{6}b^2c^2\log(c^2x^6+1) + b^2c^2\log(x)$$

```
[In] Int[(a + b*ArcTan[c*x^3])^2/x^7,x]
```

```
[Out] -1/3*(b*c*(a + b*ArcTan[c*x^3]))/x^3 - (c^2*(a + b*ArcTan[c*x^3])^2)/6 - (a
 + b*ArcTan[c*x^3])^2/(6*x^6) + b^2*c^2*Log[x] - (b^2*c^2*Log[1 + c^2*x^6])
/6
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplify
[(m + 1)/n]]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{(a+b\arctan(cx))^2}{x^3}\,dx, x, x^3\right) \\
&= -\frac{(a+b\arctan(cx^3))^2}{6x^6} + \frac{1}{3}(bc)\text{Subst}\left(\int \frac{a+b\arctan(cx)}{x^2(1+c^2x^2)}\,dx, x, x^3\right) \\
&= -\frac{(a+b\arctan(cx^3))^2}{6x^6} + \frac{1}{3}(bc)\text{Subst}\left(\int \frac{a+b\arctan(cx)}{x^2}\,dx, x, x^3\right) \\
&\quad - \frac{1}{3}(bc^3)\,\text{Subst}\left(\int \frac{a+b\arctan(cx)}{1+c^2x^2}\,dx, x, x^3\right) \\
&= -\frac{bc(a+b\arctan(cx^3))}{3x^3} - \frac{1}{6}c^2(a+b\arctan(cx^3))^2 \\
&\quad - \frac{(a+b\arctan(cx^3))^2}{6x^6} + \frac{1}{3}(b^2c^2)\,\text{Subst}\left(\int \frac{1}{x(1+c^2x^2)}\,dx, x, x^3\right) \\
&= -\frac{bc(a+b\arctan(cx^3))}{3x^3} - \frac{1}{6}c^2(a+b\arctan(cx^3))^2 \\
&\quad - \frac{(a+b\arctan(cx^3))^2}{6x^6} + \frac{1}{6}(b^2c^2)\,\text{Subst}\left(\int \frac{1}{x(1+c^2x)}\,dx, x, x^6\right) \\
&= -\frac{bc(a+b\arctan(cx^3))}{3x^3} - \frac{1}{6}c^2(a+b\arctan(cx^3))^2 - \frac{(a+b\arctan(cx^3))^2}{6x^6} \\
&\quad + \frac{1}{6}(b^2c^2)\,\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^6\right) - \frac{1}{6}(b^2c^4)\,\text{Subst}\left(\int \frac{1}{1+c^2x}\,dx, x, x^6\right) \\
&= -\frac{bc(a+b\arctan(cx^3))}{3x^3} - \frac{1}{6}c^2(a+b\arctan(cx^3))^2 \\
&\quad - \frac{(a+b\arctan(cx^3))^2}{6x^6} + b^2c^2\log(x) - \frac{1}{6}b^2c^2\log(1+c^2x^6)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.13

$$\int \frac{(a+b\arctan(cx^3))^2}{x^7}\,dx = -\frac{a^2+2abcx^3+2b(a+bcx^3+ac^2x^6)\arctan(cx^3)+b^2(1+c^2x^6)\arctan(cx^3)^2-6b^2c^2x^6\log(x)+b^2c^2x^6}{6x^6}$$

```
[In] Integrate[(a + b*ArcTan[c*x^3])^2/x^7,x]
```

```
[Out] -1/6*(a^2 + 2*a*b*c*x^3 + 2*b*(a + b*c*x^3 + a*c^2*x^6)*ArcTan[c*x^3] + b^2
*(1 + c^2*x^6)*ArcTan[c*x^3]^2 - 6*b^2*c^2*x^6*Log[x] + b^2*c^2*x^6*Log[1 +
 c^2*x^6])/x^6
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.36

method	result
default	$-\frac{a^2}{6x^6}-\frac{b^2\arctan(cx^3)^2}{6x^6}-\frac{b^2\arctan(cx^3)^2c^2}{6}-\frac{b^2c\arctan(cx^3)}{3x^3}+b^2c^2\ln(x)-\frac{b^2c^2\ln(c^2x^6+1)}{6}-\frac{ab\arctan(c x}{3x^6}$
parts	$-\frac{a^2}{6x^6}-\frac{b^2\arctan(cx^3)^2}{6x^6}-\frac{b^2\arctan(cx^3)^2c^2}{6}-\frac{b^2c\arctan(cx^3)}{3x^3}+b^2c^2\ln(x)-\frac{b^2c^2\ln(c^2x^6+1)}{6}-\frac{ab\arctan(c x}{3x^6}$
parallelrisch	$\frac{-b^2\arctan(cx^3)^2x^6c^2+6b^2c^2\ln(x)x^6-b^2c^2\ln(c^2x^6+1)x^6-2ab\arctan(cx^3)x^6c^2+a^2c^2x^6-2b^2\arctan(cx^3)x^3c-2abcx^3-b^2 a}{6x^6}$
risch	$\frac{b^2(c^2x^6+1)\ln(icx^3+1)^2}{24x^6}+\frac{ib(ibc^2x^6\ln(-icx^3+1)+2bcx^3+2a+ib\ln(-icx^3+1))\ln(icx^3+1)}{12x^6}-\frac{4i\ln((-7ibc+ac)x^3+7b+}{}$

[In] int((a+b*arctan(c*x^3))^2/x^7,x,method=_RETURNVERBOSE)

```
[Out] -1/6*a^2/x^6-1/6*b^2/x^6*arctan(c*x^3)^2-1/6*b^2*arctan(c*x^3)^2*c^2-1/3*b^
2*c*arctan(c*x^3)/x^3+b^2*c^2*ln(x)-1/6*b^2*c^2*ln(c^2*x^6+1)-1/3*a*b/x^6*a
rctan(c*x^3)-1/3*a*b*arctan(c*x^3)*c^2-1/3*a*b*c/x^3
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.17

$$\int \frac{(a+b\arctan(cx^3))^2}{x^7}\,dx = -\frac{b^2c^2x^6\log(c^2x^6+1)-6\,b^2c^2x^6\log(x)+2\,abcx^3+(b^2c^2x^6+b^2)\arctan(cx^3)^2+a^2+2\,(abc^2x^6+b^2cx^3 -}{6\,x^6}$$

[In] integrate((a+b*arctan(c*x^3))^2/x^7,x, algorithm="fricas")

```
[Out] -1/6*(b^2*c^2*x^6*log(c^2*x^6 + 1) - 6*b^2*c^2*x^6*log(x) + 2*a*b*c*x^3 + (
b^2*c^2*x^6 + b^2)*arctan(c*x^3)^2 + a^2 + 2*(a*b*c^2*x^6 + b^2*c*x^3 + a*b
)*arctan(c*x^3))/x^6
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 207 vs. 2(80) = 160.

Time = 71.42 (sec) , antiderivative size = 207, normalized size of antiderivative = 2.38

$$\int \frac{(a+b\arctan(cx^3))^2}{x^7}\,dx = \begin{cases} -\frac{a^2}{6x^6}-\frac{abc^2\operatorname{atan}(cx^3)}{3}-\frac{abc}{3x^3}-\frac{ab\operatorname{atan}(cx^3)}{3x^6}+\frac{b^2c^3\sqrt{-\frac{1}{c^2}}\operatorname{atan}(cx^3)}{3}+b^2c^2\log(x)-\frac{b^2c^2\log\left(x-\sqrt[6]{-\frac{1}{c^2}}\right)}{3}-\frac{b^2c^2\log\left(4\right.}{} \\ -\frac{a^2}{6x^6} \end{cases}$$

```
[In] integrate((a+b*atan(c*x**3))**2/x**7,x)
```

```
[Out] Piecewise((-a**2/(6*x**6) - a*b*c**2*atan(c*x**3)/3 - a*b*c/(3*x**3) - a*b*
atan(c*x**3)/(3*x**6) + b**2*c**3*sqrt(-1/c**2)*atan(c*x**3)/3 + b**2*c**2*
log(x) - b**2*c**2*log(x - (-1/c**2)**(1/6))/3 - b**2*c**2*log(4*x**2 + 4*x
*(-1/c**2)**(1/6) + 4*(-1/c**2)**(1/3))/3 - b**2*c**2*atan(c*x**3)**2/6 - b
**2*c*atan(c*x**3)/(3*x**3) - b**2*atan(c*x**3)**2/(6*x**6), Ne(c, 0)), (-a
**2/(6*x**6), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.26

$$\int \frac{(a+b\arctan(cx^3))^2}{x^7}\,dx = -\frac{1}{3}\left(\left(c\arctan(cx^3)+\frac{1}{x^3}\right)c+\frac{\arctan(cx^3)}{x^6}\right)ab + \frac{1}{6}\left(\left(\arctan(cx^3)^2-\log(c^2x^6+1)+6\log(x)\right)c^2-2\left(c\arctan(cx^3)+\frac{1}{x^3}\right)c\arctan(cx^3)\right)b^2 - \frac{b^2\arctan(cx^3)^2}{6x^6}-\frac{a^2}{6x^6}$$

```
[In] integrate((a+b*arctan(c*x^3))^2/x^7,x, algorithm="maxima")
```

```
[Out] -1/3*((c*arctan(c*x^3) + 1/x^3)*c + arctan(c*x^3)/x^6)*a*b + 1/6*((arctan(c
*x^3)^2 - log(c^2*x^6 + 1) + 6*log(x))*c^2 - 2*(c*arctan(c*x^3) + 1/x^3)*c*
arctan(c*x^3))*b^2 - 1/6*b^2*arctan(c*x^3)^2/x^6 - 1/6*a^2/x^6
```

Giac [F]

$$\int \frac{(a+b\arctan(cx^3))^2}{x^7}\,dx = \int \frac{(b\arctan(cx^3)+a)^2}{x^7}\,dx$$

```
[In] integrate((a+b*arctan(c*x^3))^2/x^7,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^3) + a)^2/x^7, x)
```

Mupad [B] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.75

$$\int \frac{(a+b\arctan(cx^3))^2}{x^7}\,dx = b^2\,c^2\,\ln(x) - \frac{b^2\,c^2\,\mathrm{atan}(c\,x^3)^2}{6} - \frac{b^2\,\mathrm{atan}(c\,x^3)^2}{6\,x^6} - \frac{b^2\,c^2\,\ln(c^2\,x^6+1)}{6} - \frac{a^2}{6\,x^6} - \frac{b^2\,c\,\mathrm{atan}(c\,x^3)}{3\,x^3} - \frac{a\,b\,c}{3\,x^3} - \frac{a\,b\,c^2\,\mathrm{atan}\left(\frac{a^2\,c\,x^3}{a^2+49\,b^2}+\frac{49\,b^2\,c\,x^3}{a^2+49\,b^2}\right)}{3} - \frac{a\,b\,\mathrm{atan}(c\,x^3)}{3\,x^6}$$

```
[In] int((a + b*atan(c*x^3))^2/x^7,x)
```

```
[Out] b^2*c^2*log(x) - (b^2*c^2*atan(c*x^3)^2)/6 - (b^2*atan(c*x^3)^2)/(6*x^6) -
(b^2*c^2*log(c^2*x^6 + 1))/6 - a^2/(6*x^6) - (b^2*c*atan(c*x^3))/(3*x^3) -
(a*b*c)/(3*x^3) - (a*b*c^2*atan((a^2*c*x^3)/(a^2 + 49*b^2) + (49*b^2*c*x^3)
/(a^2 + 49*b^2)))/3 - (a*b*atan(c*x^3))/(3*x^6)
```

3.120 $\int \frac{(a+b\arctan(cx^3))^2}{x^{10}} \, dx$

Optimal result 699
Rubi [A] (verified) 699
Mathematica [A] (verified) 702
Maple [C] (warning: unable to verify) 702
Fricas [F] 703
Sympy [F] 703
Maxima [F] 703
Giac [F] 704
Mupad [F(-1)] 704

Optimal result

Integrand size = 16, antiderivative size = 154

$$\begin{aligned}\int \frac{(a+b\arctan(cx^3))^2}{x^{10}}\,dx = & -\frac{b^2c^2}{9x^3}-\frac{1}{9}b^2c^3\arctan(cx^3)-\frac{bc(a+b\arctan(cx^3))}{9x^6} \\ & +\frac{1}{9}ic^3(a+b\arctan(cx^3))^2-\frac{(a+b\arctan(cx^3))^2}{9x^9} \\ & -\frac{2}{9}bc^3(a+b\arctan(cx^3))\log\left(2-\frac{2}{1-icx^3}\right) \\ & +\frac{1}{9}ib^2c^3\,\mathrm{PolyLog}\left(2,-1+\frac{2}{1-icx^3}\right)\end{aligned}$$

```
[Out] -1/9*b^2*c^2/x^3-1/9*b^2*c^3*arctan(c*x^3)-1/9*b*c*(a+b*arctan(c*x^3))/x^6+
1/9*I*c^3*(a+b*arctan(c*x^3))^2-1/9*(a+b*arctan(c*x^3))^2/x^9-2/9*b*c^3*(a+
b*arctan(c*x^3))*ln(2-2/(1-I*c*x^3))+1/9*I*b^2*c^3*polylog(2,-1+2/(1-I*c*x^
3))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used

$= \{4948, 4946, 5038, 331, 209, 5044, 4988, 2497\}$

$$\int \frac{(a+b\arctan(cx^3))^2}{x^{10}}\,dx = \frac{1}{9}ic^3(a+b\arctan(cx^3))^2 - \frac{2}{9}bc^3\log\left(2-\frac{2}{1-icx^3}\right)(a+b\arctan(cx^3)) - \frac{(a+b\arctan(cx^3))^2}{9x^9} - \frac{bc(a+b\arctan(cx^3))}{9x^6} - \frac{1}{9}b^2c^3\arctan(cx^3) + \frac{1}{9}ib^2c^3\,\mathrm{PolyLog}\left(2,\frac{2}{1-icx^3}-1\right) - \frac{b^2c^2}{9x^3}$$

```
[In] Int[(a + b*ArcTan[c*x^3])^2/x^10,x]
```

```
[Out] -1/9*(b^2*c^2)/x^3 - (b^2*c^3*ArcTan[c*x^3])/9 - (b*c*(a + b*ArcTan[c*x^3])
)/(9*x^6) + (I/9)*c^3*(a + b*ArcTan[c*x^3])^2 - (a + b*ArcTan[c*x^3])^2/(9*
x^9) - (2*b*c^3*(a + b*ArcTan[c*x^3])*Log[2 - 2/(1 - I*c*x^3)])/9 + (I/9)*b
^2*c^3*PolyLog[2, -1 + 2/(1 - I*c*x^3)]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplify
[(m + 1)/n]]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{(a+b\arctan(cx))^2}{x^4}\,dx, x, x^3\right)\\
&= -\frac{(a+b\arctan(cx^3))^2}{9x^9} + \frac{1}{9}(2bc)\text{Subst}\left(\int \frac{a+b\arctan(cx)}{x^3(1+c^2x^2)}\,dx, x, x^3\right)\\
&= -\frac{(a+b\arctan(cx^3))^2}{9x^9} + \frac{1}{9}(2bc)\text{Subst}\left(\int \frac{a+b\arctan(cx)}{x^3}\,dx, x, x^3\right)\\
&\quad - \frac{1}{9}(2bc^3)\,\text{Subst}\left(\int \frac{a+b\arctan(cx)}{x(1+c^2x^2)}\,dx, x, x^3\right)\\
&= -\frac{bc(a+b\arctan(cx^3))}{9x^6} + \frac{1}{9}ic^3(a+b\arctan(cx^3))^2\\
&\quad - \frac{(a+b\arctan(cx^3))^2}{9x^9} + \frac{1}{9}(b^2c^2)\,\text{Subst}\left(\int \frac{1}{x^2(1+c^2x^2)}\,dx, x, x^3\right)\\
&\quad - \frac{1}{9}(2ibc^3)\,\text{Subst}\left(\int \frac{a+b\arctan(cx)}{x(i+cx)}\,dx, x, x^3\right)
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{b^2c^2}{9x^3} - \frac{bc(a+b\arctan(cx^3))}{9x^6} + \frac{1}{9}ic^3(a+b\arctan(cx^3))^2 \\
&\quad - \frac{(a+b\arctan(cx^3))^2}{9x^9} - \frac{2}{9}bc^3(a+b\arctan(cx^3))\log\left(2-\frac{2}{1-icx^3}\right) \\
&\quad - \frac{1}{9}(b^2c^4)\,\text{Subst}\left(\int\frac{1}{1+c^2x^2}\,dx, x, x^3\right) \\
&\quad + \frac{1}{9}(2b^2c^4)\,\text{Subst}\left(\int\frac{\log\left(2-\frac{2}{1-icx}\right)}{1+c^2x^2}\,dx, x, x^3\right) \\
&= -\frac{b^2c^2}{9x^3} - \frac{1}{9}b^2c^3\arctan(cx^3) - \frac{bc(a+b\arctan(cx^3))}{9x^6} + \frac{1}{9}ic^3(a+b\arctan(cx^3))^2 \\
&\quad - \frac{(a+b\arctan(cx^3))^2}{9x^9} - \frac{2}{9}bc^3(a+b\arctan(cx^3))\log\left(2-\frac{2}{1-icx^3}\right) \\
&\quad + \frac{1}{9}ib^2c^3\,\text{PolyLog}\left(2,-1+\frac{2}{1-icx^3}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.08

$$
\int\frac{(a+b\arctan(cx^3))^2}{x^{10}}\,dx = -\frac{a^2+abcx^3+b^2c^2x^6+b^2(1-ic^3x^9)\arctan(cx^3)^2+b\arctan(cx^3)\left(2a+bcx^3+bc^3x^9+2bc^3x^9\log\left(1-\right.\right.}{9x^9}
$$

```
[In] Integrate[(a + b*ArcTan[c*x^3])^2/x^10,x]
```

```
[Out] -1/9*(a^2 + a*b*c*x^3 + b^2*c^2*x^6 + b^2*(1 - I*c^3*x^9)*ArcTan[c*x^3]^2 +
 b*ArcTan[c*x^3]*(2*a + b*c*x^3 + b*c^3*x^9 + 2*b*c^3*x^9*Log[1 - E^((2*I)*
ArcTan[c*x^3])]) + 2*a*b*c^3*x^9*Log[c*x^3] - a*b*c^3*x^9*Log[1 + c^2*x^6]
- I*b^2*c^3*x^9*PolyLog[2, E^((2*I)*ArcTan[c*x^3])])/x^9
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.34 (sec) , antiderivative size = 11496, normalized size of antiderivative = 74.65

method	result	size
default	Expression too large to display	11496
parts	Expression too large to display	11496

```
[In] int((a+b*arctan(c*x^3))^2/x^10,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [F]

$$\int \frac{(a + b\arctan(cx^3))^2}{x^{10}}\,dx = \int \frac{(b\arctan(cx^3) + a)^2}{x^{10}}\,dx$$

```
[In] integrate((a+b*arctan(c*x^3))^2/x^10,x, algorithm="fricas")
[Out] integral((b^2*arctan(c*x^3)^2 + 2*a*b*arctan(c*x^3) + a^2)/x^10, x)
```

Sympy [F]

$$\int \frac{(a + b\arctan(cx^3))^2}{x^{10}}\,dx = \int \frac{(a + b\,\mathrm{atan}(cx^3))^2}{x^{10}}\,dx$$

```
[In] integrate((a+b*atan(c*x**3))**2/x**10,x)
[Out] Integral((a + b*atan(c*x**3))**2/x**10, x)
```

Maxima [F]

$$\int \frac{(a + b\arctan(cx^3))^2}{x^{10}}\,dx = \int \frac{(b\arctan(cx^3) + a)^2}{x^{10}}\,dx$$

```
[In] integrate((a+b*arctan(c*x^3))^2/x^10,x, algorithm="maxima")
[Out] 1/9*((c^2*log(c^2*x^6 + 1) - c^2*log(x^6) - 1/x^6)*c - 2*arctan(c*x^3)/x^9)
*a*b + 1/144*(144*x^9*integrate(-1/48*(4*c^2*x^6*log(c^2*x^6 + 1) - 8*c*x^3
*arctan(c*x^3) - 36*(c^2*x^6 + 1)*arctan(c*x^3)^2 - 3*(c^2*x^6 + 1)*log(c^2
*x^6 + 1)^2)/(c^2*x^16 + x^10), x) - 4*arctan(c*x^3)^2 + log(c^2*x^6 + 1)^2
)*b^2/x^9 - 1/9*a^2/x^9
```

Giac [F]

$$\int \frac{(a+b\arctan(cx^3))^2}{x^{10}}\,dx = \int \frac{(b\arctan(cx^3)+a)^2}{x^{10}}\,dx$$

```
[In] integrate((a+b*arctan(c*x^3))^2/x^10,x, algorithm="giac")
[Out] integrate((b*arctan(c*x^3) + a)^2/x^10, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx^3))^2}{x^{10}}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x^3))^2}{x^{10}}\,dx$$

```
[In] int((a + b*atan(c*x^3))^2/x^10,x)
[Out] int((a + b*atan(c*x^3))^2/x^10, x)
```

3.121 $\int x^8(a+b\arctan(cx^3))^3\,dx$

Optimal result . . . 705
Rubi [A] (verified) . . . 706
Mathematica [A] (verified) . . . 709
Maple [F] . . . 710
Fricas [F] . . . 710
Sympy [F] . . . 710
Maxima [F] . . . 710
Giac [F] . . . 711
Mupad [F(-1)] . . . 711

Optimal result

Integrand size = 16, antiderivative size = 240

$$\int x^8(a+b\arctan(cx^3))^3\,dx=\frac{ab^2x^3}{3c^2}+\frac{b^3x^3\arctan(cx^3)}{3c^2}-\frac{b(a+b\arctan(cx^3))^2}{6c^3}-\frac{bx^6(a+b\arctan(cx^3))^2}{6c}-\frac{i(a+b\arctan(cx^3))^3}{9c^3}+\frac{1}{9}x^9(a+b\arctan(cx^3))^3-\frac{b(a+b\arctan(cx^3))^2\log\left(\frac{2}{1+icx^3}\right)}{3c^3}-\frac{b^3\log(1+c^2x^6)}{6c^3}-\frac{ib^2(a+b\arctan(cx^3))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx^3}\right)}{3c^3}-\frac{b^3\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx^3}\right)}{6c^3}$$

```
[Out] 1/3*a*b^2*x^3/c^2+1/3*b^3*x^3*arctan(c*x^3)/c^2-1/6*b*(a+b*arctan(c*x^3))^2
/c^3-1/6*b*x^6*(a+b*arctan(c*x^3))^2/c-1/9*I*(a+b*arctan(c*x^3))^3/c^3+1/9*
x^9*(a+b*arctan(c*x^3))^3-1/3*b*(a+b*arctan(c*x^3))^2*ln(2/(1+I*c*x^3))/c^3
-1/6*b^3*ln(c^2*x^6+1)/c^3-1/3*I*b^2*(a+b*arctan(c*x^3))*polylog(2,1-2/(1+I
*c*x^3))/c^3-1/6*b^3*polylog(3,1-2/(1+I*c*x^3))/c^3
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {4948, 4946, 5036, 4930, 266, 5004, 5040, 4964, 5114, 6745}

$$\int x^8\left(a+b\arctan\left(cx^3\right)\right)^3\,dx = -\frac{ib^2\operatorname{PolyLog}\left(2,1-\frac{2}{icx^3+1}\right)\left(a+b\arctan\left(cx^3\right)\right)}{3c^3} - \frac{i\left(a+b\arctan\left(cx^3\right)\right)^3}{9c^3} - \frac{b\left(a+b\arctan\left(cx^3\right)\right)^2}{6c^3} - \frac{b\log\left(\frac{2}{1+icx^3}\right)\left(a+b\arctan\left(cx^3\right)\right)^2}{3c^3} + \frac{1}{9}x^9\left(a+b\arctan\left(cx^3\right)\right)^3 - \frac{bx^6\left(a+b\arctan\left(cx^3\right)\right)^2}{6c} + \frac{ab^2x^3}{3c^2} + \frac{b^3x^3\arctan\left(cx^3\right)}{3c^2} - \frac{b^3\operatorname{PolyLog}\left(3,1-\frac{2}{icx^3+1}\right)}{6c^3} - \frac{b^3\log\left(c^2x^6+1\right)}{6c^3}$$

```
[In] Int[x^8*(a + b*ArcTan[c*x^3])^3,x]
```

```
[Out] (a*b^2*x^3)/(3*c^2) + (b^3*x^3*ArcTan[c*x^3])/(3*c^2) - (b*(a + b*ArcTan[c*
x^3])^2)/(6*c^3) - (b*x^6*(a + b*ArcTan[c*x^3])^2)/(6*c) - ((I/9)*(a + b*Ar
cTan[c*x^3])^3)/c^3 + (x^9*(a + b*ArcTan[c*x^3])^3)/9 - (b*(a + b*ArcTan[c*
x^3])^2*Log[2/(1 + I*c*x^3)])/(3*c^3) - (b^3*Log[1 + c^2*x^6])/(6*c^3) - ((
I/3)*b^2*(a + b*ArcTan[c*x^3])*PolyLog[2, 1 - 2/(1 + I*c*x^3)])/c^3 - (b^3*
PolyLog[3, 1 - 2/(1 + I*c*x^3)])/(6*c^3)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
```

```
IntegerQ[m])) && NeQ[m, -1]
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplify
[(m + 1)/n]]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int x^2(a+b\arctan(cx))^3\,dx,x,x^3\right)\\
&= \frac{1}{9}x^9\left(a+b\arctan\left(cx^3\right)\right)^3-\frac{1}{3}(bc)\text{Subst}\left(\int \frac{x^3(a+b\arctan(cx))^2}{1+c^2x^2}\,dx,x,x^3\right)\\
&= \frac{1}{9}x^9\left(a+b\arctan\left(cx^3\right)\right)^3-\frac{b\text{Subst}\left(\int x(a+b\arctan(cx))^2\,dx,x,x^3\right)}{3c}\\
&\quad+\frac{b\text{Subst}\left(\int \frac{x(a+b\arctan(cx))^2}{1+c^2x^2}\,dx,x,x^3\right)}{3c}\\
&= -\frac{bx^6\left(a+b\arctan\left(cx^3\right)\right)^2}{6c}-\frac{i\left(a+b\arctan\left(cx^3\right)\right)^3}{9c^3}+\frac{1}{9}x^9\left(a+b\arctan\left(cx^3\right)\right)^3\\
&\quad+\frac{1}{3}b^2\text{Subst}\left(\int \frac{x^2(a+b\arctan(cx))}{1+c^2x^2}\,dx,x,x^3\right)-\frac{b\text{Subst}\left(\int \frac{(a+b\arctan(cx))^2}{i-cx}\,dx,x,x^3\right)}{3c^2}\\
&= -\frac{bx^6\left(a+b\arctan\left(cx^3\right)\right)^2}{6c}-\frac{i\left(a+b\arctan\left(cx^3\right)\right)^3}{9c^3}+\frac{1}{9}x^9\left(a+b\arctan\left(cx^3\right)\right)^3\\
&\quad-\frac{b\left(a+b\arctan\left(cx^3\right)\right)^2\log\left(\frac{2}{1+icx^3}\right)}{3c^3}+\frac{b^2\text{Subst}\left(\int(a+b\arctan(cx))\,dx,x,x^3\right)}{3c^2}\\
&\quad-\frac{b^2\text{Subst}\left(\int \frac{a+b\arctan(cx)}{1+c^2x^2}\,dx,x,x^3\right)}{3c^2}+\frac{\left(2b^2\right)\text{Subst}\left(\int \frac{(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx,x,x^3\right)}{3c^2}\\
&= \frac{ab^2x^3}{3c^2}-\frac{b\left(a+b\arctan\left(cx^3\right)\right)^2}{6c^3}-\frac{bx^6\left(a+b\arctan\left(cx^3\right)\right)^2}{6c}-\frac{i\left(a+b\arctan\left(cx^3\right)\right)^3}{9c^3}\\
&\quad+\frac{1}{9}x^9\left(a+b\arctan\left(cx^3\right)\right)^3-\frac{b\left(a+b\arctan\left(cx^3\right)\right)^2\log\left(\frac{2}{1+icx^3}\right)}{3c^3}\\
&\quad-\frac{ib^2\left(a+b\arctan\left(cx^3\right)\right)\text{PolyLog}\left(2,1-\frac{2}{1+icx^3}\right)}{3c^3}\\
&\quad+\frac{\left(ib^3\right)\text{Subst}\left(\int \frac{\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx,x,x^3\right)}{3c^2}+\frac{b^3\text{Subst}\left(\int \arctan(cx)\,dx,x,x^3\right)}{3c^2}
\end{aligned}
$$

$$= \frac{ab^2x^3}{3c^2} + \frac{b^3x^3 \arctan(cx^3)}{3c^2} - \frac{b(a + b\arctan(cx^3))^2}{6c^3} - \frac{bx^6(a + b\arctan(cx^3))^2}{6c} - \frac{i(a + b\arctan(cx^3))^3}{9c^3} + \frac{1}{9}x^9(a + b\arctan(cx^3))^3 - \frac{b(a + b\arctan(cx^3))^2 \log\left(\frac{2}{1+icx^3}\right)}{3c^3} - \frac{ib^2(a + b\arctan(cx^3))\operatorname{PolyLog}\left(2, 1 - \frac{2}{1+icx^3}\right)}{3c^3} - \frac{b^3 \operatorname{PolyLog}\left(3, 1 - \frac{2}{1+icx^3}\right)}{6c^3} - \frac{b^3 \operatorname{Subst}\left(\int \frac{x}{1+c^2x^2}\,dx, x, x^3\right)}{3c}$$

$$= \frac{ab^2x^3}{3c^2} + \frac{b^3x^3 \arctan(cx^3)}{3c^2} - \frac{b(a + b\arctan(cx^3))^2}{6c^3} - \frac{bx^6(a + b\arctan(cx^3))^2}{6c} - \frac{i(a + b\arctan(cx^3))^3}{9c^3} + \frac{1}{9}x^9(a + b\arctan(cx^3))^3 - \frac{b(a + b\arctan(cx^3))^2 \log\left(\frac{2}{1+icx^3}\right)}{3c^3} - \frac{b^3 \log(1 + c^2x^6)}{6c^3} - \frac{ib^2(a + b\arctan(cx^3))\operatorname{PolyLog}\left(2, 1 - \frac{2}{1+icx^3}\right)}{3c^3} - \frac{b^3 \operatorname{PolyLog}\left(3, 1 - \frac{2}{1+icx^3}\right)}{6c^3}$$

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.44

$$\int x^8 (a + b\arctan(cx^3))^3 \, dx$$

$$= \frac{6ab^2cx^3 - 3a^2bc^2x^6 + 2a^3c^3x^9 - 6ab^2\arctan(cx^3) + 6b^3cx^3\arctan(cx^3) - 6ab^2c^2x^6\arctan(cx^3) + 6a^2bc^3}{}$$

```
[In] Integrate[x^8*(a + b*ArcTan[c*x^3])^3,x]
```

```
[Out] (6*a*b^2*c*x^3 - 3*a^2*b*c^2*x^6 + 2*a^3*c^3*x^9 - 6*a*b^2*ArcTan[c*x^3] +
6*b^3*c*x^3*ArcTan[c*x^3] - 6*a*b^2*c^2*x^6*ArcTan[c*x^3] + 6*a^2*b*c^3*x^9
*ArcTan[c*x^3] + (6*I)*a*b^2*ArcTan[c*x^3]^2 - 3*b^3*ArcTan[c*x^3]^2 - 3*b^
3*c^2*x^6*ArcTan[c*x^3]^2 + 6*a*b^2*c^3*x^9*ArcTan[c*x^3]^2 + (2*I)*b^3*Arc
Tan[c*x^3]^3 + 2*b^3*c^3*x^9*ArcTan[c*x^3]^3 - 12*a*b^2*ArcTan[c*x^3]*Log[1
 + E^((2*I)*ArcTan[c*x^3])] - 6*b^3*ArcTan[c*x^3]^2*Log[1 + E^((2*I)*ArcTan
[c*x^3])] + 3*a^2*b*Log[1 + c^2*x^6] - 3*b^3*Log[1 + c^2*x^6] + (6*I)*b^2*(
a + b*ArcTan[c*x^3])*PolyLog[2, -E^((2*I)*ArcTan[c*x^3])] - 3*b^3*PolyLog[3
, -E^((2*I)*ArcTan[c*x^3])])/(18*c^3)
```

Maple [F]

$$\int x^{8}\left(a+b\arctan\left(c\,x^{3}\right)\right)^{3}dx$$

[In] int(x^8*(a+b*arctan(c*x^3))^3,x)

[Out] int(x^8*(a+b*arctan(c*x^3))^3,x)

Fricas [F]

$$\int x^{8}\left(a+b\arctan\left(cx^{3}\right)\right)^{3}\,dx=\int\left(b\arctan\left(cx^{3}\right)+a\right)^{3}x^{8}\,dx$$

[In] integrate(x^8*(a+b*arctan(c*x^3))^3,x, algorithm="fricas")

[Out] integral(b^3*x^8*arctan(c*x^3)^3 + 3*a*b^2*x^8*arctan(c*x^3)^2 + 3*a^2*b*x^8*arctan(c*x^3) + a^3*x^8, x)

Sympy [F]

$$\int x^{8}\left(a+b\arctan\left(cx^{3}\right)\right)^{3}\,dx=\int x^{8}\left(a+b\operatorname{atan}\left(cx^{3}\right)\right)^{3}\,dx$$

[In] integrate(x**8*(a+b*atan(c*x**3))**3,x)

[Out] Integral(x**8*(a + b*atan(c*x**3))**3, x)

Maxima [F]

$$\int x^{8}\left(a+b\arctan\left(cx^{3}\right)\right)^{3}\,dx=\int\left(b\arctan\left(cx^{3}\right)+a\right)^{3}x^{8}\,dx$$

[In] integrate(x^8*(a+b*arctan(c*x^3))^3,x, algorithm="maxima")

[Out] 1/72*b^3*x^9*arctan(c*x^3)^3 - 1/96*b^3*x^9*arctan(c*x^3)*log(c^2*x^6 + 1)^2 + 1/9*a^3*x^9 + 1/6*(2*x^9*arctan(c*x^3) - (x^6/c^2 - log(c^2*x^6 + 1)/c^4)*c)*a^2*b + integrate(1/32*(4*b^3*c^2*x^14*arctan(c*x^3)*log(c^2*x^6 + 1) + 28*(b^3*c^2*x^14 + b^3*x^8)*arctan(c*x^3)^3 + 4*(24*a*b^2*c^2*x^14 - b^3*c*x^11 + 24*a*b^2*x^8)*arctan(c*x^3)^2 + (b^3*c*x^11 + 3*(b^3*c^2*x^14 + b^3*x^8)*arctan(c*x^3))*log(c^2*x^6 + 1)^2)/(c^2*x^6 + 1), x)

Giac [F]

$$\int x^8\left(a+b\arctan\left(cx^3\right)\right)^3\,dx=\int\left(b\arctan\left(cx^3\right)+a\right)^3x^8\,dx$$

```
[In] integrate(x^8*(a+b*arctan(c*x^3))^3,x, algorithm="giac")
[Out] integrate((b*arctan(c*x^3) + a)^3*x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^8\left(a+b\arctan\left(cx^3\right)\right)^3\,dx=\int x^8\,\left(a+b\,\mathrm{atan}\left(c\,x^3\right)\right)^3\,dx$$

```
[In] int(x^8*(a + b*atan(c*x^3))^3,x)
[Out] int(x^8*(a + b*atan(c*x^3))^3, x)
```

3.122 $\int x^5(a+b\arctan(cx^3))^3\,dx$

Optimal result 712
Rubi [A] (verified) 712
Mathematica [A] (verified) 715
Maple [C] (warning: unable to verify) 716
Fricas [F] 716
Sympy [F] 717
Maxima [F] 717
Giac [F] 717
Mupad [F(-1)] 717

Optimal result

Integrand size = 16, antiderivative size = 147

$$\int x^5(a+b\arctan(cx^3))^3\,dx=-\frac{ib(a+b\arctan(cx^3))^2}{2c^2}-\frac{bx^3(a+b\arctan(cx^3))^2}{2c}+\frac{(a+b\arctan(cx^3))^3}{6c^2}+\frac{1}{6}x^6(a+b\arctan(cx^3))^3-\frac{b^2(a+b\arctan(cx^3))\log\left(\frac{2}{1+icx^3}\right)}{c^2}-\frac{ib^3\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx^3}\right)}{2c^2}$$

```
[Out] -1/2*I*b*(a+b*arctan(c*x^3))^2/c^2-1/2*b*x^3*(a+b*arctan(c*x^3))^2/c+1/6*(a
+b*arctan(c*x^3))^3/c^2+1/6*x^6*(a+b*arctan(c*x^3))^3-b^2*(a+b*arctan(c*x^3
))*ln(2/(1+I*c*x^3))/c^2-1/2*I*b^3*polylog(2,1-2/(1+I*c*x^3))/c^2
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}=0.562$, Rules used = {4948, 4946, 5036, 4930, 5040, 4964, 2449, 2352, 5004}

$$\int x^5(a+b\arctan(cx^3))^3\,dx=-\frac{b^2\log\left(\frac{2}{1+icx^3}\right)(a+b\arctan(cx^3))}{c^2}+\frac{(a+b\arctan(cx^3))^3}{6c^2}-\frac{ib(a+b\arctan(cx^3))^2}{2c^2}-\frac{bx^3(a+b\arctan(cx^3))^2}{2c}+\frac{1}{6}x^6(a+b\arctan(cx^3))^3-\frac{ib^3\operatorname{PolyLog}\left(2,1-\frac{2}{icx^3+1}\right)}{2c^2}$$

```
[In] Int[x^5*(a + b*ArcTan[c*x^3])^3,x]
```

```
[Out] ((-1/2*I)*b*(a + b*ArcTan[c*x^3])^2)/c^2 - (b*x^3*(a + b*ArcTan[c*x^3])^2)/
(2*c) + (a + b*ArcTan[c*x^3])^3/(6*c^2) + (x^6*(a + b*ArcTan[c*x^3])^3)/6 -
 (b^2*(a + b*ArcTan[c*x^3])*Log[2/(1 + I*c*x^3)])/c^2 - ((I/2)*b^3*PolyLog[
2, 1 - 2/(1 + I*c*x^3)])/c^2
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplify
[(m + 1)/n]]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int x(a+b\arctan(cx))^3\,dx,x,x^3\right)\\
&= \frac{1}{6}x^6\left(a+b\arctan\left(cx^3\right)\right)^3-\frac{1}{2}(bc)\text{Subst}\left(\int\frac{x^2(a+b\arctan(cx))^2}{1+c^2x^2}\,dx,x,x^3\right)\\
&= \frac{1}{6}x^6\left(a+b\arctan\left(cx^3\right)\right)^3-\frac{b\text{Subst}\left(\int(a+b\arctan(cx))^2\,dx,x,x^3\right)}{2c}\\
&\quad+\frac{b\text{Subst}\left(\int\frac{(a+b\arctan(cx))^2}{1+c^2x^2}\,dx,x,x^3\right)}{2c}\\
&= -\frac{bx^3\left(a+b\arctan\left(cx^3\right)\right)^2}{2c}+\frac{\left(a+b\arctan\left(cx^3\right)\right)^3}{6c^2}\\
&\quad+\frac{1}{6}x^6\left(a+b\arctan\left(cx^3\right)\right)^3+b^2\text{Subst}\left(\int\frac{x(a+b\arctan(cx))}{1+c^2x^2}\,dx,x,x^3\right)\\
&= -\frac{ib\left(a+b\arctan\left(cx^3\right)\right)^2}{2c^2}-\frac{bx^3\left(a+b\arctan\left(cx^3\right)\right)^2}{2c}+\frac{\left(a+b\arctan\left(cx^3\right)\right)^3}{6c^2}\\
&\quad+\frac{1}{6}x^6\left(a+b\arctan\left(cx^3\right)\right)^3-\frac{b^2\text{Subst}\left(\int\frac{a+b\arctan(cx)}{i-cx}\,dx,x,x^3\right)}{c}
\end{aligned}
$$

$$= -\frac{ib(a+b\arctan(cx^3))^2}{2c^2} - \frac{bx^3(a+b\arctan(cx^3))^2}{2c} + \frac{(a+b\arctan(cx^3))^3}{6c^2} + \frac{1}{6}x^6(a+b\arctan(cx^3))^3 - \frac{b^2(a+b\arctan(cx^3))\log\left(\frac{2}{1+icx^3}\right)}{c^2} + \frac{b^3\text{Subst}\left(\int \frac{\log\left(\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx, x, x^3\right)}{c}$$

$$= -\frac{ib(a+b\arctan(cx^3))^2}{2c^2} - \frac{bx^3(a+b\arctan(cx^3))^2}{2c} + \frac{(a+b\arctan(cx^3))^3}{6c^2} + \frac{1}{6}x^6(a+b\arctan(cx^3))^3 - \frac{b^2(a+b\arctan(cx^3))\log\left(\frac{2}{1+icx^3}\right)}{c^2} - \frac{(ib^3)\,\text{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1+icx^3}\right)}{c^2}$$

$$= -\frac{ib(a+b\arctan(cx^3))^2}{2c^2} - \frac{bx^3(a+b\arctan(cx^3))^2}{2c} + \frac{(a+b\arctan(cx^3))^3}{6c^2} + \frac{1}{6}x^6(a+b\arctan(cx^3))^3 - \frac{b^2(a+b\arctan(cx^3))\log\left(\frac{2}{1+icx^3}\right)}{c^2} - \frac{ib^3\,\text{PolyLog}\left(2, 1-\frac{2}{1+icx^3}\right)}{2c^2}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.16

$$\int x^5(a+b\arctan(cx^3))^3\,dx$$

$$= \frac{3b^2(a+ac^2x^6+b(i-cx^3))\arctan(cx^3)^2 + b^3(1+c^2x^6)\arctan(cx^3)^3 + 3b\arctan(cx^3)\left(a(a-2bcx^3+a\right.}{}$$

```
[In] Integrate[x^5*(a + b*ArcTan[c*x^3])^3,x]
```

```
[Out] (3*b^2*(a + a*c^2*x^6 + b*(I - c*x^3))*ArcTan[c*x^3]^2 + b^3*(1 + c^2*x^6)*
ArcTan[c*x^3]^3 + 3*b*ArcTan[c*x^3]*(a*(a - 2*b*c*x^3 + a*c^2*x^6) - 2*b^2*
Log[1 + E^((2*I)*ArcTan[c*x^3])]) + a*(a*c*x^3*(-3*b + a*c*x^3) + 3*b^2*Log
[1 + c^2*x^6]) + (3*I)*b^3*PolyLog[2, -E^((2*I)*ArcTan[c*x^3])])/(6*c^2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 7.57 (sec) , antiderivative size = 935, normalized size of antiderivative = 6.36

method	result	size
risch	Expression too large to display	935
default	Expression too large to display	11515
parts	Expression too large to display	11515

```
[In] int(x^5*(a+b*arctan(c*x^3))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*b^2*(I*b*c^2*x^6*ln(1-I*c*x^3)+2*a*c^2*x^6-2*b*c*x^3+I*b*ln(1-I*c*x^3
)+2*I*b+2*a)/c^2*ln(1+I*c*x^3)^2+1/2*a*b^2/c^2*ln(c^2*x^6+1)+1/2*a^2*b/c^2*
arctan(c*x^3)-1/48*I*b^3*x^6*ln(1-I*c*x^3)^3+1/8*I/c^2*b^3*ln(c^2*x^6+1)+1/
6*a^3*x^6+1/8*I/c^2*b^3*ln(1-I*c*x^3)^2-1/2/c*a^2*b*x^3+1/8*b^3/c*x^3*ln(1-
I*c*x^3)^2+3/4*I/c*b^2*Sum(2/3*(ln(x-_alpha)*ln(1-I*c*x^3)+3*c*(-1/3*ln(x-_
alpha)*(ln((RootOf(_Z^2+_Z*RootOf(c*_Z^3-I)+RootOf(c*_Z^3-I)^2,index=1)-x+_
alpha)/RootOf(_Z^2+_Z*RootOf(c*_Z^3-I)+RootOf(c*_Z^3-I)^2,index=1))+ln((Roo
tOf(_Z^2+_Z*RootOf(c*_Z^3-I)+RootOf(c*_Z^3-I)^2,index=2)-x+_alpha)/RootOf(_
Z^2+_Z*RootOf(c*_Z^3-I)+RootOf(c*_Z^3-I)^2,index=2))+ln(1/2*(2*(I/c)^(1/3)+
x-_alpha)/(I/c)^(1/3)))/c-1/3*(dilog((RootOf(_Z^2+_Z*RootOf(c*_Z^3-I)+RootO
f(c*_Z^3-I)^2,index=1)-x+_alpha)/RootOf(_Z^2+_Z*RootOf(c*_Z^3-I)+RootOf(c*_
Z^3-I)^2,index=1))+dilog((RootOf(_Z^2+_Z*RootOf(c*_Z^3-I)+RootOf(c*_Z^3-I)^
2,index=2)-x+_alpha)/RootOf(_Z^2+_Z*RootOf(c*_Z^3-I)+RootOf(c*_Z^3-I)^2,ind
ex=2))+dilog(1/2*(2*(I/c)^(1/3)+x-_alpha)/(I/c)^(1/3)))/c))*b/c,_alpha=Root
Of(c*_Z^3-RootOf(_Z^2+1,index=1)))+1/4*I*b*a^2*x^6*ln(1-I*c*x^3)-1/8*a*b^2*
x^6*ln(1-I*c*x^3)^2-1/8/c^2*a*b^2*ln(1-I*c*x^3)^2+1/48*I*b^3*(c^2*x^6+1)/c^
2*ln(1+I*c*x^3)^3-1/2*I/c*a*b^2*x^3*ln(1-I*c*x^3)-1/4/c^2*b^3*arctan(c*x^3)
-1/48*I/c^2*b^3*ln(1-I*c*x^3)^3+(1/16*I*b^3*(c^2*x^6+1)/c^2*ln(1-I*c*x^3)^2
+1/16*b^2*(2*a*c*x^3-b)^2/c^2/a*ln(1-I*c*x^3)-1/16*b*(4*I*a^3*c^2*x^6-8*I*a
^2*b*c*x^3+4*I*ln(1-I*c*x^3)*a*b^2+4*I*a*b^2-4*ln(1-I*c*x^3)*a^2*b+ln(1-I*c
*x^3)*b^3)/a/c^2)*ln(1+I*c*x^3)
```

Fricas [F]

$$\int x^5 \left(a+b\arctan\left(cx^3\right)\right)^3\,dx=\int \left(b\arctan\left(cx^3\right)+a\right)^3x^5\,dx$$

```
[In] integrate(x^5*(a+b*arctan(c*x^3))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^5*arctan(c*x^3)^3 + 3*a*b^2*x^5*arctan(c*x^3)^2 + 3*a^2*b*x^
5*arctan(c*x^3) + a^3*x^5, x)
```

Sympy [F]

$$\int x^5 \left(a + b \arctan \left(cx^3\right)\right)^3 \, dx = \int x^5 \left(a + b \operatorname{atan} \left(cx^3\right)\right)^3 \, dx$$

```
[In] integrate(x**5*(a+b*atan(c*x**3))**3,x)
[Out] Integral(x**5*(a + b*atan(c*x**3))**3, x)
```

Maxima [F]

$$\int x^5 \left(a + b \arctan \left(cx^3\right)\right)^3 \, dx = \int \left(b \arctan \left(cx^3\right) + a\right)^3 x^5 \, dx$$

```
[In] integrate(x^5*(a+b*arctan(c*x^3))^3,x, algorithm="maxima")
[Out] 1/2*a*b^2*x^6*arctan(c*x^3)^2 + 1/6*a^3*x^6 + 1/2*(x^6*arctan(c*x^3) - c*(x
^3/c^2 - arctan(c*x^3)/c^3))*a^2*b - 1/2*(2*c*(x^3/c^2 - arctan(c*x^3)/c^3)
*arctan(c*x^3) + (arctan(c*x^3)^2 - log(6*c^5*x^6 + 6*c^3))/c^2)*a*b^2 + 1/
192*(4*x^6*arctan(c*x^3)^3 - 3*x^6*arctan(c*x^3)*log(c^2*x^6 + 1)^2 + 192*i
ntegrate(1/64*(12*c^2*x^11*arctan(c*x^3)*log(c^2*x^6 + 1) - 12*c*x^8*arctan
(c*x^3)^2 + 56*(c^2*x^11 + x^5)*arctan(c*x^3)^3 + 3*(c*x^8 + 2*(c^2*x^11 +
x^5)*arctan(c*x^3))*log(c^2*x^6 + 1)^2)/(c^2*x^6 + 1), x))*b^3
```

Giac [F]

$$\int x^5 \left(a + b \arctan \left(cx^3\right)\right)^3 \, dx = \int \left(b \arctan \left(cx^3\right) + a\right)^3 x^5 \, dx$$

```
[In] integrate(x^5*(a+b*arctan(c*x^3))^3,x, algorithm="giac")
[Out] integrate((b*arctan(c*x^3) + a)^3*x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^5 \left(a + b \arctan \left(cx^3\right)\right)^3 \, dx = \int x^5 \left(a + b \operatorname{atan}\left(c x^3\right)\right)^3 \, dx$$

```
[In] int(x^5*(a + b*atan(c*x^3))^3,x)
[Out] int(x^5*(a + b*atan(c*x^3))^3, x)
```

3.123 $\int x^2(a+b\arctan(cx^3))^3\,dx$

Optimal result . . . 718
Rubi [A] (verified) . . . 718
Mathematica [A] (verified) . . . 721
Maple [B] (verified) . . . 721
Fricas [F] . . . 722
Sympy [F] . . . 722
Maxima [F] . . . 722
Giac [F] . . . 723
Mupad [F(-1)] . . . 723

Optimal result

Integrand size = 16, antiderivative size = 139

$$\int x^2(a+b\arctan(cx^3))^3\,dx = \frac{i(a+b\arctan(cx^3))^3}{3c}+\frac{1}{3}x^3(a+b\arctan(cx^3))^3 + \frac{b(a+b\arctan(cx^3))^2\log\left(\frac{2}{1+icx^3}\right)}{c} + \frac{ib^2(a+b\arctan(cx^3))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx^3}\right)}{c} + \frac{b^3\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx^3}\right)}{2c}$$

```
[Out] 1/3*I*(a+b*arctan(c*x^3))^3/c+1/3*x^3*(a+b*arctan(c*x^3))^3+b*(a+b*arctan(c
*x^3))^2*ln(2/(1+I*c*x^3))/c+I*b^2*(a+b*arctan(c*x^3))*polylog(2,1-2/(1+I*c
*x^3))/c+1/2*b^3*polylog(3,1-2/(1+I*c*x^3))/c
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {4948, 4930, 5040, 4964, 5004, 5114, 6745}

$$\int x^2(a+b\arctan(cx^3))^3\,dx = \frac{ib^2\operatorname{PolyLog}\left(2,1-\frac{2}{icx^3+1}\right)(a+b\arctan(cx^3))}{c} + \frac{1}{3}x^3(a+b\arctan(cx^3))^3+\frac{i(a+b\arctan(cx^3))^3}{3c} + \frac{b\log\left(\frac{2}{1+icx^3}\right)(a+b\arctan(cx^3))^2}{c} + \frac{b^3\operatorname{PolyLog}\left(3,1-\frac{2}{icx^3+1}\right)}{2c}$$

```
[In] Int[x^2*(a + b*ArcTan[c*x^3])^3,x]
```

```
[Out] ((I/3)*(a + b*ArcTan[c*x^3])^3)/c + (x^3*(a + b*ArcTan[c*x^3])^3)/3 + (b*(a
 + b*ArcTan[c*x^3])^2*Log[2/(1 + I*c*x^3)])/c + (I*b^2*(a + b*ArcTan[c*x^3]
)*PolyLog[2, 1 - 2/(1 + I*c*x^3)])/c + (b^3*PolyLog[3, 1 - 2/(1 + I*c*x^3)]
)/(2*c)
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplify
[(m + 1)/n]]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
```

```
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int (a+b\arctan(cx))^3\,dx, x, x^3\right)\\
&= \frac{1}{3}x^3\left(a+b\arctan\left(cx^3\right)\right)^3-(bc)\text{Subst}\left(\int \frac{x(a+b\arctan(cx))^2}{1+c^2x^2}\,dx, x, x^3\right)\\
&= \frac{i\left(a+b\arctan\left(cx^3\right)\right)^3}{3c}+\frac{1}{3}x^3\left(a+b\arctan\left(cx^3\right)\right)^3+b\text{Subst}\left(\int \frac{(a+b\arctan(cx))^2}{i-cx}\,dx, x, x^3\right)\\
&= \frac{i\left(a+b\arctan\left(cx^3\right)\right)^3}{3c}+\frac{1}{3}x^3\left(a+b\arctan\left(cx^3\right)\right)^3\\
&\quad+\frac{b\left(a+b\arctan\left(cx^3\right)\right)^2\log\left(\frac{2}{1+icx^3}\right)}{c}\\
&\quad-\left(2b^2\right)\text{Subst}\left(\int \frac{(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx, x, x^3\right)\\
&= \frac{i\left(a+b\arctan\left(cx^3\right)\right)^3}{3c}+\frac{1}{3}x^3\left(a+b\arctan\left(cx^3\right)\right)^3\\
&\quad+\frac{b\left(a+b\arctan\left(cx^3\right)\right)^2\log\left(\frac{2}{1+icx^3}\right)}{c}\\
&\quad+\frac{ib^2\left(a+b\arctan\left(cx^3\right)\right)\text{PolyLog}\left(2,1-\frac{2}{1+icx^3}\right)}{c}\\
&\quad-\left(ib^3\right)\text{Subst}\left(\int \frac{\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx, x, x^3\right)\\
&= \frac{i\left(a+b\arctan\left(cx^3\right)\right)^3}{3c}+\frac{1}{3}x^3\left(a+b\arctan\left(cx^3\right)\right)^3+\frac{b\left(a+b\arctan\left(cx^3\right)\right)^2\log\left(\frac{2}{1+icx^3}\right)}{c}\\
&\quad+\frac{ib^2\left(a+b\arctan\left(cx^3\right)\right)\text{PolyLog}\left(2,1-\frac{2}{1+icx^3}\right)}{c}+\frac{b^3\,\text{PolyLog}\left(3,1-\frac{2}{1+icx^3}\right)}{2c}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.61

$$\int x^2 (a + b \arctan(cx^3))^3 \, dx$$
$$= \frac{2a^3cx^3 + 6a^2bcx^3 \arctan(cx^3) - 6iab^2 \arctan(cx^3)^2 + 6ab^2cx^3 \arctan(cx^3)^2 - 2ib^3 \arctan(cx^3)^3 + 2b^3cx^3}{}$$

```
[In] Integrate[x^2*(a + b*ArcTan[c*x^3])^3,x]
```

```
[Out] (2*a^3*c*x^3 + 6*a^2*b*c*x^3*ArcTan[c*x^3] - (6*I)*a*b^2*ArcTan[c*x^3]^2 +
6*a*b^2*c*x^3*ArcTan[c*x^3]^2 - (2*I)*b^3*ArcTan[c*x^3]^3 + 2*b^3*c*x^3*Arc
Tan[c*x^3]^3 + 12*a*b^2*ArcTan[c*x^3]*Log[1 + E^((2*I)*ArcTan[c*x^3])] + 6*
b^3*ArcTan[c*x^3]^2*Log[1 + E^((2*I)*ArcTan[c*x^3])] - 3*a^2*b*Log[1 + c^2*
x^6] - (6*I)*b^2*(a + b*ArcTan[c*x^3])*PolyLog[2, -E^((2*I)*ArcTan[c*x^3])]
 + 3*b^3*PolyLog[3, -E^((2*I)*ArcTan[c*x^3])])/(6*c)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 274 vs. 2(128) = 256.

Time = 9.01 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.98

method	result
derivativedivides	$\frac{a^3cx^3+b^3\left(\arctan(cx^3)^3(cx^3+i)-2i\arctan(cx^3)^3+3\arctan(cx^3)^2\ln\left(1+\frac{(icx^3+1)^2}{c^2x^6+1}\right)-3i\arctan(cx^3)\operatorname{polylog}\left(2,\right.\right.}{}$
default	$\frac{a^3cx^3+b^3\left(\arctan(cx^3)^3(cx^3+i)-2i\arctan(cx^3)^3+3\arctan(cx^3)^2\ln\left(1+\frac{(icx^3+1)^2}{c^2x^6+1}\right)-3i\arctan(cx^3)\operatorname{polylog}\left(2,\right.\right.}{}$
parts	$\frac{a^3x^3}{3}+\frac{b^3\left(\arctan(cx^3)^3(cx^3+i)-2i\arctan(cx^3)^3+3\arctan(cx^3)^2\ln\left(1+\frac{(icx^3+1)^2}{c^2x^6+1}\right)-3i\arctan(cx^3)\operatorname{polylog}\left(2\right.\right.}{3c}$

```
[In] int(x^2*(a+b*arctan(c*x^3))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/c*(a^3*c*x^3+b^3*(arctan(c*x^3)^3*(c*x^3+I)-2*I*arctan(c*x^3)^3+3*arcta
n(c*x^3)^2*ln(1+(1+I*c*x^3)^2/(c^2*x^6+1))-3*I*arctan(c*x^3)*polylog(2,-(1+
I*c*x^3)^2/(c^2*x^6+1))+3/2*polylog(3,-(1+I*c*x^3)^2/(c^2*x^6+1)))+3*a*b^2*
```

```
(arctan(c*x^3)^2*(c*x^3+I)+2*arctan(c*x^3)*ln(1+(1+I*c*x^3)^2/(c^2*x^6+1))-
2*I*arctan(c*x^3)^2-I*polylog(2,-(1+I*c*x^3)^2/(c^2*x^6+1)))+3*a^2*b*(c*x^3
*arctan(c*x^3)-1/2*ln(c^2*x^6+1)))
```

Fricas [F]

$$\int x^2(a+b\arctan(cx^3))^3\,dx=\int(b\arctan(cx^3)+a)^3x^2\,dx$$

```
[In] integrate(x^2*(a+b*arctan(c*x^3))^3,x, algorithm="fricas")
[Out] integral(b^3*x^2*arctan(c*x^3)^3 + 3*a*b^2*x^2*arctan(c*x^3)^2 + 3*a^2*b*x^
2*arctan(c*x^3) + a^3*x^2, x)
```

Sympy [F]

$$\int x^2(a+b\arctan(cx^3))^3\,dx=\int x^2(a+b\,\mathrm{atan}(cx^3))^3\,dx$$

```
[In] integrate(x**2*(a+b*atan(c*x**3))**3,x)
[Out] Integral(x**2*(a + b*atan(c*x**3))**3, x)
```

Maxima [F]

$$\int x^2(a+b\arctan(cx^3))^3\,dx=\int(b\arctan(cx^3)+a)^3x^2\,dx$$

```
[In] integrate(x^2*(a+b*arctan(c*x^3))^3,x, algorithm="maxima")
[Out] 1/24*b^3*x^3*arctan(c*x^3)^3 - 1/32*b^3*x^3*arctan(c*x^3)*log(c^2*x^6 + 1)^
2 + 1/3*a^3*x^3 + 7/96*b^3*arctan(c*x^3)^4/c + 28*b^3*c^2*integrate(1/32*x^
8*arctan(c*x^3)^3/(c^2*x^6 + 1), x) + 3*b^3*c^2*integrate(1/32*x^8*arctan(c
*x^3)*log(c^2*x^6 + 1)^2/(c^2*x^6 + 1), x) + 96*a*b^2*c^2*integrate(1/32*x^
8*arctan(c*x^3)^2/(c^2*x^6 + 1), x) + 12*b^3*c^2*integrate(1/32*x^8*arctan(
c*x^3)*log(c^2*x^6 + 1)/(c^2*x^6 + 1), x) + 1/3*a*b^2*arctan(c*x^3)^3/c - 1
2*b^3*c*integrate(1/32*x^5*arctan(c*x^3)^2/(c^2*x^6 + 1), x) + 3*b^3*c*inte
grate(1/32*x^5*log(c^2*x^6 + 1)^2/(c^2*x^6 + 1), x) + 3*b^3*integrate(1/32*
x^2*arctan(c*x^3)*log(c^2*x^6 + 1)^2/(c^2*x^6 + 1), x) + 1/2*(2*c*x^3*arcta
n(c*x^3) - log(c^2*x^6 + 1))*a^2*b/c
```

Giac [F]

$$\int x^2(a+b\arctan(cx^3))^3\,dx=\int(b\arctan(cx^3)+a)^3x^2\,dx$$

```
[In] integrate(x^2*(a+b*arctan(c*x^3))^3,x, algorithm="giac")
[Out] integrate((b*arctan(c*x^3) + a)^3*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(a+b\arctan(cx^3))^3\,dx=\int x^2\,(a+b\,\mathrm{atan}(c\,x^3))^3\,dx$$

```
[In] int(x^2*(a + b*atan(c*x^3))^3,x)
[Out] int(x^2*(a + b*atan(c*x^3))^3, x)
```

3.124 $\int \frac{(a+b\arctan(cx^3))^3}{x}\,dx$

Optimal result . . . 724
Rubi [A] (verified) . . . 725
Mathematica [A] (verified) . . . 729
Maple [F] . . . 730
Fricas [F] . . . 730
Sympy [F] . . . 730
Maxima [F] . . . 730
Giac [F] . . . 731
Mupad [F(-1)] . . . 731

Optimal result

Integrand size = 16, antiderivative size = 232

$$\int \frac{(a+b\arctan(cx^3))^3}{x}\,dx = \frac{2}{3}(a+b\arctan(cx^3))^3 \operatorname{arctanh}\left(1-\frac{2}{1+icx^3}\right) - \frac{1}{2}ib(a+b\arctan(cx^3))^2 \operatorname{PolyLog}\left(2,1-\frac{2}{1+icx^3}\right) + \frac{1}{2}ib(a+b\arctan(cx^3))^2 \operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx^3}\right) - \frac{1}{2}b^2(a+b\arctan(cx^3)) \operatorname{PolyLog}\left(3,1-\frac{2}{1+icx^3}\right) + \frac{1}{2}b^2(a+b\arctan(cx^3)) \operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx^3}\right) + \frac{1}{4}ib^3 \operatorname{PolyLog}\left(4,1-\frac{2}{1+icx^3}\right) - \frac{1}{4}ib^3 \operatorname{PolyLog}\left(4,-1+\frac{2}{1+icx^3}\right)$$

```
[Out] -2/3*(a+b*arctan(c*x^3))^3*arctanh(-1+2/(1+I*c*x^3))-1/2*I*b*(a+b*arctan(c*
x^3))^2*polylog(2,1-2/(1+I*c*x^3))+1/2*I*b*(a+b*arctan(c*x^3))^2*polylog(2,
-1+2/(1+I*c*x^3))-1/2*b^2*(a+b*arctan(c*x^3))*polylog(3,1-2/(1+I*c*x^3))+1/
2*b^2*(a+b*arctan(c*x^3))*polylog(3,-1+2/(1+I*c*x^3))+1/4*I*b^3*polylog(4,1
-2/(1+I*c*x^3))-1/4*I*b^3*polylog(4,-1+2/(1+I*c*x^3))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {4944, 4942, 5108, 5004, 5114, 5118, 6745}

$$\begin{aligned}\int \frac{(a+b\arctan(cx^3))^3}{x}\,dx = &\frac{2}{3}\text{arctanh}\left(1-\frac{2}{1+icx^3}\right)(a+b\arctan(cx^3))^3\\&-\frac{1}{2}b^2\,\text{PolyLog}\left(3,1-\frac{2}{icx^3+1}\right)(a+b\arctan(cx^3))\\&+\frac{1}{2}b^2\,\text{PolyLog}\left(3,\frac{2}{icx^3+1}-1\right)(a+b\arctan(cx^3))\\&-\frac{1}{2}ib\,\text{PolyLog}\left(2,1-\frac{2}{icx^3+1}\right)(a+b\arctan(cx^3))^2\\&+\frac{1}{2}ib\,\text{PolyLog}\left(2,\frac{2}{icx^3+1}-1\right)(a+b\arctan(cx^3))^2\\&+\frac{1}{4}ib^3\,\text{PolyLog}\left(4,1-\frac{2}{icx^3+1}\right)\\&-\frac{1}{4}ib^3\,\text{PolyLog}\left(4,\frac{2}{icx^3+1}-1\right)\end{aligned}$$

```
[In] Int[(a + b*ArcTan[c*x^3])^3/x,x]
```

```
[Out] (2*(a + b*ArcTan[c*x^3])^3*ArcTanh[1 - 2/(1 + I*c*x^3)])/3 - (I/2)*b*(a + b
*ArcTan[c*x^3])^2*PolyLog[2, 1 - 2/(1 + I*c*x^3)] + (I/2)*b*(a + b*ArcTan[c
*x^3])^2*PolyLog[2, -1 + 2/(1 + I*c*x^3)] - (b^2*(a + b*ArcTan[c*x^3])*Poly
Log[3, 1 - 2/(1 + I*c*x^3)])/2 + (b^2*(a + b*ArcTan[c*x^3])*PolyLog[3, -1 +
 2/(1 + I*c*x^3)])/2 + (I/4)*b^3*PolyLog[4, 1 - 2/(1 + I*c*x^3)] - (I/4)*b^
3*PolyLog[4, -1 + 2/(1 + I*c*x^3)]
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1
/n, Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[p, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5108

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e*
x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5118

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k + 1,
 u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{(a+b\arctan(cx))^3}{x}\,dx, x, x^3\right)\\ &= \frac{2}{3}\left(a+b\arctan\left(cx^3\right)\right)^3\text{arctanh}\left(1-\frac{2}{1+icx^3}\right)\\ &\quad-(2bc)\text{Subst}\left(\int \frac{(a+b\arctan(cx))^2\text{arctanh}\left(1-\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx, x, x^3\right)\end{aligned}$$

$$
\begin{aligned}
&= \frac{2}{3}\left(a+b\arctan\left(cx^3\right)\right)^3 \operatorname{arctanh}\left(1-\frac{2}{1+icx^3}\right) \\
&\quad + (bc)\text{Subst}\left(\int \frac{(a+b\arctan(cx))^2 \log\left(\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx, x, x^3\right) \\
&\quad - (bc)\text{Subst}\left(\int \frac{(a+b\arctan(cx))^2 \log\left(2-\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx, x, x^3\right) \\
&= \frac{2}{3}\left(a+b\arctan\left(cx^3\right)\right)^3 \operatorname{arctanh}\left(1-\frac{2}{1+icx^3}\right) \\
&\quad - \frac{1}{2}ib\left(a+b\arctan\left(cx^3\right)\right)^2 \text{PolyLog}\left(2,1-\frac{2}{1+icx^3}\right) \\
&\quad + \frac{1}{2}ib\left(a+b\arctan\left(cx^3\right)\right)^2 \text{PolyLog}\left(2,-1+\frac{2}{1+icx^3}\right) \\
&\quad + \left(ib^2c\right)\text{Subst}\left(\int \frac{(a+b\arctan(cx))\,\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx, x, x^3\right) \\
&\quad - \left(ib^2c\right)\text{Subst}\left(\int \frac{(a+b\arctan(cx))\,\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx, x, x^3\right) \\
&= \frac{2}{3}\left(a+b\arctan\left(cx^3\right)\right)^3 \operatorname{arctanh}\left(1-\frac{2}{1+icx^3}\right) \\
&\quad - \frac{1}{2}ib\left(a+b\arctan\left(cx^3\right)\right)^2 \text{PolyLog}\left(2,1-\frac{2}{1+icx^3}\right) \\
&\quad + \frac{1}{2}ib\left(a+b\arctan\left(cx^3\right)\right)^2 \text{PolyLog}\left(2,-1+\frac{2}{1+icx^3}\right) \\
&\quad - \frac{1}{2}b^2\left(a+b\arctan\left(cx^3\right)\right) \text{PolyLog}\left(3,1-\frac{2}{1+icx^3}\right) \\
&\quad + \frac{1}{2}b^2\left(a+b\arctan\left(cx^3\right)\right) \text{PolyLog}\left(3,-1+\frac{2}{1+icx^3}\right) \\
&\quad + \frac{1}{2}\left(b^3c\right)\text{Subst}\left(\int \frac{\text{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx, x, x^3\right) \\
&\quad - \frac{1}{2}\left(b^3c\right)\text{Subst}\left(\int \frac{\text{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx, x, x^3\right)
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2}{3}\left(a+b\arctan\left(cx^3\right)\right)^3 \operatorname{arctanh}\left(1-\frac{2}{1+icx^3}\right) \\
&\quad -\frac{1}{2} ib\left(a+b\arctan\left(cx^3\right)\right)^2 \operatorname{PolyLog}\left(2,1-\frac{2}{1+icx^3}\right) \\
&\quad +\frac{1}{2} ib\left(a+b\arctan\left(cx^3\right)\right)^2 \operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx^3}\right) \\
&\quad -\frac{1}{2} b^2\left(a+b\arctan\left(cx^3\right)\right) \operatorname{PolyLog}\left(3,1-\frac{2}{1+icx^3}\right) \\
&\quad +\frac{1}{2} b^2\left(a+b\arctan\left(cx^3\right)\right) \operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx^3}\right) \\
&\quad +\frac{1}{4} ib^3 \operatorname{PolyLog}\left(4,1-\frac{2}{1+icx^3}\right)-\frac{1}{4} ib^3 \operatorname{PolyLog}\left(4,-1+\frac{2}{1+icx^3}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.81

$$\int \frac{(a+b\arctan(cx^3))^3}{x}\,dx = a^3\log(x)+\frac{1}{2}ia^2b\left(\text{PolyLog}\left(2,-icx^3\right)-\text{PolyLog}\left(2,icx^3\right)\right) + ab^2\left(-\frac{i\pi^3}{24}+\frac{2}{3}i\arctan\left(cx^3\right)^3 + \arctan\left(cx^3\right)^2\log\left(1-e^{-2i\arctan(cx^3)}\right) - \arctan\left(cx^3\right)^2\log\left(1+e^{2i\arctan(cx^3)}\right) + i\arctan\left(cx^3\right)\text{PolyLog}\left(2,e^{-2i\arctan(cx^3)}\right) + i\arctan\left(cx^3\right)\text{PolyLog}\left(2,-e^{2i\arctan(cx^3)}\right) + \frac{1}{2}\text{PolyLog}\left(3,e^{-2i\arctan(cx^3)}\right) - \frac{1}{2}\text{PolyLog}\left(3,-e^{2i\arctan(cx^3)}\right)\right) - \frac{1}{192}ib^3\left(\pi^4 - 32\arctan\left(cx^3\right)^4+64i\arctan\left(cx^3\right)^3\log\left(1-e^{-2i\arctan(cx^3)}\right) - 64i\arctan\left(cx^3\right)^3\log\left(1+e^{2i\arctan(cx^3)}\right) - 96\arctan\left(cx^3\right)^2\text{PolyLog}\left(2,e^{-2i\arctan(cx^3)}\right) - 96\arctan\left(cx^3\right)^2\text{PolyLog}\left(2,-e^{2i\arctan(cx^3)}\right) + 96i\arctan\left(cx^3\right)\text{PolyLog}\left(3,e^{-2i\arctan(cx^3)}\right) - 96i\arctan\left(cx^3\right)\text{PolyLog}\left(3,-e^{2i\arctan(cx^3)}\right) + 48\,\text{PolyLog}\left(4,e^{-2i\arctan(cx^3)}\right) + 48\,\text{PolyLog}\left(4,-e^{2i\arctan(cx^3)}\right)\right)$$

```
[In] Integrate[(a + b*ArcTan[c*x^3])^3/x,x]
```

```
[Out] a^3*Log[x] + (I/2)*a^2*b*(PolyLog[2, (-I)*c*x^3] - PolyLog[2, I*c*x^3]) + a
*b^2*((-1/24*I)*Pi^3 + ((2*I)/3)*ArcTan[c*x^3]^3 + ArcTan[c*x^3]^2*Log[1 -
E^((-2*I)*ArcTan[c*x^3])] - ArcTan[c*x^3]^2*Log[1 + E^((2*I)*ArcTan[c*x^3])
] + I*ArcTan[c*x^3]*PolyLog[2, E^((-2*I)*ArcTan[c*x^3])] + I*ArcTan[c*x^3]*
PolyLog[2, -E^((2*I)*ArcTan[c*x^3])] + PolyLog[3, E^((-2*I)*ArcTan[c*x^3])]
/2 - PolyLog[3, -E^((2*I)*ArcTan[c*x^3])]/2) - (I/192)*b^3*(Pi^4 - 32*ArcTa
n[c*x^3]^4 + (64*I)*ArcTan[c*x^3]^3*Log[1 - E^((-2*I)*ArcTan[c*x^3])] - (64
*I)*ArcTan[c*x^3]^3*Log[1 + E^((2*I)*ArcTan[c*x^3])] - 96*ArcTan[c*x^3]^2*P
olyLog[2, E^((-2*I)*ArcTan[c*x^3])] - 96*ArcTan[c*x^3]^2*PolyLog[2, -E^((2*
```

```
I)*ArcTan[c*x^3])] + (96*I)*ArcTan[c*x^3]*PolyLog[3, E^((-2*I)*ArcTan[c*x^3
])] - (96*I)*ArcTan[c*x^3]*PolyLog[3, -E^((2*I)*ArcTan[c*x^3])] + 48*PolyLo
g[4, E^((-2*I)*ArcTan[c*x^3])] + 48*PolyLog[4, -E^((2*I)*ArcTan[c*x^3])])
```

Maple [F]

$$\int \frac{(a+b\arctan(c\,x^3))^3}{x}dx$$

[In] int((a+b*arctan(c*x^3))^3/x,x)

[Out] int((a+b*arctan(c*x^3))^3/x,x)

Fricas [F]

$$\int \frac{(a+b\arctan(cx^3))^3}{x}\,dx = \int \frac{(b\arctan(cx^3)+a)^3}{x}\,dx$$

[In] integrate((a+b*arctan(c*x^3))^3/x,x, algorithm="fricas")

[Out] integral((b^3*arctan(c*x^3)^3 + 3*a*b^2*arctan(c*x^3)^2 + 3*a^2*b*arctan(c*
x^3) + a^3)/x, x)

Sympy [F]

$$\int \frac{(a+b\arctan(cx^3))^3}{x}\,dx = \int \frac{(a+b\,\mathrm{atan}(cx^3))^3}{x}\,dx$$

[In] integrate((a+b*atan(c*x**3))**3/x,x)

[Out] Integral((a + b*atan(c*x**3))**3/x, x)

Maxima [F]

$$\int \frac{(a+b\arctan(cx^3))^3}{x}\,dx = \int \frac{(b\arctan(cx^3)+a)^3}{x}\,dx$$

[In] integrate((a+b*arctan(c*x^3))^3/x,x, algorithm="maxima")

[Out] a^3*log(x) + 1/32*integrate((28*b^3*arctan(c*x^3)^3 + 3*b^3*arctan(c*x^3)*l
og(c^2*x^6 + 1)^2 + 96*a*b^2*arctan(c*x^3)^2 + 96*a^2*b*arctan(c*x^3))/x, x
)

Giac [F]

$$\int \frac{(a+b\arctan(cx^3))^3}{x}\,dx = \int \frac{(b\arctan(cx^3)+a)^3}{x}\,dx$$

```
[In] integrate((a+b*arctan(c*x^3))^3/x,x, algorithm="giac")
[Out] integrate((b*arctan(c*x^3) + a)^3/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx^3))^3}{x}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x^3))^3}{x}\,dx$$

```
[In] int((a + b*atan(c*x^3))^3/x,x)
[Out] int((a + b*atan(c*x^3))^3/x, x)
```

3.125 $\int \frac{(a+b\arctan(cx^3))^3}{x^4}\,dx$

Optimal result 732
Rubi [A] (verified) 733
Mathematica [A] (verified) 735
Maple [F] 736
Fricas [F] 736
Sympy [F] 736
Maxima [F] 736
Giac [F] 737
Mupad [F(-1)] 737

Optimal result

Integrand size = 16, antiderivative size = 133

$$\int \frac{(a+b\arctan(cx^3))^3}{x^4}\,dx = -\frac{1}{3}ic(a+b\arctan(cx^3))^3-\frac{(a+b\arctan(cx^3))^3}{3x^3} + bc(a+b\arctan(cx^3))^2\log\left(2-\frac{2}{1-icx^3}\right) - ib^2c(a+b\arctan(cx^3))\operatorname{PolyLog}\left(2,-1+\frac{2}{1-icx^3}\right) + \frac{1}{2}b^3c\operatorname{PolyLog}\left(3,-1+\frac{2}{1-icx^3}\right)$$

```
[Out] -1/3*I*c*(a+b*arctan(c*x^3))^3-1/3*(a+b*arctan(c*x^3))^3/x^3+b*c*(a+b*arcta
n(c*x^3))^2*ln(2-2/(1-I*c*x^3))-I*b^2*c*(a+b*arctan(c*x^3))*polylog(2,-1+2/
(1-I*c*x^3))+1/2*b^3*c*polylog(3,-1+2/(1-I*c*x^3))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {4948, 4946, 5044, 4988, 5004, 5112, 6745}

$$\int \frac{(a+b\arctan(cx^3))^3}{x^4}\,dx = -ib^2c\,\text{PolyLog}\left(2,\frac{2}{1-icx^3}-1\right)(a+b\arctan(cx^3)) - \frac{1}{3}ic\left(a+b\arctan(cx^3)\right)^3 - \frac{(a+b\arctan(cx^3))^3}{3x^3} + bc\log\left(2-\frac{2}{1-icx^3}\right)\left(a+b\arctan(cx^3)\right)^2 + \frac{1}{2}b^3c\,\text{PolyLog}\left(3,\frac{2}{1-icx^3}-1\right)$$

```
[In] Int[(a + b*ArcTan[c*x^3])^3/x^4,x]
```

```
[Out] (-1/3*I)*c*(a + b*ArcTan[c*x^3])^3 - (a + b*ArcTan[c*x^3])^3/(3*x^3) + b*c*
(a + b*ArcTan[c*x^3])^2*Log[2 - 2/(1 - I*c*x^3)] - I*b^2*c*(a + b*ArcTan[c*
x^3])*PolyLog[2, -1 + 2/(1 - I*c*x^3)] + (b^3*c*PolyLog[3, -1 + 2/(1 - I*c*
x^3)])/2
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplify
[(m + 1)/n]]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5112

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{(a+b\arctan(cx))^3}{x^2}\,dx, x, x^3\right)\\
&= -\frac{\left(a+b\arctan\left(cx^3\right)\right)^3}{3x^3} + (bc)\text{Subst}\left(\int \frac{(a+b\arctan(cx))^2}{x\left(1+c^2x^2\right)}\,dx, x, x^3\right)\\
&= -\frac{1}{3}ic\left(a+b\arctan\left(cx^3\right)\right)^3 - \frac{\left(a+b\arctan\left(cx^3\right)\right)^3}{3x^3}\\
&\quad + (ibc)\text{Subst}\left(\int \frac{(a+b\arctan(cx))^2}{x(i+cx)}\,dx, x, x^3\right)\\
&= -\frac{1}{3}ic\left(a+b\arctan\left(cx^3\right)\right)^3 - \frac{\left(a+b\arctan\left(cx^3\right)\right)^3}{3x^3}\\
&\quad + bc\left(a+b\arctan\left(cx^3\right)\right)^2\log\left(2-\frac{2}{1-icx^3}\right)\\
&\quad - \left(2b^2c^2\right)\text{Subst}\left(\int \frac{(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right)}{1+c^2x^2}\,dx, x, x^3\right)
\end{aligned}$$

$$= -\frac{1}{3}ic\left(a+b\arctan\left(cx^3\right)\right)^3 - \frac{\left(a+b\arctan\left(cx^3\right)\right)^3}{3x^3} + bc\left(a+b\arctan\left(cx^3\right)\right)^2\log\left(2-\frac{2}{1-icx^3}\right) - ib^2c\left(a+b\arctan\left(cx^3\right)\right)\text{PolyLog}\left(2,-1+\frac{2}{1-icx^3}\right) + \left(ib^3c^2\right)\text{Subst}\left(\int\frac{\text{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)}{1+c^2x^2}dx,x,x^3\right)$$

$$= -\frac{1}{3}ic\left(a+b\arctan\left(cx^3\right)\right)^3 - \frac{\left(a+b\arctan\left(cx^3\right)\right)^3}{3x^3} + bc\left(a+b\arctan\left(cx^3\right)\right)^2\log\left(2-\frac{2}{1-icx^3}\right) - ib^2c\left(a+b\arctan\left(cx^3\right)\right)\text{PolyLog}\left(2,-1+\frac{2}{1-icx^3}\right) + \frac{1}{2}b^3c\,\text{PolyLog}\left(3,-1+\frac{2}{1-icx^3}\right)$$

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.80

$$\int\frac{\left(a+b\arctan\left(cx^3\right)\right)^3}{x^4}dx = -\frac{a^3}{3x^3} - \frac{a^2b\arctan\left(cx^3\right)}{x^3} + 3a^2bc\log(x) - \frac{1}{2}a^2bc\log\left(1+c^2x^6\right) + ab^2c\left(\arctan\left(cx^3\right)\left(\left(-i-\frac{1}{cx^3}\right)\arctan\left(cx^3\right) + 2\log\left(1-e^{2i\arctan\left(cx^3\right)}\right)\right) - i\,\text{PolyLog}\left(2,e^{2i\arctan\left(cx^3\right)}\right)\right) + \frac{1}{3}b^3c\left(-\frac{i\pi^3}{8} + i\arctan\left(cx^3\right)^3 - \frac{\arctan\left(cx^3\right)^3}{cx^3} + 3\arctan\left(cx^3\right)^2\log\left(1-e^{-2i\arctan\left(cx^3\right)}\right) + 3i\arctan\left(cx^3\right)\text{PolyLog}\left(2,e^{-2i\arctan\left(cx^3\right)}\right) + \frac{3}{2}\text{PolyLog}\left(3,e^{-2i\arctan\left(cx^3\right)}\right)\right)$$

```
[In] Integrate[(a + b*ArcTan[c*x^3])^3/x^4,x]

[Out] -1/3*a^3/x^3 - (a^2*b*ArcTan[c*x^3])/x^3 + 3*a^2*b*c*Log[x] - (a^2*b*c*Log[
1 + c^2*x^6])/2 + a*b^2*c*(ArcTan[c*x^3]*((-I - 1/(c*x^3))*ArcTan[c*x^3] +
2*Log[1 - E^((2*I)*ArcTan[c*x^3])]) - I*PolyLog[2, E^((2*I)*ArcTan[c*x^3])]
) + (b^3*c*((-1/8*I)*Pi^3 + I*ArcTan[c*x^3]^3 - ArcTan[c*x^3]^3/(c*x^3) + 3
```

```
*ArcTan[c*x^3]^2*Log[1 - E^((-2*I)*ArcTan[c*x^3])] + (3*I)*ArcTan[c*x^3]*Po
lyLog[2, E^((-2*I)*ArcTan[c*x^3])] + (3*PolyLog[3, E^((-2*I)*ArcTan[c*x^3])
])/2))/3
```

Maple [F]

$$\int \frac{(a+b\arctan(c\,x^3))^3}{x^4}dx$$

```
[In] int((a+b*arctan(c*x^3))^3/x^4,x)
[Out] int((a+b*arctan(c*x^3))^3/x^4,x)
```

Fricas [F]

$$\int \frac{(a+b\arctan(cx^3))^3}{x^4}\,dx = \int \frac{(b\arctan(cx^3)+a)^3}{x^4}\,dx$$

```
[In] integrate((a+b*arctan(c*x^3))^3/x^4,x, algorithm="fricas")
[Out] integral((b^3*arctan(c*x^3)^3 + 3*a*b^2*arctan(c*x^3)^2 + 3*a^2*b*arctan(c*
x^3) + a^3)/x^4, x)
```

Sympy [F]

$$\int \frac{(a+b\arctan(cx^3))^3}{x^4}\,dx = \int \frac{(a+b\operatorname{atan}(cx^3))^3}{x^4}\,dx$$

```
[In] integrate((a+b*atan(c*x**3))**3/x**4,x)
[Out] Integral((a + b*atan(c*x**3))**3/x**4, x)
```

Maxima [F]

$$\int \frac{(a+b\arctan(cx^3))^3}{x^4}\,dx = \int \frac{(b\arctan(cx^3)+a)^3}{x^4}\,dx$$

```
[In] integrate((a+b*arctan(c*x^3))^3/x^4,x, algorithm="maxima")
[Out] -1/2*(c*(log(c^2*x^6 + 1) - log(x^6)) + 2*arctan(c*x^3)/x^3)*a^2*b - 1/3*a^
3/x^3 - 1/96*(4*b^3*arctan(c*x^3)^3 - 3*b^3*arctan(c*x^3)*log(c^2*x^6 + 1)^
2 - 96*x^3*integrate(-1/32*(12*b^3*c^2*x^6*arctan(c*x^3)*log(c^2*x^6 + 1) -
 28*(b^3*c^2*x^6 + b^3)*arctan(c*x^3)^3 - 12*(8*a*b^2*c^2*x^6 + b^3*c*x^3 +
 8*a*b^2)*arctan(c*x^3)^2 + 3*(b^3*c*x^3 - (b^3*c^2*x^6 + b^3)*arctan(c*x^3
))*log(c^2*x^6 + 1)^2)/(c^2*x^10 + x^4), x))/x^3
```

Giac [F]

$$\int \frac{(a+b\arctan(cx^3))^3}{x^4}\,dx = \int \frac{(b\arctan(cx^3)+a)^3}{x^4}\,dx$$

```
[In] integrate((a+b*arctan(c*x^3))^3/x^4,x, algorithm="giac")
[Out] integrate((b*arctan(c*x^3) + a)^3/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx^3))^3}{x^4}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x^3))^3}{x^4}\,dx$$

```
[In] int((a + b*atan(c*x^3))^3/x^4,x)
[Out] int((a + b*atan(c*x^3))^3/x^4, x)
```

3.126 $\int \frac{(a+b\arctan(cx^3))^3}{x^7}\,dx$

Optimal result 738
Rubi [A] (verified) 738
Mathematica [A] (verified) 741
Maple [C] (warning: unable to verify) 741
Fricas [F] 741
Sympy [F] 742
Maxima [F] 742
Giac [F] 742
Mupad [F(-1)] 742

Optimal result

Integrand size = 16, antiderivative size = 146

$$\begin{aligned}\int \frac{(a+b\arctan(cx^3))^3}{x^7}\,dx &= -\frac{1}{2}ibc^2\left(a+b\arctan\left(cx^3\right)\right)^2-\frac{bc(a+b\arctan(cx^3))^2}{2x^3}\\ &\quad -\frac{1}{6}c^2\left(a+b\arctan\left(cx^3\right)\right)^3-\frac{(a+b\arctan(cx^3))^3}{6x^6}\\ &\quad +b^2c^2\left(a+b\arctan\left(cx^3\right)\right)\log\left(2-\frac{2}{1-icx^3}\right)\\ &\quad -\frac{1}{2}ib^3c^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1-icx^3}\right)\end{aligned}$$

```
[Out] -1/2*I*b*c^2*(a+b*arctan(c*x^3))^2-1/2*b*c*(a+b*arctan(c*x^3))^2/x^3-1/6*c^
2*(a+b*arctan(c*x^3))^3-1/6*(a+b*arctan(c*x^3))^3/x^6+b^2*c^2*(a+b*arctan(c
*x^3))*ln(2-2/(1-I*c*x^3))-1/2*I*b^3*c^2*polylog(2,-1+2/(1-I*c*x^3))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {4948, 4946, 5038, 5044, 4988, 2497, 5004}

$$\begin{aligned}\int \frac{(a+b\arctan(cx^3))^3}{x^7}\,dx &= b^2c^2\log\left(2-\frac{2}{1-icx^3}\right)\left(a+b\arctan\left(cx^3\right)\right)\\ &\quad -\frac{1}{2}ibc^2\left(a+b\arctan\left(cx^3\right)\right)^2-\frac{1}{6}c^2\left(a+b\arctan\left(cx^3\right)\right)^3\\ &\quad -\frac{bc(a+b\arctan(cx^3))^2}{2x^3}-\frac{(a+b\arctan(cx^3))^3}{6x^6}\\ &\quad -\frac{1}{2}ib^3c^2\operatorname{PolyLog}\left(2,\frac{2}{1-icx^3}-1\right)\end{aligned}$$

```
[In] Int[(a + b*ArcTan[c*x^3])^3/x^7,x]
```

```
[Out] (-1/2*I)*b*c^2*(a + b*ArcTan[c*x^3])^2 - (b*c*(a + b*ArcTan[c*x^3])^2)/(2*x
^3) - (c^2*(a + b*ArcTan[c*x^3])^3)/6 - (a + b*ArcTan[c*x^3])^3/(6*x^6) + b
^2*c^2*(a + b*ArcTan[c*x^3])*Log[2 - 2/(1 - I*c*x^3)] - (I/2)*b^3*c^2*PolyL
og[2, -1 + 2/(1 - I*c*x^3)]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplify
[(m + 1)/n]]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
```

```
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}\text{Subst}\left(\int \frac{(a+b\arctan(cx))^3}{x^3}\,dx,x,x^3\right)\\
&= -\frac{(a+b\arctan(cx^3))^3}{6x^6}+\frac{1}{2}(bc)\text{Subst}\left(\int \frac{(a+b\arctan(cx))^2}{x^2(1+c^2x^2)}\,dx,x,x^3\right)\\
&= -\frac{(a+b\arctan(cx^3))^3}{6x^6}+\frac{1}{2}(bc)\text{Subst}\left(\int \frac{(a+b\arctan(cx))^2}{x^2}\,dx,x,x^3\right)\\
&\quad -\frac{1}{2}(bc^3)\,\text{Subst}\left(\int \frac{(a+b\arctan(cx))^2}{1+c^2x^2}\,dx,x,x^3\right)\\
&= -\frac{bc(a+b\arctan(cx^3))^2}{2x^3}-\frac{1}{6}c^2(a+b\arctan(cx^3))^3\\
&\quad -\frac{(a+b\arctan(cx^3))^3}{6x^6}+(b^2c^2)\,\text{Subst}\left(\int \frac{a+b\arctan(cx)}{x(1+c^2x^2)}\,dx,x,x^3\right)\\
&= -\frac{1}{2}ibc^2(a+b\arctan(cx^3))^2-\frac{bc(a+b\arctan(cx^3))^2}{2x^3}-\frac{1}{6}c^2(a+b\arctan(cx^3))^3\\
&\quad -\frac{(a+b\arctan(cx^3))^3}{6x^6}+(ib^2c^2)\,\text{Subst}\left(\int \frac{a+b\arctan(cx)}{x(i+cx)}\,dx,x,x^3\right)\\
&= -\frac{1}{2}ibc^2(a+b\arctan(cx^3))^2-\frac{bc(a+b\arctan(cx^3))^2}{2x^3}-\frac{1}{6}c^2(a+b\arctan(cx^3))^3\\
&\quad -\frac{(a+b\arctan(cx^3))^3}{6x^6}+b^2c^2(a+b\arctan(cx^3))\log\left(2-\frac{2}{1-icx^3}\right)\\
&\quad -(b^3c^3)\,\text{Subst}\left(\int \frac{\log\left(2-\frac{2}{1-icx}\right)}{1+c^2x^2}\,dx,x,x^3\right)\\
&= -\frac{1}{2}ibc^2(a+b\arctan(cx^3))^2-\frac{bc(a+b\arctan(cx^3))^2}{2x^3}\\
&\quad -\frac{1}{6}c^2(a+b\arctan(cx^3))^3-\frac{(a+b\arctan(cx^3))^3}{6x^6}\\
&\quad +b^2c^2(a+b\arctan(cx^3))\log\left(2-\frac{2}{1-icx^3}\right)-\frac{1}{2}ib^3c^2\,\text{PolyLog}\left(2,-1+\frac{2}{1-icx^3}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.34

$$\int \frac{(a+b\arctan(cx^3))^3}{x^7}\,dx =$$
$$-\frac{3b^2(a+ac^2x^6+bcx^3(1+icx^3))\arctan(cx^3)^2+b^3(1+c^2x^6)\arctan(cx^3)^3+3b\arctan(cx^3)\left(a(a+2bc\right.}{}$$

```
[In] Integrate[(a + b*ArcTan[c*x^3])^3/x^7,x]
```

```
[Out] -1/6*(3*b^2*(a + a*c^2*x^6 + b*c*x^3*(1 + I*c*x^3))*ArcTan[c*x^3]^2 + b^3*(
1 + c^2*x^6)*ArcTan[c*x^3]^3 + 3*b*ArcTan[c*x^3]*(a*(a + 2*b*c*x^3 + a*c^2*
x^6) - 2*b^2*c^2*x^6*Log[1 - E^((2*I)*ArcTan[c*x^3])]) + a*(a*(a + 3*b*c*x^
3) - 6*b^2*c^2*x^6*Log[(c*x^3)/Sqrt[1 + c^2*x^6]]) + (3*I)*b^3*c^2*x^6*Poly
Log[2, E^((2*I)*ArcTan[c*x^3])])/x^6
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.88 (sec) , antiderivative size = 11581, normalized size of antiderivative = 79.32

method	result	size
default	Expression too large to display	11581
parts	Expression too large to display	11581

```
[In] int((a+b*arctan(c*x^3))^3/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [F]

$$\int \frac{(a+b\arctan(cx^3))^3}{x^7}\,dx = \int \frac{(b\arctan(cx^3)+a)^3}{x^7}\,dx$$

```
[In] integrate((a+b*arctan(c*x^3))^3/x^7,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(c*x^3)^3 + 3*a*b^2*arctan(c*x^3)^2 + 3*a^2*b*arctan(c*
x^3) + a^3)/x^7, x)
```

Sympy [F]

$$\int \frac{(a+b\arctan(cx^3))^3}{x^7}\,dx=\int \frac{(a+b\,\text{atan}\,(cx^3))^3}{x^7}\,dx$$

```
[In] integrate((a+b*atan(c*x**3))**3/x**7,x)
```

```
[Out] Integral((a + b*atan(c*x**3))**3/x**7, x)
```

Maxima [F]

$$\int \frac{(a+b\arctan(cx^3))^3}{x^7}\,dx=\int \frac{(b\arctan(cx^3)+a)^3}{x^7}\,dx$$

```
[In] integrate((a+b*arctan(c*x^3))^3/x^7,x, algorithm="maxima")
```

```
[Out] -1/2*((c*arctan(c*x^3) + 1/x^3)*c + arctan(c*x^3)/x^6)*a^2*b + 1/2*((arctan
(c*x^3)^2 - log(c^2*x^6 + 1) + 6*log(x))*c^2 - 2*(c*arctan(c*x^3) + 1/x^3)*
c*arctan(c*x^3))*a*b^2 - 1/2*a*b^2*arctan(c*x^3)^2/x^6 + 1/192*(192*x^6*int
egrate(-1/64*(12*c^2*x^6*arctan(c*x^3)*log(c^2*x^6 + 1) - 12*c*x^3*arctan(c
*x^3)^2 - 56*(c^2*x^6 + 1)*arctan(c*x^3)^3 + 3*(c*x^3 - 2*(c^2*x^6 + 1)*arc
tan(c*x^3))*log(c^2*x^6 + 1)^2)/(c^2*x^13 + x^7), x) - 4*arctan(c*x^3)^3 +
3*arctan(c*x^3)*log(c^2*x^6 + 1)^2)*b^3/x^6 - 1/6*a^3/x^6
```

Giac [F]

$$\int \frac{(a+b\arctan(cx^3))^3}{x^7}\,dx=\int \frac{(b\arctan(cx^3)+a)^3}{x^7}\,dx$$

```
[In] integrate((a+b*arctan(c*x^3))^3/x^7,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^3) + a)^3/x^7, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx^3))^3}{x^7}\,dx=\int \frac{(a+b\,\text{atan}(c\,x^3))^3}{x^7}\,dx$$

```
[In] int((a + b*atan(c*x^3))^3/x^7,x)
```

```
[Out] int((a + b*atan(c*x^3))^3/x^7, x)
```

3.127 $\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right)^3\,dx$

Optimal result 743
Rubi [N/A] 743
Mathematica [N/A] 744
Maple [N/A] (verified) 744
Fricas [N/A] 744
Sympy [F(-1)] 744
Maxima [N/A] 745
Giac [N/A] 745
Mupad [N/A] 745

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right)^3\,dx=\operatorname{Int}\left((dx)^m\left(a+b\arctan\left(cx^3\right)\right)^3,x\right)$$

```
[Out] Unintegrable((d*x)^m*(a+b*arctan(c*x^3))^3,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}=0.000$, Rules used = {}

$$\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right)^3\,dx=\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right)^3\,dx$$

```
[In] Int[(d*x)^m*(a + b*ArcTan[c*x^3])^3,x]
```

```
[Out] Defer[Int][(d*x)^m*(a + b*ArcTan[c*x^3])^3, x]
```

Rubi steps

$$\text{integral}=\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right)^3\,dx$$

Mathematica [N/A]

Not integrable

Time = 2.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right)^3\,dx=\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right)^3\,dx$$

```
[In] Integrate[(d*x)^m*(a + b*ArcTan[c*x^3])^3,x]
[Out] Integrate[(d*x)^m*(a + b*ArcTan[c*x^3])^3, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (dx)^m \left(a+b\arctan\left(c\,x^3\right)\right)^3dx$$

```
[In] int((d*x)^m*(a+b*arctan(c*x^3))^3,x)
[Out] int((d*x)^m*(a+b*arctan(c*x^3))^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.78

$$\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right)^3\,dx=\int \left(b\arctan\left(cx^3\right)+a\right)^3(dx)^m\,dx$$

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^3))^3,x, algorithm="fricas")
[Out] integral((b^3*arctan(c*x^3)^3 + 3*a*b^2*arctan(c*x^3)^2 + 3*a^2*b*arctan(c*
x^3) + a^3)*(d*x)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right)^3\,dx=\text{Timed out}$$

```
[In] integrate((d*x)**m*(a+b*atan(c*x**3))**3,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 4.06 (sec) , antiderivative size = 407, normalized size of antiderivative = 22.61

$$\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right)^3\,dx=\int \left(b\arctan\left(cx^3\right)+a\right)^3(dx)^m\,dx$$

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^3))^3,x, algorithm="maxima")
```

```
[Out] (d*x)^(m + 1)*a^3/(d*(m + 1)) + 1/32*(4*b^3*d^m*x*x^m*arctan(c*x^3)^3 - 3*b
^3*d^m*x*x^m*arctan(c*x^3)*log(c^2*x^6 + 1)^2 + 32*(m + 1)*integrate(1/32*(
36*b^3*c^2*d^m*x^6*x^m*arctan(c*x^3)*log(c^2*x^6 + 1) + 28*((b^3*c^2*d^m*m
+ b^3*c^2*d^m)*x^6 + b^3*d^m*m + b^3*d^m)*x^m*arctan(c*x^3)^3 - 12*(3*b^3*c
*d^m*x^3 - 8*(a*b^2*c^2*d^m*m + a*b^2*c^2*d^m)*x^6 - 8*a*b^2*d^m*m - 8*a*b^
2*d^m)*x^m*arctan(c*x^3)^2 + 96*((a^2*b*c^2*d^m*m + a^2*b*c^2*d^m)*x^6 + a^
2*b*d^m*m + a^2*b*d^m)*x^m*arctan(c*x^3) + 3*(3*b^3*c*d^m*x^3*x^m + ((b^3*c
^2*d^m*m + b^3*c^2*d^m)*x^6 + b^3*d^m*m + b^3*d^m)*x^m*arctan(c*x^3))*log(c
^2*x^6 + 1)^2)/((c^2*m + c^2)*x^6 + m + 1), x))/(m + 1)
```

Giac [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right)^3\,dx=\int \left(b\arctan\left(cx^3\right)+a\right)^3(dx)^m\,dx$$

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^3))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^3) + a)^3*(d*x)^m, x)
```

Mupad [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right)^3\,dx=\int {\left(d\,x\right)}^m\,{\left(a+b\,\mathrm{atan}\left(c\,x^3\right)\right)}^3 \,d x$$

```
[In] int((d*x)^m*(a + b*atan(c*x^3))^3,x)
```

```
[Out] int((d*x)^m*(a + b*atan(c*x^3))^3, x)
```

3.128 $\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right)^2 dx$

Optimal result 746
Rubi [N/A] 746
Mathematica [N/A] 747
Maple [N/A] (verified) 747
Fricas [N/A] 747
Sympy [F(-1)] 747
Maxima [N/A] 748
Giac [N/A] 748
Mupad [N/A] 748

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right)^2 dx = \operatorname{Int}\left((dx)^m \left(a+b\arctan\left(cx^3\right)\right)^2, x\right)$$

```
[Out] Unintegrable((d*x)^m*(a+b*arctan(c*x^3))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right)^2 dx = \int (dx)^m \left(a+b\arctan\left(cx^3\right)\right)^2 dx$$

```
[In] Int[(d*x)^m*(a + b*ArcTan[c*x^3])^2,x]
[Out] Defer[Int][(d*x)^m*(a + b*ArcTan[c*x^3])^2, x]
```

Rubi steps

$$\text{integral} = \int (dx)^m \left(a+b\arctan\left(cx^3\right)\right)^2 dx$$

Mathematica [N/A]

Not integrable

Time = 1.53 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right)^2\,dx=\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right)^2\,dx$$

```
[In] Integrate[(d*x)^m*(a + b*ArcTan[c*x^3])^2,x]
[Out] Integrate[(d*x)^m*(a + b*ArcTan[c*x^3])^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (dx)^m \left(a+b\arctan\left(c\,x^3\right)\right)^2dx$$

```
[In] int((d*x)^m*(a+b*arctan(c*x^3))^2,x)
[Out] int((d*x)^m*(a+b*arctan(c*x^3))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.89

$$\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right)^2\,dx=\int \left(b\arctan\left(cx^3\right)+a\right)^2(dx)^m\,dx$$

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^3))^2,x, algorithm="fricas")
[Out] integral((b^2*arctan(c*x^3)^2 + 2*a*b*arctan(c*x^3) + a^2)*(d*x)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right)^2\,dx=\text{Timed out}$$

```
[In] integrate((d*x)**m*(a+b*atan(c*x**3))**2,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 2.37 (sec) , antiderivative size = 304, normalized size of antiderivative = 16.89

$$\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right)^2 \, dx = \int \left(b\arctan\left(cx^3\right)+a\right)^2 (dx)^m \, dx$$

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^3))^2,x, algorithm="maxima")
```

```
[Out] (d*x)^(m + 1)*a^2/(d*(m + 1)) + 1/16*(4*b^2*d^m*x*x^m*arctan(c*x^3)^2 - b^2
*d^m*x*x^m*log(c^2*x^6 + 1)^2 + 16*(m + 1)*integrate(1/16*(12*b^2*c^2*d^m*x
^6*x^m*log(c^2*x^6 + 1) + 12*((b^2*c^2*d^m*m + b^2*c^2*d^m)*x^6 + b^2*d^m*m
 + b^2*d^m)*x^m*arctan(c*x^3)^2 + ((b^2*c^2*d^m*m + b^2*c^2*d^m)*x^6 + b^2*
d^m*m + b^2*d^m)*x^m*log(c^2*x^6 + 1)^2 - 8*(3*b^2*c*d^m*x^3 - 4*(a*b*c^2*d
^m*m + a*b*c^2*d^m)*x^6 - 4*a*b*d^m*m - 4*a*b*d^m)*x^m*arctan(c*x^3))/((c^2
*m + c^2)*x^6 + m + 1), x))/(m + 1)
```

Giac [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right)^2 \, dx = \int \left(b\arctan\left(cx^3\right)+a\right)^2 (dx)^m \, dx$$

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^3))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^3) + a)^2*(d*x)^m, x)
```

Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right)^2 \, dx = \int (d\,x)^m \left(a+b\,\mathrm{atan}\left(c\,x^3\right)\right)^2 dx$$

```
[In] int((d*x)^m*(a + b*atan(c*x^3))^2,x)
```

```
[Out] int((d*x)^m*(a + b*atan(c*x^3))^2, x)
```

3.129 $\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right) dx$

Optimal result . . . 749
Rubi [A] (verified) . . . 749
Mathematica [A] (verified) . . . 750
Maple [F] . . . 750
Fricas [F] . . . 751
Sympy [F(-1)] . . . 751
Maxima [F] . . . 751
Giac [F] . . . 751
Mupad [F(-1)] . . . 752

Optimal result

Integrand size = 16, antiderivative size = 75

$$\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right) dx = \frac{(dx)^{1+m}\left(a+b\arctan\left(cx^3\right)\right)}{d(1+m)} - \frac{3bc(dx)^{4+m}\,\text{Hypergeometric2F1}\left(1,\frac{4+m}{6},\frac{10+m}{6},-c^2x^6\right)}{d^4(1+m)(4+m)}$$

```
[Out] (d*x)^(1+m)*(a+b*arctan(c*x^3))/d/(1+m)-3*b*c*(d*x)^(4+m)*hypergeom([1, 2/3
+1/6*m],[5/3+1/6*m],-c^2*x^6)/d^4/(1+m)/(4+m)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4958, 371}

$$\int (dx)^m \left(a+b\arctan\left(cx^3\right)\right) dx = \frac{(dx)^{m+1}\left(a+b\arctan\left(cx^3\right)\right)}{d(m+1)} - \frac{3bc(dx)^{m+4}\,\text{Hypergeometric2F1}\left(1,\frac{m+4}{6},\frac{m+10}{6},-c^2x^6\right)}{d^4(m+1)(m+4)}$$

```
[In] Int[(d*x)^m*(a + b*ArcTan[c*x^3]),x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*ArcTan[c*x^3]))/(d*(1 + m)) - (3*b*c*(d*x)^(4 + m)*Hy
pergeometric2F1[1, (4 + m)/6, (10 + m)/6, -(c^2*x^6)])/(d^4*(1 + m)*(4 + m)
)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4958

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))*((d_)*(x_))^(m_), x_Symbol] :>
 Simp[(d*x)^(m + 1)*((a + b*ArcTan[c*x^n])/(d*(m + 1))), x] - Dist[b*c*(n/(
d^n*(m + 1))), Int[(d*x)^(m + n)/(1 + c^2*x^(2*n)), x], x] /; FreeQ[{a, b,
c, d, m, n}, x] && IntegerQ[n] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(dx)^{1+m}\left(a+b\arctan\left(cx^3\right)\right)}{d(1+m)}-\frac{(3bc)\int\frac{(dx)^{3+m}}{1+c^2x^6}\,dx}{d^3(1+m)}\\ &= \frac{(dx)^{1+m}\left(a+b\arctan\left(cx^3\right)\right)}{d(1+m)}-\frac{3bc(dx)^{4+m}\,\text{Hypergeometric2F1}\left(1,\frac{4+m}{6},\frac{10+m}{6},-c^2x^6\right)}{d^4(1+m)(4+m)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.87

$$\int (dx)^m\left(a+b\arctan\left(cx^3\right)\right)\,dx = -\frac{x(dx)^m\left(-((4+m)\left(a+b\arctan\left(cx^3\right)\right))+3bcx^3\,\text{Hypergeometric2F1}\left(1,\frac{4+m}{6},\frac{10+m}{6},-c^2x^6\right)\right)}{(1+m)(4+m)}$$

```
[In] Integrate[(d*x)^m*(a + b*ArcTan[c*x^3]),x]
[Out] -((x*(d*x)^m*(-((4 + m)*(a + b*ArcTan[c*x^3])) + 3*b*c*x^3*Hypergeometric2F
1[1, (4 + m)/6, (10 + m)/6, -(c^2*x^6)]))/((1 + m)*(4 + m)))
```

Maple [F]

$$\int (dx)^m\left(a+b\arctan\left(c\,x^3\right)\right)dx$$

```
[In] int((d*x)^m*(a+b*arctan(c*x^3)),x)
[Out] int((d*x)^m*(a+b*arctan(c*x^3)),x)
```

Fricas [F]

$$\int (dx)^m \left(a + b \arctan\left(cx^3\right)\right) \, dx = \int \left(b \arctan\left(cx^3\right) + a\right) (dx)^m \, dx$$

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^3)),x, algorithm="fricas")
[Out] integral((b*arctan(c*x^3) + a)*(d*x)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (dx)^m \left(a + b \arctan\left(cx^3\right)\right) \, dx = \text{Timed out}$$

```
[In] integrate((d*x)**m*(a+b*atan(c*x**3)),x)
[Out] Timed out
```

Maxima [F]

$$\int (dx)^m \left(a + b \arctan\left(cx^3\right)\right) \, dx = \int \left(b \arctan\left(cx^3\right) + a\right) (dx)^m \, dx$$

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^3)),x, algorithm="maxima")
[Out] (d^m*x*x^m*arctan(c*x^3) - 3*(c*d^m*m + c*d^m)*integrate(x^3*x^m/((c^2*m +
c^2)*x^6 + m + 1), x))*b/(m + 1) + (d*x)^(m + 1)*a/(d*(m + 1))
```

Giac [F]

$$\int (dx)^m \left(a + b \arctan\left(cx^3\right)\right) \, dx = \int \left(b \arctan\left(cx^3\right) + a\right) (dx)^m \, dx$$

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^3)),x, algorithm="giac")
[Out] integrate((b*arctan(c*x^3) + a)*(d*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^m \left(a + b \arctan \left(cx^3\right)\right) \, dx = \int (d\,x)^m \left(a + b\,\mathrm{atan}\left(c\,x^3\right)\right) \, dx$$

```
[In] int((d*x)^m*(a + b*atan(c*x^3)),x)
[Out] int((d*x)^m*(a + b*atan(c*x^3)), x)
```

3.130 $\int \frac{(dx)^m}{a+b\arctan(cx^3)}\,dx$

Optimal result 753
Rubi [N/A] 753
Mathematica [N/A] 754
Maple [N/A] (verified) 754
Fricas [N/A] 754
Sympy [F(-1)] 754
Maxima [N/A] 755
Giac [N/A] 755
Mupad [N/A] 755

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{(dx)^m}{a+b\arctan(cx^3)}\,dx = \text{Int}\left(\frac{(dx)^m}{a+b\arctan(cx^3)},x\right)$$

```
[Out] Unintegrable((d*x)^m/(a+b*arctan(c*x^3)),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(dx)^m}{a+b\arctan(cx^3)}\,dx = \int \frac{(dx)^m}{a+b\arctan(cx^3)}\,dx$$

```
[In] Int[(d*x)^m/(a + b*ArcTan[c*x^3]),x]
[Out] Defer[Int][(d*x)^m/(a + b*ArcTan[c*x^3]), x]
```

Rubi steps

$$\text{integral} = \int \frac{(dx)^m}{a+b\arctan(cx^3)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{a+b\arctan(cx^3)}\,dx=\int \frac{(dx)^m}{a+b\arctan(cx^3)}\,dx$$

```
[In] Integrate[(d*x)^m/(a + b*ArcTan[c*x^3]),x]
[Out] Integrate[(d*x)^m/(a + b*ArcTan[c*x^3]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^m}{a+b\arctan(c\,x^3)}dx$$

```
[In] int((d*x)^m/(a+b*arctan(c*x^3)),x)
[Out] int((d*x)^m/(a+b*arctan(c*x^3)),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{a+b\arctan(cx^3)}\,dx=\int \frac{(dx)^m}{b\arctan(cx^3)+a}\,dx$$

```
[In] integrate((d*x)^m/(a+b*arctan(c*x^3)),x, algorithm="fricas")
[Out] integral((d*x)^m/(b*arctan(c*x^3) + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{a+b\arctan(cx^3)}\,dx=\text{Timed out}$$

```
[In] integrate((d*x)**m/(a+b*atan(c*x**3)),x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{a+b\arctan(cx^3)}\,dx=\int \frac{(dx)^m}{b\arctan(cx^3)+a}\,dx$$

```
[In] integrate((d*x)^m/(a+b*arctan(c*x^3)),x, algorithm="maxima")
[Out] integrate((d*x)^m/(b*arctan(c*x^3) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{a+b\arctan(cx^3)}\,dx=\int \frac{(dx)^m}{b\arctan(cx^3)+a}\,dx$$

```
[In] integrate((d*x)^m/(a+b*arctan(c*x^3)),x, algorithm="giac")
[Out] integrate((d*x)^m/(b*arctan(c*x^3) + a), x)
```

Mupad [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{a+b\arctan(cx^3)}\,dx=\int \frac{(d\,x)^m}{a+b\,\mathrm{atan}\left(c\,x^3\right)}\,dx$$

```
[In] int((d*x)^m/(a + b*atan(c*x^3)),x)
[Out] int((d*x)^m/(a + b*atan(c*x^3)), x)
```

3.131 $\int \frac{(dx)^m}{(a+b\arctan(cx^3))^2}\,dx$

Optimal result 756
Rubi [N/A] 756
Mathematica [N/A] 757
Maple [N/A] (verified) 757
Fricas [N/A] 757
Sympy [F(-1)] 757
Maxima [N/A] 758
Giac [N/A] 758
Mupad [N/A] 758

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{(dx)^m}{(a+b\arctan(cx^3))^2}\,dx = \text{Int}\left(\frac{(dx)^m}{(a+b\arctan(cx^3))^2},x\right)$$

```
[Out] Unintegrable((d*x)^m/(a+b*arctan(c*x^3))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(dx)^m}{(a+b\arctan(cx^3))^2}\,dx = \int \frac{(dx)^m}{(a+b\arctan(cx^3))^2}\,dx$$

```
[In] Int[(d*x)^m/(a + b*ArcTan[c*x^3])^2,x]
[Out] Defer[Int][(d*x)^m/(a + b*ArcTan[c*x^3])^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{(dx)^m}{(a+b\arctan(cx^3))^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{(a+b\arctan(cx^3))^2}\,dx = \int \frac{(dx)^m}{(a+b\arctan(cx^3))^2}\,dx$$

```
[In] Integrate[(d*x)^m/(a + b*ArcTan[c*x^3])^2,x]
[Out] Integrate[(d*x)^m/(a + b*ArcTan[c*x^3])^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^m}{(a+b\arctan(c\,x^3))^2}dx$$

```
[In] int((d*x)^m/(a+b*arctan(c*x^3))^2,x)
[Out] int((d*x)^m/(a+b*arctan(c*x^3))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.00

$$\int \frac{(dx)^m}{(a+b\arctan(cx^3))^2}\,dx = \int \frac{(dx)^m}{(b\arctan(cx^3)+a)^2}\,dx$$

```
[In] integrate((d*x)^m/(a+b*arctan(c*x^3))^2,x, algorithm="fricas")
[Out] integral((d*x)^m/(b^2*arctan(c*x^3)^2 + 2*a*b*arctan(c*x^3) + a^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{(a+b\arctan(cx^3))^2}\,dx = \text{Timed out}$$

```
[In] integrate((d*x)**m/(a+b*atan(c*x**3))**2,x)
[Out] Timed out
```

Maxima [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 132, normalized size of antiderivative = 7.33

$$\int \frac{(dx)^m}{(a+b\arctan(cx^3))^2}\,dx = \int \frac{(dx)^m}{(b\arctan(cx^3)+a)^2}\,dx$$

```
[In] integrate((d*x)^m/(a+b*arctan(c*x^3))^2,x, algorithm="maxima")
[Out] -1/3*((c^2*d^m*x^6 + d^m)*x^m - 3*(b^2*c*x^2*arctan(c*x^3) + a*b*c*x^2)*int
egrate(1/3*((c^2*d^m*m + 4*c^2*d^m)*x^6 + d^m*m - 2*d^m)*x^m/(b^2*c*x^3*arc
tan(c*x^3) + a*b*c*x^3), x))/(b^2*c*x^2*arctan(c*x^3) + a*b*c*x^2)
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{(a+b\arctan(cx^3))^2}\,dx = \int \frac{(dx)^m}{(b\arctan(cx^3)+a)^2}\,dx$$

```
[In] integrate((d*x)^m/(a+b*arctan(c*x^3))^2,x, algorithm="giac")
[Out] integrate((d*x)^m/(b*arctan(c*x^3) + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{(a+b\arctan(cx^3))^2}\,dx = \int \frac{(d\,x)^m}{(a+b\,\mathrm{atan}(c\,x^3))^2}\,dx$$

```
[In] int((d*x)^m/(a + b*atan(c*x^3))^2,x)
[Out] int((d*x)^m/(a + b*atan(c*x^3))^2, x)
```

3.132 $\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx$

Optimal result 759
Rubi [A] (verified) 759
Mathematica [A] (verified) 760
Maple [A] (verified) 761
Fricas [A] (verification not implemented) 761
Sympy [A] (verification not implemented) 761
Maxima [A] (verification not implemented) 762
Giac [C] (verification not implemented) 762
Mupad [B] (verification not implemented) 762

Optimal result

Integrand size = 14, antiderivative size = 50

$$\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=-\frac{1}{4}bc^3x+\frac{1}{12}bcx^3+\frac{1}{4}x^4\left(a+b\arctan\left(\frac{c}{x}\right)\right)+\frac{1}{4}bc^4\arctan\left(\frac{x}{c}\right)$$

```
[Out] -1/4*b*c^3*x+1/12*b*c*x^3+1/4*x^4*(a+b*arctan(c/x))+1/4*b*c^4*arctan(x/c)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4946, 269, 308, 209}

$$\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=\frac{1}{4}x^4\left(a+b\arctan\left(\frac{c}{x}\right)\right)+\frac{1}{4}bc^4\arctan\left(\frac{x}{c}\right)-\frac{1}{4}bc^3x+\frac{1}{12}bcx^3$$

```
[In] Int[x^3*(a + b*ArcTan[c/x]),x]
```

```
[Out] -1/4*(b*c^3*x) + (b*c*x^3)/12 + (x^4*(a + b*ArcTan[c/x]))/4 + (b*c^4*ArcTan
[x/c])/4
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}x^4\left(a+b\arctan\left(\frac{c}{x}\right)\right)+\frac{1}{4}(bc)\int\frac{x^2}{1+\frac{c^2}{x^2}}\,dx\\
&= \frac{1}{4}x^4\left(a+b\arctan\left(\frac{c}{x}\right)\right)+\frac{1}{4}(bc)\int\frac{x^4}{c^2+x^2}\,dx\\
&= \frac{1}{4}x^4\left(a+b\arctan\left(\frac{c}{x}\right)\right)+\frac{1}{4}(bc)\int\left(-c^2+x^2+\frac{c^4}{c^2+x^2}\right)dx\\
&= -\frac{1}{4}bc^3x+\frac{1}{12}bcx^3+\frac{1}{4}x^4\left(a+b\arctan\left(\frac{c}{x}\right)\right)+\frac{1}{4}\left(bc^5\right)\int\frac{1}{c^2+x^2}\,dx\\
&= -\frac{1}{4}bc^3x+\frac{1}{12}bcx^3+\frac{1}{4}x^4\left(a+b\arctan\left(\frac{c}{x}\right)\right)+\frac{1}{4}bc^4\arctan\left(\frac{x}{c}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.10

$$\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=-\frac{1}{4}bc^3x+\frac{1}{12}bcx^3+\frac{ax^4}{4}-\frac{1}{4}bc^4\arctan\left(\frac{c}{x}\right)+\frac{1}{4}bx^4\arctan\left(\frac{c}{x}\right)$$

```
[In] Integrate[x^3*(a + b*ArcTan[c/x]),x]
[Out] -1/4*(b*c^3*x) + (b*c*x^3)/12 + (a*x^4)/4 - (b*c^4*ArcTan[c/x])/4 + (b*x^4*
ArcTan[c/x])/4
```

Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

method	result	size
parallelrisch	$\frac{x^4\arctan\left(\frac{c}{x}\right)b}{4}-\frac{\arctan\left(\frac{c}{x}\right)b\,c^4}{4}+\frac{a\,x^4}{4}+\frac{bc\,x^3}{12}-\frac{b\,c^3x}{4}$	46
parts	$\frac{a\,x^4}{4}+\frac{x^4\arctan\left(\frac{c}{x}\right)b}{4}+\frac{bc\,x^3}{12}-\frac{b\,c^3x}{4}+\frac{b\,c^4\arctan\left(\frac{x}{c}\right)}{4}$	46
derivativedivides	$-c^4\left(-\frac{a\,x^4}{4c^4}+b\left(-\frac{x^4\arctan\left(\frac{c}{x}\right)}{4c^4}-\frac{x^3}{12c^3}+\frac{x}{4c}+\frac{\arctan\left(\frac{c}{x}\right)}{4}\right)\right)$	55
default	$-c^4\left(-\frac{a\,x^4}{4c^4}+b\left(-\frac{x^4\arctan\left(\frac{c}{x}\right)}{4c^4}-\frac{x^3}{12c^3}+\frac{x}{4c}+\frac{\arctan\left(\frac{c}{x}\right)}{4}\right)\right)$	55
risch	Expression too large to display	697

```
[In] int(x^3*(a+b*arctan(c/x)),x,method=_RETURNVERBOSE)
[Out] 1/4*x^4*arctan(c/x)*b-1/4*arctan(c/x)*b*c^4+1/4*a*x^4+1/12*b*c*x^3-1/4*b*c^
3*x
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.82

$$\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=-\frac{1}{4}\,bc^3x+\frac{1}{12}\,bcx^3+\frac{1}{4}\,ax^4-\frac{1}{4}\left(bc^4-bx^4\right)\arctan\left(\frac{c}{x}\right)$$

```
[In] integrate(x^3*(a+b*arctan(c/x)),x, algorithm="fricas")
[Out] -1/4*b*c^3*x + 1/12*b*c*x^3 + 1/4*a*x^4 - 1/4*(b*c^4 - b*x^4)*arctan(c/x)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=\frac{ax^4}{4}-\frac{bc^4\operatorname{atan}\left(\frac{c}{x}\right)}{4}-\frac{bc^3x}{4}+\frac{bcx^3}{12}+\frac{bx^4\operatorname{atan}\left(\frac{c}{x}\right)}{4}$$

```
[In] integrate(x**3*(a+b*atan(c/x)),x)
[Out] a*x**4/4 - b*c**4*atan(c/x)/4 - b*c**3*x/4 + b*c*x**3/12 + b*x**4*atan(c/x)
/4
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

$$\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=\frac{1}{4}\,ax^4+\frac{1}{12}\left(3\,x^4\arctan\left(\frac{c}{x}\right)+\left(3\,c^3\arctan\left(\frac{x}{c}\right)-3\,c^2x+x^3\right)c\right)b$$

```
[In] integrate(x^3*(a+b*arctan(c/x)),x, algorithm="maxima")
```

```
[Out] 1/4*a*x^4 + 1/12*(3*x^4*arctan(c/x) + (3*c^3*arctan(x/c) - 3*c^2*x + x^3)*c
)*b
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.62

$$\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=\frac{\left(6\,bc^5\arctan\left(\frac{c}{x}\right)-\frac{3i\,bc^9\log\left(\frac{i\,c}{x}-1\right)}{x^4}+\frac{3i\,bc^9\log\left(-\frac{i\,c}{x}-1\right)}{x^4}+6\,ac^5-\frac{6\,bc^8}{x^3}+\frac{2\,bc^6}{x}\right)x^4}{24\,c^5}$$

```
[In] integrate(x^3*(a+b*arctan(c/x)),x, algorithm="giac")
```

```
[Out] 1/24*(6*b*c^5*arctan(c/x) - 3*I*b*c^9*log(I*c/x - 1)/x^4 + 3*I*b*c^9*log(-I
*c/x - 1)/x^4 + 6*a*c^5 - 6*b*c^8/x^3 + 2*b*c^6/x)*x^4/c^5
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

$$\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=\frac{a\,x^4}{4}-\frac{b\,c^4\,\mathrm{atan}\left(\frac{c}{x}\right)}{4}+\frac{b\,x^4\,\mathrm{atan}\left(\frac{c}{x}\right)}{4}+\frac{b\,c\,x^3}{12}-\frac{b\,c^3\,x}{4}$$

```
[In] int(x^3*(a + b*atan(c/x)),x)
```

```
[Out] (a*x^4)/4 - (b*c^4*atan(c/x))/4 + (b*x^4*atan(c/x))/4 + (b*c*x^3)/12 - (b*c
^3*x)/4
```

3.133 $\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx$

Optimal result 763
Rubi [A] (verified) 763
Mathematica [A] (verified) 764
Maple [A] (verified) 765
Fricas [A] (verification not implemented) 765
Sympy [A] (verification not implemented) 765
Maxima [A] (verification not implemented) 766
Giac [A] (verification not implemented) 766
Mupad [B] (verification not implemented) 766

Optimal result

Integrand size = 14, antiderivative size = 43

$$\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=\frac{1}{6}bcx^2+\frac{1}{3}x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)-\frac{1}{6}bc^3\log\left(c^2+x^2\right)$$

```
[Out] 1/6*b*c*x^2+1/3*x^3*(a+b*arctan(c/x))-1/6*b*c^3*ln(c^2+x^2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4946, 269, 272, 45}

$$\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=\frac{1}{3}x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)-\frac{1}{6}bc^3\log\left(c^2+x^2\right)+\frac{1}{6}bcx^2$$

```
[In] Int[x^2*(a + b*ArcTan[c/x]),x]
```

```
[Out] (b*c*x^2)/6 + (x^3*(a + b*ArcTan[c/x]))/3 - (b*c^3*Log[c^2 + x^2])/6
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)+\frac{1}{3}(bc)\int\frac{x}{1+\frac{c^2}{x^2}}\,dx\\
&= \frac{1}{3}x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)+\frac{1}{3}(bc)\int\frac{x^3}{c^2+x^2}\,dx\\
&= \frac{1}{3}x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)+\frac{1}{6}(bc)\text{Subst}\left(\int\frac{x}{c^2+x}\,dx,x,x^2\right)\\
&= \frac{1}{3}x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)+\frac{1}{6}(bc)\text{Subst}\left(\int\left(1-\frac{c^2}{c^2+x}\right)\,dx,x,x^2\right)\\
&= \frac{1}{6}bcx^2+\frac{1}{3}x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)-\frac{1}{6}bc^3\log\left(c^2+x^2\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.12

$$\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=\frac{1}{6}bcx^2+\frac{ax^3}{3}+\frac{1}{3}bx^3\arctan\left(\frac{c}{x}\right)-\frac{1}{6}bc^3\log\left(c^2+x^2\right)$$

```
[In] Integrate[x^2*(a + b*ArcTan[c/x]),x]
[Out] (b*c*x^2)/6 + (a*x^3)/3 + (b*x^3*ArcTan[c/x])/3 - (b*c^3*Log[c^2 + x^2])/6
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.09

method	result	size
parallelrisch	$-\frac{b\,c^{3}\ln\left(c^{2}+x^{2}\right)}{6}+\frac{b\,x^{3}\arctan\left(\frac{c}{x}\right)}{3}+\frac{x^{3}a}{3}+\frac{b c\,x^{2}}{6}-\frac{b\,c^{3}}{6}$	47
parts	$\frac{x^{3}a}{3}-b\,c^{3}\left(-\frac{x^{3}\arctan\left(\frac{c}{x}\right)}{3c^{3}}+\frac{\ln\left(1+\frac{c^{2}}{x^{2}}\right)}{6}-\frac{x^{2}}{6c^{2}}-\frac{\ln\left(\frac{c}{x}\right)}{3}\right)$	57
derivativedivides	$-c^{3}\left(-\frac{a\,x^{3}}{3c^{3}}+b\left(-\frac{x^{3}\arctan\left(\frac{c}{x}\right)}{3c^{3}}+\frac{\ln\left(1+\frac{c^{2}}{x^{2}}\right)}{6}-\frac{x^{2}}{6c^{2}}-\frac{\ln\left(\frac{c}{x}\right)}{3}\right)\right)$	61
default	$-c^{3}\left(-\frac{a\,x^{3}}{3c^{3}}+b\left(-\frac{x^{3}\arctan\left(\frac{c}{x}\right)}{3c^{3}}+\frac{\ln\left(1+\frac{c^{2}}{x^{2}}\right)}{6}-\frac{x^{2}}{6c^{2}}-\frac{\ln\left(\frac{c}{x}\right)}{3}\right)\right)$	61
risch	Expression too large to display	692

```
[In] int(x^2*(a+b*arctan(c/x)),x,method=_RETURNVERBOSE)
[Out] -1/6*b*c^3*ln(c^2+x^2)+1/3*b*x^3*arctan(c/x)+1/3*x^3*a+1/6*b*c*x^2-1/6*b*c^
3
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=\frac{1}{3}\,bx^3\arctan\left(\frac{c}{x}\right)-\frac{1}{6}\,bc^3\log\left(c^2+x^2\right)+\frac{1}{6}\,bcx^2+\frac{1}{3}\,ax^3$$

```
[In] integrate(x^2*(a+b*arctan(c/x)),x, algorithm="fricas")
[Out] 1/3*b*x^3*arctan(c/x) - 1/6*b*c^3*log(c^2 + x^2) + 1/6*b*c*x^2 + 1/3*a*x^3
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=\frac{ax^3}{3}-\frac{bc^3\log\left(c^2+x^2\right)}{6}+\frac{bcx^2}{6}+\frac{bx^3\operatorname{atan}\left(\frac{c}{x}\right)}{3}$$

```
[In] integrate(x**2*(a+b*atan(c/x)),x)
[Out] a*x**3/3 - b*c**3*log(c**2 + x**2)/6 + b*c*x**2/6 + b*x**3*atan(c/x)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=\frac{1}{3}\,ax^3+\frac{1}{6}\left(2\,x^3\arctan\left(\frac{c}{x}\right)-\left(c^2\log\left(c^2+x^2\right)-x^2\right)c\right)b$$

```
[In] integrate(x^2*(a+b*arctan(c/x)),x, algorithm="maxima")
[Out] 1/3*a*x^3 + 1/6*(2*x^3*arctan(c/x) - (c^2*log(c^2 + x^2) - x^2)*c)*b
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.60

$$\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=\frac{\left(2\,bc^4\arctan\left(\frac{c}{x}\right)-\frac{bc^7\log\left(\frac{c^2}{x^2}+1\right)}{x^3}+\frac{2\,bc^7\log\left(\frac{c}{x}\right)}{x^3}+2\,ac^4+\frac{bc^5}{x}\right)x^3}{6\,c^4}$$

```
[In] integrate(x^2*(a+b*arctan(c/x)),x, algorithm="giac")
[Out] 1/6*(2*b*c^4*arctan(c/x) - b*c^7*log(c^2/x^2 + 1)/x^3 + 2*b*c^7*log(c/x)/x^
3 + 2*a*c^4 + b*c^5/x)*x^3/c^4
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=\frac{a\,x^3}{3}+\frac{b\,x^3\,\mathrm{atan}\left(\frac{c}{x}\right)}{3}-\frac{b\,c^3\,\ln\left(c^2+x^2\right)}{6}+\frac{b\,c\,x^2}{6}$$

```
[In] int(x^2*(a + b*atan(c/x)),x)
[Out] (a*x^3)/3 + (b*x^3*atan(c/x))/3 - (b*c^3*log(c^2 + x^2))/6 + (b*c*x^2)/6
```

3.134 $\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx$

Optimal result 767
Rubi [A] (verified) 767
Mathematica [A] (verified) 768
Maple [A] (verified) 769
Fricas [A] (verification not implemented) 769
Sympy [A] (verification not implemented) 769
Maxima [A] (verification not implemented) 770
Giac [C] (verification not implemented) 770
Mupad [B] (verification not implemented) 770

Optimal result

Integrand size = 12, antiderivative size = 39

$$\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=\frac{bcx}{2}+\frac{1}{2}x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)-\frac{1}{2}bc^2\arctan\left(\frac{x}{c}\right)$$

```
[Out] 1/2*b*c*x+1/2*x^2*(a+b*arctan(c/x))-1/2*b*c^2*arctan(x/c)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4946, 199, 327, 209}

$$\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=\frac{1}{2}x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)-\frac{1}{2}bc^2\arctan\left(\frac{x}{c}\right)+\frac{bcx}{2}$$

```
[In] Int[x*(a + b*ArcTan[c/x]),x]
```

```
[Out] (b*c*x)/2 + (x^2*(a + b*ArcTan[c/x]))/2 - (b*c^2*ArcTan[x/c])/2
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)+\frac{1}{2}(bc)\int\frac{1}{1+\frac{c^2}{x^2}}\,dx\\
&= \frac{1}{2}x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)+\frac{1}{2}(bc)\int\frac{x^2}{c^2+x^2}\,dx\\
&= \frac{bcx}{2}+\frac{1}{2}x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)-\frac{1}{2}\left(bc^3\right)\int\frac{1}{c^2+x^2}\,dx\\
&= \frac{bcx}{2}+\frac{1}{2}x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)-\frac{1}{2}bc^2\arctan\left(\frac{x}{c}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.13

$$\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=\frac{bcx}{2}+\frac{ax^2}{2}+\frac{1}{2}bc^2\arctan\left(\frac{c}{x}\right)+\frac{1}{2}bx^2\arctan\left(\frac{c}{x}\right)$$

```
[In] Integrate[x*(a + b*ArcTan[c/x]),x]
[Out] (b*c*x)/2 + (a*x^2)/2 + (b*c^2*ArcTan[c/x])/2 + (b*x^2*ArcTan[c/x])/2
```

Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

method	result	size
parts	$\frac{a\,x^{2}}{2}+\frac{\arctan\left(\frac{c}{x}\right) b\,x^{2}}{2}-\frac{b\,c^{2} \arctan\left(\frac{x}{c}\right)}{2}+\frac{x b c}{2}$	37
parallelrisch	$\frac{\arctan\left(\frac{c}{x}\right) b\,x^{2}}{2}+\frac{\arctan\left(\frac{c}{x}\right) b\,c^{2}}{2}+\frac{a\,x^{2}}{2}+\frac{x b c}{2}-\frac{a\,c^{2}}{2}$	43
derivativedivides	$-c^{2} \left(-\frac{a\,x^{2}}{2 c^{2}}+b \left(-\frac{x^{2} \arctan\left(\frac{c}{x}\right)}{2 c^{2}}-\frac{x}{2 c}-\frac{\arctan\left(\frac{c}{x}\right)}{2}\right)\right)$	47
default	$-c^{2} \left(-\frac{a\,x^{2}}{2 c^{2}}+b \left(-\frac{x^{2} \arctan\left(\frac{c}{x}\right)}{2 c^{2}}-\frac{x}{2 c}-\frac{\arctan\left(\frac{c}{x}\right)}{2}\right)\right)$	47
risch	Expression too large to display	688

```
[In] int(x*(a+b*arctan(c/x)),x,method=_RETURNVERBOSE)
[Out] 1/2*a*x^2+1/2*arctan(c/x)*b*x^2-1/2*b*c^2*arctan(x/c)+1/2*x*b*c
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=\frac{1}{2}\,bcx+\frac{1}{2}\,ax^{2}+\frac{1}{2}\,\left(bc^{2}+bx^{2}\right)\arctan\left(\frac{c}{x}\right)$$

```
[In] integrate(x*(a+b*arctan(c/x)),x, algorithm="fricas")
[Out] 1/2*b*c*x + 1/2*a*x^2 + 1/2*(b*c^2 + b*x^2)*arctan(c/x)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=\frac{ax^{2}}{2}+\frac{bc^{2}\operatorname{atan}\left(\frac{c}{x}\right)}{2}+\frac{bcx}{2}+\frac{bx^{2}\operatorname{atan}\left(\frac{c}{x}\right)}{2}$$

```
[In] integrate(x*(a+b*atan(c/x)),x)
[Out] a*x**2/2 + b*c**2*atan(c/x)/2 + b*c*x/2 + b*x**2*atan(c/x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=\frac{1}{2}\,ax^2+\frac{1}{2}\left(x^2\arctan\left(\frac{c}{x}\right)-\left(c\arctan\left(\frac{x}{c}\right)-x\right)c\right)b$$

```
[In] integrate(x*(a+b*arctan(c/x)),x, algorithm="maxima")
[Out] 1/2*a*x^2 + 1/2*(x^2*arctan(c/x) - (c*arctan(x/c) - x)*c)*b
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.85

$$\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=\frac{\left(2\,bc^3\arctan\left(\frac{c}{x}\right)-\frac{i\,bc^5\log\left(\frac{i\,c}{x}+1\right)}{x^2}+\frac{i\,bc^5\log\left(-\frac{i\,c}{x}+1\right)}{x^2}+2\,ac^3+\frac{2\,bc^4}{x}\right)x^2}{4\,c^3}$$

```
[In] integrate(x*(a+b*arctan(c/x)),x, algorithm="giac")
[Out] 1/4*(2*b*c^3*arctan(c/x) - I*b*c^5*log(I*c/x + 1)/x^2 + I*b*c^5*log(-I*c/x
+ 1)/x^2 + 2*a*c^3 + 2*b*c^4/x)*x^2/c^3
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=\frac{a\,x^2}{2}+\frac{b\,c^2\,\mathrm{atan}\left(\frac{c}{x}\right)}{2}+\frac{b\,x^2\,\mathrm{atan}\left(\frac{c}{x}\right)}{2}+\frac{b\,c\,x}{2}$$

```
[In] int(x*(a + b*atan(c/x)),x)
[Out] (a*x^2)/2 + (b*c^2*atan(c/x))/2 + (b*x^2*atan(c/x))/2 + (b*c*x)/2
```

3.135 $\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx$

Optimal result . . . 771
Rubi [A] (verified) . . . 771
Mathematica [A] (verified) . . . 772
Maple [A] (verified) . . . 772
Fricas [A] (verification not implemented) . . . 773
Sympy [A] (verification not implemented) . . . 773
Maxima [A] (verification not implemented) . . . 773
Giac [A] (verification not implemented) . . . 774
Mupad [B] (verification not implemented) . . . 774

Optimal result

Integrand size = 10, antiderivative size = 27

$$\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=ax+bx\arctan\left(\frac{c}{x}\right)+\frac{1}{2}bc\log\left(c^2+x^2\right)$$

```
[Out] a*x+b*x*arctan(c/x)+1/2*b*c*ln(c^2+x^2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4930, 269, 266}

$$\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=ax+bx\arctan\left(\frac{c}{x}\right)+\frac{1}{2}bc\log\left(c^2+x^2\right)$$

```
[In] Int[a + b*ArcTan[c/x],x]
```

```
[Out] a*x + b*x*ArcTan[c/x] + (b*c*Log[c^2 + x^2])/2
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rubi steps

$$\begin{aligned}\text{integral} &= ax + b\int \arctan\left(\frac{c}{x}\right)\,dx\\&= ax + bx\arctan\left(\frac{c}{x}\right) + (bc)\int \frac{1}{\left(1+\frac{c^2}{x^2}\right)x}\,dx\\&= ax + bx\arctan\left(\frac{c}{x}\right) + (bc)\int \frac{x}{c^2+x^2}\,dx\\&= ax + bx\arctan\left(\frac{c}{x}\right) + \frac{1}{2}bc\log\left(c^2+x^2\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx = ax + bx\arctan\left(\frac{c}{x}\right) + \frac{1}{2}bc\log\left(c^2+x^2\right)$$

```
[In] Integrate[a + b*ArcTan[c/x],x]
[Out] a*x + b*x*ArcTan[c/x] + (b*c*Log[c^2 + x^2])/2
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

method	result	size
parallelrisch	$b\left(\frac{c\ln(c^2+x^2)}{2}+\arctan\left(\frac{c}{x}\right)x\right)+ax$	27
default	$ax-bc\left(-\frac{x\arctan(\frac{c}{x})}{c}-\frac{\ln\left(1+\frac{c^2}{x^2}\right)}{2}+\ln\left(\frac{c}{x}\right)\right)$	40
parts	$ax-bc\left(-\frac{x\arctan(\frac{c}{x})}{c}-\frac{\ln\left(1+\frac{c^2}{x^2}\right)}{2}+\ln\left(\frac{c}{x}\right)\right)$	40
derivativedivides	$-c\left(-\frac{ax}{c}+b\left(-\frac{x\arctan(\frac{c}{x})}{c}-\frac{\ln\left(1+\frac{c^2}{x^2}\right)}{2}+\ln\left(\frac{c}{x}\right)\right)\right)$	45
risch	Expression too large to display	642

```
[In] int(a+b*arctan(c/x),x,method=_RETURNVERBOSE)
[Out] b*(1/2*c*ln(c^2+x^2)+arctan(c/x)*x)+a*x
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx = bx\arctan\left(\frac{c}{x}\right)+\frac{1}{2}\,bc\log\left(c^2+x^2\right)+ax$$

```
[In] integrate(a+b*arctan(c/x),x, algorithm="fricas")
[Out] b*x*arctan(c/x) + 1/2*b*c*log(c^2 + x^2) + a*x
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx = ax+b\left(\frac{c\log\left(c^2+x^2\right)}{2}+x\,\mathrm{atan}\left(\frac{c}{x}\right)\right)$$

```
[In] integrate(a+b*atan(c/x),x)
[Out] a*x + b*(c*log(c**2 + x**2)/2 + x*atan(c/x))
```

Maxima [A] (verification not implemented)

none

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx = \frac{1}{2}\left(2\,x\arctan\left(\frac{c}{x}\right)+c\log\left(c^2+x^2\right)\right)b+ax$$

```
[In] integrate(a+b*arctan(c/x),x, algorithm="maxima")
[Out] 1/2*(2*x*arctan(c/x) + c*log(c^2 + x^2))*b + a*x
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.70

$$\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=ax+\frac{\left(c^2\left(\log\left(\frac{c^2}{x^2}+1\right)-\log\left(\frac{c^2}{x^2}\right)\right)+2\,cx\arctan\left(\frac{c}{x}\right)\right)b}{2\,c}$$

```
[In] integrate(a+b*arctan(c/x),x, algorithm="giac")
[Out] a*x + 1/2*(c^2*(log(c^2/x^2 + 1) - log(c^2/x^2)) + 2*c*x*arctan(c/x))*b/c
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)\,dx=a\,x+b\,x\,\mathrm{atan}\left(\frac{c}{x}\right)+\frac{b\,c\,\ln\left(c^2+x^2\right)}{2}$$

```
[In] int(a + b*atan(c/x),x)
[Out] a*x + b*x*atan(c/x) + (b*c*log(c^2 + x^2))/2
```

3.136 $\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x}\,dx$

Optimal result 775
Rubi [A] (verified) 775
Mathematica [A] (verified) 776
Maple [B] (verified) 776
Fricas [F] 777
Sympy [F] 777
Maxima [F] 777
Giac [B] (verification not implemented) 777
Mupad [B] (verification not implemented) 778

Optimal result

Integrand size = 14, antiderivative size = 39

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x}\,dx = a\log(x)-\frac{1}{2}ib\,\text{PolyLog}\left(2,-\frac{ic}{x}\right)+\frac{1}{2}ib\,\text{PolyLog}\left(2,\frac{ic}{x}\right)$$

```
[Out] a*ln(x)-1/2*I*b*polylog(2,-I*c/x)+1/2*I*b*polylog(2,I*c/x)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4944, 4940, 2438}

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x}\,dx = a\log(x)-\frac{1}{2}ib\,\text{PolyLog}\left(2,-\frac{ic}{x}\right)+\frac{1}{2}ib\,\text{PolyLog}\left(2,\frac{ic}{x}\right)$$

```
[In] Int[(a + b*ArcTan[c/x])/x,x]
```

```
[Out] a*Log[x] - (I/2)*b*PolyLog[2, ((-I)*c)/x] + (I/2)*b*PolyLog[2, (I*c)/x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1
/n, Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\text{Subst}\left(\int \frac{a+b\arctan(cx)}{x}\,dx, x, \frac{1}{x}\right)\\ &= a\log(x)-\frac{1}{2}(ib)\text{Subst}\left(\int \frac{\log(1-icx)}{x}\,dx, x, \frac{1}{x}\right)+\frac{1}{2}(ib)\text{Subst}\left(\int \frac{\log(1+icx)}{x}\,dx, x, \frac{1}{x}\right)\\ &= a\log(x)-\frac{1}{2}ib\,\text{PolyLog}\left(2,-\frac{ic}{x}\right)+\frac{1}{2}ib\,\text{PolyLog}\left(2,\frac{ic}{x}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x}\,dx = a\log(x)-\frac{1}{2}ib\,\text{PolyLog}\left(2,-\frac{ic}{x}\right)+\frac{1}{2}ib\,\text{PolyLog}\left(2,\frac{ic}{x}\right)$$

```
[In] Integrate[(a + b*ArcTan[c/x])/x,x]
[Out] a*Log[x] - (I/2)*b*PolyLog[2, ((-I)*c)/x] + (I/2)*b*PolyLog[2, (I*c)/x]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 86 vs. 2(31) = 62.

Time = 1.33 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.23

method	result
parts	$a\ln(x)+b\Big(-\ln\left(\frac{c}{x}\right)\arctan\left(\frac{c}{x}\right)-\frac{i\ln\left(\frac{c}{x}\right)\ln\left(1+\frac{ic}{x}\right)}{2}+\frac{i\ln\left(\frac{c}{x}\right)\ln\left(1-\frac{ic}{x}\right)}{2}-\frac{i\,\text{dilog}\left(1+\frac{ic}{x}\right)}{2}+\frac{i\,\text{dilog}(1-}{2}$
derivativedivides	$-a\ln\left(\frac{c}{x}\right)-b\Big(\ln\left(\frac{c}{x}\right)\arctan\left(\frac{c}{x}\right)+\frac{i\ln\left(\frac{c}{x}\right)\ln\left(1+\frac{ic}{x}\right)}{2}-\frac{i\ln\left(\frac{c}{x}\right)\ln\left(1-\frac{ic}{x}\right)}{2}+\frac{i\,\text{dilog}\left(1+\frac{ic}{x}\right)}{2}-\frac{i\,\text{dilog}(1-}{2}$
default	$-a\ln\left(\frac{c}{x}\right)-b\Big(\ln\left(\frac{c}{x}\right)\arctan\left(\frac{c}{x}\right)+\frac{i\ln\left(\frac{c}{x}\right)\ln\left(1+\frac{ic}{x}\right)}{2}-\frac{i\ln\left(\frac{c}{x}\right)\ln\left(1-\frac{ic}{x}\right)}{2}+\frac{i\,\text{dilog}\left(1+\frac{ic}{x}\right)}{2}-\frac{i\,\text{dilog}(1-}{2}$
risch	Expression too large to display

```
[In] int((a+b*arctan(c/x))/x,x,method=_RETURNVERBOSE)
```

```
[Out] a*ln(x)+b*(-ln(c/x)*arctan(c/x)-1/2*I*ln(c/x)*ln(1+I*c/x)+1/2*I*ln(c/x)*ln(
1-I*c/x)-1/2*I*dilog(1+I*c/x)+1/2*I*dilog(1-I*c/x))
```

Fricas [F]

$$\int \frac{a + b \arctan\left(\frac{c}{x}\right)}{x}\,dx = \int \frac{b \arctan\left(\frac{c}{x}\right) + a}{x}\,dx$$

```
[In] integrate((a+b*arctan(c/x))/x,x, algorithm="fricas")
[Out] integral((b*arctan(c/x) + a)/x, x)
```

Sympy [F]

$$\int \frac{a + b \arctan\left(\frac{c}{x}\right)}{x}\,dx = \int \frac{a + b \operatorname{atan}\left(\frac{c}{x}\right)}{x}\,dx$$

```
[In] integrate((a+b*atan(c/x))/x,x)
[Out] Integral((a + b*atan(c/x))/x, x)
```

Maxima [F]

$$\int \frac{a + b \arctan\left(\frac{c}{x}\right)}{x}\,dx = \int \frac{b \arctan\left(\frac{c}{x}\right) + a}{x}\,dx$$

```
[In] integrate((a+b*arctan(c/x))/x,x, algorithm="maxima")
[Out] b*integrate(arctan2(c, x)/x, x) + a*log(x)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(25) = 50$.

Time = 0.31 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.85

$$\int \frac{a + b \arctan\left(\frac{c}{x}\right)}{x}\,dx = -\frac{\left(2\,bc^4 \arctan\left(\frac{c}{x}\right) - \frac{i\,bc^6 \log\left(\frac{i\,c}{x}+1\right)}{x^2} + \frac{i\,bc^6 \log\left(-\frac{i\,c}{x}+1\right)}{x^2} + 2\,ac^4 + \frac{2\,bc^5}{x}\right)x^2}{4\,c^5}$$

```
[In] integrate((a+b*arctan(c/x))/x,x, algorithm="giac")
[Out] -1/4*(2*b*c^4*arctan(c/x) - I*b*c^6*log(I*c/x + 1)/x^2 + I*b*c^6*log(-I*c/x
 + 1)/x^2 + 2*a*c^4 + 2*b*c^5/x)*x^2/c^5
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x}\,dx = a\,\ln\left(x\right)+\frac{b\,\left(\mathrm{Li}_2\left(1-\frac{c\,1\mathrm{i}}{x}\right)-\mathrm{Li}_2\left(1+\frac{c\,1\mathrm{i}}{x}\right)\right)\,1\mathrm{i}}{2}$$

```
[In] int((a + b*atan(c/x))/x,x)
[Out] (b*(dilog(1 - (c*1i)/x) - dilog((c*1i)/x + 1))*1i)/2 + a*log(x)
```

3.137 $\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^2}\,dx$

Optimal result 779
Rubi [A] (verified) 779
Mathematica [A] (verified) 780
Maple [A] (verified) 780
Fricas [A] (verification not implemented) 781
Sympy [A] (verification not implemented) 781
Maxima [A] (verification not implemented) 781
Giac [A] (verification not implemented) 782
Mupad [B] (verification not implemented) 782

Optimal result

Integrand size = 14, antiderivative size = 34

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^2}\,dx = -\frac{a+b\arctan\left(\frac{c}{x}\right)}{x}+\frac{b\log\left(1+\frac{c^2}{x^2}\right)}{2c}$$

```
[Out] (-a-b*arctan(c/x))/x+1/2*b*ln(1+c^2/x^2)/c
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4946, 266}

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^2}\,dx = \frac{b\log\left(\frac{c^2}{x^2}+1\right)}{2c}-\frac{a+b\arctan\left(\frac{c}{x}\right)}{x}$$

```
[In] Int[(a + b*ArcTan[c/x])/x^2,x]
```

```
[Out] -((a + b*ArcTan[c/x])/x) + (b*Log[1 + c^2/x^2])/(2*c)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
```

```
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{a+b\arctan\left(\frac{c}{x}\right)}{x} - (bc)\int \frac{1}{\left(1+\frac{c^2}{x^2}\right)x^3}\,dx$$
$$= -\frac{a+b\arctan\left(\frac{c}{x}\right)}{x} + \frac{b\log\left(1+\frac{c^2}{x^2}\right)}{2c}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.09

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^2}\,dx = -\frac{a}{x} - \frac{b\arctan\left(\frac{c}{x}\right)}{x} + \frac{b\log\left(1+\frac{c^2}{x^2}\right)}{2c}$$

```
[In] Integrate[(a + b*ArcTan[c/x])/x^2,x]
[Out] -(a/x) - (b*ArcTan[c/x])/x + (b*Log[1 + c^2/x^2])/(2*c)
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

method	result	size
parts	$-\frac{a}{x}-\frac{b\arctan\left(\frac{c}{x}\right)}{x}+\frac{b\ln\left(1+\frac{c^2}{x^2}\right)}{2c}$	36
derivativedivides	$-\frac{\frac{ca}{x}+b\left(\frac{c\arctan\left(\frac{c}{x}\right)}{x}-\frac{\ln\left(1+\frac{c^2}{x^2}\right)}{2}\right)}{c}$	39
default	$-\frac{\frac{ca}{x}+b\left(\frac{c\arctan\left(\frac{c}{x}\right)}{x}-\frac{\ln\left(1+\frac{c^2}{x^2}\right)}{2}\right)}{c}$	39
parallelrisch	$-\frac{2xb\ln(x)-b\ln(c^2+x^2)x+2\arctan\left(\frac{c}{x}\right)bc+2ac}{2cx}$	42
risch	Expression too large to display	652

```
[In] int((a+b*arctan(c/x))/x^2,x,method=_RETURNVERBOSE)
[Out] -a/x-b/x*arctan(c/x)+1/2*b*ln(1+c^2/x^2)/c
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.21

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^2}\,dx = -\frac{2\,bc\arctan\left(\frac{c}{x}\right) - bx\log\left(c^2+x^2\right) + 2\,bx\log\left(x\right) + 2\,ac}{2\,cx}$$

```
[In] integrate((a+b*arctan(c/x))/x^2,x, algorithm="fricas")
[Out] -1/2*(2*b*c*arctan(c/x) - b*x*log(c^2 + x^2) + 2*b*x*log(x) + 2*a*c)/(c*x)
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^2}\,dx = \begin{cases} -\frac{a}{x} - \frac{b\,\mathrm{atan}\left(\frac{c}{x}\right)}{x} - \frac{b\log\left(x\right)}{c} + \frac{b\log\left(c^2+x^2\right)}{2c} & \text{for}\ c \neq 0 \\ -\frac{a}{x} & \text{otherwise} \end{cases}$$

```
[In] integrate((a+b*atan(c/x))/x**2,x)
[Out] Piecewise((-a/x - b*atan(c/x)/x - b*log(x)/c + b*log(c**2 + x**2)/(2*c), Ne
(c, 0)), (-a/x, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^2}\,dx = -\frac{b\left(\frac{2\,c\arctan\left(\frac{c}{x}\right)}{x} - \log\left(\frac{c^2}{x^2}+1\right)\right)}{2\,c} - \frac{a}{x}$$

```
[In] integrate((a+b*arctan(c/x))/x^2,x, algorithm="maxima")
[Out] -1/2*b*(2*c*arctan(c/x)/x - log(c^2/x^2 + 1))/c - a/x
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.15

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^2}\,dx = -\frac{\frac{2\,bc\arctan\left(\frac{c}{x}\right)}{x} - b\log\left(\frac{c^2}{x^2}+1\right) + \frac{2\,ac}{x}}{2\,c}$$

```
[In] integrate((a+b*arctan(c/x))/x^2,x, algorithm="giac")
[Out] -1/2*(2*b*c*arctan(c/x)/x - b*log(c^2/x^2 + 1) + 2*a*c/x)/c
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.26

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^2}\,dx = \frac{\frac{b\,\ln\left(c^2+x^2\right)}{2} - b\,\ln\left(x\right)}{c} - \frac{a\,c+b\,c\,\mathrm{atan}\left(\frac{c}{x}\right)}{c\,x}$$

```
[In] int((a + b*atan(c/x))/x^2,x)
[Out] ((b*log(c^2 + x^2))/2 - b*log(x))/c - (a*c + b*c*atan(c/x))/(c*x)
```

3.138 $\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^3}\,dx$

Optimal result . . . 783
Rubi [A] (verified) . . . 783
Mathematica [A] (verified) . . . 784
Maple [A] (verified) . . . 785
Fricas [A] (verification not implemented) . . . 785
Sympy [A] (verification not implemented) . . . 785
Maxima [A] (verification not implemented) . . . 786
Giac [C] (verification not implemented) . . . 786
Mupad [B] (verification not implemented) . . . 786

Optimal result

Integrand size = 14, antiderivative size = 43

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^3}\,dx=\frac{b}{2cx}-\frac{a+b\arctan\left(\frac{c}{x}\right)}{2x^2}+\frac{b\arctan\left(\frac{x}{c}\right)}{2c^2}$$

```
[Out] 1/2*b/c/x+1/2*(-a-b*arctan(c/x))/x^2+1/2*b*arctan(x/c)/c^2
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4946, 269, 331, 209}

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^3}\,dx=-\frac{a+b\arctan\left(\frac{c}{x}\right)}{2x^2}+\frac{b\arctan\left(\frac{x}{c}\right)}{2c^2}+\frac{b}{2cx}$$

```
[In] Int[(a + b*ArcTan[c/x])/x^3,x]
```

```
[Out] b/(2*c*x) - (a + b*ArcTan[c/x])/(2*x^2) + (b*ArcTan[x/c])/(2*c^2)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{a+b\arctan\left(\frac{c}{x}\right)}{2x^2}-\frac{1}{2}(bc)\int\frac{1}{\left(1+\frac{c^2}{x^2}\right)x^4}\,dx\\
&= -\frac{a+b\arctan\left(\frac{c}{x}\right)}{2x^2}-\frac{1}{2}(bc)\int\frac{1}{x^2\left(c^2+x^2\right)}\,dx\\
&= \frac{b}{2cx}-\frac{a+b\arctan\left(\frac{c}{x}\right)}{2x^2}+\frac{b\int\frac{1}{c^2+x^2}\,dx}{2c}\\
&= \frac{b}{2cx}-\frac{a+b\arctan\left(\frac{c}{x}\right)}{2x^2}+\frac{b\arctan\left(\frac{x}{c}\right)}{2c^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.12

$$
\int\frac{a+b\arctan\left(\frac{c}{x}\right)}{x^3}\,dx=-\frac{a}{2x^2}+\frac{b}{2cx}-\frac{b\arctan\left(\frac{c}{x}\right)}{2x^2}+\frac{b\arctan\left(\frac{x}{c}\right)}{2c^2}
$$

```
[In] Integrate[(a + b*ArcTan[c/x])/x^3,x]
[Out] -1/2*a/x^2 + b/(2*c*x) - (b*ArcTan[c/x])/(2*x^2) + (b*ArcTan[x/c])/(2*c^2)
```

Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

method	result	size
parts	$-\frac{a}{2x^2}-\frac{b\arctan\left(\frac{c}{x}\right)}{2x^2}+\frac{b}{2cx}+\frac{b\arctan\left(\frac{x}{c}\right)}{2c^2}$	41
parallelrisch	$-\frac{\arctan\left(\frac{c}{x}\right)b\,x^2+\arctan\left(\frac{c}{x}\right)b\,c^2-xbc+a\,c^2}{2x^2c^2}$	42
derivativedivides	$-\frac{\frac{a\,c^2}{2x^2}+b\left(\frac{c^2\arctan\left(\frac{c}{x}\right)}{2x^2}-\frac{c}{2x}+\frac{\arctan\left(\frac{c}{x}\right)}{2}\right)}{c^2}$	47
default	$-\frac{\frac{a\,c^2}{2x^2}+b\left(\frac{c^2\arctan\left(\frac{c}{x}\right)}{2x^2}-\frac{c}{2x}+\frac{\arctan\left(\frac{c}{x}\right)}{2}\right)}{c^2}$	47
risch	Expression too large to display	709

```
[In] int((a+b*arctan(c/x))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a/x^2-1/2*b/x^2*arctan(c/x)+1/2*b/c/x+1/2*b*arctan(x/c)/c^2
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^3}\,dx=-\frac{ac^2-bcx+(bc^2+bx^2)\arctan\left(\frac{c}{x}\right)}{2\,c^2x^2}$$

```
[In] integrate((a+b*arctan(c/x))/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(a*c^2 - b*c*x + (b*c^2 + b*x^2)*arctan(c/x))/(c^2*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.02

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^3}\,dx=\begin{cases}-\frac{a}{2x^2}-\frac{b\operatorname{atan}\left(\frac{c}{x}\right)}{2x^2}+\frac{b}{2cx}-\frac{b\operatorname{atan}\left(\frac{c}{x}\right)}{2c^2} & \text{for } c\neq 0\\ -\frac{a}{2x^2} & \text{otherwise}\end{cases}$$

```
[In] integrate((a+b*atan(c/x))/x**3,x)
```

```
[Out] Piecewise((-a/(2*x**2) - b*atan(c/x)/(2*x**2) + b/(2*c*x) - b*atan(c/x)/(2*
c**2), Ne(c, 0)), (-a/(2*x**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int \frac{a + b\arctan\left(\frac{c}{x}\right)}{x^3}\,dx = \frac{1}{2}\left(c\left(\frac{\arctan\left(\frac{x}{c}\right)}{c^3} + \frac{1}{c^2x}\right) - \frac{\arctan\left(\frac{c}{x}\right)}{x^2}\right)b - \frac{a}{2\,x^2}$$

```
[In] integrate((a+b*arctan(c/x))/x^3,x, algorithm="maxima")
[Out] 1/2*(c*(arctan(x/c)/c^3 + 1/(c^2*x)) - arctan(c/x)/x^2)*b - 1/2*a/x^2
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.40

$$\int \frac{a + b\arctan\left(\frac{c}{x}\right)}{x^3}\,dx = -\frac{i\left(-\frac{2i\,bc^2\arctan\left(\frac{c}{x}\right)}{x^2} + b\log\left(\frac{i\,c}{x} - 1\right) - b\log\left(-\frac{i\,c}{x} - 1\right) - \frac{2i\,ac^2}{x^2} + \frac{2i\,bc}{x}\right)}{4\,c^2}$$

```
[In] integrate((a+b*arctan(c/x))/x^3,x, algorithm="giac")
[Out] -1/4*I*(-2*I*b*c^2*arctan(c/x)/x^2 + b*log(I*c/x - 1) - b*log(-I*c/x - 1) -
 2*I*a*c^2/x^2 + 2*I*b*c/x)/c^2
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.16

$$\int \frac{a + b\arctan\left(\frac{c}{x}\right)}{x^3}\,dx = \frac{b\,c\,\mathrm{atan}\left(\frac{x}{\sqrt{c^2}}\right)}{2\,{(c^2)}^{3/2}} - \frac{\frac{a\,c^2}{2} + \frac{b\,c^2\,\mathrm{atan}\left(\frac{c}{x}\right)}{2} - \frac{b\,c\,x}{2}}{c^2\,x^2}$$

```
[In] int((a + b*atan(c/x))/x^3,x)
[Out] (b*c*atan(x/(c^2)^(1/2)))/(2*(c^2)^(3/2)) - ((a*c^2)/2 + (b*c^2*atan(c/x))/
2 - (b*c*x)/2)/(c^2*x^2)
```

3.139 $\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^4}\,dx$

Optimal result 787
Rubi [A] (verified) 787
Mathematica [A] (verified) 788
Maple [A] (verified) 789
Fricas [A] (verification not implemented) 789
Sympy [A] (verification not implemented) 789
Maxima [A] (verification not implemented) 790
Giac [A] (verification not implemented) 790
Mupad [B] (verification not implemented) 790

Optimal result

Integrand size = 14, antiderivative size = 55

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^4}\,dx=\frac{b}{6cx^2}-\frac{a+b\arctan\left(\frac{c}{x}\right)}{3x^3}+\frac{b\log(x)}{3c^3}-\frac{b\log\left(c^2+x^2\right)}{6c^3}$$

```
[Out] 1/6*b/c/x^2+1/3*(-a-b*arctan(c/x))/x^3+1/3*b*ln(x)/c^3-1/6*b*ln(c^2+x^2)/c^
3
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4946, 269, 272, 46}

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^4}\,dx=-\frac{a+b\arctan\left(\frac{c}{x}\right)}{3x^3}+\frac{b\log(x)}{3c^3}-\frac{b\log\left(c^2+x^2\right)}{6c^3}+\frac{b}{6cx^2}$$

```
[In] Int[(a + b*ArcTan[c/x])/x^4,x]
```

```
[Out] b/(6*c*x^2) - (a + b*ArcTan[c/x])/(3*x^3) + (b*Log[x])/(3*c^3) - (b*Log[c^2
 + x^2])/(6*c^3)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{a+b\arctan\left(\frac{c}{x}\right)}{3x^3}-\frac{1}{3}(bc)\int\frac{1}{\left(1+\frac{c^2}{x^2}\right)x^5}\,dx\\
&= -\frac{a+b\arctan\left(\frac{c}{x}\right)}{3x^3}-\frac{1}{3}(bc)\int\frac{1}{x^3\left(c^2+x^2\right)}\,dx\\
&= -\frac{a+b\arctan\left(\frac{c}{x}\right)}{3x^3}-\frac{1}{6}(bc)\text{Subst}\left(\int\frac{1}{x^2\left(c^2+x\right)}\,dx,x,x^2\right)\\
&= -\frac{a+b\arctan\left(\frac{c}{x}\right)}{3x^3}-\frac{1}{6}(bc)\text{Subst}\left(\int\left(\frac{1}{c^2x^2}-\frac{1}{c^4x}+\frac{1}{c^4\left(c^2+x\right)}\right)\,dx,x,x^2\right)\\
&= \frac{b}{6cx^2}-\frac{a+b\arctan\left(\frac{c}{x}\right)}{3x^3}+\frac{b\log(x)}{3c^3}-\frac{b\log\left(c^2+x^2\right)}{6c^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.09

$$
\int\frac{a+b\arctan\left(\frac{c}{x}\right)}{x^4}\,dx=-\frac{a}{3x^3}+\frac{b}{6cx^2}-\frac{b\arctan\left(\frac{c}{x}\right)}{3x^3}+\frac{b\log(x)}{3c^3}-\frac{b\log\left(c^2+x^2\right)}{6c^3}
$$

```
[In] Integrate[(a + b*ArcTan[c/x])/x^4,x]
```

```
[Out] -1/3*a/x^3 + b/(6*c*x^2) - (b*ArcTan[c/x])/(3*x^3) + (b*Log[x])/(3*c^3) - (
b*Log[c^2 + x^2])/(6*c^3)
```

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.82

method	result	size
parts	$-\frac{a}{3x^3}-\frac{b\arctan\left(\frac{c}{x}\right)}{3x^3}+\frac{b}{6c\,x^2}-\frac{b\ln\left(1+\frac{c^2}{x^2}\right)}{6c^3}$	45
derivativedivides	$-\frac{\frac{a\,c^3}{3x^3}+b\left(\frac{c^3\arctan\left(\frac{c}{x}\right)}{3x^3}-\frac{c^2}{6x^2}+\frac{\ln\left(1+\frac{c^2}{x^2}\right)}{6}\right)}{c^3}$	53
default	$-\frac{\frac{a\,c^3}{3x^3}+b\left(\frac{c^3\arctan\left(\frac{c}{x}\right)}{3x^3}-\frac{c^2}{6x^2}+\frac{\ln\left(1+\frac{c^2}{x^2}\right)}{6}\right)}{c^3}$	53
parallelrisch	$\frac{2b\ln(x)x^3-b\ln(c^2+x^2)x^3-2b\,c^3\arctan\left(\frac{c}{x}\right)+b\,c^2x-2a\,c^3}{6x^3c^3}$	56
risch	Expression too large to display	705

```
[In] int((a+b*arctan(c/x))/x^4,x,method=_RETURNVERBOSE)
[Out] -1/3*a/x^3-1/3*b/x^3*arctan(c/x)+1/6*b/c/x^2-1/6*b/c^3*ln(1+c^2/x^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^4}\,dx=-\frac{2\,bc^3\arctan\left(\frac{c}{x}\right)+bx^3\log\left(c^2+x^2\right)-2\,bx^3\log\left(x\right)+2\,ac^3-bc^2x}{6\,c^3x^3}$$

```
[In] integrate((a+b*arctan(c/x))/x^4,x, algorithm="fricas")
[Out] -1/6*(2*b*c^3*arctan(c/x) + b*x^3*log(c^2 + x^2) - 2*b*x^3*log(x) + 2*a*c^3
 - b*c^2*x)/(c^3*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.09

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^4}\,dx=\begin{cases}-\frac{a}{3x^3}-\frac{b\operatorname{atan}\left(\frac{c}{x}\right)}{3x^3}+\frac{b}{6cx^2}+\frac{b\log\left(x\right)}{3c^3}-\frac{b\log\left(c^2+x^2\right)}{6c^3} & \text{for}\ c\neq 0\\ -\frac{a}{3x^3} & \text{otherwise}\end{cases}$$

```
[In] integrate((a+b*atan(c/x))/x**4,x)
[Out] Piecewise((-a/(3*x**3) - b*atan(c/x)/(3*x**3) + b/(6*c*x**2) + b*log(x)/(3*
c**3) - b*log(c**2 + x**2)/(6*c**3), Ne(c, 0)), (-a/(3*x**3), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.98

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^4}\,dx = -\frac{1}{6}\left(c\left(\frac{\log\left(c^2+x^2\right)}{c^4}-\frac{\log\left(x^2\right)}{c^4}-\frac{1}{c^2x^2}\right)+\frac{2\arctan\left(\frac{c}{x}\right)}{x^3}\right)b-\frac{a}{3\,x^3}$$

```
[In] integrate((a+b*arctan(c/x))/x^4,x, algorithm="maxima")
[Out] -1/6*(c*(log(c^2 + x^2)/c^4 - log(x^2)/c^4 - 1/(c^2*x^2)) + 2*arctan(c/x)/x
^3)*b - 1/3*a/x^3
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^4}\,dx = -\frac{\frac{2\,bc^3\arctan\left(\frac{c}{x}\right)}{x^3}+b\log\left(\frac{c^2}{x^2}+1\right)+\frac{2\,ac^3}{x^3}-\frac{bc^2}{x^2}}{6\,c^3}$$

```
[In] integrate((a+b*arctan(c/x))/x^4,x, algorithm="giac")
[Out] -1/6*(2*b*c^3*arctan(c/x)/x^3 + b*log(c^2/x^2 + 1) + 2*a*c^3/x^3 - b*c^2/x^
2)/c^3
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int \frac{a+b\arctan\left(\frac{c}{x}\right)}{x^4}\,dx = \frac{\frac{b\,x^3\,\ln(x)}{3}+\frac{b\,c^2\,x}{6}-\frac{b\,x^3\,\ln\left(c^2+x^2\right)}{6}}{c^3\,x^3}-\frac{\frac{a}{3}+\frac{b\,\mathrm{atan}\left(\frac{c}{x}\right)}{3}}{x^3}$$

```
[In] int((a + b*atan(c/x))/x^4,x)
[Out] ((b*x^3*log(x))/3 + (b*c^2*x)/6 - (b*x^3*log(c^2 + x^2))/6)/(c^3*x^3) - (a/
3 + (b*atan(c/x))/3)/x^3
```

3.140 $\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx$

Optimal result 791
Rubi [A] (verified) 791
Mathematica [A] (verified) 794
Maple [A] (verified) 794
Fricas [A] (verification not implemented) 795
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Giac [F] 797
Mupad [B] (verification not implemented) 797

Optimal result

Integrand size = 16, antiderivative size = 122

$$\begin{aligned}\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx=&\ \frac{1}{12}b^2c^2x^2-\frac{1}{2}bc^3x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\\&+\frac{1}{6}bcx^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\\&-\frac{1}{4}c^4\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{1}{4}x^4\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\\&-\frac{1}{3}b^2c^4\log\left(1+\frac{c^2}{x^2}\right)-\frac{2}{3}b^2c^4\log(x)\end{aligned}$$

```
[Out] 1/12*b^2*c^2*x^2-1/2*b*c^3*x*(a+b*arccot(x/c))+1/6*b*c*x^3*(a+b*arccot(x/c)
)-1/4*c^4*(a+b*arccot(x/c))^2+1/4*x^4*(a+b*arccot(x/c))^2-1/3*b^2*c^4*ln(1+
c^2/x^2)-2/3*b^2*c^4*ln(x)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {4948, 4946, 5038, 272, 46, 36, 29, 31, 5004}

$$\begin{aligned}\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx=&-\frac{1}{4}c^4\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2-\frac{1}{2}bc^3x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\\&+\frac{1}{4}x^4\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{1}{6}bcx^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\\&-\frac{2}{3}b^2c^4\log(x)+\frac{1}{12}b^2c^2x^2-\frac{1}{3}b^2c^4\log\left(\frac{c^2}{x^2}+1\right)\end{aligned}$$

```
[In] Int[x^3*(a + b*ArcTan[c/x])^2,x]
```

```
[Out] (b^2*c^2*x^2)/12 - (b*c^3*x*(a + b*ArcCot[x/c]))/2 + (b*c*x^3*(a + b*ArcCot
[x/c]))/6 - (c^4*(a + b*ArcCot[x/c])^2)/4 + (x^4*(a + b*ArcCot[x/c])^2)/4 -
 (b^2*c^4*Log[1 + c^2/x^2])/3 - (2*b^2*c^4*Log[x])/3
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplify
[(m + 1)/n]]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{(a+b\arctan(cx))^2}{x^5}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{4}x^4\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2 - \frac{1}{2}(bc)\text{Subst}\left(\int \frac{a+b\arctan(cx)}{x^4(1+c^2x^2)}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{4}x^4\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2 - \frac{1}{2}(bc)\text{Subst}\left(\int \frac{a+b\arctan(cx)}{x^4}\,dx, x, \frac{1}{x}\right)\\
&\quad + \frac{1}{2}\left(bc^3\right)\text{Subst}\left(\int \frac{a+b\arctan(cx)}{x^2(1+c^2x^2)}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{6}bcx^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right) + \frac{1}{4}x^4\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\\
&\quad - \frac{1}{6}\left(b^2c^2\right)\text{Subst}\left(\int \frac{1}{x^3(1+c^2x^2)}\,dx, x, \frac{1}{x}\right)\\
&\quad + \frac{1}{2}\left(bc^3\right)\text{Subst}\left(\int \frac{a+b\arctan(cx)}{x^2}\,dx, x, \frac{1}{x}\right)\\
&\quad - \frac{1}{2}\left(bc^5\right)\text{Subst}\left(\int \frac{a+b\arctan(cx)}{1+c^2x^2}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{1}{2}bc^3x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right) + \frac{1}{6}bcx^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right) - \frac{1}{4}c^4\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\\
&\quad + \frac{1}{4}x^4\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2 - \frac{1}{12}\left(b^2c^2\right)\text{Subst}\left(\int \frac{1}{x^2(1+c^2x)}\,dx, x, \frac{1}{x^2}\right)\\
&\quad + \frac{1}{2}\left(b^2c^4\right)\text{Subst}\left(\int \frac{1}{x(1+c^2x^2)}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{1}{2}bc^3x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right) + \frac{1}{6}bcx^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right) - \frac{1}{4}c^4\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\\
&\quad + \frac{1}{4}x^4\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2 - \frac{1}{12}\left(b^2c^2\right)\text{Subst}\left(\int \left(\frac{1}{x^2}-\frac{c^2}{x}+\frac{c^4}{1+c^2x}\right)dx, x, \frac{1}{x^2}\right)\\
&\quad + \frac{1}{4}\left(b^2c^4\right)\text{Subst}\left(\int \frac{1}{x(1+c^2x)}\,dx, x, \frac{1}{x^2}\right)
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{1}{12}b^2c^2x^2 - \frac{1}{2}bc^3x\left(a + b\cot^{-1}\left(\frac{x}{c}\right)\right) + \frac{1}{6}bcx^3\left(a + b\cot^{-1}\left(\frac{x}{c}\right)\right) \\
&\quad - \frac{1}{4}c^4\left(a + b\cot^{-1}\left(\frac{x}{c}\right)\right)^2 + \frac{1}{4}x^4\left(a + b\cot^{-1}\left(\frac{x}{c}\right)\right)^2 \\
&\quad - \frac{1}{12}b^2c^4\log\left(1 + \frac{c^2}{x^2}\right) - \frac{1}{6}b^2c^4\log(x) + \frac{1}{4}\left(b^2c^4\right)\text{Subst}\left(\int \frac{1}{x}\,dx, x, \frac{1}{x^2}\right) \\
&\quad - \frac{1}{4}\left(b^2c^6\right)\text{Subst}\left(\int \frac{1}{1+c^2x}\,dx, x, \frac{1}{x^2}\right) \\
&= \frac{1}{12}b^2c^2x^2 - \frac{1}{2}bc^3x\left(a + b\cot^{-1}\left(\frac{x}{c}\right)\right) + \frac{1}{6}bcx^3\left(a + b\cot^{-1}\left(\frac{x}{c}\right)\right) \\
&\quad - \frac{1}{4}c^4\left(a + b\cot^{-1}\left(\frac{x}{c}\right)\right)^2 + \frac{1}{4}x^4\left(a + b\cot^{-1}\left(\frac{x}{c}\right)\right)^2 \\
&\quad - \frac{1}{3}b^2c^4\log\left(1 + \frac{c^2}{x^2}\right) - \frac{2}{3}b^2c^4\log(x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.91

$$
\begin{aligned}
\int x^3\left(a + b\arctan\left(\frac{c}{x}\right)\right)^2\,dx = \frac{1}{12}\Big(& x\left(b^2c^2x + 3a^2x^3 + 2abc\left(-3c^2 + x^2\right)\right) \\
&+ 2b\left(bcx\left(-3c^2 + x^2\right) + 3a\left(-c^4 + x^4\right)\right)\arctan\left(\frac{c}{x}\right) \\
&+ 3b^2\left(-c^4 + x^4\right)\arctan\left(\frac{c}{x}\right)^2 - 4b^2c^4\log\left(c^2 + x^2\right)\Big)
\end{aligned}
$$

```
[In] Integrate[x^3*(a + b*ArcTan[c/x])^2,x]
```

```
[Out] (x*(b^2*c^2*x + 3*a^2*x^3 + 2*a*b*c*(-3*c^2 + x^2)) + 2*b*(b*c*x*(-3*c^2 +
x^2) + 3*a*(-c^4 + x^4))*ArcTan[c/x] + 3*b^2*(-c^4 + x^4)*ArcTan[c/x]^2 - 4
*b^2*c^4*Log[c^2 + x^2])/12
```

Maple [A] (verified)

Time = 3.22 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.16

method	result
parts	$\frac{a^2x^4}{4}-b^2c^4\left(-\frac{x^4\arctan\left(\frac{c}{x}\right)^2}{4c^4}+\frac{\arctan\left(\frac{c}{x}\right)^2}{4}-\frac{x^3\arctan\left(\frac{c}{x}\right)}{6c^3}+\frac{x\arctan\left(\frac{c}{x}\right)}{2c}+\frac{\ln\left(1+\frac{c^2}{x^2}\right)}{3}-\frac{x^2}{12c^2}-\frac{2\ln}{3}\right.$
derivativedivides	$-c^4\left(-\frac{a^2x^4}{4c^4}+b^2\left(-\frac{x^4\arctan\left(\frac{c}{x}\right)^2}{4c^4}+\frac{\arctan\left(\frac{c}{x}\right)^2}{4}-\frac{x^3\arctan\left(\frac{c}{x}\right)}{6c^3}+\frac{x\arctan\left(\frac{c}{x}\right)}{2c}+\frac{\ln\left(1+\frac{c^2}{x^2}\right)}{3}-\frac{x^2}{12c^2}\right.\right.$
default	$-c^4\left(-\frac{a^2x^4}{4c^4}+b^2\left(-\frac{x^4\arctan\left(\frac{c}{x}\right)^2}{4c^4}+\frac{\arctan\left(\frac{c}{x}\right)^2}{4}-\frac{x^3\arctan\left(\frac{c}{x}\right)}{6c^3}+\frac{x\arctan\left(\frac{c}{x}\right)}{2c}+\frac{\ln\left(1+\frac{c^2}{x^2}\right)}{3}-\frac{x^2}{12c^2}\right.\right.$
parallelrisch	$\frac{x^4\arctan\left(\frac{c}{x}\right)^2b^2}{4}-\frac{\arctan\left(\frac{c}{x}\right)^2b^2c^4}{4}-\frac{b^2c^4\ln\left(c^2+x^2\right)}{3}+\frac{x^4\arctan\left(\frac{c}{x}\right)ab}{2}+\frac{x^3\arctan\left(\frac{c}{x}\right)b^2c}{6}-\frac{x\arctan\left(\frac{c}{x}\right)b^2c}{2}$
risch	Expression too large to display

```
[In] int(x^3*(a+b*arctan(c/x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a^2*x^4-b^2*c^4*(-1/4/c^4*x^4*arctan(c/x)^2+1/4*arctan(c/x)^2-1/6/c^3*x
^3*arctan(c/x)+1/2/c*x*arctan(c/x)+1/3*ln(1+c^2/x^2)-1/12/c^2*x^2-2/3*ln(c/
x))+1/2*x^4*arctan(c/x)*a*b+1/6*a*b*c*x^3-1/2*a*b*c^3*x+1/2*a*b*c^4*arctan(
x/c)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.02

$$\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx=\frac{1}{2}\,abc^4\arctan\left(\frac{x}{c}\right)-\frac{1}{3}\,b^2c^4\log\left(c^2+x^2\right)-\frac{1}{2}\,abc^3x+\frac{1}{12}\,b^2c^2x^2+\frac{1}{6}\,abcx^3+\frac{1}{4}\,a^2x^4-\frac{1}{4}\left(b^2c^4-b^2x^4\right)\arctan\left(\frac{c}{x}\right)^2-\frac{1}{6}\left(3\,b^2c^3x-b^2cx^3-3\,abx^4\right)\arctan\left(\frac{c}{x}\right)$$

```
[In] integrate(x^3*(a+b*arctan(c/x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*a*b*c^4*arctan(x/c) - 1/3*b^2*c^4*log(c^2 + x^2) - 1/2*a*b*c^3*x + 1/12
*b^2*c^2*x^2 + 1/6*a*b*c*x^3 + 1/4*a^2*x^4 - 1/4*(b^2*c^4 - b^2*x^4)*arctan
(c/x)^2 - 1/6*(3*b^2*c^3*x - b^2*c*x^3 - 3*a*b*x^4)*arctan(c/x)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.18

$$\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx=\frac{a^2x^4}{4}-\frac{abc^4\operatorname{atan}\left(\frac{c}{x}\right)}{2}-\frac{abc^3x}{2}+\frac{abcx^3}{6}+\frac{abx^4\operatorname{atan}\left(\frac{c}{x}\right)}{2}-\frac{b^2c^4\log\left(c^2+x^2\right)}{3}-\frac{b^2c^4\operatorname{atan}^2\left(\frac{c}{x}\right)}{4}-\frac{b^2c^3x\operatorname{atan}\left(\frac{c}{x}\right)}{2}+\frac{b^2c^2x^2}{12}+\frac{b^2cx^3\operatorname{atan}\left(\frac{c}{x}\right)}{6}+\frac{b^2x^4\operatorname{atan}^2\left(\frac{c}{x}\right)}{4}$$

```
[In] integrate(x**3*(a+b*atan(c/x))**2,x)
```

```
[Out] a**2*x**4/4 - a*b*c**4*atan(c/x)/2 - a*b*c**3*x/2 + a*b*c*x**3/6 + a*b*x**4
*atan(c/x)/2 - b**2*c**4*log(c**2 + x**2)/3 - b**2*c**4*atan(c/x)**2/4 - b*
*2*c**3*x*atan(c/x)/2 + b**2*c**2*x**2/12 + b**2*c*x**3*atan(c/x)/6 + b**2*
x**4*atan(c/x)**2/4
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.10

$$\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx=\frac{1}{4}\,b^2x^4\arctan\left(\frac{c}{x}\right)^2+\frac{1}{4}\,a^2x^4+\frac{1}{6}\left(3\,x^4\arctan\left(\frac{c}{x}\right)+\left(3\,c^3\arctan\left(\frac{x}{c}\right)-3\,c^2x+x^3\right)c\right)ab+\frac{1}{12}\left(\left(3\,c^2\arctan\left(\frac{x}{c}\right)^2-4\,c^2\log\left(c^2+x^2\right)+x^2\right)c^2+2\left(3\,c^3\arctan\left(\frac{x}{c}\right)-3\,c^2x+x^3\right)c\arctan\left(\frac{c}{x}\right)\right)b^2$$

```
[In] integrate(x^3*(a+b*arctan(c/x))^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*x^4*arctan(c/x)^2 + 1/4*a^2*x^4 + 1/6*(3*x^4*arctan(c/x) + (3*c^3*a
rctan(x/c) - 3*c^2*x + x^3)*c)*a*b + 1/12*((3*c^2*arctan(x/c)^2 - 4*c^2*log
(c^2 + x^2) + x^2)*c^2 + 2*(3*c^3*arctan(x/c) - 3*c^2*x + x^3)*c*arctan(c/x
))*b^2
```

Giac [F]

$$\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx=\int\left(b\arctan\left(\frac{c}{x}\right)+a\right)^2x^3\,dx$$

```
[In] integrate(x^3*(a+b*arctan(c/x))^2,x, algorithm="giac")
[Out] integrate((b*arctan(c/x) + a)^2*x^3, x)
```

Mupad [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.15

$$\begin{aligned}\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx&=\frac{a^2\,x^4}{4}-\frac{b^2\,c^4\,\mathrm{atan}\left(\frac{c}{x}\right)^2}{4}-\frac{b^2\,c^4\,\ln\left(c^2+x^2\right)}{3}+\frac{b^2\,x^4\,\mathrm{atan}\left(\frac{c}{x}\right)^2}{4}\\&\quad+\frac{b^2\,c^2\,x^2}{12}+\frac{b^2\,c\,x^3\,\mathrm{atan}\left(\frac{c}{x}\right)}{6}-\frac{b^2\,c^3\,x\,\mathrm{atan}\left(\frac{c}{x}\right)}{2}\\&\quad+\frac{a\,b\,c\,x^3}{6}-\frac{a\,b\,c^3\,x}{2}-\frac{a\,b\,c^4\,\mathrm{atan}\left(\frac{c}{x}\right)}{2}+\frac{a\,b\,x^4\,\mathrm{atan}\left(\frac{c}{x}\right)}{2}\end{aligned}$$

```
[In] int(x^3*(a + b*atan(c/x))^2,x)
[Out] (a^2*x^4)/4 - (b^2*c^4*atan(c/x)^2)/4 - (b^2*c^4*log(c^2 + x^2))/3 + (b^2*x
^4*atan(c/x)^2)/4 + (b^2*c^2*x^2)/12 + (b^2*c*x^3*atan(c/x))/6 - (b^2*c^3*x
*atan(c/x))/2 + (a*b*c*x^3)/6 - (a*b*c^3*x)/2 - (a*b*c^4*atan(c/x))/2 + (a*
b*x^4*atan(c/x))/2
```

3.141 $\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx$

Optimal result 798
Rubi [A] (verified) 799
Mathematica [A] (verified) 801
Maple [B] (verified) 802
Fricas [F] 802
Sympy [F] 803
Maxima [F] 803
Giac [F] 803
Mupad [F(-1)] 803

Optimal result

Integrand size = 16, antiderivative size = 152

$$\begin{aligned}\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx &= \frac{1}{3}b^2c^2x+\frac{1}{3}b^2c^3\cot^{-1}\left(\frac{x}{c}\right)+\frac{1}{3}bcx^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\\&\quad-\frac{1}{3}ic^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{1}{3}x^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\\&\quad+\frac{2}{3}bc^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\log\left(2-\frac{2}{1-\frac{ic}{x}}\right)\\&\quad-\frac{1}{3}ib^2c^3\,\mathrm{PolyLog}\left(2,-1+\frac{2}{1-\frac{ic}{x}}\right)\end{aligned}$$

```
[Out] 1/3*b^2*c^2*x+1/3*b^2*c^3*arccot(x/c)+1/3*b*c*x^2*(a+b*arccot(x/c))-1/3*I*c
^3*(a+b*arccot(x/c))^2+1/3*x^3*(a+b*arccot(x/c))^2+2/3*b*c^3*(a+b*arccot(x/
c))*ln(2-2/(1-I*c/x))-1/3*I*b^2*c^3*polylog(2,-1+2/(1-I*c/x))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4948, 4946, 5038, 331, 209, 5044, 4988, 2497}

$$\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2dx = -\frac{1}{3}ic^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2 +\frac{2}{3}bc^3\log\left(2-\frac{2}{1-\frac{ic}{x}}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right) +\frac{1}{3}x^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{1}{3}bcx^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right) -\frac{1}{3}ib^2c^3\operatorname{PolyLog}\left(2,\frac{2}{1-\frac{ic}{x}}-1\right) +\frac{1}{3}b^2c^3\cot^{-1}\left(\frac{x}{c}\right)+\frac{1}{3}b^2c^2x$$

```
[In] Int[x^2*(a + b*ArcTan[c/x])^2,x]
```

```
[Out] (b^2*c^2*x)/3 + (b^2*c^3*ArcCot[x/c])/3 + (b*c*x^2*(a + b*ArcCot[x/c]))/3 -
 (I/3)*c^3*(a + b*ArcCot[x/c])^2 + (x^3*(a + b*ArcCot[x/c])^2)/3 + (2*b*c^3
*(a + b*ArcCot[x/c])*Log[2 - 2/(1 - (I*c)/x)])/3 - (I/3)*b^2*c^3*PolyLog[2,
 -1 + 2/(1 - (I*c)/x)]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplify
[(m + 1)/n]]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{(a+b\arctan(cx))^2}{x^4}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{3}x^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2 - \frac{1}{3}(2bc)\text{Subst}\left(\int \frac{a+b\arctan(cx)}{x^3\left(1+c^2x^2\right)}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{3}x^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2 - \frac{1}{3}(2bc)\text{Subst}\left(\int \frac{a+b\arctan(cx)}{x^3}\,dx, x, \frac{1}{x}\right)\\
&\quad + \frac{1}{3}\left(2bc^3\right)\text{Subst}\left(\int \frac{a+b\arctan(cx)}{x\left(1+c^2x^2\right)}\,dx, x, \frac{1}{x}\right)
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{1}{3}bcx^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right) - \frac{1}{3}ic^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2 \\
&\quad + \frac{1}{3}x^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2 - \frac{1}{3}\left(b^2c^2\right)\text{Subst}\left(\int \frac{1}{x^2\left(1+c^2x^2\right)}\,dx, x, \frac{1}{x}\right) \\
&\quad + \frac{1}{3}\left(2ibc^3\right)\text{Subst}\left(\int \frac{a+b\arctan(cx)}{x(i+cx)}\,dx, x, \frac{1}{x}\right) \\
&= \frac{1}{3}b^2c^2x + \frac{1}{3}bcx^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right) - \frac{1}{3}ic^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2 \\
&\quad + \frac{1}{3}x^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2 + \frac{2}{3}bc^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\log\left(2-\frac{2}{1-\frac{ic}{x}}\right) \\
&\quad + \frac{1}{3}\left(b^2c^4\right)\text{Subst}\left(\int \frac{1}{1+c^2x^2}\,dx, x, \frac{1}{x}\right) \\
&\quad - \frac{1}{3}\left(2b^2c^4\right)\text{Subst}\left(\int \frac{\log\left(2-\frac{2}{1-icx}\right)}{1+c^2x^2}\,dx, x, \frac{1}{x}\right) \\
&= \frac{1}{3}b^2c^2x + \frac{1}{3}b^2c^3\cot^{-1}\left(\frac{x}{c}\right) + \frac{1}{3}bcx^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right) \\
&\quad - \frac{1}{3}ic^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2 + \frac{1}{3}x^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2 \\
&\quad + \frac{2}{3}bc^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\log\left(2-\frac{2}{1-\frac{ic}{x}}\right) - \frac{1}{3}ib^2c^3\,\text{PolyLog}\left(2, -1+\frac{2}{1-\frac{ic}{x}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00

$$
\begin{aligned}
\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2 dx = \frac{1}{3}\Bigg(& b^2c^2x + abcx^2 + a^2x^3 + b^2\left(-ic^3+x^3\right)\arctan\left(\frac{c}{x}\right)^2 \\
&+ b\arctan\left(\frac{c}{x}\right)\left(2ax^3 + bc\left(c^2+x^2\right)\right. \\
&\left.+ 2bc^3\log\left(1-e^{2i\arctan\left(\frac{c}{x}\right)}\right)\right) - abc^3\log\left(1+\frac{c^2}{x^2}\right) \\
&+ 2abc^3\log\left(\frac{c}{x}\right) - ib^2c^3\,\text{PolyLog}\left(2, e^{2i\arctan\left(\frac{c}{x}\right)}\right)\Bigg)
\end{aligned}
$$

```
[In] Integrate[x^2*(a + b*ArcTan[c/x])^2,x]
```

```
[Out] (b^2*c^2*x + a*b*c*x^2 + a^2*x^3 + b^2*((-I)*c^3 + x^3)*ArcTan[c/x]^2 + b*A
rcTan[c/x]*(2*a*x^3 + b*c*(c^2 + x^2) + 2*b*c^3*Log[1 - E^((2*I)*ArcTan[c/x
])]) - a*b*c^3*Log[1 + c^2/x^2] + 2*a*b*c^3*Log[c/x] - I*b^2*c^3*PolyLog[2,
 E^((2*I)*ArcTan[c/x])])/3
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 357 vs. $2(134) = 268$.

Time = 3.78 (sec) , antiderivative size = 358, normalized size of antiderivative = 2.36

method	result
derivativedivides	$-c^3\left(-\frac{a^2x^3}{3c^3}+b^2\left(-\frac{x^3\arctan\left(\frac{c}{x}\right)^2}{3c^3}+\frac{\arctan\left(\frac{c}{x}\right)\ln\left(1+\frac{c^2}{x^2}\right)}{3}-\frac{x^2\arctan\left(\frac{c}{x}\right)}{3c^2}-\frac{2\ln\left(\frac{c}{x}\right)\arctan\left(\frac{c}{x}\right)}{3}+\frac{i\left(\ln\left(\right.\right.}{}\right.\right.$
default	$-c^3\left(-\frac{a^2x^3}{3c^3}+b^2\left(-\frac{x^3\arctan\left(\frac{c}{x}\right)^2}{3c^3}+\frac{\arctan\left(\frac{c}{x}\right)\ln\left(1+\frac{c^2}{x^2}\right)}{3}-\frac{x^2\arctan\left(\frac{c}{x}\right)}{3c^2}-\frac{2\ln\left(\frac{c}{x}\right)\arctan\left(\frac{c}{x}\right)}{3}+\frac{i\left(\ln\left(\right.\right.}{}\right.\right.$
parts	$\frac{a^2x^3}{3}+\frac{b^2x^3\arctan\left(\frac{c}{x}\right)^2}{3}+\frac{c\,b^2x^2\arctan\left(\frac{c}{x}\right)}{3}+\frac{2c^3b^2\ln\left(\frac{c}{x}\right)\arctan\left(\frac{c}{x}\right)}{3}-\frac{c^3b^2\arctan\left(\frac{c}{x}\right)\ln\left(1+\frac{c^2}{x^2}\right)}{3}-\frac{ic^3b^2\ln\left(\frac{c}{x}\right.}{12}$
risch	Expression too large to display

```
[In] int(x^2*(a+b*arctan(c/x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -c^3*(-1/3*a^2/c^3*x^3+b^2*(-1/3/c^3*x^3*arctan(c/x)^2+1/3*arctan(c/x)*ln(1
+c^2/x^2)-1/3/c^2*x^2*arctan(c/x)-2/3*ln(c/x)*arctan(c/x)+1/6*I*(ln(c/x-I)*
ln(1+c^2/x^2)-1/2*ln(c/x-I)^2-dilog(-1/2*I*(c/x+I))-ln(c/x-I)*ln(-1/2*I*(c/
x+I)))-1/6*I*(ln(c/x+I)*ln(1+c^2/x^2)-1/2*ln(c/x+I)^2-dilog(1/2*I*(c/x-I))-
ln(c/x+I)*ln(1/2*I*(c/x-I)))-1/3*arctan(c/x)-1/3*x/c-1/3*I*ln(c/x)*ln(1+I*c
/x)+1/3*I*ln(c/x)*ln(1-I*c/x)-1/3*I*dilog(1+I*c/x)+1/3*I*dilog(1-I*c/x))+2*
a*b*(-1/3/c^3*x^3*arctan(c/x)+1/6*ln(1+c^2/x^2)-1/6/c^2*x^2-1/3*ln(c/x)))
```

Fricas [F]

$$\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx=\int\left(b\arctan\left(\frac{c}{x}\right)+a\right)^2x^2\,dx$$

```
[In] integrate(x^2*(a+b*arctan(c/x))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^2*arctan(c/x)^2 + 2*a*b*x^2*arctan(c/x) + a^2*x^2, x)
```

Sympy [F]

$$\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx=\int x^2\left(a+b\operatorname{atan}\left(\frac{c}{x}\right)\right)^2\,dx$$

```
[In] integrate(x**2*(a+b*atan(c/x))**2,x)
[Out] Integral(x**2*(a + b*atan(c/x))**2, x)
```

Maxima [F]

$$\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx=\int\left(b\arctan\left(\frac{c}{x}\right)+a\right)^2x^2\,dx$$

```
[In] integrate(x^2*(a+b*arctan(c/x))^2,x, algorithm="maxima")
[Out] 1/3*a^2*x^3 + 1/3*(2*x^3*arctan(c/x) - (c^2*log(c^2 + x^2) - x^2)*c)*a*b +
1/48*(4*x^3*arctan2(c, x)^2 - x^3*log(c^2 + x^2)^2 + 48*integrate(1/48*(36*
c^2*x^2*arctan2(c, x)^2 + 36*x^4*arctan2(c, x)^2 + 8*c*x^3*arctan2(c, x) +
4*x^4*log(c^2 + x^2) + 3*(c^2*x^2 + x^4)*log(c^2 + x^2)^2)/(c^2 + x^2), x))
*b^2
```

Giac [F]

$$\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx=\int\left(b\arctan\left(\frac{c}{x}\right)+a\right)^2x^2\,dx$$

```
[In] integrate(x^2*(a+b*arctan(c/x))^2,x, algorithm="giac")
[Out] integrate((b*arctan(c/x) + a)^2*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx=\int x^2\left(a+b\operatorname{atan}\left(\frac{c}{x}\right)\right)^2\,dx$$

```
[In] int(x^2*(a + b*atan(c/x))^2,x)
[Out] int(x^2*(a + b*atan(c/x))^2, x)
```

3.142 $\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx$

Optimal result 804
Rubi [A] (verified) 804
Mathematica [A] (verified) 806
Maple [A] (verified) 807
Fricas [A] (verification not implemented) 807
Sympy [A] (verification not implemented) 808
Maxima [A] (verification not implemented) 808
Giac [F] 808
Mupad [B] (verification not implemented) 809

Optimal result

Integrand size = 14, antiderivative size = 82

$$\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx=bcx\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)+\frac{1}{2}c^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{1}{2}x^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{1}{2}b^2c^2\log\left(1+\frac{c^2}{x^2}\right)+b^2c^2\log(x)$$

```
[Out] b*c*x*(a+b*arccot(x/c))+1/2*c^2*(a+b*arccot(x/c))^2+1/2*x^2*(a+b*arccot(x/c
))^2+1/2*b^2*c^2*ln(1+c^2/x^2)+b^2*c^2*ln(x)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {4948, 4946, 5038, 272, 36, 29, 31, 5004}

$$\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx=\frac{1}{2}c^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{1}{2}x^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+bcx\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)+\frac{1}{2}b^2c^2\log\left(\frac{c^2}{x^2}+1\right)+b^2c^2\log(x)$$

```
[In] Int[x*(a + b*ArcTan[c/x])^2,x]
```

```
[Out] b*c*x*(a + b*ArcCot[x/c]) + (c^2*(a + b*ArcCot[x/c])^2)/2 + (x^2*(a + b*Arc
Cot[x/c])^2)/2 + (b^2*c^2*Log[1 + c^2/x^2])/2 + b^2*c^2*Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplify
[(m + 1)/n]]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{(a+b\arctan(cx))^2}{x^3}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{2}x^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2-(bc)\text{Subst}\left(\int \frac{a+b\arctan(cx)}{x^2\,(1+c^2x^2)}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{2}x^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2-(bc)\text{Subst}\left(\int \frac{a+b\arctan(cx)}{x^2}\,dx, x, \frac{1}{x}\right)\\
&\quad+\left(bc^3\right)\text{Subst}\left(\int \frac{a+b\arctan(cx)}{1+c^2x^2}\,dx, x, \frac{1}{x}\right)\\
&= bcx\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)+\frac{1}{2}c^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\\
&\quad+\frac{1}{2}x^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2-\left(b^2c^2\right)\text{Subst}\left(\int \frac{1}{x\,(1+c^2x^2)}\,dx, x, \frac{1}{x}\right)\\
&= bcx\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)+\frac{1}{2}c^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\\
&\quad+\frac{1}{2}x^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2-\frac{1}{2}\left(b^2c^2\right)\text{Subst}\left(\int \frac{1}{x\,(1+c^2x)}\,dx, x, \frac{1}{x^2}\right)\\
&= bcx\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)+\frac{1}{2}c^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{1}{2}x^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\\
&\quad-\frac{1}{2}\left(b^2c^2\right)\text{Subst}\left(\int \frac{1}{x}\,dx, x, \frac{1}{x^2}\right)+\frac{1}{2}\left(b^2c^4\right)\text{Subst}\left(\int \frac{1}{1+c^2x}\,dx, x, \frac{1}{x^2}\right)\\
&= bcx\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)+\frac{1}{2}c^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\\
&\quad+\frac{1}{2}x^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{1}{2}b^2c^2\log\left(1+\frac{c^2}{x^2}\right)+b^2c^2\log(x)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.89

$$
\begin{aligned}
\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx &= \frac{1}{2}\left(ax(2bc+ax)+2b\left(bcx+a\left(c^2+x^2\right)\right)\arctan\left(\frac{c}{x}\right)\right.\\
&\quad\left.+b^2\left(c^2+x^2\right)\arctan\left(\frac{c}{x}\right)^2+b^2c^2\log\left(c^2+x^2\right)\right)
\end{aligned}
$$

```
[In] Integrate[x*(a + b*ArcTan[c/x])^2,x]
```

```
[Out] (a*x*(2*b*c + a*x) + 2*b*(b*c*x + a*(c^2 + x^2))*ArcTan[c/x] + b^2*(c^2 + x
^2)*ArcTan[c/x]^2 + b^2*c^2*Log[c^2 + x^2])/2
```

Maple [A] (verified)

Time = 3.17 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.28

method	result
parts	$\frac{a^2x^2}{2}-b^2c^2\left(-\frac{x^2\arctan\left(\frac{c}{x}\right)^2}{2c^2}-\frac{\arctan\left(\frac{c}{x}\right)^2}{2}-\frac{x\arctan\left(\frac{c}{x}\right)}{c}-\frac{\ln\left(1+\frac{c^2}{x^2}\right)}{2}+\ln\left(\frac{c}{x}\right)\right)-ab\,c^2\arctan$
parallelrisch	$\frac{x^2\arctan\left(\frac{c}{x}\right)^2b^2}{2}+\frac{\arctan\left(\frac{c}{x}\right)^2b^2c^2}{2}+\frac{b^2c^2\ln(c^2+x^2)}{2}+x^2\arctan\left(\frac{c}{x}\right)ab+x\arctan\left(\frac{c}{x}\right)b^2c+\mathrm{arct}$
derivativedivides	$-c^2\left(-\frac{a^2x^2}{2c^2}+b^2\left(-\frac{x^2\arctan\left(\frac{c}{x}\right)^2}{2c^2}-\frac{\arctan\left(\frac{c}{x}\right)^2}{2}-\frac{x\arctan\left(\frac{c}{x}\right)}{c}-\frac{\ln\left(1+\frac{c^2}{x^2}\right)}{2}+\ln\left(\frac{c}{x}\right)\right)-\frac{ab\,x^2\,\mathrm{arc}}{c^2}\right.$
default	$-c^2\left(-\frac{a^2x^2}{2c^2}+b^2\left(-\frac{x^2\arctan\left(\frac{c}{x}\right)^2}{2c^2}-\frac{\arctan\left(\frac{c}{x}\right)^2}{2}-\frac{x\arctan\left(\frac{c}{x}\right)}{c}-\frac{\ln\left(1+\frac{c^2}{x^2}\right)}{2}+\ln\left(\frac{c}{x}\right)\right)-\frac{ab\,x^2\,\mathrm{arc}}{c^2}\right.$
risch	Expression too large to display

```
[In] int(x*(a+b*arctan(c/x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a^2*x^2-b^2*c^2*(-1/2/c^2*x^2*arctan(c/x)^2-1/2*arctan(c/x)^2-1/c*x*arc
tan(c/x)-1/2*ln(1+c^2/x^2)+ln(c/x))-a*b*c^2*arctan(x/c)+x^2*arctan(c/x)*a*b
+a*b*c*x
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.07

$$\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx=-abc^2\arctan\left(\frac{x}{c}\right)+\frac{1}{2}\,b^2c^2\log\left(c^2+x^2\right)+abcx+\frac{1}{2}\,a^2x^2+\frac{1}{2}\left(b^2c^2+b^2x^2\right)\arctan\left(\frac{c}{x}\right)^2+\left(b^2cx+abx^2\right)\arctan\left(\frac{c}{x}\right)$$

```
[In] integrate(x*(a+b*arctan(c/x))^2,x, algorithm="fricas")
```

```
[Out] -a*b*c^2*arctan(x/c) + 1/2*b^2*c^2*log(c^2 + x^2) + a*b*c*x + 1/2*a^2*x^2 +
 1/2*(b^2*c^2 + b^2*x^2)*arctan(c/x)^2 + (b^2*c*x + a*b*x^2)*arctan(c/x)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.18

$$\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2dx=\frac{a^2x^2}{2}+abc^2\operatorname{atan}\left(\frac{c}{x}\right)+abcx+abx^2\operatorname{atan}\left(\frac{c}{x}\right)+\frac{b^2c^2\log\left(c^2+x^2\right)}{2}+\frac{b^2c^2\operatorname{atan}^2\left(\frac{c}{x}\right)}{2}+b^2cx\operatorname{atan}\left(\frac{c}{x}\right)+\frac{b^2x^2\operatorname{atan}^2\left(\frac{c}{x}\right)}{2}$$

```
[In] integrate(x*(a+b*atan(c/x))**2,x)
```

```
[Out] a**2*x**2/2 + a*b*c**2*atan(c/x) + a*b*c*x + a*b*x**2*atan(c/x) + b**2*c**2
*log(c**2 + x**2)/2 + b**2*c**2*atan(c/x)**2/2 + b**2*c*x*atan(c/x) + b**2*
x**2*atan(c/x)**2/2
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.27

$$\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2dx=\frac{1}{2}b^2x^2\arctan\left(\frac{c}{x}\right)^2+\frac{1}{2}a^2x^2+\left(x^2\arctan\left(\frac{c}{x}\right)-\left(c\arctan\left(\frac{x}{c}\right)-x\right)c\right)ab-\frac{1}{2}\left(\left(\arctan\left(\frac{x}{c}\right)^2-\log\left(c^2+x^2\right)\right)c^2+2\left(c\arctan\left(\frac{x}{c}\right)-x\right)c\arctan\left(\frac{c}{x}\right)\right)b^2$$

```
[In] integrate(x*(a+b*arctan(c/x))^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^2*arctan(c/x)^2 + 1/2*a^2*x^2 + (x^2*arctan(c/x) - (c*arctan(x/c)
 - x)*c)*a*b - 1/2*((arctan(x/c)^2 - log(c^2 + x^2))*c^2 + 2*(c*arctan(x/c)
 - x)*c*arctan(c/x))*b^2
```

Giac [F]

$$\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2dx=\int\left(b\arctan\left(\frac{c}{x}\right)+a\right)^2x\,dx$$

```
[In] integrate(x*(a+b*arctan(c/x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c/x) + a)^2*x, x)
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.20

$$\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx=\frac{a^2\,x^2}{2}+\frac{b^2\,c^2\,\mathrm{atan}\left(\frac{c}{x}\right)^2}{2}+\frac{b^2\,c^2\,\ln\left(c^2+x^2\right)}{2}+\frac{b^2\,x^2\,\mathrm{atan}\left(\frac{c}{x}\right)^2}{2}+a\,b\,c^2\,\mathrm{atan}\left(\frac{c}{x}\right)+a\,b\,x^2\,\mathrm{atan}\left(\frac{c}{x}\right)+b^2\,c\,x\,\mathrm{atan}\left(\frac{c}{x}\right)+a\,b\,c\,x$$

```
[In] int(x*(a + b*atan(c/x))^2,x)
[Out] (a^2*x^2)/2 + (b^2*c^2*atan(c/x)^2)/2 + (b^2*c^2*log(c^2 + x^2))/2 + (b^2*x
^2*atan(c/x)^2)/2 + a*b*c^2*atan(c/x) + a*b*x^2*atan(c/x) + b^2*c*x*atan(c/
x) + a*b*c*x
```

3.143 $\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)^2 dx$

Optimal result . . . 810
Rubi [A] (verified) . . . 810
Mathematica [A] (verified) . . . 812
Maple [B] (verified) . . . 812
Fricas [F] . . . 813
Sympy [F] . . . 813
Maxima [F] . . . 814
Giac [F] . . . 814
Mupad [F(-1)] . . . 814

Optimal result

Integrand size = 12, antiderivative size = 83

$$\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)^2 dx = ic\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2 - 2bc\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\log\left(\frac{2c}{c+ix}\right) + ib^2c\,\text{PolyLog}\left(2,1-\frac{2c}{c+ix}\right)$$

```
[Out] I*c*(a+b*arccot(x/c))^2+x*(a+b*arccot(x/c))^2-2*b*c*(a+b*arccot(x/c))*ln(2*
c/(c+I*x))+I*b^2*c*polylog(2,1-2*c/(c+I*x))
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4934, 4931, 5041, 4965, 2449, 2352}

$$\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)^2 dx = ic\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2 - 2bc\log\left(\frac{2c}{c+ix}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right) + ib^2c\,\text{PolyLog}\left(2,1-\frac{2c}{c+ix}\right)$$

```
[In] Int[(a + b*ArcTan[c/x])^2,x]
```

```
[Out] I*c*(a + b*ArcCot[x/c])^2 + x*(a + b*ArcCot[x/c])^2 - 2*b*c*(a + b*ArcCot[x
/c])*Log[(2*c)/(c + I*x)] + I*b^2*c*PolyLog[2, 1 - (2*c)/(c + I*x)]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4931

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Dist[b*c*n*p, Int[x^n*((a + b*ArcCot[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4934

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_), x_Symbol] :> Int[(a + b*A
rcCot[1/(x^n*c)])^p, x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1] && ILtQ[n, 0]
```

Rule 4965

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcCot[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] - Dist[b*c*(
p/e), Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5041

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*e*(p + 1))), x] - Dist[
1/(c*d), Int[(a + b*ArcCot[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\text{integral} = \int \left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\,dx$$

$$= x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{(2b)\int\frac{x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)}{1+\frac{x^2}{c^2}}\,dx}{c}$$

$$= ic\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2-(2b)\int\frac{a+b\cot^{-1}\left(\frac{x}{c}\right)}{i-\frac{x}{c}}\,dx$$

$$
\begin{aligned}
&= ic\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2 \\
&\quad -2bc\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\log\left(\frac{2c}{c+ix}\right)-\left(2b^2\right)\int\frac{\log\left(\frac{2}{1+\frac{ix}{c}}\right)}{1+\frac{x^2}{c^2}}\,dx \\
&= ic\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2 \\
&\quad -2bc\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\log\left(\frac{2c}{c+ix}\right)+\left(2ib^2c\right)\text{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx,x,\frac{1}{1+\frac{ix}{c}}\right) \\
&= ic\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2 \\
&\quad -2bc\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\log\left(\frac{2c}{c+ix}\right)+ib^2c\,\text{PolyLog}\left(2,1-\frac{2c}{c+ix}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.27

$$
\begin{aligned}
\int\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2dx &= b^2(ic+x)\arctan\left(\frac{c}{x}\right)^2 \\
&\quad +2b\arctan\left(\frac{c}{x}\right)\left(ax-bc\log\left(1-e^{2i\arctan\left(\frac{c}{x}\right)}\right)\right) \\
&\quad +a\left(ax+bc\log\left(1+\frac{c^2}{x^2}\right)-2bc\log\left(\frac{c}{x}\right)\right) \\
&\quad +ib^2c\,\text{PolyLog}\left(2,e^{2i\arctan\left(\frac{c}{x}\right)}\right)
\end{aligned}
$$

```
[In] Integrate[(a + b*ArcTan[c/x])^2,x]
```

```
[Out] b^2*(I*c + x)*ArcTan[c/x]^2 + 2*b*ArcTan[c/x]*(a*x - b*c*Log[1 - E^((2*I)*A
rcTan[c/x])]) + a*(a*x + b*c*Log[1 + c^2/x^2] - 2*b*c*Log[c/x]) + I*b^2*c*P
olyLog[2, E^((2*I)*ArcTan[c/x])]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 307 vs. 2(79) = 158.

Time = 3.62 (sec) , antiderivative size = 308, normalized size of antiderivative = 3.71

method	result
parts	$a^2x - b^2c\left(-\frac{x\arctan(\frac{c}{x})^2}{c} + 2\ln\left(\frac{c}{x}\right)\arctan\left(\frac{c}{x}\right) - \arctan\left(\frac{c}{x}\right)\ln\left(1+\frac{c^2}{x^2}\right) + i\ln\left(\frac{c}{x}\right)\ln(1\right.$
derivativedivides	$-c\left(-\frac{a^2x}{c} + b^2\left(-\frac{x\arctan(\frac{c}{x})^2}{c} + 2\ln\left(\frac{c}{x}\right)\arctan\left(\frac{c}{x}\right) - \arctan\left(\frac{c}{x}\right)\ln\left(1+\frac{c^2}{x^2}\right) + i\ln\left(\frac{c}{x}\right)\right.\right.$
default	$-c\left(-\frac{a^2x}{c} + b^2\left(-\frac{x\arctan(\frac{c}{x})^2}{c} + 2\ln\left(\frac{c}{x}\right)\arctan\left(\frac{c}{x}\right) - \arctan\left(\frac{c}{x}\right)\ln\left(1+\frac{c^2}{x^2}\right) + i\ln\left(\frac{c}{x}\right)\right.\right.$
risch	Expression too large to display

```
[In] int((a+b*arctan(c/x))^2,x,method=_RETURNVERBOSE)
[Out] a^2*x-b^2*c*(-1/c*x*arctan(c/x)^2+2*ln(c/x)*arctan(c/x)-arctan(c/x)*ln(1+c^
2/x^2)+I*ln(c/x)*ln(1+I*c/x)-I*ln(c/x)*ln(1-I*c/x)+I*dilog(1+I*c/x)-I*dilog
(1-I*c/x)-1/2*I*(ln(c/x-I)*ln(1+c^2/x^2)-1/2*ln(c/x-I)^2-dilog(-1/2*I*(c/x+
I))-ln(c/x-I)*ln(-1/2*I*(c/x+I)))+1/2*I*(ln(c/x+I)*ln(1+c^2/x^2)-1/2*ln(c/x
+I)^2-dilog(1/2*I*(c/x-I))-ln(c/x+I)*ln(1/2*I*(c/x-I))))-2*a*b*c*(-1/c*x*ar
ctan(c/x)-1/2*ln(1+c^2/x^2)+ln(c/x))
```

Fricas [F]

$$\int\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx=\int\left(b\arctan\left(\frac{c}{x}\right)+a\right)^2\,dx$$

```
[In] integrate((a+b*arctan(c/x))^2,x, algorithm="fricas")
[Out] integral(b^2*arctan(c/x)^2 + 2*a*b*arctan(c/x) + a^2, x)
```

Sympy [F]

$$\int\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx=\int\left(a+b\operatorname{atan}\left(\frac{c}{x}\right)\right)^2\,dx$$

```
[In] integrate((a+b*atan(c/x))**2,x)
[Out] Integral((a + b*atan(c/x))**2, x)
```

Maxima [F]

$$\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx=\int \left(b\arctan\left(\frac{c}{x}\right)+a\right)^2\,dx$$

[In] integrate((a+b*arctan(c/x))^2,x, algorithm="maxima")

[Out] (2*x*arctan(c/x) + c*log(c^2 + x^2))*a*b + 1/16*(12*c*arctan(c/x)^2*arctan(x/c) + 4*(3*arctan(c/x)*arctan(x/c)^2/c + arctan(x/c)^3/c)*c^2 + 4*x*arctan2(c, x)^2 + 16*c^2*integrate(1/16*log(c^2 + x^2)^2/(c^2 + x^2), x) - x*log(c^2 + x^2)^2 + 128*c*integrate(1/16*x*arctan(c/x)/(c^2 + x^2), x) + 192*integrate(1/16*x^2*arctan(c/x)^2/(c^2 + x^2), x) + 16*integrate(1/16*x^2*log(c^2 + x^2)^2/(c^2 + x^2), x) + 64*integrate(1/16*x^2*log(c^2 + x^2)/(c^2 + x^2), x))*b^2 + a^2*x

Giac [F]

$$\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx=\int \left(b\arctan\left(\frac{c}{x}\right)+a\right)^2\,dx$$

[In] integrate((a+b*arctan(c/x))^2,x, algorithm="giac")

[Out] integrate((b*arctan(c/x) + a)^2, x)

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)^2\,dx=\int \left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^2\,dx$$

[In] int((a + b*atan(c/x))^2,x)

[Out] int((a + b*atan(c/x))^2, x)

3.144 $\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x}\,dx$

Optimal result 815
Rubi [A] (verified) 815
Mathematica [A] (verified) 818
Maple [C] (warning: unable to verify) 819
Fricas [F] 819
Sympy [F] 820
Maxima [F] 820
Giac [F] 820
Mupad [F(-1)] 820

Optimal result

Integrand size = 16, antiderivative size = 148

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x}\,dx = -2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2 \operatorname{arctanh}\left(1-\frac{2}{1+\frac{ic}{x}}\right) + ib\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\operatorname{PolyLog}\left(2,1-\frac{2}{1+\frac{ic}{x}}\right) - ib\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\operatorname{PolyLog}\left(2,-1+\frac{2}{1+\frac{ic}{x}}\right) + \frac{1}{2}b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+\frac{ic}{x}}\right) - \frac{1}{2}b^2\operatorname{PolyLog}\left(3,-1+\frac{2}{1+\frac{ic}{x}}\right)$$

```
[Out] 2*(a+b*arccot(x/c))^2*arctanh(-1+2/(1+I*c/x))+I*b*(a+b*arccot(x/c))*polylog
(2,1-2/(1+I*c/x))-I*b*(a+b*arccot(x/c))*polylog(2,-1+2/(1+I*c/x))+1/2*b^2*p
olylog(3,1-2/(1+I*c/x))-1/2*b^2*polylog(3,-1+2/(1+I*c/x))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used

$= \{4944, 4942, 5108, 5004, 5114, 6745\}$

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x}\,dx = -2\text{arctanh}\left(1-\frac{2}{1+\frac{ic}{x}}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2 + ib\,\text{PolyLog}\left(2,1-\frac{2}{\frac{ic}{x}+1}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right) - ib\,\text{PolyLog}\left(2,\frac{2}{\frac{ic}{x}+1}-1\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right) + \frac{1}{2}b^2\,\text{PolyLog}\left(3,1-\frac{2}{\frac{ic}{x}+1}\right) - \frac{1}{2}b^2\,\text{PolyLog}\left(3,\frac{2}{\frac{ic}{x}+1}-1\right)$$

```
[In] Int[(a + b*ArcTan[c/x])^2/x,x]
```

```
[Out] -2*(a + b*ArcCot[x/c])^2*ArcTanh[1 - 2/(1 + (I*c)/x)] + I*b*(a + b*ArcCot[x
/c])*PolyLog[2, 1 - 2/(1 + (I*c)/x)] - I*b*(a + b*ArcCot[x/c])*PolyLog[2, -
1 + 2/(1 + (I*c)/x)] + (b^2*PolyLog[3, 1 - 2/(1 + (I*c)/x)])/2 - (b^2*PolyL
og[3, -1 + 2/(1 + (I*c)/x)])/2
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1
/n, Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[p, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5108

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e*
x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{(a+b\arctan(cx))^2}{x}\,dx, x, \frac{1}{x}\right)$$

$$= -2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2 \operatorname{arctanh}\left(1-\frac{2}{1+\frac{ic}{x}}\right) + (4bc)\text{Subst}\left(\int \frac{(a+b\arctan(cx))\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx, x, \frac{1}{x}\right)$$

$$= -2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2 \operatorname{arctanh}\left(1-\frac{2}{1+\frac{ic}{x}}\right) - (2bc)\text{Subst}\left(\int \frac{(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx, x, \frac{1}{x}\right) + (2bc)\text{Subst}\left(\int \frac{(a+b\arctan(cx))\log\left(2-\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx, x, \frac{1}{x}\right)$$

$$= -2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2 \operatorname{arctanh}\left(1-\frac{2}{1+\frac{ic}{x}}\right) + ib\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\text{PolyLog}\left(2,1-\frac{2}{1+\frac{ic}{x}}\right) - ib\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\text{PolyLog}\left(2,-1+\frac{2}{1+\frac{ic}{x}}\right) - \left(ib^2c\right)\text{Subst}\left(\int \frac{\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx, x, \frac{1}{x}\right) + \left(ib^2c\right)\text{Subst}\left(\int \frac{\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx, x, \frac{1}{x}\right)$$

$$= -2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2 \operatorname{arctanh}\left(1-\frac{2}{1+\frac{ic}{x}}\right) + ib\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right) \operatorname{PolyLog}\left(2,1-\frac{2}{1+\frac{ic}{x}}\right) - ib\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right) \operatorname{PolyLog}\left(2,-1+\frac{2}{1+\frac{ic}{x}}\right) + \frac{1}{2}b^2 \operatorname{PolyLog}\left(3,1-\frac{2}{1+\frac{ic}{x}}\right) - \frac{1}{2}b^2 \operatorname{PolyLog}\left(3,-1+\frac{2}{1+\frac{ic}{x}}\right)$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.37

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x}\,dx = a^2\log(x) - iab\left(\operatorname{PolyLog}\left(2,-\frac{ic}{x}\right) - \operatorname{PolyLog}\left(2,\frac{ic}{x}\right)\right) + b^2\left(\frac{i\pi^3}{24} - \frac{2}{3}i\arctan\left(\frac{c}{x}\right)^3 - \arctan\left(\frac{c}{x}\right)^2\log\left(1-e^{-2i\arctan\left(\frac{c}{x}\right)}\right) + \arctan\left(\frac{c}{x}\right)^2\log\left(1+e^{2i\arctan\left(\frac{c}{x}\right)}\right) - i\arctan\left(\frac{c}{x}\right)\operatorname{PolyLog}\left(2,e^{-2i\arctan\left(\frac{c}{x}\right)}\right) - i\arctan\left(\frac{c}{x}\right)\operatorname{PolyLog}\left(2,-e^{2i\arctan\left(\frac{c}{x}\right)}\right) - \frac{1}{2}\operatorname{PolyLog}\left(3,e^{-2i\arctan\left(\frac{c}{x}\right)}\right) + \frac{1}{2}\operatorname{PolyLog}\left(3,-e^{2i\arctan\left(\frac{c}{x}\right)}\right)\right)$$

```
[In] Integrate[(a + b*ArcTan[c/x])^2/x,x]
```

```
[Out] a^2*Log[x] - I*a*b*(PolyLog[2, ((-I)*c)/x] - PolyLog[2, (I*c)/x]) + b^2*((I
/24)*Pi^3 - ((2*I)/3)*ArcTan[c/x]^3 - ArcTan[c/x]^2*Log[1 - E^((-2*I)*ArcTa
n[c/x])] + ArcTan[c/x]^2*Log[1 + E^((2*I)*ArcTan[c/x])] - I*ArcTan[c/x]*Pol
yLog[2, E^((-2*I)*ArcTan[c/x])] - I*ArcTan[c/x]*PolyLog[2, -E^((2*I)*ArcTan
[c/x])] - PolyLog[3, E^((-2*I)*ArcTan[c/x])]/2 + PolyLog[3, -E^((2*I)*ArcTa
n[c/x])]/2)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.55 (sec) , antiderivative size = 1106, normalized size of antiderivative = 7.47

Expression too large to display

```
[In] int((a+b*arctan(c/x))^2/x,x)
```

```
[Out] -a^2*ln(c/x)-b^2*(ln(c/x)*arctan(c/x)^2+I*arctan(c/x)*polylog(2,-(1+I*c/x)^
2/(1+c^2/x^2))-1/2*polylog(3,-(1+I*c/x)^2/(1+c^2/x^2))-arctan(c/x)^2*ln((1+
I*c/x)^2/(1+c^2/x^2)-1)+arctan(c/x)^2*ln(1-(1+I*c/x)/(1+c^2/x^2)^(1/2))-2*I
*arctan(c/x)*polylog(2,(1+I*c/x)/(1+c^2/x^2)^(1/2))+2*polylog(3,(1+I*c/x)/(
1+c^2/x^2)^(1/2))+arctan(c/x)^2*ln((1+I*c/x)/(1+c^2/x^2)^(1/2)+1)-2*I*arcta
n(c/x)*polylog(2,-(1+I*c/x)/(1+c^2/x^2)^(1/2))+2*polylog(3,-(1+I*c/x)/(1+c^
2/x^2)^(1/2))+1/2*I*Pi*(csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+
c^2/x^2)+1))*csgn(((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))-
csgn(((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))^2-csgn(I/((1+
I*c/x)^2/(1+c^2/x^2)+1))*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1
+c^2/x^2)+1))^2+csgn(I/((1+I*c/x)^2/(1+c^2/x^2)+1))*csgn(I*((1+I*c/x)^2/(1+
c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)
)+csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))^3-csgn(I*
((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))^2*csgn(I*((1+I*c/x
)^2/(1+c^2/x^2)-1))-csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/
x^2)+1))*csgn(((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))^2+cs
gn(((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))^3+1)*arctan(c/x
)^2)-2*a*b*(ln(c/x)*arctan(c/x)+1/2*I*ln(c/x)*ln(1+I*c/x)-1/2*I*ln(c/x)*ln(
1-I*c/x)+1/2*I*dilog(1+I*c/x)-1/2*I*dilog(1-I*c/x))
```

Fricas [F]

$$\int \frac{\left(a + b \arctan\left(\frac{c}{x}\right)\right)^2}{x}\,dx = \int \frac{\left(b \arctan\left(\frac{c}{x}\right) + a\right)^2}{x}\,dx$$

```
[In] integrate((a+b*arctan(c/x))^2/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c/x)^2 + 2*a*b*arctan(c/x) + a^2)/x, x)
```

Sympy [F]

$$\int \frac{\left(a + b\arctan\left(\frac{c}{x}\right)\right)^2}{x}\,dx = \int \frac{\left(a + b\operatorname{atan}\left(\frac{c}{x}\right)\right)^2}{x}\,dx$$

```
[In] integrate((a+b*atan(c/x))**2/x,x)
[Out] Integral((a + b*atan(c/x))**2/x, x)
```

Maxima [F]

$$\int \frac{\left(a + b\arctan\left(\frac{c}{x}\right)\right)^2}{x}\,dx = \int \frac{\left(b\arctan\left(\frac{c}{x}\right) + a\right)^2}{x}\,dx$$

```
[In] integrate((a+b*arctan(c/x))^2/x,x, algorithm="maxima")
[Out] a^2*log(x) + 1/16*integrate((12*b^2*arctan2(c, x)^2 + b^2*log(c^2 + x^2)^2
+ 32*a*b*arctan2(c, x))/x, x)
```

Giac [F]

$$\int \frac{\left(a + b\arctan\left(\frac{c}{x}\right)\right)^2}{x}\,dx = \int \frac{\left(b\arctan\left(\frac{c}{x}\right) + a\right)^2}{x}\,dx$$

```
[In] integrate((a+b*arctan(c/x))^2/x,x, algorithm="giac")
[Out] integrate((b*arctan(c/x) + a)^2/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a + b\arctan\left(\frac{c}{x}\right)\right)^2}{x}\,dx = \int \frac{\left(a + b\operatorname{atan}\left(\frac{c}{x}\right)\right)^2}{x}\,dx$$

```
[In] int((a + b*atan(c/x))^2/x,x)
[Out] int((a + b*atan(c/x))^2/x, x)
```

3.145 $\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x^2}\,dx$

Optimal result 821
Rubi [A] (verified) 821
Mathematica [A] (verified) 823
Maple [A] (verified) 823
Fricas [F] 824
Sympy [F] 824
Maxima [F] 825
Giac [F] 825
Mupad [F(-1)] 825

Optimal result

Integrand size = 16, antiderivative size = 96

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x^2}\,dx = -\frac{i\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{c} - \frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{x} - \frac{2b\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\log\left(\frac{2}{1+\frac{ic}{x}}\right)}{c} - \frac{ib^2\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+\frac{ic}{x}}\right)}{c}$$

```
[Out] -I*(a+b*arccot(x/c))^2/c-(a+b*arccot(x/c))^2/x-2*b*(a+b*arccot(x/c))*ln(2/(
1+I*c/x))/c-I*b^2*polylog(2,1-2/(1+I*c/x))/c
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4948, 4930, 5040, 4964, 2449, 2352}

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x^2}\,dx = -\frac{i\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{c} - \frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{x} - \frac{2b\log\left(\frac{2}{1+\frac{ic}{x}}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)}{c} - \frac{ib^2\,\mathrm{PolyLog}\left(2,1-\frac{2}{\frac{ic}{x}+1}\right)}{c}$$

```
[In] Int[(a + b*ArcTan[c/x])^2/x^2,x]
```

```
[Out] ((-I)*(a + b*ArcCot[x/c])^2)/c - (a + b*ArcCot[x/c])^2/x - (2*b*(a + b*ArcC
ot[x/c])*Log[2/(1 + (I*c)/x)])/c - (I*b^2*PolyLog[2, 1 - 2/(1 + (I*c)/x)])/
c
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplify
[(m + 1)/n]]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int (a+b\arctan(cx))^2\,dx, x, \frac{1}{x}\right)\\
&= -\frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{x} + (2bc)\text{Subst}\left(\int \frac{x(a+b\arctan(cx))}{1+c^2x^2}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{i\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{c} - \frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{x} - (2b)\text{Subst}\left(\int \frac{a+b\arctan(cx)}{i-cx}\,dx, x, \frac{1}{x}\right)
\end{aligned}$$

$$= -\frac{i\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{c} - \frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{x} - \frac{2b\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\log\left(\frac{2}{1+\frac{ic}{x}}\right)}{c} + (2b^2)\,\text{Subst}\left(\int \frac{\log\left(\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{i\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{c} - \frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{x} - \frac{2b\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\log\left(\frac{2}{1+\frac{ic}{x}}\right)}{c} - \frac{(2ib^2)\,\text{Subst}\left(\int \frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1+\frac{ic}{x}}\right)}{c}$$

$$= -\frac{i\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{c} - \frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{x} - \frac{2b\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\log\left(\frac{2}{1+\frac{ic}{x}}\right)}{c} - \frac{ib^2\,\text{PolyLog}\left(2, 1-\frac{2}{1+\frac{ic}{x}}\right)}{c}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.11

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x^2}\,dx = -\frac{b^2(c-ix)\arctan\left(\frac{c}{x}\right)^2 + 2b\arctan\left(\frac{c}{x}\right)\left(ac+bx\log\left(1+e^{2i\arctan\left(\frac{c}{x}\right)}\right)\right) + a\left(ac+2bx\log\left(\frac{1}{\sqrt{1+\frac{c^2}{x^2}}}\right)\right) -}{cx}$$

```
[In] Integrate[(a + b*ArcTan[c/x])^2/x^2,x]
```

```
[Out] -((b^2*(c - I*x)*ArcTan[c/x]^2 + 2*b*ArcTan[c/x]*(a*c + b*x*Log[1 + E^((2*I
)*ArcTan[c/x])]) + a*(a*c + 2*b*x*Log[1/Sqrt[1 + c^2/x^2]]) - I*b^2*x*PolyL
og[2, -E^((2*I)*ArcTan[c/x])])/(c*x))
```

Maple [A] (verified)

Time = 6.11 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.48

method	result
derivativedivides	$-\frac{\frac{c\,a^2}{x}-i\arctan\left(\frac{c}{x}\right)^2b^2+\frac{\arctan\left(\frac{c}{x}\right)^2b^2c}{x}+2\arctan\left(\frac{c}{x}\right)\ln\left(\frac{\left(1+\frac{ic}{x}\right)^2}{1+\frac{c^2}{x^2}}+1\right)b^2-i\,\mathrm{polylog}\left(2,-\frac{\left(1+\frac{ic}{x}\right)^2}{1+\frac{c^2}{x^2}}\right)b^2+\frac{2abc\arctan\left(\frac{c}{x}\right)}{x}}{c}$
default	$-\frac{\frac{c\,a^2}{x}-i\arctan\left(\frac{c}{x}\right)^2b^2+\frac{\arctan\left(\frac{c}{x}\right)^2b^2c}{x}+2\arctan\left(\frac{c}{x}\right)\ln\left(\frac{\left(1+\frac{ic}{x}\right)^2}{1+\frac{c^2}{x^2}}+1\right)b^2-i\,\mathrm{polylog}\left(2,-\frac{\left(1+\frac{ic}{x}\right)^2}{1+\frac{c^2}{x^2}}\right)b^2+\frac{2abc\arctan\left(\frac{c}{x}\right)}{x}}{c}$
parts	$-\frac{a^2}{x}-\frac{b^2\arctan\left(\frac{c}{x}\right)^2}{x}+\frac{ib^2\arctan\left(\frac{c}{x}\right)^2}{c}-\frac{2b^2\arctan\left(\frac{c}{x}\right)\ln\left(\frac{\left(1+\frac{ic}{x}\right)^2}{1+\frac{c^2}{x^2}}+1\right)}{c}+\frac{ib^2\,\mathrm{polylog}\left(2,-\frac{\left(1+\frac{ic}{x}\right)^2}{1+\frac{c^2}{x^2}}\right)}{c}-\frac{2ab}{}$

```
[In] int((a+b*arctan(c/x))^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/c*(c/x*a^2-I*arctan(c/x)^2*b^2+arctan(c/x)^2*b^2*c/x+2*arctan(c/x)*ln((1
+I*c/x)^2/(1+c^2/x^2)+1)*b^2-I*polylog(2,-(1+I*c/x)^2/(1+c^2/x^2))*b^2+2*a*
b*c/x*arctan(c/x)-a*b*ln(1+c^2/x^2))
```

Fricas [F]

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x^2}\,dx=\int \frac{\left(b\arctan\left(\frac{c}{x}\right)+a\right)^2}{x^2}\,dx$$

```
[In] integrate((a+b*arctan(c/x))^2/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c/x)^2 + 2*a*b*arctan(c/x) + a^2)/x^2, x)
```

Sympy [F]

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x^2}\,dx=\int \frac{\left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^2}{x^2}\,dx$$

```
[In] integrate((a+b*atan(c/x))**2/x**2,x)
```

```
[Out] Integral((a + b*atan(c/x))**2/x**2, x)
```

Maxima [F]

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x^2}\,dx=\int \frac{\left(b\arctan\left(\frac{c}{x}\right)+a\right)^2}{x^2}\,dx$$

```
[In] integrate((a+b*arctan(c/x))^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/16*(4*(48*c^2*integrate(1/16*arctan(c/x)^2/(c^2*x^2 + x^4), x) + 4*c^2*in
tegrate(1/16*log(c^2 + x^2)^2/(c^2*x^2 + x^4), x) + 3*arctan(c/x)^2*arctan(
x/c)/c + 3*arctan(c/x)*arctan(x/c)^2/c + arctan(x/c)^3/c - 32*c*integrate(1
/16*x*arctan(c/x)/(c^2*x^2 + x^4), x) + 4*integrate(1/16*x^2*log(c^2 + x^2)
^2/(c^2*x^2 + x^4), x) - 16*integrate(1/16*x^2*log(c^2 + x^2)/(c^2*x^2 + x^
4), x))*x - 4*arctan2(c, x)^2 + log(c^2 + x^2)^2)*b^2/x - a*b*(2*c*arctan(c
/x)/x - log(c^2/x^2 + 1))/c - a^2/x
```

Giac [F]

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x^2}\,dx=\int \frac{\left(b\arctan\left(\frac{c}{x}\right)+a\right)^2}{x^2}\,dx$$

```
[In] integrate((a+b*arctan(c/x))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c/x) + a)^2/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x^2}\,dx=\int \frac{\left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^2}{x^2}\,dx$$

```
[In] int((a + b*atan(c/x))^2/x^2,x)
```

```
[Out] int((a + b*atan(c/x))^2/x^2, x)
```

3.146 $\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x^3}\,dx$

Optimal result 826
Rubi [A] (verified) 826
Mathematica [A] (verified) 828
Maple [A] (verified) 829
Fricas [A] (verification not implemented) 829
Sympy [A] (verification not implemented) 830
Maxima [A] (verification not implemented) 830
Giac [C] (verification not implemented) 831
Mupad [B] (verification not implemented) 831

Optimal result

Integrand size = 16, antiderivative size = 84

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x^3}\,dx=\frac{ab}{cx}+\frac{b^2\cot^{-1}\left(\frac{x}{c}\right)}{cx}-\frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{2c^2}-\frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{2x^2}-\frac{b^2\log\left(1+\frac{c^2}{x^2}\right)}{2c^2}$$

```
[Out] a*b/c/x+b^2*arccot(x/c)/c/x-1/2*(a+b*arccot(x/c))^2/c^2-1/2*(a+b*arccot(x/c
))^2/x^2-1/2*b^2*ln(1+c^2/x^2)/c^2
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4948, 4946, 5036, 4930, 266, 5004}

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x^3}\,dx=-\frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{2c^2}-\frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{2x^2}+\frac{ab}{cx}-\frac{b^2\log\left(\frac{c^2}{x^2}+1\right)}{2c^2}+\frac{b^2\cot^{-1}\left(\frac{x}{c}\right)}{cx}$$

```
[In] Int[(a + b*ArcTan[c/x])^2/x^3,x]
```

```
[Out] (a*b)/(c*x) + (b^2*ArcCot[x/c])/(c*x) - (a + b*ArcCot[x/c])^2/(2*c^2) - (a
+ b*ArcCot[x/c])^2/(2*x^2) - (b^2*Log[1 + c^2/x^2])/(2*c^2)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplify
[(m + 1)/n]]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int x(a+b\arctan(cx))^2\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{2x^2} + (bc)\text{Subst}\left(\int \frac{x^2(a+b\arctan(cx))}{1+c^2x^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{2x^2} + \frac{b\text{Subst}\left(\int (a+b\arctan(cx))\,dx, x, \frac{1}{x}\right)}{c} - \frac{b\text{Subst}\left(\int \frac{a+b\arctan(cx)}{1+c^2x^2}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{ab}{cx} - \frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{2c^2} - \frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{2x^2} + \frac{b^2\text{Subst}\left(\int \arctan(cx)\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{ab}{cx} + \frac{b^2\cot^{-1}\left(\frac{x}{c}\right)}{cx} - \frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{2c^2} - \frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{2x^2} - b^2\text{Subst}\left(\int \frac{x}{1+c^2x^2}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{ab}{cx} + \frac{b^2\cot^{-1}\left(\frac{x}{c}\right)}{cx} - \frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{2c^2} - \frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{2x^2} - \frac{b^2\log\left(1+\frac{c^2}{x^2}\right)}{2c^2}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.18

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x^3}\,dx = -\frac{a^2c^2 - 2abcx + 2bc(ac-bx)\arctan\left(\frac{c}{x}\right) + b^2(c^2+x^2)\arctan\left(\frac{c}{x}\right)^2 - 2abx^2\arctan\left(\frac{x}{c}\right) - 2b^2x^2\log(x) + b}{2c^2x^2}$$

```
[In] Integrate[(a + b*ArcTan[c/x])^2/x^3,x]
```

```
[Out] -1/2*(a^2*c^2 - 2*a*b*c*x + 2*b*c*(a*c - b*x)*ArcTan[c/x] + b^2*(c^2 + x^2)
*ArcTan[c/x]^2 - 2*a*b*x^2*ArcTan[x/c] - 2*b^2*x^2*Log[x] + b^2*x^2*Log[c^2
 + x^2])/(c^2*x^2)
```

Maple [A] (verified)

Time = 3.08 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.23

method	result
parts	$-\frac{a^2}{2x^2}-\frac{b^2\left(\frac{c^2\arctan\left(\frac{c}{x}\right)^2}{2x^2}+\frac{\arctan\left(\frac{c}{x}\right)^2}{2}-\frac{c\arctan\left(\frac{c}{x}\right)}{x}+\frac{\ln\left(1+\frac{c^2}{x^2}\right)}{2}\right)}{c^2}-\frac{ab\arctan\left(\frac{c}{x}\right)}{x^2}+\frac{ab}{cx}+\frac{ab\arctan\left(\frac{x}{c}\right)}{c^2}$
derivativedivides	$-\frac{\frac{a^2c^2}{2x^2}+b^2\left(\frac{c^2\arctan\left(\frac{c}{x}\right)^2}{2x^2}+\frac{\arctan\left(\frac{c}{x}\right)^2}{2}-\frac{c\arctan\left(\frac{c}{x}\right)}{x}+\frac{\ln\left(1+\frac{c^2}{x^2}\right)}{2}\right)+2ab\left(\frac{c^2\arctan\left(\frac{c}{x}\right)}{2x^2}-\frac{c}{2x}+\frac{\arctan\left(\frac{c}{x}\right)}{2}\right)}{c^2}$
default	$-\frac{\frac{a^2c^2}{2x^2}+b^2\left(\frac{c^2\arctan\left(\frac{c}{x}\right)^2}{2x^2}+\frac{\arctan\left(\frac{c}{x}\right)^2}{2}-\frac{c\arctan\left(\frac{c}{x}\right)}{x}+\frac{\ln\left(1+\frac{c^2}{x^2}\right)}{2}\right)+2ab\left(\frac{c^2\arctan\left(\frac{c}{x}\right)}{2x^2}-\frac{c}{2x}+\frac{\arctan\left(\frac{c}{x}\right)}{2}\right)}{c^2}$
parallelrisch	$\frac{-x^2\arctan\left(\frac{c}{x}\right)^2b^2-\arctan\left(\frac{c}{x}\right)^2b^2c^2+2b^2\ln(x)x^2-b^2\ln(c^2+x^2)x^2-2x^2\arctan\left(\frac{c}{x}\right)ab+2x\arctan\left(\frac{c}{x}\right)b^2c-2\arctan\left(\frac{c}{x}\right)a}{2x^2c^2}$
risch	Expression too large to display

[In] int((a+b*arctan(c/x))^2/x^3,x,method=_RETURNVERBOSE)

[Out] -1/2*a^2/x^2-b^2/c^2*(1/2*c^2/x^2*arctan(c/x)^2+1/2*arctan(c/x)^2-c/x*arcta n(c/x)+1/2*ln(1+c^2/x^2))-a*b*arctan(c/x)/x^2+a*b/c/x+a*b/c^2*arctan(x/c)

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.30

$$\int\frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x^3}\,dx$$
$$=\frac{2\,abx^2\arctan\left(\frac{x}{c}\right)-b^2x^2\log\left(c^2+x^2\right)+2\,b^2x^2\log\left(x\right)-a^2c^2+2\,abcx-\left(b^2c^2+b^2x^2\right)\arctan\left(\frac{c}{x}\right)^2-2\,(ab}{2\,c^2x^2}$$

[In] integrate((a+b*arctan(c/x))^2/x^3,x, algorithm="fricas")

[Out] 1/2*(2*a*b*x^2*arctan(x/c) - b^2*x^2*log(c^2 + x^2) + 2*b^2*x^2*log(x) - a^ 2*c^2 + 2*a*b*c*x - (b^2*c^2 + b^2*x^2)*arctan(c/x)^2 - 2*(a*b*c^2 - b^2*c* x)*arctan(c/x))/(c^2*x^2)

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.39

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x^3}\,dx$$
$$=\begin{cases} -\frac{a^2}{2x^2}-\frac{ab\operatorname{atan}\left(\frac{c}{x}\right)}{x^2}+\frac{ab}{cx}-\frac{ab\operatorname{atan}\left(\frac{c}{x}\right)}{c^2}-\frac{b^2\operatorname{atan}^2\left(\frac{c}{x}\right)}{2x^2}+\frac{b^2\operatorname{atan}\left(\frac{c}{x}\right)}{cx}+\frac{b^2\log\left(x\right)}{c^2}-\frac{b^2\log\left(c^2+x^2\right)}{2c^2}-\frac{b^2\operatorname{atan}^2\left(\frac{c}{x}\right)}{2c^2} & \text{for } c \neq \\ -\frac{a^2}{2x^2} & \text{otherwi} \end{cases}$$

```
[In] integrate((a+b*atan(c/x))**2/x**3,x)
```

```
[Out] Piecewise((-a**2/(2*x**2) - a*b*atan(c/x)/x**2 + a*b/(c*x) - a*b*atan(c/x)/
c**2 - b**2*atan(c/x)**2/(2*x**2) + b**2*atan(c/x)/(c*x) + b**2*log(x)/c**2
 - b**2*log(c**2 + x**2)/(2*c**2) - b**2*atan(c/x)**2/(2*c**2), Ne(c, 0)),
(-a**2/(2*x**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.43

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x^3}\,dx$$
$$=\left(c\left(\frac{\arctan\left(\frac{x}{c}\right)}{c^3}+\frac{1}{c^2x}\right)-\frac{\arctan\left(\frac{c}{x}\right)}{x^2}\right)ab$$
$$+\frac{1}{2}\left(2\,c\left(\frac{\arctan\left(\frac{x}{c}\right)}{c^3}+\frac{1}{c^2x}\right)\arctan\left(\frac{c}{x}\right)+\frac{\arctan\left(\frac{x}{c}\right)^2-\log\left(c^2+x^2\right)+2\,\log\left(x\right)}{c^2}\right)b^2$$
$$-\frac{b^2\arctan\left(\frac{c}{x}\right)^2}{2\,x^2}-\frac{a^2}{2\,x^2}$$

```
[In] integrate((a+b*arctan(c/x))^2/x^3,x, algorithm="maxima")
```

```
[Out] (c*(arctan(x/c)/c^3 + 1/(c^2*x)) - arctan(c/x)/x^2)*a*b + 1/2*(2*c*(arctan(
x/c)/c^3 + 1/(c^2*x))*arctan(c/x) + (arctan(x/c)^2 - log(c^2 + x^2) + 2*log
(x))/c^2)*b^2 - 1/2*b^2*arctan(c/x)^2/x^2 - 1/2*a^2/x^2
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.63

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x^3}\,dx = -\frac{b^2\arctan\left(\frac{c}{x}\right)^2+\frac{b^2c^2\arctan\left(\frac{c}{x}\right)^2}{x^2}+\frac{2abc^2\arctan\left(\frac{c}{x}\right)}{x^2}-\frac{2b^2c\arctan\left(\frac{c}{x}\right)}{x}+iab\log\left(\frac{ic}{x}-1\right)+b^2\log\left(\frac{ic}{x}-1\right)-iab\text{l}}{2c^2}$$

```
[In] integrate((a+b*arctan(c/x))^2/x^3,x, algorithm="giac")
```

```
[Out] -1/2*(b^2*arctan(c/x)^2 + b^2*c^2*arctan(c/x)^2/x^2 + 2*a*b*c^2*arctan(c/x)
/x^2 - 2*b^2*c*arctan(c/x)/x + I*a*b*log(I*c/x - 1) + b^2*log(I*c/x - 1) -
I*a*b*log(-I*c/x - 1) + b^2*log(-I*c/x - 1) + a^2*c^2/x^2 - 2*a*b*c/x)/c^2
```

Mupad [B] (verification not implemented)

Time = 2.93 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.70

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^2}{x^3}\,dx = \frac{b^2\ln(x)-\frac{b^2\ln(x+c\,1\mathrm{i})}{2}-\frac{b^2\,\mathrm{atan}\left(\frac{c}{x}\right)^2}{2}+\frac{b^2\ln\left(\frac{1}{-x+c\,1\mathrm{i}}\right)}{2}+\frac{a\,b\ln(x+c\,1\mathrm{i})\,1\mathrm{i}}{2}-\frac{a\,b\ln(-x+c\,1\mathrm{i})\,1\mathrm{i}}{2}}{c^2} - \frac{\frac{a^2c^2}{2}-x\left(c\,\mathrm{atan}\left(\frac{c}{x}\right)\,b^2+a\,c\,b\right)+\frac{b^2c^2\,\mathrm{atan}\left(\frac{c}{x}\right)^2}{2}+a\,b\,c^2\,\mathrm{atan}\left(\frac{c}{x}\right)}{c^2x^2}$$

```
[In] int((a + b*atan(c/x))^2/x^3,x)
```

```
[Out] (b^2*log(x) - (b^2*log(c*1i + x))/2 - (b^2*atan(c/x)^2)/2 + (b^2*log(1/(c*1
i - x)))/2 + (a*b*log(c*1i + x)*1i)/2 - (a*b*log(c*1i - x)*1i)/2)/c^2 - ((a
^2*c^2)/2 - x*(b^2*c*atan(c/x) + a*b*c) + (b^2*c^2*atan(c/x)^2)/2 + a*b*c^2
*atan(c/x))/(c^2*x^2)
```

3.147 $\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx$

Optimal result 832
Rubi [A] (verified) 833
Mathematica [A] (verified) 836
Maple [B] (verified) 837
Fricas [F] 838
Sympy [F] 838
Maxima [F] 838
Giac [F] 839
Mupad [F(-1)] 839

Optimal result

Integrand size = 16, antiderivative size = 214

$$\begin{aligned}\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx=&\ \frac{1}{4}b^3c^3x+\frac{1}{4}b^3c^4\cot^{-1}\left(\frac{x}{c}\right)+\frac{1}{4}b^2c^2x^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\\&-ibc^4\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2-\frac{3}{4}bc^3x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\\&+\frac{1}{4}bcx^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\\&-\frac{1}{4}c^4\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3+\frac{1}{4}x^4\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3\\&+2b^2c^4\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\log\left(2-\frac{2}{1-\frac{ic}{x}}\right)\\&-ib^3c^4\,\mathrm{PolyLog}\left(2,-1+\frac{2}{1-\frac{ic}{x}}\right)\end{aligned}$$

```
[Out] 1/4*b^3*c^3*x+1/4*b^3*c^4*arccot(x/c)+1/4*b^2*c^2*x^2*(a+b*arccot(x/c))-I*b
*c^4*(a+b*arccot(x/c))^2-3/4*b*c^3*x*(a+b*arccot(x/c))^2+1/4*b*c*x^3*(a+b*a
rccot(x/c))^2-1/4*c^4*(a+b*arccot(x/c))^3+1/4*x^4*(a+b*arccot(x/c))^3+2*b^2
*c^4*(a+b*arccot(x/c))*ln(2-2/(1-I*c/x))-I*b^3*c^4*polylog(2,-1+2/(1-I*c/x)
)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {4948, 4946, 5038, 331, 209, 5044, 4988, 2497, 5004}

$$\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3dx=2b^2c^4\log\left(2-\frac{2}{1-\frac{ic}{x}}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)+\frac{1}{4}b^2c^2x^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)-\frac{1}{4}c^4\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3-ibc^4\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2-\frac{3}{4}bc^3x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{1}{4}x^4\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3+\frac{1}{4}bcx^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2-ib^3c^4\,\mathrm{PolyLog}\left(2,\frac{2}{1-\frac{ic}{x}}-1\right)+\frac{1}{4}b^3c^4\cot^{-1}\left(\frac{x}{c}\right)+\frac{1}{4}b^3c^3x$$

```
[In] Int[x^3*(a + b*ArcTan[c/x])^3,x]
```

```
[Out] (b^3*c^3*x)/4 + (b^3*c^4*ArcCot[x/c])/4 + (b^2*c^2*x^2*(a + b*ArcCot[x/c]))
/4 - I*b*c^4*(a + b*ArcCot[x/c])^2 - (3*b*c^3*x*(a + b*ArcCot[x/c])^2)/4 +
(b*c*x^3*(a + b*ArcCot[x/c])^2)/4 - (c^4*(a + b*ArcCot[x/c])^3)/4 + (x^4*(a
 + b*ArcCot[x/c])^3)/4 + 2*b^2*c^4*(a + b*ArcCot[x/c])*Log[2 - 2/(1 - (I*c)
/x)] - I*b^3*c^4*PolyLog[2, -1 + 2/(1 - (I*c)/x)]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplify
[(m + 1)/n]]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int \frac{(a+b\arctan(cx))^3}{x^5}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{4}x^4\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3-\frac{1}{4}(3bc)\text{Subst}\left(\int\frac{(a+b\arctan(cx))^2}{x^4\,(1+c^2x^2)}\,dx,x,\frac{1}{x}\right)$$

$$= \frac{1}{4}x^4\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3-\frac{1}{4}(3bc)\text{Subst}\left(\int\frac{(a+b\arctan(cx))^2}{x^4}\,dx,x,\frac{1}{x}\right)$$
$$+\frac{1}{4}\left(3bc^3\right)\text{Subst}\left(\int\frac{(a+b\arctan(cx))^2}{x^2\,(1+c^2x^2)}\,dx,x,\frac{1}{x}\right)$$

$$= \frac{1}{4}bcx^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{1}{4}x^4\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3$$
$$-\frac{1}{2}\left(b^2c^2\right)\text{Subst}\left(\int\frac{a+b\arctan(cx)}{x^3\,(1+c^2x^2)}\,dx,x,\frac{1}{x}\right)$$
$$+\frac{1}{4}\left(3bc^3\right)\text{Subst}\left(\int\frac{(a+b\arctan(cx))^2}{x^2}\,dx,x,\frac{1}{x}\right)$$
$$-\frac{1}{4}\left(3bc^5\right)\text{Subst}\left(\int\frac{(a+b\arctan(cx))^2}{1+c^2x^2}\,dx,x,\frac{1}{x}\right)$$

$$= -\frac{3}{4}bc^3x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{1}{4}bcx^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2-\frac{1}{4}c^4\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3$$
$$+\frac{1}{4}x^4\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3-\frac{1}{2}\left(b^2c^2\right)\text{Subst}\left(\int\frac{a+b\arctan(cx)}{x^3}\,dx,x,\frac{1}{x}\right)$$
$$+\frac{1}{2}\left(b^2c^4\right)\text{Subst}\left(\int\frac{a+b\arctan(cx)}{x\,(1+c^2x^2)}\,dx,x,\frac{1}{x}\right)$$
$$+\frac{1}{2}\left(3b^2c^4\right)\text{Subst}\left(\int\frac{a+b\arctan(cx)}{x\,(1+c^2x^2)}\,dx,x,\frac{1}{x}\right)$$

$$= \frac{1}{4}b^2c^2x^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)-ibc^4\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2-\frac{3}{4}bc^3x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2$$
$$+\frac{1}{4}bcx^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2-\frac{1}{4}c^4\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3$$
$$+\frac{1}{4}x^4\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3-\frac{1}{4}\left(b^3c^3\right)\text{Subst}\left(\int\frac{1}{x^2\,(1+c^2x^2)}\,dx,x,\frac{1}{x}\right)$$
$$+\frac{1}{2}\left(ib^2c^4\right)\text{Subst}\left(\int\frac{a+b\arctan(cx)}{x(i+cx)}\,dx,x,\frac{1}{x}\right)$$
$$+\frac{1}{2}\left(3ib^2c^4\right)\text{Subst}\left(\int\frac{a+b\arctan(cx)}{x(i+cx)}\,dx,x,\frac{1}{x}\right)$$

$$
\begin{aligned}
&= \frac{1}{4}b^3c^3x + \frac{1}{4}b^2c^2x^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right) - ibc^4\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2 \\
&\quad - \frac{3}{4}bc^3x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2 + \frac{1}{4}bcx^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2 \\
&\quad - \frac{1}{4}c^4\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3 + \frac{1}{4}x^4\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3 \\
&\quad + 2b^2c^4\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\log\left(2-\frac{2}{1-\frac{ic}{x}}\right) + \frac{1}{4}\left(b^3c^5\right)\text{Subst}\left(\int\frac{1}{1+c^2x^2}\,dx, x, \frac{1}{x}\right) \\
&\quad - \frac{1}{2}\left(b^3c^5\right)\text{Subst}\left(\int\frac{\log\left(2-\frac{2}{1-icx}\right)}{1+c^2x^2}\,dx, x, \frac{1}{x}\right) \\
&\quad - \frac{1}{2}\left(3b^3c^5\right)\text{Subst}\left(\int\frac{\log\left(2-\frac{2}{1-icx}\right)}{1+c^2x^2}\,dx, x, \frac{1}{x}\right) \\
&= \frac{1}{4}b^3c^3x + \frac{1}{4}b^3c^4\cot^{-1}\left(\frac{x}{c}\right) + \frac{1}{4}b^2c^2x^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right) \\
&\quad - ibc^4\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2 - \frac{3}{4}bc^3x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2 \\
&\quad + \frac{1}{4}bcx^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2 - \frac{1}{4}c^4\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3 + \frac{1}{4}x^4\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3 \\
&\quad + 2b^2c^4\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\log\left(2-\frac{2}{1-\frac{ic}{x}}\right) - ib^3c^4\,\text{PolyLog}\left(2, -1+\frac{2}{1-\frac{ic}{x}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.18

$$
\begin{aligned}
\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3 dx = \frac{1}{4}\Bigg(&ab^2c^4 - 3a^2bc^3x + b^3c^3x + ab^2c^2x^2 + a^2bcx^3 + a^3x^4 \\
&+ b^2\left(bc\left(-4ic^3-3c^2x+x^3\right)+3a\left(-c^4+x^4\right)\right)\arctan\left(\frac{c}{x}\right)^2 \\
&+ b^3\left(-c^4+x^4\right)\arctan\left(\frac{c}{x}\right)^3 \\
&+ b\arctan\left(\frac{c}{x}\right)\left(2abcx\left(-3c^2+x^2\right)+b^2c^2\left(c^2+x^2\right)\right. \\
&\left.+ 3a^2\left(-c^4+x^4\right) + 8b^2c^4\log\left(1-e^{2i\arctan\left(\frac{c}{x}\right)}\right)\right) \\
&+ 8ab^2c^4\log\left(\frac{c}{\sqrt{1+\frac{c^2}{x^2}}\,x}\right) \\
&- 4ib^3c^4\,\text{PolyLog}\left(2, e^{2i\arctan\left(\frac{c}{x}\right)}\right)\Bigg)
\end{aligned}
$$

```
[In] Integrate[x^3*(a + b*ArcTan[c/x])^3,x]
```

```
[Out] (a*b^2*c^4 - 3*a^2*b*c^3*x + b^3*c^3*x + a*b^2*c^2*x^2 + a^2*b*c*x^3 + a^3*
x^4 + b^2*(b*c*((-4*I)*c^3 - 3*c^2*x + x^3) + 3*a*(-c^4 + x^4))*ArcTan[c/x]
^2 + b^3*(-c^4 + x^4)*ArcTan[c/x]^3 + b*ArcTan[c/x]*(2*a*b*c*x*(-3*c^2 + x^
2) + b^2*c^2*(c^2 + x^2) + 3*a^2*(-c^4 + x^4) + 8*b^2*c^4*Log[1 - E^((2*I)*
ArcTan[c/x])]) + 8*a*b^2*c^4*Log[c/(Sqrt[1 + c^2/x^2]*x)] - (4*I)*b^3*c^4*P
olyLog[2, E^((2*I)*ArcTan[c/x])])/4
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 479 vs. $2(196) = 392$.

Time = 14.52 (sec) , antiderivative size = 480, normalized size of antiderivative = 2.24

method	result
derivativedivides	$-c^4\left(-\frac{a^3x^4}{4c^4}+b^3\left(-\frac{x^4\arctan\left(\frac{c}{x}\right)^3}{4c^4}+\frac{\arctan\left(\frac{c}{x}\right)^3}{4}-\frac{x^3\arctan\left(\frac{c}{x}\right)^2}{4c^3}+\frac{3x\arctan\left(\frac{c}{x}\right)^2}{4c}+\arctan\left(\frac{c}{x}\right)\ln\right.\right.$
default	$-c^4\left(-\frac{a^3x^4}{4c^4}+b^3\left(-\frac{x^4\arctan\left(\frac{c}{x}\right)^3}{4c^4}+\frac{\arctan\left(\frac{c}{x}\right)^3}{4}-\frac{x^3\arctan\left(\frac{c}{x}\right)^2}{4c^3}+\frac{3x\arctan\left(\frac{c}{x}\right)^2}{4c}+\arctan\left(\frac{c}{x}\right)\ln\right.\right.$
parts	$\frac{a^2bc\,x^3}{4}+\frac{a^3x^4}{4}+\frac{b^3c^3x}{4}-\frac{c^4b^3\arctan\left(\frac{x}{c}\right)}{4}-ic^4b^3\ln\left(\frac{c}{x}\right)\ln\left(1-\frac{ic}{x}\right)-\frac{ic^4b^3\ln\left(\frac{c}{x}-i\right)\ln\left(1+\frac{c^2}{x^2}\right)}{2}+ic$
risch	Expression too large to display

```
[In] int(x^3*(a+b*arctan(c/x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -c^4*(-1/4*a^3/c^4*x^4+b^3*(-1/4/c^4*x^4*arctan(c/x)^3+1/4*arctan(c/x)^3-1/
4/c^3*x^3*arctan(c/x)^2+3/4/c*x*arctan(c/x)^2+arctan(c/x)*ln(1+c^2/x^2)-1/4
/c^2*x^2*arctan(c/x)-2*ln(c/x)*arctan(c/x)-1/4*arctan(c/x)-1/4*x/c-I*ln(c/x
)*ln(1+I*c/x)+I*ln(c/x)*ln(1-I*c/x)-I*dilog(1+I*c/x)+I*dilog(1-I*c/x)+1/2*I
*(ln(c/x-I)*ln(1+c^2/x^2)-1/2*ln(c/x-I)^2-dilog(-1/2*I*(c/x+I))-ln(c/x-I)*l
n(-1/2*I*(c/x+I)))-1/2*I*(ln(c/x+I)*ln(1+c^2/x^2)-1/2*ln(c/x+I)^2-dilog(1/2
*I*(c/x-I))-ln(c/x+I)*ln(1/2*I*(c/x-I)))))+3*a*b^2*(-1/4/c^4*x^4*arctan(c/x)
^2+1/4*arctan(c/x)^2-1/6/c^3*x^3*arctan(c/x)+1/2/c*x*arctan(c/x)+1/3*ln(1+c
^2/x^2)-1/12/c^2*x^2-2/3*ln(c/x))+3*a^2*b*(-1/4/c^4*x^4*arctan(c/x)-1/12/c^
3*x^3+1/4*x/c+1/4*arctan(c/x)))
```

Fricas [F]

$$\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx=\int\left(b\arctan\left(\frac{c}{x}\right)+a\right)^3x^3\,dx$$

```
[In] integrate(x^3*(a+b*arctan(c/x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^3*arctan(c/x)^3 + 3*a*b^2*x^3*arctan(c/x)^2 + 3*a^2*b*x^3*ar
ctan(c/x) + a^3*x^3, x)
```

Sympy [F]

$$\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx=\int x^3\left(a+b\operatorname{atan}\left(\frac{c}{x}\right)\right)^3\,dx$$

```
[In] integrate(x**3*(a+b*atan(c/x))**3,x)
```

```
[Out] Integral(x**3*(a + b*atan(c/x))**3, x)
```

Maxima [F]

$$\int x^3\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx=\int\left(b\arctan\left(\frac{c}{x}\right)+a\right)^3x^3\,dx$$

```
[In] integrate(x^3*(a+b*arctan(c/x))^3,x, algorithm="maxima")
```

```
[Out] 3/4*a*b^2*x^4*arctan(c/x)^2 + 1/4*a^3*x^4 + 1/4*(3*x^4*arctan(c/x) + (3*c^3
*arctan(x/c) - 3*c^2*x + x^3)*c)*a^2*b + 1/4*((3*c^2*arctan(x/c)^2 - 4*c^2*
log(c^2 + x^2) + x^2)*c^2 + 2*(3*c^3*arctan(x/c) - 3*c^2*x + x^3)*c*arctan(
c/x))*a*b^2 - 1/64*(12*c^4*arctan(c/x)^2*arctan(x/c) + 8*c^4*arctan2(c, x)^
3 - 8*x^4*arctan2(c, x)^3 + 4*(3*arctan(c/x)*arctan(x/c)^2/c + arctan(x/c)^
3/c)*c^5 + 12*c^3*x*arctan2(c, x)^2 - 4*c*x^3*arctan2(c, x)^2 + 192*c^5*int
egrate(1/64*log(c^2 + x^2)^2/(c^2 + x^2), x) + 1536*c^4*integrate(1/64*x*ar
ctan(c/x)/(c^2 + x^2), x) + 768*c^3*integrate(1/64*x^2*log(c^2 + x^2)/(c^2
+ x^2), x) - 2048*c^2*integrate(1/64*x^3*arctan(c/x)^3/(c^2 + x^2), x) - 51
2*c^2*integrate(1/64*x^3*arctan(c/x)/(c^2 + x^2), x) - (3*c^3*x - c*x^3)*lo
g(c^2 + x^2)^2 - 768*c*integrate(1/64*x^4*arctan(c/x)^2/(c^2 + x^2), x) - 1
92*c*integrate(1/64*x^4*log(c^2 + x^2)^2/(c^2 + x^2), x) - 256*c*integrate(
1/64*x^4*log(c^2 + x^2)/(c^2 + x^2), x) - 2048*integrate(1/64*x^5*arctan(c/
x)^3/(c^2 + x^2), x))*b^3
```

Giac [F]

$$\int x^3 \left(a+b\arctan\left(\frac{c}{x}\right)\right)^3 dx = \int \left(b\arctan\left(\frac{c}{x}\right)+a\right)^3 x^3 dx$$

```
[In] integrate(x^3*(a+b*arctan(c/x))^3,x, algorithm="giac")
[Out] integrate((b*arctan(c/x) + a)^3*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \left(a+b\arctan\left(\frac{c}{x}\right)\right)^3 dx = \int x^3 \left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^3 dx$$

```
[In] int(x^3*(a + b*atan(c/x))^3,x)
[Out] int(x^3*(a + b*atan(c/x))^3, x)
```

3.148 $\int x^2 \left(a+b \arctan \left(\frac{c}{x}\right)\right)^3 \, dx$

Optimal result 840
Rubi [A] (verified) 841
Mathematica [A] (verified) 845
Maple [C] (warning: unable to verify) 846
Fricas [F] 847
Sympy [F] 847
Maxima [F] 848
Giac [F] 848
Mupad [F(-1)] 848

Optimal result

Integrand size = 16, antiderivative size = 229

$$\begin{aligned}\int x^2 \left(a+b \arctan \left(\frac{c}{x}\right)\right)^3 \, dx =\ & b^2 c^2 x \left(a+b \cot^{-1}\left(\frac{x}{c}\right)\right)+\frac{1}{2} b c^3 \left(a+b \cot^{-1}\left(\frac{x}{c}\right)\right)^2 \\ & +\frac{1}{2} b c x^2 \left(a+b \cot^{-1}\left(\frac{x}{c}\right)\right)^2 \\ & -\frac{1}{3} i c^3 \left(a+b \cot^{-1}\left(\frac{x}{c}\right)\right)^3+\frac{1}{3} x^3 \left(a+b \cot^{-1}\left(\frac{x}{c}\right)\right)^3 \\ & +b c^3 \left(a+b \cot^{-1}\left(\frac{x}{c}\right)\right)^2 \log \left(2-\frac{2}{1-\frac{i c}{x}}\right) \\ & +\frac{1}{2} b^3 c^3 \log \left(1+\frac{c^2}{x^2}\right)+b^3 c^3 \log (x) \\ & -i b^2 c^3 \left(a+b \cot^{-1}\left(\frac{x}{c}\right)\right) \text{PolyLog}\left(2,-1+\frac{2}{1-\frac{i c}{x}}\right) \\ & +\frac{1}{2} b^3 c^3 \text{PolyLog}\left(3,-1+\frac{2}{1-\frac{i c}{x}}\right)\end{aligned}$$

```
[Out] b^2*c^2*x*(a+b*arccot(x/c))+1/2*b*c^3*(a+b*arccot(x/c))^2+1/2*b*c*x^2*(a+b*
arccot(x/c))^2-1/3*I*c^3*(a+b*arccot(x/c))^3+1/3*x^3*(a+b*arccot(x/c))^3+b*
c^3*(a+b*arccot(x/c))^2*ln(2-2/(1-I*c/x))+1/2*b^3*c^3*ln(1+c^2/x^2)+b^3*c^3
*ln(x)-I*b^2*c^3*(a+b*arccot(x/c))*polylog(2,-1+2/(1-I*c/x))+1/2*b^3*c^3*po
lylog(3,-1+2/(1-I*c/x))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {4948, 4946, 5038, 272, 36, 29, 31, 5004, 5044, 4988, 5112, 6745}

$$\int x^2 \left(a+b\arctan\left(\frac{c}{x}\right)\right)^3 dx = -ib^2c^3\operatorname{PolyLog}\left(2,\frac{2}{1-\frac{ic}{x}}-1\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right) + b^2c^2x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right) - \frac{1}{3}ic^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3 + \frac{1}{2}bc^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2 + bc^3\log\left(2-\frac{2}{1-\frac{ic}{x}}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2 + \frac{1}{3}x^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3 + \frac{1}{2}bcx^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2 + \frac{1}{2}b^3c^3\operatorname{PolyLog}\left(3,\frac{2}{1-\frac{ic}{x}}-1\right) + b^3c^3\log(x) + \frac{1}{2}b^3c^3\log\left(\frac{c^2}{x^2}+1\right)$$

```
[In] Int[x^2*(a + b*ArcTan[c/x])^3,x]
```

```
[Out] b^2*c^2*x*(a + b*ArcCot[x/c]) + (b*c^3*(a + b*ArcCot[x/c])^2)/2 + (b*c*x^2*
(a + b*ArcCot[x/c])^2)/2 - (I/3)*c^3*(a + b*ArcCot[x/c])^3 + (x^3*(a + b*Ar
cCot[x/c])^3)/3 + b*c^3*(a + b*ArcCot[x/c])^2*Log[2 - 2/(1 - (I*c)/x)] + (b
^3*c^3*Log[1 + c^2/x^2])/2 + b^3*c^3*Log[x] - I*b^2*c^3*(a + b*ArcCot[x/c])
*PolyLog[2, -1 + 2/(1 - (I*c)/x)] + (b^3*c^3*PolyLog[3, -1 + 2/(1 - (I*c)/x
)])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplify
[(m + 1)/n]]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5112

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{(a+b\arctan(cx))^3}{x^4}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{3}x^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3-(bc)\text{Subst}\left(\int \frac{(a+b\arctan(cx))^2}{x^3\left(1+c^2x^2\right)}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{3}x^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3-(bc)\text{Subst}\left(\int \frac{(a+b\arctan(cx))^2}{x^3}\,dx, x, \frac{1}{x}\right)\\
&\quad+\left(bc^3\right)\text{Subst}\left(\int \frac{(a+b\arctan(cx))^2}{x\left(1+c^2x^2\right)}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{2}bcx^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2-\frac{1}{3}ic^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3\\
&\quad+\frac{1}{3}x^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3-\left(b^2c^2\right)\text{Subst}\left(\int \frac{a+b\arctan(cx)}{x^2\left(1+c^2x^2\right)}\,dx, x, \frac{1}{x}\right)\\
&\quad+\left(ibc^3\right)\text{Subst}\left(\int \frac{(a+b\arctan(cx))^2}{x(i+cx)}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{2}bcx^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2-\frac{1}{3}ic^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3\\
&\quad+\frac{1}{3}x^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3+bc^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\log\left(2-\frac{2}{1-\frac{ic}{x}}\right)\\
&\quad-\left(b^2c^2\right)\text{Subst}\left(\int \frac{a+b\arctan(cx)}{x^2}\,dx, x, \frac{1}{x}\right)\\
&\quad+\left(b^2c^4\right)\text{Subst}\left(\int \frac{a+b\arctan(cx)}{1+c^2x^2}\,dx, x, \frac{1}{x}\right)\\
&\quad-\left(2b^2c^4\right)\text{Subst}\left(\int \frac{(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right)}{1+c^2x^2}\,dx, x, \frac{1}{x}\right)
\end{aligned}
$$

$$
\begin{aligned}
&= b^2c^2x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)+\frac{1}{2}bc^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2 \\
&\quad+\frac{1}{2}bcx^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2-\frac{1}{3}ic^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3 \\
&\quad+\frac{1}{3}x^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3+bc^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\log\left(2-\frac{2}{1-\frac{ic}{x}}\right) \\
&\quad-ib^2c^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\text{PolyLog}\left(2,-1+\frac{2}{1-\frac{ic}{x}}\right) \\
&\quad-\left(b^3c^3\right)\text{Subst}\left(\int\frac{1}{x\left(1+c^2x^2\right)}\,dx,x,\frac{1}{x}\right) \\
&\quad+\left(ib^3c^4\right)\text{Subst}\left(\int\frac{\text{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)}{1+c^2x^2}\,dx,x,\frac{1}{x}\right)
\end{aligned}
$$

$$
\begin{aligned}
&= b^2c^2x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)+\frac{1}{2}bc^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2 \\
&\quad+\frac{1}{2}bcx^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2-\frac{1}{3}ic^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3 \\
&\quad+\frac{1}{3}x^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3+bc^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\log\left(2-\frac{2}{1-\frac{ic}{x}}\right) \\
&\quad-ib^2c^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\text{PolyLog}\left(2,-1+\frac{2}{1-\frac{ic}{x}}\right) \\
&\quad+\frac{1}{2}b^3c^3\,\text{PolyLog}\left(3,-1+\frac{2}{1-\frac{ic}{x}}\right)-\frac{1}{2}\left(b^3c^3\right)\text{Subst}\left(\int\frac{1}{x\left(1+c^2x\right)}\,dx,x,\frac{1}{x^2}\right)
\end{aligned}
$$

$$
\begin{aligned}
&= b^2c^2x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)+\frac{1}{2}bc^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2 \\
&\quad+\frac{1}{2}bcx^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2-\frac{1}{3}ic^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3 \\
&\quad+\frac{1}{3}x^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3+bc^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\log\left(2-\frac{2}{1-\frac{ic}{x}}\right) \\
&\quad-ib^2c^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\text{PolyLog}\left(2,-1+\frac{2}{1-\frac{ic}{x}}\right) \\
&\quad+\frac{1}{2}b^3c^3\,\text{PolyLog}\left(3,-1+\frac{2}{1-\frac{ic}{x}}\right)-\frac{1}{2}\left(b^3c^3\right)\text{Subst}\left(\int\frac{1}{x}\,dx,x,\frac{1}{x^2}\right) \\
&\quad+\frac{1}{2}\left(b^3c^5\right)\text{Subst}\left(\int\frac{1}{1+c^2x}\,dx,x,\frac{1}{x^2}\right)
\end{aligned}
$$

$$
\begin{aligned}
&= b^2c^2x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)+\frac{1}{2}bc^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{1}{2}bcx^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2 \\
&\quad -\frac{1}{3}ic^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3+\frac{1}{3}x^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3 \\
&\quad +bc^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\log\left(2-\frac{2}{1-\frac{ic}{x}}\right)+\frac{1}{2}b^3c^3\log\left(1+\frac{c^2}{x^2}\right) \\
&\quad +b^3c^3\log(x)-ib^2c^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\operatorname{PolyLog}\left(2,-1+\frac{2}{1-\frac{ic}{x}}\right) \\
&\quad +\frac{1}{2}b^3c^3\operatorname{PolyLog}\left(3,-1+\frac{2}{1-\frac{ic}{x}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.47

$$
\begin{aligned}
\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3 dx &= \frac{1}{6}\Bigg(3a^2bcx^2+2a^3x^3+6a^2bx^3\arctan\left(\frac{c}{x}\right)-3a^2bc^3\log\left(c^2+x^2\right) \\
&\quad +6ab^2\Bigg(c^2x+\left(-ic^3+x^3\right)\arctan\left(\frac{c}{x}\right)^2 \\
&\quad +c\arctan\left(\frac{c}{x}\right)\left(c^2+x^2+2c^2\log\left(1-e^{2i\arctan\left(\frac{c}{x}\right)}\right)\right) \\
&\quad -ic^3\operatorname{PolyLog}\left(2,e^{2i\arctan\left(\frac{c}{x}\right)}\right)\Bigg) \\
&\quad +\frac{1}{4}b^3\Bigg(-ic^3\pi^3+24c^2x\arctan\left(\frac{c}{x}\right)+12c^3\arctan\left(\frac{c}{x}\right)^2 \\
&\quad +12cx^2\arctan\left(\frac{c}{x}\right)^2+8ic^3\arctan\left(\frac{c}{x}\right)^3+8x^3\arctan\left(\frac{c}{x}\right)^3 \\
&\quad +24c^3\arctan\left(\frac{c}{x}\right)^2\log\left(1-e^{-2i\arctan\left(\frac{c}{x}\right)}\right) \\
&\quad -24c^3\log\left(\frac{1}{\sqrt{1+\frac{c^2}{x^2}}}\right)-24c^3\log\left(\frac{c}{x}\right) \\
&\quad +24ic^3\arctan\left(\frac{c}{x}\right)\operatorname{PolyLog}\left(2,e^{-2i\arctan\left(\frac{c}{x}\right)}\right) \\
&\quad +12c^3\operatorname{PolyLog}\left(3,e^{-2i\arctan\left(\frac{c}{x}\right)}\right)\Bigg)\Bigg)
\end{aligned}
$$

```
[In] Integrate[x^2*(a + b*ArcTan[c/x])^3,x]
```

```
[Out] (3*a^2*b*c*x^2 + 2*a^3*x^3 + 6*a^2*b*x^3*ArcTan[c/x] - 3*a^2*b*c^3*Log[c^2
+ x^2] + 6*a*b^2*(c^2*x + ((-I)*c^3 + x^3)*ArcTan[c/x]^2 + c*ArcTan[c/x]*(c
```

```
^2 + x^2 + 2*c^2*Log[1 - E^((2*I)*ArcTan[c/x])]) - I*c^3*PolyLog[2, E^((2*I
)*ArcTan[c/x])]) + (b^3*((-I)*c^3*Pi^3 + 24*c^2*x*ArcTan[c/x] + 12*c^3*ArcT
an[c/x]^2 + 12*c*x^2*ArcTan[c/x]^2 + (8*I)*c^3*ArcTan[c/x]^3 + 8*x^3*ArcTan
[c/x]^3 + 24*c^3*ArcTan[c/x]^2*Log[1 - E^((-2*I)*ArcTan[c/x])] - 24*c^3*Log
[1/Sqrt[1 + c^2/x^2]] - 24*c^3*Log[c/x] + (24*I)*c^3*ArcTan[c/x]*PolyLog[2,
 E^((-2*I)*ArcTan[c/x])] + 12*c^3*PolyLog[3, E^((-2*I)*ArcTan[c/x])]))/4)/6
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.89 (sec) , antiderivative size = 2634, normalized size of antiderivative = 11.50

Expression too large to display

```
[In] int(x^2*(a+b*arctan(c/x))^3,x)
```

```
[Out] 1/4*I*c^3*b^3*arctan(c/x)^2*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)+1)^2)*csgn(I*((
1+I*c/x)^2/(1+c^2/x^2)+1))^2*Pi-1/2*I*c^3*b^3*arctan(c/x)^2*csgn(I*((1+I*c/
x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))*csgn(((1+I*c/x)^2/(1+c^2/x
^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))^2*Pi+1/2*I*c^3*b^3*arctan(c/x)^2*csgn(I
*(1+I*c/x)/(1+c^2/x^2)^(1/2))*csgn(I*(1+I*c/x)^2/(1+c^2/x^2))^2*Pi+1/2*I*c^
3*b^3*arctan(c/x)^2*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/
x^2)+1))*csgn(((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))*Pi+1
/4*I*c^3*b^3*arctan(c/x)^2*csgn(I*(1+I*c/x)^2/(1+c^2/x^2)/((1+I*c/x)^2/(1+c
^2/x^2)+1)^2)^2*csgn(I/((1+I*c/x)^2/(1+c^2/x^2)+1)^2)*Pi-1/2*I*c^3*b^3*arct
an(c/x)^2*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))^2
*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1))*Pi+a*b^2*c^2*x+1/3*a^3*x^3-1/2*I*c^3*b
^3*arctan(c/x)^2*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2
)+1))^2*csgn(I/((1+I*c/x)^2/(1+c^2/x^2)+1))*Pi-1/4*I*c^3*b^3*arctan(c/x)^2*
csgn(I*(1+I*c/x)/(1+c^2/x^2)^(1/2))^2*csgn(I*(1+I*c/x)^2/(1+c^2/x^2))*Pi+1/
2*a^2*b*c*x^2-c^3*a*b^2*arctan(x/c)+c^3*a^2*b*ln(c/x)-1/2*c^3*a^2*b*ln(1+c^
2/x^2)+I*c^3*b^3*arctan(c/x)+c^3*b^3*arctan(c/x)^2*ln(2)+c^3*b^3*ln(c/x)*ar
ctan(c/x)^2-1/2*c^3*b^3*arctan(c/x)^2*ln(1+c^2/x^2)+c^3*b^3*arctan(c/x)^2*l
n(1-(1+I*c/x)/(1+c^2/x^2)^(1/2))+c^3*b^3*arctan(c/x)^2*ln((1+I*c/x)/(1+c^2/
x^2)^(1/2))-c^3*b^3*arctan(c/x)^2*ln((1+I*c/x)^2/(1+c^2/x^2)-1)+c^3*b^3*arc
tan(c/x)^2*ln((1+I*c/x)/(1+c^2/x^2)^(1/2)+1)-1/3*I*c^3*b^3*arctan(c/x)^3+c^
2*b^3*arctan(c/x)*x+1/2*c*b^3*arctan(c/x)^2*x^2-1/2*I*c^3*b^3*arctan(c/x)^2
*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)+1)^2)^2*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)+1)
)*Pi+1/4*I*c^3*b^3*arctan(c/x)^2*csgn(I*(1+I*c/x)^2/(1+c^2/x^2)/((1+I*c/x)^
2/(1+c^2/x^2)+1)^2)^2*csgn(I*(1+I*c/x)^2/(1+c^2/x^2))*Pi+a*b^2*x^3*arctan(c
/x)^2+x^3*a^2*b*arctan(c/x)+1/3*x^3*b^3*arctan(c/x)^3+I*c^3*a*b^2*dilog(1+I
*c/x)+1/2*I*c^3*b^3*arctan(c/x)^2*Pi-2*I*c^3*b^3*arctan(c/x)*polylog(2,(1+I
*c/x)/(1+c^2/x^2)^(1/2))-2*I*c^3*b^3*arctan(c/x)*polylog(2,-(1+I*c/x)/(1+c^
2/x^2)^(1/2))-1/4*I*c^3*a*b^2*ln(c/x+I)^2+1/2*I*c^3*a*b^2*dilog(-1/2*I*(c/x
+I))+1/4*I*c^3*a*b^2*ln(c/x-I)^2-I*c^3*a*b^2*dilog(1-I*c/x)-1/2*I*c^3*a*b^2
*dilog(1/2*I*(c/x-I))+c*a*b^2*x^2*arctan(c/x)+2*c^3*a*b^2*ln(c/x)*arctan(c/
```

```
x)-c^3*a*b^2*arctan(c/x)*ln(1+c^2/x^2)+1/2*I*c^3*b^3*arctan(c/x)^2*csgn(I*(
(1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))*csgn(I/((1+I*c/x)^2
/(1+c^2/x^2)+1))*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1))*Pi-1/4*I*c^3*b^3*arcta
n(c/x)^2*csgn(I*(1+I*c/x)^2/(1+c^2/x^2)/((1+I*c/x)^2/(1+c^2/x^2)+1)^2)*csgn
(I/((1+I*c/x)^2/(1+c^2/x^2)+1)^2)*csgn(I*(1+I*c/x)^2/(1+c^2/x^2))*Pi+I*c^3*
a*b^2*ln(c/x)*ln(1+I*c/x)-1/2*I*c^3*b^3*arctan(c/x)^2*csgn(((1+I*c/x)^2/(1+
c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))^2*Pi+1/2*I*c^3*b^3*arctan(c/x)^2*c
sgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))^3*Pi-1/4*I*c
^3*b^3*arctan(c/x)^2*csgn(I*(1+I*c/x)^2/(1+c^2/x^2))^3*Pi-1/4*I*c^3*b^3*arc
tan(c/x)^2*csgn(I*(1+I*c/x)^2/(1+c^2/x^2)/((1+I*c/x)^2/(1+c^2/x^2)+1)^2)^3*
Pi+1/2*I*c^3*b^3*arctan(c/x)^2*csgn(((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^
2/(1+c^2/x^2)+1))^3*Pi+1/2*I*c^3*a*b^2*ln(c/x+I)*ln(1+c^2/x^2)-I*c^3*a*b^2*
ln(c/x)*ln(1-I*c/x)+1/2*I*c^3*a*b^2*ln(c/x-I)*ln(-1/2*I*(c/x+I))-1/2*I*c^3*
a*b^2*ln(c/x+I)*ln(1/2*I*(c/x-I))-1/2*I*c^3*a*b^2*ln(c/x-I)*ln(1+c^2/x^2)+1
/4*I*c^3*b^3*arctan(c/x)^2*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)+1)^2)^3*Pi+2*c^3
*b^3*polylog(3,-(1+I*c/x)/(1+c^2/x^2)^(1/2))-c^3*b^3*ln((1+I*c/x)/(1+c^2/x^
2)^(1/2)-1)-c^3*b^3*ln((1+I*c/x)/(1+c^2/x^2)^(1/2)+1)+2*c^3*b^3*polylog(3,(
1+I*c/x)/(1+c^2/x^2)^(1/2))+1/2*c^3*b^3*arctan(c/x)^2
```

Fricas [F]

$$\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx=\int\left(b\arctan\left(\frac{c}{x}\right)+a\right)^3x^2\,dx$$

```
[In] integrate(x^2*(a+b*arctan(c/x))^3,x, algorithm="fricas")
[Out] integral(b^3*x^2*arctan(c/x)^3 + 3*a*b^2*x^2*arctan(c/x)^2 + 3*a^2*b*x^2*ar
ctan(c/x) + a^3*x^2, x)
```

Sympy [F]

$$\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx=\int x^2\left(a+b\operatorname{atan}\left(\frac{c}{x}\right)\right)^3\,dx$$

```
[In] integrate(x**2*(a+b*atan(c/x))**3,x)
[Out] Integral(x**2*(a + b*atan(c/x))**3, x)
```

Maxima [F]

$$\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx=\int\left(b\arctan\left(\frac{c}{x}\right)+a\right)^3x^2\,dx$$

[In] integrate(x^2*(a+b*arctan(c/x))^3,x, algorithm="maxima")

[Out] 1/24*b^3*x^3*arctan2(c, x)^3 - 1/32*b^3*x^3*arctan2(c, x)*log(c^2 + x^2)^2 + 1/3*a^3*x^3 + 1/2*(2*x^3*arctan(c/x) - (c^2*log(c^2 + x^2) - x^2)*c)*a^2*b + integrate(1/32*(4*b^3*c*x^3*arctan2(c, x)^2 + 4*b^3*x^4*arctan2(c, x)*log(c^2 + x^2) + 4*(7*b^3*arctan2(c, x)^3 + 24*a*b^2*arctan2(c, x)^2)*x^4 + 4*(7*b^3*c^2*arctan2(c, x)^3 + 24*a*b^2*c^2*arctan2(c, x)^2)*x^2 + (3*b^3*c^2*x^2*arctan2(c, x) + 3*b^3*x^4*arctan2(c, x) - b^3*c*x^3)*log(c^2 + x^2)^2)/(c^2 + x^2), x)

Giac [F]

$$\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx=\int\left(b\arctan\left(\frac{c}{x}\right)+a\right)^3x^2\,dx$$

[In] integrate(x^2*(a+b*arctan(c/x))^3,x, algorithm="giac")

[Out] integrate((b*arctan(c/x) + a)^3*x^2, x)

Mupad [F(-1)]

Timed out.

$$\int x^2\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx=\int x^2\,\left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^3\,dx$$

[In] int(x^2*(a + b*atan(c/x))^3,x)

[Out] int(x^2*(a + b*atan(c/x))^3, x)

3.149 $\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx$

Optimal result . . . 849
Rubi [A] (verified) . . . 849
Mathematica [A] (verified) . . . 852
Maple [B] (verified) . . . 852
Fricas [F] . . . 853
Sympy [F] . . . 854
Maxima [F] . . . 854
Giac [F] . . . 854
Mupad [F(-1)] . . . 855

Optimal result

Integrand size = 14, antiderivative size = 145

$$\begin{aligned}\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx &= \frac{3}{2}ibc^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{3}{2}bcx\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\\&\quad+\frac{1}{2}c^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3+\frac{1}{2}x^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3\\&\quad-3b^2c^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\log\left(2-\frac{2}{1-\frac{ic}{x}}\right)\\&\quad+\frac{3}{2}ib^3c^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1-\frac{ic}{x}}\right)\end{aligned}$$

```
[Out] 3/2*I*b*c^2*(a+b*arccot(x/c))^2+3/2*b*c*x*(a+b*arccot(x/c))^2+1/2*c^2*(a+b*
arccot(x/c))^3+1/2*x^2*(a+b*arccot(x/c))^3-3*b^2*c^2*(a+b*arccot(x/c))*ln(2
-2/(1-I*c/x))+3/2*I*b^3*c^2*polylog(2,-1+2/(1-I*c/x))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4948, 4946, 5038, 5044, 4988, 2497, 5004}

$$\begin{aligned}\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx &= -3b^2c^2\log\left(2-\frac{2}{1-\frac{ic}{x}}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\\&\quad+\frac{3}{2}ibc^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{1}{2}c^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3\\&\quad+\frac{1}{2}x^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3+\frac{3}{2}bcx\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\\&\quad+\frac{3}{2}ib^3c^2\operatorname{PolyLog}\left(2,\frac{2}{1-\frac{ic}{x}}-1\right)\end{aligned}$$

```
[In] Int[x*(a + b*ArcTan[c/x])^3,x]
```

```
[Out] ((3*I)/2)*b*c^2*(a + b*ArcCot[x/c])^2 + (3*b*c*x*(a + b*ArcCot[x/c])^2)/2 +
 (c^2*(a + b*ArcCot[x/c])^3)/2 + (x^2*(a + b*ArcCot[x/c])^3)/2 - 3*b^2*c^2*
(a + b*ArcCot[x/c])*Log[2 - 2/(1 - (I*c)/x)] + ((3*I)/2)*b^3*c^2*PolyLog[2,
 -1 + 2/(1 - (I*c)/x)]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplify
[(m + 1)/n]]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
```

```
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{(a+b\arctan(cx))^3}{x^3}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{2}x^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3 - \frac{1}{2}(3bc)\text{Subst}\left(\int \frac{(a+b\arctan(cx))^2}{x^2(1+c^2x^2)}\,dx, x, \frac{1}{x}\right)\\
&= \frac{1}{2}x^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3 - \frac{1}{2}(3bc)\text{Subst}\left(\int \frac{(a+b\arctan(cx))^2}{x^2}\,dx, x, \frac{1}{x}\right)\\
&\quad + \frac{1}{2}\left(3bc^3\right)\text{Subst}\left(\int \frac{(a+b\arctan(cx))^2}{1+c^2x^2}\,dx, x, \frac{1}{x}\right)\\
&= \frac{3}{2}bcx\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2 + \frac{1}{2}c^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3\\
&\quad + \frac{1}{2}x^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3 - \left(3b^2c^2\right)\text{Subst}\left(\int \frac{a+b\arctan(cx)}{x(1+c^2x^2)}\,dx, x, \frac{1}{x}\right)\\
&= \frac{3}{2}ibc^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2 + \frac{3}{2}bcx\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2 + \frac{1}{2}c^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3\\
&\quad + \frac{1}{2}x^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3 - \left(3ib^2c^2\right)\text{Subst}\left(\int \frac{a+b\arctan(cx)}{x(i+cx)}\,dx, x, \frac{1}{x}\right)\\
&= \frac{3}{2}ibc^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2 + \frac{3}{2}bcx\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2 + \frac{1}{2}c^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3\\
&\quad + \frac{1}{2}x^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3 - 3b^2c^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\log\left(2-\frac{2}{1-\frac{ic}{x}}\right)\\
&\quad + \left(3b^3c^3\right)\text{Subst}\left(\int \frac{\log\left(2-\frac{2}{1-icx}\right)}{1+c^2x^2}\,dx, x, \frac{1}{x}\right)\\
&= \frac{3}{2}ibc^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2 + \frac{3}{2}bcx\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\\
&\quad + \frac{1}{2}c^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3 + \frac{1}{2}x^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3\\
&\quad - 3b^2c^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\log\left(2-\frac{2}{1-\frac{ic}{x}}\right) + \frac{3}{2}ib^3c^2\,\text{PolyLog}\left(2,-1+\frac{2}{1-\frac{ic}{x}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.20

$$\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3dx=\frac{1}{2}\left(3b^2\left(bc(ic+x)+a\left(c^2+x^2\right)\right)\arctan\left(\frac{c}{x}\right)^2+b^3\left(c^2+x^2\right)\arctan\left(\frac{c}{x}\right)^3+3b\arctan\left(\frac{c}{x}\right)\left(a\left(2bcx+a\left(c^2+x^2\right)\right)-2b^2c^2\log\left(1-e^{2i\arctan\left(\frac{c}{x}\right)}\right)\right)+a\left(ax(3bc+ax)-6b^2c^2\log\left(\frac{c}{\sqrt{1+\frac{c^2}{x^2}}x}\right)\right)+3ib^3c^2\,\text{PolyLog}\left(2,e^{2i\arctan\left(\frac{c}{x}\right)}\right)\right)$$

```
[In] Integrate[x*(a + b*ArcTan[c/x])^3,x]
```

```
[Out] (3*b^2*(b*c*(I*c + x) + a*(c^2 + x^2))*ArcTan[c/x]^2 + b^3*(c^2 + x^2)*ArcT
an[c/x]^3 + 3*b*ArcTan[c/x]*(a*(2*b*c*x + a*(c^2 + x^2)) - 2*b^2*c^2*Log[1
- E^((2*I)*ArcTan[c/x])]) + a*(a*x*(3*b*c + a*x) - 6*b^2*c^2*Log[c/(Sqrt[1
+ c^2/x^2]*x)]) + (3*I)*b^3*c^2*PolyLog[2, E^((2*I)*ArcTan[c/x])])/2
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 404 vs. 2(131) = 262.

Time = 8.16 (sec) , antiderivative size = 405, normalized size of antiderivative = 2.79

method	result
derivativedivides	$-c^2\left(-\frac{a^3x^2}{2c^2}+b^3\left(-\frac{x^2\arctan\left(\frac{c}{x}\right)^3}{2c^2}-\frac{3x\arctan\left(\frac{c}{x}\right)^2}{2c}-\frac{\arctan\left(\frac{c}{x}\right)^3}{2}+3\ln\left(\frac{c}{x}\right)\arctan\left(\frac{c}{x}\right)-\frac{3\arctan}{}\right.\right.$
default	$-c^2\left(-\frac{a^3x^2}{2c^2}+b^3\left(-\frac{x^2\arctan\left(\frac{c}{x}\right)^3}{2c^2}-\frac{3x\arctan\left(\frac{c}{x}\right)^2}{2c}-\frac{\arctan\left(\frac{c}{x}\right)^3}{2}+3\ln\left(\frac{c}{x}\right)\arctan\left(\frac{c}{x}\right)-\frac{3\arctan}{}\right.\right.$
parts	$\frac{a^3x^2}{2}-b^3c^2\left(-\frac{x^2\arctan\left(\frac{c}{x}\right)^3}{2c^2}-\frac{3x\arctan\left(\frac{c}{x}\right)^2}{2c}-\frac{\arctan\left(\frac{c}{x}\right)^3}{2}+3\ln\left(\frac{c}{x}\right)\arctan\left(\frac{c}{x}\right)-\frac{3\arctan\left(\frac{c}{x}\right)\ln}{2}\right.$
risch	Expression too large to display

```
[In] int(x*(a+b*arctan(c/x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -c^2*(-1/2*a^3/c^2*x^2+b^3*(-1/2/c^2*x^2*arctan(c/x)^3-3/2/c*x*arctan(c/x)^
2-1/2*arctan(c/x)^3+3*ln(c/x)*arctan(c/x)-3/2*arctan(c/x)*ln(1+c^2/x^2)+3/2
*I*ln(c/x)*ln(1+I*c/x)-3/2*I*ln(c/x)*ln(1-I*c/x)+3/2*I*dilog(1+I*c/x)-3/2*I
*dilog(1-I*c/x)-3/4*I*(ln(c/x-I)*ln(1+c^2/x^2)-1/2*ln(c/x-I)^2-dilog(-1/2*I
*(c/x+I))-ln(c/x-I)*ln(-1/2*I*(c/x+I)))+3/4*I*(ln(c/x+I)*ln(1+c^2/x^2)-1/2*
ln(c/x+I)^2-dilog(1/2*I*(c/x-I))-ln(c/x+I)*ln(1/2*I*(c/x-I))))+3*a*b^2*(-1/
2/c^2*x^2*arctan(c/x)^2-1/2*arctan(c/x)^2-1/c*x*arctan(c/x)-1/2*ln(1+c^2/x^
2)+ln(c/x))+3*a^2*b*(-1/2/c^2*x^2*arctan(c/x)-1/2*x/c-1/2*arctan(c/x)))
```

Fricas [F]

$$\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx=\int\left(b\arctan\left(\frac{c}{x}\right)+a\right)^3x\,dx$$

```
[In] integrate(x*(a+b*arctan(c/x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x*arctan(c/x)^3 + 3*a*b^2*x*arctan(c/x)^2 + 3*a^2*b*x*arctan(c
/x) + a^3*x, x)
```

Sympy [F]

$$\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx=\int x\left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^3\,dx$$

```
[In] integrate(x*(a+b*atan(c/x))**3,x)
[Out] Integral(x*(a + b*atan(c/x))**3, x)
```

Maxima [F]

$$\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx=\int\left(b\arctan\left(\frac{c}{x}\right)+a\right)^3x\,dx$$

```
[In] integrate(x*(a+b*arctan(c/x))^3,x, algorithm="maxima")
[Out] 3/2*a*b^2*x^2*arctan(c/x)^2 + 1/2*a^3*x^2 + 3/2*(x^2*arctan(c/x) - (c*arcta
n(x/c) - x)*c)*a^2*b - 3/2*((arctan(x/c)^2 - log(c^2 + x^2))*c^2 + 2*(c*arc
tan(x/c) - x)*c*arctan(c/x))*a*b^2 + 1/32*(12*c^2*arctan(c/x)^2*arctan(x/c)
 + 8*c^2*arctan2(c, x)^3 + 8*x^2*arctan2(c, x)^3 + 4*(3*arctan(c/x)*arctan(
x/c)^2/c + arctan(x/c)^3/c)*c^3 + 12*c*x*arctan2(c, x)^2 + 96*c^3*integrate
(1/32*log(c^2 + x^2)^2/(c^2 + x^2), x) - 3*c*x*log(c^2 + x^2)^2 + 512*c^2*i
ntegrate(1/32*x*arctan(c/x)^3/(c^2 + x^2), x) + 768*c^2*integrate(1/32*x*ar
ctan(c/x)/(c^2 + x^2), x) + 384*c*integrate(1/32*x^2*arctan(c/x)^2/(c^2 + x
^2), x) + 96*c*integrate(1/32*x^2*log(c^2 + x^2)^2/(c^2 + x^2), x) + 384*c*
integrate(1/32*x^2*log(c^2 + x^2)/(c^2 + x^2), x) + 512*integrate(1/32*x^3*
arctan(c/x)^3/(c^2 + x^2), x))*b^3
```

Giac [F]

$$\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx=\int\left(b\arctan\left(\frac{c}{x}\right)+a\right)^3x\,dx$$

```
[In] integrate(x*(a+b*arctan(c/x))^3,x, algorithm="giac")
[Out] integrate((b*arctan(c/x) + a)^3*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx=\int x\,\left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^3\,dx$$

```
[In] int(x*(a + b*atan(c/x))^3,x)
[Out] int(x*(a + b*atan(c/x))^3, x)
```

3.150 $\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)^3 \, dx$

Optimal result 856
Rubi [A] (verified) 856
Mathematica [A] (verified) 859
Maple [C] (warning: unable to verify) 859
Fricas [F] 860
Sympy [F] 861
Maxima [F] 861
Giac [F] 861
Mupad [F(-1)] 862

Optimal result

Integrand size = 12, antiderivative size = 119

$$\begin{aligned}\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)^3 dx &= ic\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3+x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3\\ &\quad-3bc\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\log\left(\frac{2c}{c+ix}\right)\\ &\quad+3ib^2c\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\text{PolyLog}\left(2,1-\frac{2c}{c+ix}\right)\\ &\quad-\frac{3}{2}b^3c\,\text{PolyLog}\left(3,1-\frac{2c}{c+ix}\right)\end{aligned}$$

```
[Out] I*c*(a+b*arccot(x/c))^3+x*(a+b*arccot(x/c))^3-3*b*c*(a+b*arccot(x/c))^2*ln(
2*c/(c+I*x))+3*I*b^2*c*(a+b*arccot(x/c))*polylog(2,1-2*c/(c+I*x))-3/2*b^3*c
*polylog(3,1-2*c/(c+I*x))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {4934, 4931, 5041, 4965, 5005, 5115, 6745}

$$\begin{aligned}\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)^3 dx &= 3ib^2c\,\text{PolyLog}\left(2,1-\frac{2c}{c+ix}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\\ &\quad+ic\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3+x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3\\ &\quad-3bc\log\left(\frac{2c}{c+ix}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\\ &\quad-\frac{3}{2}b^3c\,\text{PolyLog}\left(3,1-\frac{2c}{c+ix}\right)\end{aligned}$$

```
[In] Int[(a + b*ArcTan[c/x])^3,x]
```

```
[Out] I*c*(a + b*ArcCot[x/c])^3 + x*(a + b*ArcCot[x/c])^3 - 3*b*c*(a + b*ArcCot[x
/c])^2*Log[(2*c)/(c + I*x)] + (3*I)*b^2*c*(a + b*ArcCot[x/c])*PolyLog[2, 1
- (2*c)/(c + I*x)] - (3*b^3*c*PolyLog[3, 1 - (2*c)/(c + I*x)])/2
```

Rule 4931

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Dist[b*c*n*p, Int[x^n*((a + b*ArcCot[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4934

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_), x_Symbol] :> Int[(a + b*A
rcCot[1/(x^n*c)])^p, x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1] && ILtQ[n, 0]
```

Rule 4965

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcCot[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] - Dist[b*c*(
p/e), Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5005

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5041

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*e*(p + 1))), x] - Dist[
1/(c*d), Int[(a + b*ArcCot[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5115

```
Int[(Log[u_]*((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcCot[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] - Dist[b*p*(I/2), Int[(a + b*ArcCot[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\text{integral} = \int \left(a + b\cot^{-1}\left(\frac{x}{c}\right)\right)^3 dx$$

$$= x\left(a + b\cot^{-1}\left(\frac{x}{c}\right)\right)^3 + \frac{(3b)\int \frac{x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{1+\frac{x^2}{c^2}}\,dx}{c}$$

$$= ic\left(a + b\cot^{-1}\left(\frac{x}{c}\right)\right)^3 + x\left(a + b\cot^{-1}\left(\frac{x}{c}\right)\right)^3 - (3b)\int \frac{\left(a + b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{i - \frac{x}{c}}\,dx$$

$$= ic\left(a + b\cot^{-1}\left(\frac{x}{c}\right)\right)^3 + x\left(a + b\cot^{-1}\left(\frac{x}{c}\right)\right)^3 - 3bc\left(a + b\cot^{-1}\left(\frac{x}{c}\right)\right)^2 \log\left(\frac{2c}{c+ix}\right) - \left(6b^2\right)\int \frac{\left(a + b\cot^{-1}\left(\frac{x}{c}\right)\right)\log\left(\frac{2}{1+\frac{ix}{c}}\right)}{1+\frac{x^2}{c^2}}\,dx$$

$$= ic\left(a + b\cot^{-1}\left(\frac{x}{c}\right)\right)^3 + x\left(a + b\cot^{-1}\left(\frac{x}{c}\right)\right)^3 - 3bc\left(a + b\cot^{-1}\left(\frac{x}{c}\right)\right)^2 \log\left(\frac{2c}{c+ix}\right) + 3ib^2c\left(a + b\cot^{-1}\left(\frac{x}{c}\right)\right)\text{PolyLog}\left(2, 1 - \frac{2c}{c+ix}\right) + \left(3ib^3\right)\int \frac{\text{PolyLog}\left(2, 1 - \frac{2}{1+\frac{ix}{c}}\right)}{1+\frac{x^2}{c^2}}\,dx$$

$$= ic\left(a + b\cot^{-1}\left(\frac{x}{c}\right)\right)^3 + x\left(a + b\cot^{-1}\left(\frac{x}{c}\right)\right)^3 - 3bc\left(a + b\cot^{-1}\left(\frac{x}{c}\right)\right)^2 \log\left(\frac{2c}{c+ix}\right) + 3ib^2c\left(a + b\cot^{-1}\left(\frac{x}{c}\right)\right)\text{PolyLog}\left(2, 1 - \frac{2c}{c+ix}\right) - \frac{3}{2}b^3c\,\text{PolyLog}\left(3, 1 - \frac{2c}{c+ix}\right)$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.81

$$\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)^3 dx = a^3x+3a^2bx\arctan\left(\frac{c}{x}\right)+\frac{3}{2}a^2bc\log\left(c^2+x^2\right) - 3ab^2\left(-\left((ic+x)\arctan\left(\frac{c}{x}\right)^2\right) + 2c\arctan\left(\frac{c}{x}\right)\log\left(1-e^{2i\arctan\left(\frac{c}{x}\right)}\right) - ic\,\mathrm{PolyLog}\left(2,e^{2i\arctan\left(\frac{c}{x}\right)}\right)\right) - \frac{1}{8}b^3\left(-ic\pi^3+8ic\arctan\left(\frac{c}{x}\right)^3-8x\arctan\left(\frac{c}{x}\right)^3 + 24c\arctan\left(\frac{c}{x}\right)^2\log\left(1-e^{-2i\arctan\left(\frac{c}{x}\right)}\right) + 24ic\arctan\left(\frac{c}{x}\right)\mathrm{PolyLog}\left(2,e^{-2i\arctan\left(\frac{c}{x}\right)}\right) + 12c\,\mathrm{PolyLog}\left(3,e^{-2i\arctan\left(\frac{c}{x}\right)}\right)\right)$$

```
[In] Integrate[(a + b*ArcTan[c/x])^3,x]
```

```
[Out] a^3*x + 3*a^2*b*x*ArcTan[c/x] + (3*a^2*b*c*Log[c^2 + x^2])/2 - 3*a*b^2*(-((
I*c + x)*ArcTan[c/x]^2) + 2*c*ArcTan[c/x]*Log[1 - E^((2*I)*ArcTan[c/x])] -
I*c*PolyLog[2, E^((2*I)*ArcTan[c/x])]) - (b^3*((-I)*c*Pi^3 + (8*I)*c*ArcTan
[c/x]^3 - 8*x*ArcTan[c/x]^3 + 24*c*ArcTan[c/x]^2*Log[1 - E^((-2*I)*ArcTan[c
/x])] + (24*I)*c*ArcTan[c/x]*PolyLog[2, E^((-2*I)*ArcTan[c/x])] + 12*c*Poly
Log[3, E^((-2*I)*ArcTan[c/x])]))/8
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 102.88 (sec) , antiderivative size = 2028, normalized size of antiderivative = 17.04

method	result	size
parts	Expression too large to display	2028
derivativedivides	Expression too large to display	2031
default	Expression too large to display	2031

```
[In] int((a+b*arctan(c/x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] x*a^3-b^3*c*(-1/c*x*arctan(c/x)^3+3*ln(c/x)*arctan(c/x)^2-3/2*arctan(c/x)^2
*ln(1+c^2/x^2)+3*arctan(c/x)^2*ln((1+I*c/x)/(1+c^2/x^2)^(1/2))-3*arctan(c/x
```

```
)^2*ln((1+I*c/x)^2/(1+c^2/x^2)-1)-I*arctan(c/x)^3+3/4*(-I*Pi*csgn(I/((1+I*c
/x)^2/(1+c^2/x^2)+1)^2)*csgn(I*(1+I*c/x)^2/(1+c^2/x^2))*csgn(I*(1+I*c/x)^2/
(1+c^2/x^2)/((1+I*c/x)^2/(1+c^2/x^2)+1)^2)+2*I*Pi*csgn(((1+I*c/x)^2/(1+c^2/
x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))^3+2*I*Pi*csgn(I*((1+I*c/x)^2/(1+c^2/x^
2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))^3+2*I*Pi*csgn(I*(1+I*c/x)/(1+c^2/x^2)^(1
/2))*csgn(I*(1+I*c/x)^2/(1+c^2/x^2))^2-2*I*Pi*csgn(I*((1+I*c/x)^2/(1+c^2/x^
2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))*csgn(((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c
/x)^2/(1+c^2/x^2)+1))^2-2*I*Pi*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1))*csgn(I*(
(1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))^2-I*Pi*csgn(I*(1+I*
c/x)/(1+c^2/x^2)^(1/2))^2*csgn(I*(1+I*c/x)^2/(1+c^2/x^2))+I*Pi*csgn(I*((1+I
*c/x)^2/(1+c^2/x^2)+1))^2*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)+1)^2)+I*Pi*csgn(I
*((1+I*c/x)^2/(1+c^2/x^2)+1)^2)^3-2*I*Pi*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)+1)
)*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)+1)^2)^2-2*I*Pi*csgn(((1+I*c/x)^2/(1+c^2/x
^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))^2+2*I*Pi*csgn(I*((1+I*c/x)^2/(1+c^2/x^2
)-1))*csgn(I/((1+I*c/x)^2/(1+c^2/x^2)+1))*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1
)/((1+I*c/x)^2/(1+c^2/x^2)+1))+2*I*Pi+I*Pi*csgn(I*(1+I*c/x)^2/(1+c^2/x^2))*
csgn(I*(1+I*c/x)^2/(1+c^2/x^2)/((1+I*c/x)^2/(1+c^2/x^2)+1)^2)^2-I*Pi*csgn(I
*(1+I*c/x)^2/(1+c^2/x^2))^3+2*I*Pi*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I
*c/x)^2/(1+c^2/x^2)+1))*csgn(((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^
2/x^2)+1))-2*I*Pi*csgn(I/((1+I*c/x)^2/(1+c^2/x^2)+1))*csgn(I*((1+I*c/x)^2/(
1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))^2-I*Pi*csgn(I*(1+I*c/x)^2/(1+c^2
/x^2)/((1+I*c/x)^2/(1+c^2/x^2)+1)^2)^3+I*Pi*csgn(I/((1+I*c/x)^2/(1+c^2/x^2)
+1)^2)*csgn(I*(1+I*c/x)^2/(1+c^2/x^2)/((1+I*c/x)^2/(1+c^2/x^2)+1)^2)^2+4*ln
(2))*arctan(c/x)^2+3*arctan(c/x)^2*ln(1-(1+I*c/x)/(1+c^2/x^2)^(1/2))-6*I*ar
ctan(c/x)*polylog(2,(1+I*c/x)/(1+c^2/x^2)^(1/2))+6*polylog(3,(1+I*c/x)/(1+c
^2/x^2)^(1/2))+3*arctan(c/x)^2*ln((1+I*c/x)/(1+c^2/x^2)^(1/2)+1)-6*I*arctan
(c/x)*polylog(2,-(1+I*c/x)/(1+c^2/x^2)^(1/2))+6*polylog(3,-(1+I*c/x)/(1+c^2
/x^2)^(1/2)))-3*a*b^2*c*(-1/c*x*arctan(c/x)^2+2*ln(c/x)*arctan(c/x)-arctan(
c/x)*ln(1+c^2/x^2)+I*ln(c/x)*ln(1+I*c/x)-I*ln(c/x)*ln(1-I*c/x)+I*dilog(1+I*
c/x)-I*dilog(1-I*c/x)-1/2*I*(ln(c/x-I)*ln(1+c^2/x^2)-1/2*ln(c/x-I)^2-dilog(
-1/2*I*(c/x+I))-ln(c/x-I)*ln(-1/2*I*(c/x+I)))+1/2*I*(ln(c/x+I)*ln(1+c^2/x^2
)-1/2*ln(c/x+I)^2-dilog(1/2*I*(c/x-I))-ln(c/x+I)*ln(1/2*I*(c/x-I))))-3*a^2*
b*c*(-1/c*x*arctan(c/x)-1/2*ln(1+c^2/x^2)+ln(c/x))
```

Fricas [F]

$$\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx=\int \left(b\arctan\left(\frac{c}{x}\right)+a\right)^3\,dx$$

```
[In] integrate((a+b*arctan(c/x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*arctan(c/x)^3 + 3*a*b^2*arctan(c/x)^2 + 3*a^2*b*arctan(c/x) +
a^3, x)
```

Sympy [F]

$$\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx=\int \left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^3\,dx$$

```
[In] integrate((a+b*atan(c/x))**3,x)
[Out] Integral((a + b*atan(c/x))**3, x)
```

Maxima [F]

$$\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx=\int \left(b\arctan\left(\frac{c}{x}\right)+a\right)^3\,dx$$

```
[In] integrate((a+b*arctan(c/x))^3,x, algorithm="maxima")
[Out] 7/8*b^3*c*arctan(c/x)^3*arctan(x/c) + 3*a*b^2*c*arctan(c/x)^2*arctan(x/c) +
 1/8*b^3*x*arctan2(c, x)^3 - 3/32*b^3*x*arctan2(c, x)*log(c^2 + x^2)^2 + (3
*arctan(c/x)*arctan(x/c)^2/c + arctan(x/c)^3/c)*a*b^2*c^2 + 7/32*(6*arctan(
c/x)^2*arctan(x/c)^2/c + 4*arctan(c/x)*arctan(x/c)^3/c + arctan(x/c)^4/c)*b
^3*c^2 + 3*b^3*c^2*integrate(1/32*arctan(c/x)*log(c^2 + x^2)^2/(c^2 + x^2),
 x) + 12*b^3*c*integrate(1/32*x*arctan(c/x)^2/(c^2 + x^2), x) - 3*b^3*c*int
egrate(1/32*x*log(c^2 + x^2)^2/(c^2 + x^2), x) + 3/2*(2*x*arctan(c/x) + c*l
og(c^2 + x^2))*a^2*b + a^3*x + 28*b^3*integrate(1/32*x^2*arctan(c/x)^3/(c^2
 + x^2), x) + 3*b^3*integrate(1/32*x^2*arctan(c/x)*log(c^2 + x^2)^2/(c^2 +
x^2), x) + 96*a*b^2*integrate(1/32*x^2*arctan(c/x)^2/(c^2 + x^2), x) + 12*b
^3*integrate(1/32*x^2*arctan(c/x)*log(c^2 + x^2)/(c^2 + x^2), x)
```

Giac [F]

$$\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx=\int \left(b\arctan\left(\frac{c}{x}\right)+a\right)^3\,dx$$

```
[In] integrate((a+b*arctan(c/x))^3,x, algorithm="giac")
[Out] integrate((b*arctan(c/x) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\arctan\left(\frac{c}{x}\right)\right)^3\,dx=\int \left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^3\,dx$$

```
[In] int((a + b*atan(c/x))^3,x)
[Out] int((a + b*atan(c/x))^3, x)
```

3.151 $\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x}\,dx$

Optimal result 863
Rubi [A] (verified) 864
Mathematica [A] (verified) 868
Maple [C] (warning: unable to verify) 869
Fricas [F] 870
Sympy [F] 870
Maxima [F] 870
Giac [F] 871
Mupad [F(-1)] 871

Optimal result

Integrand size = 16, antiderivative size = 230

$$\begin{aligned}\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x}\,dx = & -2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3\operatorname{arctanh}\left(1-\frac{2}{1+\frac{ic}{x}}\right)\\ & +\frac{3}{2}ib\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+\frac{ic}{x}}\right)\\ & -\frac{3}{2}ib\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1+\frac{ic}{x}}\right)\\ & +\frac{3}{2}b^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\operatorname{PolyLog}\left(3,1-\frac{2}{1+\frac{ic}{x}}\right)\\ & -\frac{3}{2}b^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\operatorname{PolyLog}\left(3,-1+\frac{2}{1+\frac{ic}{x}}\right)\\ & -\frac{3}{4}ib^3\operatorname{PolyLog}\left(4,1-\frac{2}{1+\frac{ic}{x}}\right)\\ & +\frac{3}{4}ib^3\operatorname{PolyLog}\left(4,-1+\frac{2}{1+\frac{ic}{x}}\right)\end{aligned}$$

```
[Out] 2*(a+b*arccot(x/c))^3*arctanh(-1+2/(1+I*c/x))+3/2*I*b*(a+b*arccot(x/c))^2*p
olylog(2,1-2/(1+I*c/x))-3/2*I*b*(a+b*arccot(x/c))^2*polylog(2,-1+2/(1+I*c/x
))+3/2*b^2*(a+b*arccot(x/c))*polylog(3,1-2/(1+I*c/x))-3/2*b^2*(a+b*arccot(x
/c))*polylog(3,-1+2/(1+I*c/x))-3/4*I*b^3*polylog(4,1-2/(1+I*c/x))+3/4*I*b^3
*polylog(4,-1+2/(1+I*c/x))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {4944, 4942, 5108, 5004, 5114, 5118, 6745}

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x}\,dx = -2\text{arctanh}\left(1-\frac{2}{1+\frac{ic}{x}}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3 + \frac{3}{2}b^2\,\text{PolyLog}\left(3,1-\frac{2}{\frac{ic}{x}+1}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right) - \frac{3}{2}b^2\,\text{PolyLog}\left(3,\frac{2}{\frac{ic}{x}+1}-1\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right) + \frac{3}{2}ib\,\text{PolyLog}\left(2,1-\frac{2}{\frac{ic}{x}+1}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2 - \frac{3}{2}ib\,\text{PolyLog}\left(2,\frac{2}{\frac{ic}{x}+1}-1\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2 - \frac{3}{4}ib^3\,\text{PolyLog}\left(4,1-\frac{2}{\frac{ic}{x}+1}\right) + \frac{3}{4}ib^3\,\text{PolyLog}\left(4,\frac{2}{\frac{ic}{x}+1}-1\right)$$

```
[In] Int[(a + b*ArcTan[c/x])^3/x,x]
```

```
[Out] -2*(a + b*ArcCot[x/c])^3*ArcTanh[1 - 2/(1 + (I*c)/x)] + ((3*I)/2)*b*(a + b*
ArcCot[x/c])^2*PolyLog[2, 1 - 2/(1 + (I*c)/x)] - ((3*I)/2)*b*(a + b*ArcCot[
x/c])^2*PolyLog[2, -1 + 2/(1 + (I*c)/x)] + (3*b^2*(a + b*ArcCot[x/c])*PolyL
og[3, 1 - 2/(1 + (I*c)/x)])/2 - (3*b^2*(a + b*ArcCot[x/c])*PolyLog[3, -1 +
2/(1 + (I*c)/x)])/2 - ((3*I)/4)*b^3*PolyLog[4, 1 - 2/(1 + (I*c)/x)] + ((3*I
)/4)*b^3*PolyLog[4, -1 + 2/(1 + (I*c)/x)]
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1
/n, Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[p, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
```

```
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5108

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e*
x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5118

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k + 1,
 u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{(a+b\arctan(cx))^3}{x}\,dx, x, \frac{1}{x}\right)\\
&= -2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3 \operatorname{arctanh}\left(1-\frac{2}{1+\frac{ic}{x}}\right)\\
&\quad + (6bc)\text{Subst}\left(\int \frac{(a+b\arctan(cx))^2 \operatorname{arctanh}\left(1-\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx, x, \frac{1}{x}\right)
\end{aligned}$$

$$= -2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3\operatorname{arctanh}\left(1-\frac{2}{1+\frac{ic}{x}}\right)$$
$$-(3bc)\text{Subst}\left(\int\frac{(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{1+c^2x^2}dx,x,\frac{1}{x}\right)$$
$$+(3bc)\text{Subst}\left(\int\frac{(a+b\arctan(cx))^2\log\left(2-\frac{2}{1+icx}\right)}{1+c^2x^2}dx,x,\frac{1}{x}\right)$$
$$= -2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3\operatorname{arctanh}\left(1-\frac{2}{1+\frac{ic}{x}}\right)$$
$$+\frac{3}{2}ib\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\text{PolyLog}\left(2,1-\frac{2}{1+\frac{ic}{x}}\right)$$
$$-\frac{3}{2}ib\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\text{PolyLog}\left(2,-1+\frac{2}{1+\frac{ic}{x}}\right)$$
$$-\left(3ib^2c\right)\text{Subst}\left(\int\frac{(a+b\arctan(cx))\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{1+c^2x^2}dx,x,\frac{1}{x}\right)$$
$$+\left(3ib^2c\right)\text{Subst}\left(\int\frac{(a+b\arctan(cx))\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)}{1+c^2x^2}dx,x,\frac{1}{x}\right)$$
$$= -2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3\operatorname{arctanh}\left(1-\frac{2}{1+\frac{ic}{x}}\right)$$
$$+\frac{3}{2}ib\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\text{PolyLog}\left(2,1-\frac{2}{1+\frac{ic}{x}}\right)$$
$$-\frac{3}{2}ib\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\text{PolyLog}\left(2,-1+\frac{2}{1+\frac{ic}{x}}\right)$$
$$+\frac{3}{2}b^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\text{PolyLog}\left(3,1-\frac{2}{1+\frac{ic}{x}}\right)$$
$$-\frac{3}{2}b^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\text{PolyLog}\left(3,-1+\frac{2}{1+\frac{ic}{x}}\right)$$
$$-\frac{1}{2}\left(3b^3c\right)\text{Subst}\left(\int\frac{\text{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{1+c^2x^2}dx,x,\frac{1}{x}\right)$$
$$+\frac{1}{2}\left(3b^3c\right)\text{Subst}\left(\int\frac{\text{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)}{1+c^2x^2}dx,x,\frac{1}{x}\right)$$

$$
\begin{aligned}
&= -2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3 \operatorname{arctanh}\left(1-\frac{2}{1+\frac{ic}{x}}\right) \\
&\quad+\frac{3}{2}ib\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2 \operatorname{PolyLog}\left(2,1-\frac{2}{1+\frac{ic}{x}}\right) \\
&\quad-\frac{3}{2}ib\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2 \operatorname{PolyLog}\left(2,-1+\frac{2}{1+\frac{ic}{x}}\right) \\
&\quad+\frac{3}{2}b^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right) \operatorname{PolyLog}\left(3,1-\frac{2}{1+\frac{ic}{x}}\right) \\
&\quad-\frac{3}{2}b^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right) \operatorname{PolyLog}\left(3,-1+\frac{2}{1+\frac{ic}{x}}\right) \\
&\quad-\frac{3}{4}ib^3 \operatorname{PolyLog}\left(4,1-\frac{2}{1+\frac{ic}{x}}\right)+\frac{3}{4}ib^3 \operatorname{PolyLog}\left(4,-1+\frac{2}{1+\frac{ic}{x}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 422, normalized size of antiderivative = 1.83

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x}\,dx = a^3\log(x)-\frac{3}{2}ia^2b\left(\text{PolyLog}\left(2,-\frac{ic}{x}\right)-\text{PolyLog}\left(2,\frac{ic}{x}\right)\right) + 3ab^2\left(\frac{i\pi^3}{24}-\frac{2}{3}i\arctan\left(\frac{c}{x}\right)^3 - \arctan\left(\frac{c}{x}\right)^2\log\left(1-e^{-2i\arctan\left(\frac{c}{x}\right)}\right) + \arctan\left(\frac{c}{x}\right)^2\log\left(1+e^{2i\arctan\left(\frac{c}{x}\right)}\right) - i\arctan\left(\frac{c}{x}\right)\text{PolyLog}\left(2,e^{-2i\arctan\left(\frac{c}{x}\right)}\right) - i\arctan\left(\frac{c}{x}\right)\text{PolyLog}\left(2,-e^{2i\arctan\left(\frac{c}{x}\right)}\right) - \frac{1}{2}\text{PolyLog}\left(3,e^{-2i\arctan\left(\frac{c}{x}\right)}\right)+\frac{1}{2}\text{PolyLog}\left(3,-e^{2i\arctan\left(\frac{c}{x}\right)}\right)\right) + \frac{1}{64}ib^3\left(\pi^4-32\arctan\left(\frac{c}{x}\right)^4 + 64i\arctan\left(\frac{c}{x}\right)^3\log\left(1-e^{-2i\arctan\left(\frac{c}{x}\right)}\right) - 64i\arctan\left(\frac{c}{x}\right)^3\log\left(1+e^{2i\arctan\left(\frac{c}{x}\right)}\right) - 96\arctan\left(\frac{c}{x}\right)^2\text{PolyLog}\left(2,e^{-2i\arctan\left(\frac{c}{x}\right)}\right) - 96\arctan\left(\frac{c}{x}\right)^2\text{PolyLog}\left(2,-e^{2i\arctan\left(\frac{c}{x}\right)}\right) + 96i\arctan\left(\frac{c}{x}\right)\text{PolyLog}\left(3,e^{-2i\arctan\left(\frac{c}{x}\right)}\right) - 96i\arctan\left(\frac{c}{x}\right)\text{PolyLog}\left(3,-e^{2i\arctan\left(\frac{c}{x}\right)}\right) + 48\,\text{PolyLog}\left(4,e^{-2i\arctan\left(\frac{c}{x}\right)}\right)+48\,\text{PolyLog}\left(4,-e^{2i\arctan\left(\frac{c}{x}\right)}\right)\right)$$

```
[In] Integrate[(a + b*ArcTan[c/x])^3/x,x]
```

```
[Out] a^3*Log[x] - ((3*I)/2)*a^2*b*(PolyLog[2, ((-I)*c)/x] - PolyLog[2, (I*c)/x])
 + 3*a*b^2*((I/24)*Pi^3 - ((2*I)/3)*ArcTan[c/x]^3 - ArcTan[c/x]^2*Log[1 - E
^((-2*I)*ArcTan[c/x])] + ArcTan[c/x]^2*Log[1 + E^((2*I)*ArcTan[c/x])] - I*A
rcTan[c/x]*PolyLog[2, E^((-2*I)*ArcTan[c/x])] - I*ArcTan[c/x]*PolyLog[2, -E
^((2*I)*ArcTan[c/x])] - PolyLog[3, E^((-2*I)*ArcTan[c/x])]/2 + PolyLog[3, -
E^((2*I)*ArcTan[c/x])]/2) + (I/64)*b^3*(Pi^4 - 32*ArcTan[c/x]^4 + (64*I)*Ar
cTan[c/x]^3*Log[1 - E^((-2*I)*ArcTan[c/x])] - (64*I)*ArcTan[c/x]^3*Log[1 +
E^((2*I)*ArcTan[c/x])] - 96*ArcTan[c/x]^2*PolyLog[2, E^((-2*I)*ArcTan[c/x])
] - 96*ArcTan[c/x]^2*PolyLog[2, -E^((2*I)*ArcTan[c/x])] + (96*I)*ArcTan[c/x
```

```
]*PolyLog[3, E^((-2*I)*ArcTan[c/x])] - (96*I)*ArcTan[c/x]*PolyLog[3, -E^((2
*I)*ArcTan[c/x])] + 48*PolyLog[4, E^((-2*I)*ArcTan[c/x])] + 48*PolyLog[4, -
E^((2*I)*ArcTan[c/x])])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 26.33 (sec) , antiderivative size = 2225, normalized size of antiderivative = 9.67

method	result	size
parts	Expression too large to display	2225
derivativedivides	Expression too large to display	2226
default	Expression too large to display	2226

```
[In] int((a+b*arctan(c/x))^3/x,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*ln(x)+b^3*(-ln(c/x)*arctan(c/x)^3+arctan(c/x)^3*ln((1+I*c/x)^2/(1+c^2/x
^2)-1)-arctan(c/x)^3*ln(1-(1+I*c/x)/(1+c^2/x^2)^(1/2))+3*I*arctan(c/x)^2*po
lylog(2,(1+I*c/x)/(1+c^2/x^2)^(1/2))-6*arctan(c/x)*polylog(3,(1+I*c/x)/(1+c
^2/x^2)^(1/2))-6*I*polylog(4,(1+I*c/x)/(1+c^2/x^2)^(1/2))-arctan(c/x)^3*ln(
(1+I*c/x)/(1+c^2/x^2)^(1/2)+1)+3*I*arctan(c/x)^2*polylog(2,-(1+I*c/x)/(1+c^
2/x^2)^(1/2))-6*arctan(c/x)*polylog(3,-(1+I*c/x)/(1+c^2/x^2)^(1/2))-6*I*pol
ylog(4,-(1+I*c/x)/(1+c^2/x^2)^(1/2))-1/2*I*Pi*(csgn(I*((1+I*c/x)^2/(1+c^2/x
^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))*csgn(((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*
c/x)^2/(1+c^2/x^2)+1))-csgn(((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2
/x^2)+1))^2-csgn(I/((1+I*c/x)^2/(1+c^2/x^2)+1))*csgn(I*((1+I*c/x)^2/(1+c^2/
x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))^2+csgn(I/((1+I*c/x)^2/(1+c^2/x^2)+1))*
csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))*csgn(I*((1+
I*c/x)^2/(1+c^2/x^2)-1))+csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1
+c^2/x^2)+1))^3-csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)
+1))^2*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1))-csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-
1)/((1+I*c/x)^2/(1+c^2/x^2)+1))*csgn(((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)
^2/(1+c^2/x^2)+1))^2+csgn(((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x
^2)+1))^3+1)*arctan(c/x)^3-3/2*I*arctan(c/x)^2*polylog(2,-(1+I*c/x)^2/(1+c^
2/x^2))+3/2*arctan(c/x)*polylog(3,-(1+I*c/x)^2/(1+c^2/x^2))+3/4*I*polylog(4
,-(1+I*c/x)^2/(1+c^2/x^2)))+3*a^2*b*(-ln(c/x)*arctan(c/x)-1/2*I*ln(c/x)*ln(
1+I*c/x)+1/2*I*ln(c/x)*ln(1-I*c/x)-1/2*I*dilog(1+I*c/x)+1/2*I*dilog(1-I*c/x
))+3*a*b^2*(-ln(c/x)*arctan(c/x)^2-I*arctan(c/x)*polylog(2,-(1+I*c/x)^2/(1+
c^2/x^2))+1/2*polylog(3,-(1+I*c/x)^2/(1+c^2/x^2))+arctan(c/x)^2*ln((1+I*c/x
)^2/(1+c^2/x^2)-1)-arctan(c/x)^2*ln(1-(1+I*c/x)/(1+c^2/x^2)^(1/2))+2*I*arct
an(c/x)*polylog(2,(1+I*c/x)/(1+c^2/x^2)^(1/2))-2*polylog(3,(1+I*c/x)/(1+c^2
/x^2)^(1/2))-arctan(c/x)^2*ln((1+I*c/x)/(1+c^2/x^2)^(1/2)+1)+2*I*arctan(c/x
)*polylog(2,-(1+I*c/x)/(1+c^2/x^2)^(1/2))-2*polylog(3,-(1+I*c/x)/(1+c^2/x^2
)^(1/2))-1/2*I*Pi*(csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x
```

```
^2)+1))*csgn(((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))-csgn(
((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))^2-csgn(I/((1+I*c/x
)^2/(1+c^2/x^2)+1))*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/
x^2)+1))^2+csgn(I/((1+I*c/x)^2/(1+c^2/x^2)+1))*csgn(I*((1+I*c/x)^2/(1+c^2/x
^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1))+csg
n(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))^3-csgn(I*((1+I
*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))^2*csgn(I*((1+I*c/x)^2/(
1+c^2/x^2)-1))-csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+
1))*csgn(((1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))^2+csgn(((
1+I*c/x)^2/(1+c^2/x^2)-1)/((1+I*c/x)^2/(1+c^2/x^2)+1))^3+1)*arctan(c/x)^2)
```

Fricas [F]

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x}\,dx=\int \frac{\left(b\arctan\left(\frac{c}{x}\right)+a\right)^3}{x}\,dx$$

```
[In] integrate((a+b*arctan(c/x))^3/x,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(c/x)^3 + 3*a*b^2*arctan(c/x)^2 + 3*a^2*b*arctan(c/x) +
 a^3)/x, x)
```

Sympy [F]

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x}\,dx=\int \frac{\left(a+b\operatorname{atan}\left(\frac{c}{x}\right)\right)^3}{x}\,dx$$

```
[In] integrate((a+b*atan(c/x))**3/x,x)
```

```
[Out] Integral((a + b*atan(c/x))**3/x, x)
```

Maxima [F]

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x}\,dx=\int \frac{\left(b\arctan\left(\frac{c}{x}\right)+a\right)^3}{x}\,dx$$

```
[In] integrate((a+b*arctan(c/x))^3/x,x, algorithm="maxima")
```

```
[Out] a^3*log(x) + 1/32*integrate((28*b^3*arctan2(c, x)^3 + 3*b^3*arctan2(c, x)*l
og(c^2 + x^2)^2 + 96*a*b^2*arctan2(c, x)^2 + 96*a^2*b*arctan2(c, x))/x, x)
```

Giac [F]

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x}\,dx = \int \frac{\left(b\arctan\left(\frac{c}{x}\right)+a\right)^3}{x}\,dx$$

```
[In] integrate((a+b*arctan(c/x))^3/x,x, algorithm="giac")
[Out] integrate((b*arctan(c/x) + a)^3/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x}\,dx = \int \frac{\left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^3}{x}\,dx$$

```
[In] int((a + b*atan(c/x))^3/x,x)
[Out] int((a + b*atan(c/x))^3/x, x)
```

3.152 $\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x^2}\,dx$

Optimal result 872
Rubi [A] (verified) 872
Mathematica [A] (verified) 875
Maple [B] (verified) 875
Fricas [F] 876
Sympy [F] 876
Maxima [F] 876
Giac [F] 877
Mupad [F(-1)] 877

Optimal result

Integrand size = 16, antiderivative size = 136

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x^2}\,dx = -\frac{i\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3}{c} - \frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3}{x} - \frac{3b\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\log\left(\frac{2}{1+\frac{ic}{x}}\right)}{c} - \frac{3ib^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\operatorname{PolyLog}\left(2,1-\frac{2}{1+\frac{ic}{x}}\right)}{c} - \frac{3b^3\operatorname{PolyLog}\left(3,1-\frac{2}{1+\frac{ic}{x}}\right)}{2c}$$

```
[Out] -I*(a+b*arccot(x/c))^3/c-(a+b*arccot(x/c))^3/x-3*b*(a+b*arccot(x/c))^2*ln(2
/(1+I*c/x))/c-3*I*b^2*(a+b*arccot(x/c))*polylog(2,1-2/(1+I*c/x))/c-3/2*b^3*
polylog(3,1-2/(1+I*c/x))/c
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used

= {4948, 4930, 5040, 4964, 5004, 5114, 6745}

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x^2}\,dx = -\frac{3ib^2\,\text{PolyLog}\left(2,1-\frac{2}{\frac{ic}{x}+1}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)}{c} - \frac{i\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3}{c} - \frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3}{x} - \frac{3b\log\left(\frac{2}{1+\frac{ic}{x}}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{c} - \frac{3b^3\,\text{PolyLog}\left(3,1-\frac{2}{\frac{ic}{x}+1}\right)}{2c}$$

```
[In] Int[(a + b*ArcTan[c/x])^3/x^2,x]
```

```
[Out] ((-I)*(a + b*ArcCot[x/c])^3)/c - (a + b*ArcCot[x/c])^3/x - (3*b*(a + b*ArcC
ot[x/c])^2*Log[2/(1 + (I*c)/x)])/c - ((3*I)*b^2*(a + b*ArcCot[x/c])*PolyLog
[2, 1 - 2/(1 + (I*c)/x)])/c - (3*b^3*PolyLog[3, 1 - 2/(1 + (I*c)/x)])/(2*c)
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplify
[(m + 1)/n]]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
```

```
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int (a+b\arctan(cx))^3\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3}{x} + (3bc)\text{Subst}\left(\int \frac{x(a+b\arctan(cx))^2}{1+c^2x^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{i\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3}{c} - \frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3}{x} - (3b)\text{Subst}\left(\int \frac{(a+b\arctan(cx))^2}{i-cx}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{i\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3}{c} - \frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3}{x} - \frac{3b\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\log\left(\frac{2}{1+\frac{ic}{x}}\right)}{c} + \left(6b^2\right)\text{Subst}\left(\int \frac{(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{i\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3}{c} - \frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3}{x} - \frac{3b\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\log\left(\frac{2}{1+\frac{ic}{x}}\right)}{c} - \frac{3ib^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\text{PolyLog}\left(2,1-\frac{2}{1+\frac{ic}{x}}\right)}{c} + \left(3ib^3\right)\text{Subst}\left(\int \frac{\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{i\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3}{c} - \frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3}{x} - \frac{3b\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\log\left(\frac{2}{1+\frac{ic}{x}}\right)}{c} - \frac{3ib^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\text{PolyLog}\left(2,1-\frac{2}{1+\frac{ic}{x}}\right)}{c} - \frac{3b^3\,\text{PolyLog}\left(3,1-\frac{2}{1+\frac{ic}{x}}\right)}{2c}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.63

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x^2}\,dx = -\frac{2a^3c+6a^2bc\arctan\left(\frac{c}{x}\right)+6ab^2c\arctan\left(\frac{c}{x}\right)^2-6iab^2x\arctan\left(\frac{c}{x}\right)^2+2b^3c\arctan\left(\frac{c}{x}\right)^3-2ib^3x\arctan\left(\frac{c}{x}\right.}{}$$

```
[In] Integrate[(a + b*ArcTan[c/x])^3/x^2,x]
```

```
[Out] -1/2*(2*a^3*c + 6*a^2*b*c*ArcTan[c/x] + 6*a*b^2*c*ArcTan[c/x]^2 - (6*I)*a*b
^2*x*ArcTan[c/x]^2 + 2*b^3*c*ArcTan[c/x]^3 - (2*I)*b^3*x*ArcTan[c/x]^3 + 12
*a*b^2*x*ArcTan[c/x]*Log[1 + E^((2*I)*ArcTan[c/x])] + 6*b^3*x*ArcTan[c/x]^2
*Log[1 + E^((2*I)*ArcTan[c/x])] - 3*a^2*b*x*Log[1 + c^2/x^2] - (6*I)*b^2*x*
(a + b*ArcTan[c/x])*PolyLog[2, -E^((2*I)*ArcTan[c/x])] + 3*b^3*x*PolyLog[3,
 -E^((2*I)*ArcTan[c/x])])/(c*x)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 274 vs. 2(129) = 258.

Time = 9.31 (sec) , antiderivative size = 275, normalized size of antiderivative = 2.02

method	result
derivativedivides	$-\frac{\frac{c\,a^3}{x}+b^3\left(\arctan\left(\frac{c}{x}\right)^3\left(\frac{c}{x}+i\right)-2i\arctan\left(\frac{c}{x}\right)^3+3\arctan\left(\frac{c}{x}\right)^2\ln\left(\frac{\left(1+\frac{ic}{x}\right)^2}{1+\frac{c^2}{x^2}}+1\right)-3i\arctan\left(\frac{c}{x}\right)\operatorname{polylog}\left(2,-\frac{\left(1+\frac{ic}{x}\right)^2}{1+\frac{c^2}{x^2}}\right)\right.}{}$
default	$-\frac{\frac{c\,a^3}{x}+b^3\left(\arctan\left(\frac{c}{x}\right)^3\left(\frac{c}{x}+i\right)-2i\arctan\left(\frac{c}{x}\right)^3+3\arctan\left(\frac{c}{x}\right)^2\ln\left(\frac{\left(1+\frac{ic}{x}\right)^2}{1+\frac{c^2}{x^2}}+1\right)-3i\arctan\left(\frac{c}{x}\right)\operatorname{polylog}\left(2,-\frac{\left(1+\frac{ic}{x}\right)^2}{1+\frac{c^2}{x^2}}\right)\right.}{}$
parts	$-\frac{a^3}{x}-\frac{b^3\left(\arctan\left(\frac{c}{x}\right)^3\left(\frac{c}{x}+i\right)-2i\arctan\left(\frac{c}{x}\right)^3+3\arctan\left(\frac{c}{x}\right)^2\ln\left(\frac{\left(1+\frac{ic}{x}\right)^2}{1+\frac{c^2}{x^2}}+1\right)-3i\arctan\left(\frac{c}{x}\right)\operatorname{polylog}\left(2,-\frac{\left(1+\frac{ic}{x}\right)^2}{1+\frac{c^2}{x^2}}\right)\right.}{c}$

```
[In] int((a+b*arctan(c/x))^3/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/c*(c/x*a^3+b^3*(arctan(c/x)^3*(c/x+I)-2*I*arctan(c/x)^3+3*arctan(c/x)^2*
ln((1+I*c/x)^2/(1+c^2/x^2)+1)-3*I*arctan(c/x)*polylog(2,-(1+I*c/x)^2/(1+c^2
/x^2))+3/2*polylog(3,-(1+I*c/x)^2/(1+c^2/x^2)))+3*a*b^2*(arctan(c/x)^2*(c/x
```

```
+I)+2*arctan(c/x)*ln((1+I*c/x)^2/(1+c^2/x^2)+1)-2*I*arctan(c/x)^2-I*polylog
(2,-(1+I*c/x)^2/(1+c^2/x^2)))+3*a^2*b*(c/x*arctan(c/x)-1/2*ln(1+c^2/x^2)))
```

Fricas [F]

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x^2}\,dx=\int \frac{\left(b\arctan\left(\frac{c}{x}\right)+a\right)^3}{x^2}\,dx$$

```
[In] integrate((a+b*arctan(c/x))^3/x^2,x, algorithm="fricas")
[Out] integral((b^3*arctan(c/x)^3 + 3*a*b^2*arctan(c/x)^2 + 3*a^2*b*arctan(c/x) +
 a^3)/x^2, x)
```

Sympy [F]

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x^2}\,dx=\int \frac{\left(a+b\operatorname{atan}\left(\frac{c}{x}\right)\right)^3}{x^2}\,dx$$

```
[In] integrate((a+b*atan(c/x))**3/x**2,x)
[Out] Integral((a + b*atan(c/x))**3/x**2, x)
```

Maxima [F]

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x^2}\,dx=\int \frac{\left(b\arctan\left(\frac{c}{x}\right)+a\right)^3}{x^2}\,dx$$

```
[In] integrate((a+b*arctan(c/x))^3/x^2,x, algorithm="maxima")
[Out] -3/2*a^2*b*(2*c*arctan(c/x)/x - log(c^2/x^2 + 1))/c - a^3/x - 1/32*(4*b^3*a
rctan2(c, x)^3 - 3*b^3*arctan2(c, x)*log(c^2 + x^2)^2 - (28*b^3*arctan(c/x)
^3*arctan(x/c)/c + 896*b^3*c^2*integrate(1/32*arctan(c/x)^3/(c^2*x^2 + x^4)
, x) + 96*b^3*c^2*integrate(1/32*arctan(c/x)*log(c^2 + x^2)^2/(c^2*x^2 + x^
4), x) + 3072*a*b^2*c^2*integrate(1/32*arctan(c/x)^2/(c^2*x^2 + x^4), x) +
96*a*b^2*arctan(c/x)^2*arctan(x/c)/c - 384*b^3*c*integrate(1/32*x*arctan(c/
x)^2/(c^2*x^2 + x^4), x) + 96*b^3*c*integrate(1/32*x*log(c^2 + x^2)^2/(c^2*
x^2 + x^4), x) + 32*(3*arctan(c/x)*arctan(x/c)^2/c + arctan(x/c)^3/c)*a*b^2
 + 7*(6*arctan(c/x)^2*arctan(x/c)^2/c + 4*arctan(c/x)*arctan(x/c)^3/c + arc
tan(x/c)^4/c)*b^3 + 96*b^3*integrate(1/32*x^2*arctan(c/x)*log(c^2 + x^2)^2/
(c^2*x^2 + x^4), x) - 384*b^3*integrate(1/32*x^2*arctan(c/x)*log(c^2 + x^2)
/(c^2*x^2 + x^4), x))*x)/x
```

Giac [F]

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x^2}\,dx=\int \frac{\left(b\arctan\left(\frac{c}{x}\right)+a\right)^3}{x^2}\,dx$$

```
[In] integrate((a+b*arctan(c/x))^3/x^2,x, algorithm="giac")
[Out] integrate((b*arctan(c/x) + a)^3/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x^2}\,dx=\int \frac{\left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^3}{x^2}\,dx$$

```
[In] int((a + b*atan(c/x))^3/x^2,x)
[Out] int((a + b*atan(c/x))^3/x^2, x)
```

3.153 $\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x^3}\,dx$

Optimal result 878
Rubi [A] (verified) 878
Mathematica [A] (verified) 881
Maple [B] (verified) 882
Fricas [F] 882
Sympy [F] 883
Maxima [F] 883
Giac [F] 883
Mupad [F(-1)] 884

Optimal result

Integrand size = 16, antiderivative size = 147

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x^3}\,dx = \frac{3ib\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{2c^2}+\frac{3b\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{2cx} - \frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3}{2c^2}-\frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3}{2x^2} + \frac{3b^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\log\left(\frac{2}{1+\frac{ic}{x}}\right)}{c^2}+\frac{3ib^3\operatorname{PolyLog}\left(2,1-\frac{2}{1+\frac{ic}{x}}\right)}{2c^2}$$

```
[Out] 3/2*I*b*(a+b*arccot(x/c))^2/c^2+3/2*b*(a+b*arccot(x/c))^2/c/x-1/2*(a+b*arcc
ot(x/c))^3/c^2-1/2*(a+b*arccot(x/c))^3/x^2+3*b^2*(a+b*arccot(x/c))*ln(2/(1+
I*c/x))/c^2+3/2*I*b^3*polylog(2,1-2/(1+I*c/x))/c^2
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {4948, 4946, 5036, 4930, 5040, 4964, 2449, 2352, 5004}

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x^3}\,dx = \frac{3b^2\log\left(\frac{2}{1+\frac{ic}{x}}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)}{c^2}+\frac{3ib\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{2c^2} - \frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3}{2c^2}-\frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3}{2x^2} + \frac{3b\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{2cx}+\frac{3ib^3\operatorname{PolyLog}\left(2,1-\frac{2}{\frac{ic}{x}+1}\right)}{2c^2}$$

```
[In] Int[(a + b*ArcTan[c/x])^3/x^3,x]
```

```
[Out] (((3*I)/2)*b*(a + b*ArcCot[x/c])^2)/c^2 + (3*b*(a + b*ArcCot[x/c])^2)/(2*c*
x) - (a + b*ArcCot[x/c])^3/(2*c^2) - (a + b*ArcCot[x/c])^3/(2*x^2) + (3*b^2
*(a + b*ArcCot[x/c])*Log[2/(1 + (I*c)/x)])/c^2 + (((3*I)/2)*b^3*PolyLog[2,
1 - 2/(1 + (I*c)/x)])/c^2
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplify
[(m + 1)/n]]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int x(a+b\arctan(cx))^3\,dx,x,\frac{1}{x}\right)\\
&= -\frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3}{2x^2}+\frac{1}{2}(3bc)\text{Subst}\left(\int \frac{x^2(a+b\arctan(cx))^2}{1+c^2x^2}\,dx,x,\frac{1}{x}\right)\\
&= -\frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3}{2x^2}+\frac{(3b)\text{Subst}\left(\int (a+b\arctan(cx))^2\,dx,x,\frac{1}{x}\right)}{2c}\\
&\quad -\frac{(3b)\text{Subst}\left(\int \frac{(a+b\arctan(cx))^2}{1+c^2x^2}\,dx,x,\frac{1}{x}\right)}{2c}\\
&= \frac{3b\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{2cx}-\frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3}{2c^2}-\frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3}{2x^2}\\
&\quad -\left(3b^2\right)\text{Subst}\left(\int \frac{x(a+b\arctan(cx))}{1+c^2x^2}\,dx,x,\frac{1}{x}\right)\\
&= \frac{3ib\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{2c^2}+\frac{3b\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{2cx}-\frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3}{2c^2}\\
&\quad -\frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3}{2x^2}+\frac{\left(3b^2\right)\text{Subst}\left(\int \frac{a+b\arctan(cx)}{i-cx}\,dx,x,\frac{1}{x}\right)}{c}
\end{aligned}
$$

$$= \frac{3ib\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{2c^2} + \frac{3b\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{2cx} - \frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3}{2c^2} - \frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3}{2x^2} + \frac{3b^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\log\left(\frac{2}{1+\frac{ic}{x}}\right)}{c^2} - \frac{(3b^3)\,\text{Subst}\left(\int\frac{\log\left(\frac{2}{1+icx}\right)}{1+c^2x^2}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{3ib\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{2c^2} + \frac{3b\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{2cx} - \frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3}{2c^2} - \frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3}{2x^2} + \frac{3b^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\log\left(\frac{2}{1+\frac{ic}{x}}\right)}{c^2} + \frac{(3ib^3)\,\text{Subst}\left(\int\frac{\log(2x)}{1-2x}\,dx, x, \frac{1}{1+\frac{ic}{x}}\right)}{c^2}$$

$$= \frac{3ib\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{2c^2} + \frac{3b\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{2cx} - \frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3}{2c^2} - \frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3}{2x^2} + \frac{3b^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\log\left(\frac{2}{1+\frac{ic}{x}}\right)}{c^2} + \frac{3ib^3\,\text{PolyLog}\left(2, 1-\frac{2}{1+\frac{ic}{x}}\right)}{2c^2}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.21

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x^3}\,dx$$

$$= \frac{3b^2(c-ix)(-a(c+ix)+bx)\arctan\left(\frac{c}{x}\right)^2 - b^3(c^2+x^2)\arctan\left(\frac{c}{x}\right)^3 - 3b\arctan\left(\frac{c}{x}\right)\left(a(-2bcx+a(c^2+x\right.}{}$$

```
[In] Integrate[(a + b*ArcTan[c/x])^3/x^3,x]
```

```
[Out] (3*b^2*(c - I*x)*(-(a*(c + I*x)) + b*x)*ArcTan[c/x]^2 - b^3*(c^2 + x^2)*Arc
Tan[c/x]^3 - 3*b*ArcTan[c/x]*(a*(-2*b*c*x + a*(c^2 + x^2)) - 2*b^2*x^2*Log[
1 + E^((2*I)*ArcTan[c/x])]) + a*(a*c*(-(a*c) + 3*b*x) + 6*b^2*x^2*Log[1/Sqr
t[1 + c^2/x^2]]) - (3*I)*b^3*x^2*PolyLog[2, -E^((2*I)*ArcTan[c/x])])/(2*c^2
*x^2)
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 320 vs. 2(133) = 266.

Time = 14.68 (sec) , antiderivative size = 321, normalized size of antiderivative = 2.18

method	result
derivativedivides	$-\frac{\frac{a^3c^2}{2x^2}+b^3\left(\frac{c^2\arctan\left(\frac{c}{x}\right)^3}{2x^2}+\frac{\arctan\left(\frac{c}{x}\right)^3}{2}-\frac{3\arctan\left(\frac{c}{x}\right)^2c}{2x}+\frac{3\arctan\left(\frac{c}{x}\right)\ln\left(1+\frac{c^2}{x^2}\right)}{2}+\frac{3i\left(\ln\left(\frac{c}{x}-i\right)\ln\left(1+\frac{c^2}{x^2}\right)-\frac{\ln\left(\frac{c}{x}-i\right)^2}{2}-\mathrm{dil}\cdots\right.}{\cdots}\right.}{\cdots}$
default	$-\frac{\frac{a^3c^2}{2x^2}+b^3\left(\frac{c^2\arctan\left(\frac{c}{x}\right)^3}{2x^2}+\frac{\arctan\left(\frac{c}{x}\right)^3}{2}-\frac{3\arctan\left(\frac{c}{x}\right)^2c}{2x}+\frac{3\arctan\left(\frac{c}{x}\right)\ln\left(1+\frac{c^2}{x^2}\right)}{2}+\frac{3i\left(\ln\left(\frac{c}{x}-i\right)\ln\left(1+\frac{c^2}{x^2}\right)-\frac{\ln\left(\frac{c}{x}-i\right)^2}{2}-\mathrm{dil}\cdots\right.}{\cdots}\right.}{\cdots}$
parts	$-\frac{a^3}{2x^2}-\frac{b^3\left(\frac{c^2\arctan\left(\frac{c}{x}\right)^3}{2x^2}+\frac{\arctan\left(\frac{c}{x}\right)^3}{2}-\frac{3\arctan\left(\frac{c}{x}\right)^2c}{2x}+\frac{3\arctan\left(\frac{c}{x}\right)\ln\left(1+\frac{c^2}{x^2}\right)}{2}+\frac{3i\left(\ln\left(\frac{c}{x}-i\right)\ln\left(1+\frac{c^2}{x^2}\right)-\frac{\ln\left(\frac{c}{x}-i\right)^2}{2}-\mathrm{d}\cdots\right.}{\cdots}\right.}{\cdots}$

```
[In] int((a+b*arctan(c/x))^3/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/c^2*(1/2*a^3*c^2/x^2+b^3*(1/2*c^2/x^2*arctan(c/x)^3+1/2*arctan(c/x)^3-3/
2*arctan(c/x)^2*c/x+3/2*arctan(c/x)*ln(1+c^2/x^2)+3/4*I*(ln(c/x-I)*ln(1+c^2
/x^2)-1/2*ln(c/x-I)^2-dilog(-1/2*I*(c/x+I))-ln(c/x-I)*ln(-1/2*I*(c/x+I)))-3
/4*I*(ln(c/x+I)*ln(1+c^2/x^2)-1/2*ln(c/x+I)^2-dilog(1/2*I*(c/x-I))-ln(c/x+I
)*ln(1/2*I*(c/x-I))))+3*a*b^2*(1/2*c^2/x^2*arctan(c/x)^2+1/2*arctan(c/x)^2-
c/x*arctan(c/x)+1/2*ln(1+c^2/x^2))+3*a^2*b*(1/2*c^2/x^2*arctan(c/x)-1/2*c/x
+1/2*arctan(c/x)))
```

Fricas [F]

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x^3}\,dx=\int \frac{\left(b\arctan\left(\frac{c}{x}\right)+a\right)^3}{x^3}\,dx$$

```
[In] integrate((a+b*arctan(c/x))^3/x^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(c/x)^3 + 3*a*b^2*arctan(c/x)^2 + 3*a^2*b*arctan(c/x) +
 a^3)/x^3, x)
```

Sympy [F]

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x^3}\,dx=\int \frac{\left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^3}{x^3}\,dx$$

```
[In] integrate((a+b*atan(c/x))**3/x**3,x)
[Out] Integral((a + b*atan(c/x))**3/x**3, x)
```

Maxima [F]

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x^3}\,dx=\int \frac{\left(b\arctan\left(\frac{c}{x}\right)+a\right)^3}{x^3}\,dx$$

```
[In] integrate((a+b*arctan(c/x))^3/x^3,x, algorithm="maxima")
[Out] 3/2*(c*(arctan(x/c)/c^3 + 1/(c^2*x)) - arctan(c/x)/x^2)*a^2*b + 3/2*(2*c*(a
rctan(x/c)/c^3 + 1/(c^2*x))*arctan(c/x) + (arctan(x/c)^2 - log(c^2 + x^2) +
 2*log(x))/c^2)*a*b^2 - 3/2*a*b^2*arctan(c/x)^2/x^2 - 1/2*a^3/x^2 + 1/32*(4
*(128*c^3*integrate(1/32*arctan(c/x)^3/(c^3*x^3 + c*x^5), x) - 96*c^2*integ
rate(1/32*x*arctan(c/x)^2/(c^3*x^3 + c*x^5), x) - 24*c^2*integrate(1/32*x*l
og(c^2 + x^2)^2/(c^3*x^3 + c*x^5), x) + 128*c*integrate(1/32*x^2*arctan(c/x
)^3/(c^3*x^3 + c*x^5), x) + 192*c*integrate(1/32*x^2*arctan(c/x)/(c^3*x^3 +
 c*x^5), x) - 3*arctan(c/x)^2*arctan(x/c)/c^2 - 3*arctan(c/x)*arctan(x/c)^2
/c^2 - arctan(x/c)^3/c^2 - 24*integrate(1/32*x^3*log(c^2 + x^2)^2/(c^3*x^3
+ c*x^5), x) + 96*integrate(1/32*x^3*log(c^2 + x^2)/(c^3*x^3 + c*x^5), x))*
c^2*x^2 - 8*c^2*arctan2(c, x)^3 - 8*x^2*arctan2(c, x)^3 + 12*c*x*arctan2(c,
 x)^2 - 3*c*x*log(c^2 + x^2)^2)*b^3/(c^2*x^2)
```

Giac [F]

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x^3}\,dx=\int \frac{\left(b\arctan\left(\frac{c}{x}\right)+a\right)^3}{x^3}\,dx$$

```
[In] integrate((a+b*arctan(c/x))^3/x^3,x, algorithm="giac")
[Out] integrate((b*arctan(c/x) + a)^3/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\arctan\left(\frac{c}{x}\right)\right)^3}{x^3}\,dx=\int \frac{\left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^3}{x^3}\,dx$$

```
[In] int((a + b*atan(c/x))^3/x^3,x)
[Out] int((a + b*atan(c/x))^3/x^3, x)
```

3.154 $\int x^2 \arctan\left(\sqrt{x}\right)\,dx$

Optimal result 885
Rubi [A] (verified) 885
Mathematica [A] (verified) 887
Maple [A] (verified) 887
Fricas [A] (verification not implemented) 887
Sympy [A] (verification not implemented) 888
Maxima [A] (verification not implemented) 888
Giac [A] (verification not implemented) 888
Mupad [B] (verification not implemented) 889

Optimal result

Integrand size = 10, antiderivative size = 51

$$\int x^2 \arctan\left(\sqrt{x}\right)\,dx = -\frac{\sqrt{x}}{3} + \frac{x^{3/2}}{9} - \frac{x^{5/2}}{15} + \frac{\arctan\left(\sqrt{x}\right)}{3} + \frac{1}{3}x^3 \arctan\left(\sqrt{x}\right)$$

```
[Out] 1/9*x^(3/2)-1/15*x^(5/2)+1/3*arctan(x^(1/2))+1/3*x^3*arctan(x^(1/2))-1/3*x^
(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4946, 52, 65, 209}

$$\int x^2 \arctan\left(\sqrt{x}\right)\,dx = \frac{1}{3}x^3 \arctan\left(\sqrt{x}\right) + \frac{\arctan\left(\sqrt{x}\right)}{3} - \frac{x^{5/2}}{15} + \frac{x^{3/2}}{9} - \frac{\sqrt{x}}{3}$$

```
[In] Int[x^2*ArcTan[Sqrt[x]],x]
```

```
[Out] -1/3*Sqrt[x] + x^(3/2)/9 - x^(5/2)/15 + ArcTan[Sqrt[x]]/3 + (x^3*ArcTan[Sqr
t[x]])/3
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\arctan\left(\sqrt{x}\right) - \frac{1}{6}\int \frac{x^{5/2}}{1+x}\,dx \\
&= -\frac{x^{5/2}}{15} + \frac{1}{3}x^3\arctan\left(\sqrt{x}\right) + \frac{1}{6}\int \frac{x^{3/2}}{1+x}\,dx \\
&= \frac{x^{3/2}}{9} - \frac{x^{5/2}}{15} + \frac{1}{3}x^3\arctan\left(\sqrt{x}\right) - \frac{1}{6}\int \frac{\sqrt{x}}{1+x}\,dx \\
&= -\frac{\sqrt{x}}{3} + \frac{x^{3/2}}{9} - \frac{x^{5/2}}{15} + \frac{1}{3}x^3\arctan\left(\sqrt{x}\right) + \frac{1}{6}\int \frac{1}{\sqrt{x}(1+x)}\,dx \\
&= -\frac{\sqrt{x}}{3} + \frac{x^{3/2}}{9} - \frac{x^{5/2}}{15} + \frac{1}{3}x^3\arctan\left(\sqrt{x}\right) + \frac{1}{3}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{x}\right) \\
&= -\frac{\sqrt{x}}{3} + \frac{x^{3/2}}{9} - \frac{x^{5/2}}{15} + \frac{\arctan\left(\sqrt{x}\right)}{3} + \frac{1}{3}x^3\arctan\left(\sqrt{x}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.67

$$\int x^2 \arctan\left(\sqrt{x}\right)\,dx = \frac{1}{45}\left(\sqrt{x}\left(-15+5x-3x^2\right)+15\left(1+x^3\right)\arctan\left(\sqrt{x}\right)\right)$$

```
[In] Integrate[x^2*ArcTan[Sqrt[x]],x]
[Out] (Sqrt[x]*(-15 + 5*x - 3*x^2) + 15*(1 + x^3)*ArcTan[Sqrt[x]])/45
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.59

method	result	size
meijerg	$-\frac{\sqrt{x}\,(21x^2-35x+105)}{315}+\frac{(7x^3+7)\arctan(\sqrt{x})}{21}$	30
derivativedivides	$\frac{x^{\frac{3}{2}}}{9}-\frac{x^{\frac{5}{2}}}{15}+\frac{\arctan(\sqrt{x})}{3}+\frac{x^3\arctan(\sqrt{x})}{3}-\frac{\sqrt{x}}{3}$	32
default	$\frac{x^{\frac{3}{2}}}{9}-\frac{x^{\frac{5}{2}}}{15}+\frac{\arctan(\sqrt{x})}{3}+\frac{x^3\arctan(\sqrt{x})}{3}-\frac{\sqrt{x}}{3}$	32
parts	$\frac{x^{\frac{3}{2}}}{9}-\frac{x^{\frac{5}{2}}}{15}+\frac{\arctan(\sqrt{x})}{3}+\frac{x^3\arctan(\sqrt{x})}{3}-\frac{\sqrt{x}}{3}$	32

```
[In] int(x^2*arctan(x^(1/2)),x,method=_RETURNVERBOSE)
[Out] -1/315*x^(1/2)*(21*x^2-35*x+105)+1/21*(7*x^3+7)*arctan(x^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.53

$$\int x^2 \arctan\left(\sqrt{x}\right)\,dx = \frac{1}{3}\left(x^3+1\right)\arctan\left(\sqrt{x}\right)-\frac{1}{45}\left(3\,x^2-5\,x+15\right)\sqrt{x}$$

```
[In] integrate(x^2*arctan(x^(1/2)),x, algorithm="fricas")
[Out] 1/3*(x^3 + 1)*arctan(sqrt(x)) - 1/45*(3*x^2 - 5*x + 15)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 1.13 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

$$\int x^2 \arctan\left(\sqrt{x}\right)\,dx = -\frac{x^{\frac{5}{2}}}{15} + \frac{x^{\frac{3}{2}}}{9} - \frac{\sqrt{x}}{3} + \frac{x^3 \operatorname{atan}\left(\sqrt{x}\right)}{3} + \frac{\operatorname{atan}\left(\sqrt{x}\right)}{3}$$

```
[In] integrate(x**2*atan(x**(1/2)),x)
[Out] -x**(5/2)/15 + x**(3/2)/9 - sqrt(x)/3 + x**3*atan(sqrt(x))/3 + atan(sqrt(x)
)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.61

$$\int x^2 \arctan\left(\sqrt{x}\right)\,dx = \frac{1}{3}\,x^3 \arctan\left(\sqrt{x}\right) - \frac{1}{15}\,x^{\frac{5}{2}} + \frac{1}{9}\,x^{\frac{3}{2}} - \frac{1}{3}\,\sqrt{x} + \frac{1}{3}\,\arctan\left(\sqrt{x}\right)$$

```
[In] integrate(x^2*arctan(x^(1/2)),x, algorithm="maxima")
[Out] 1/3*x^3*arctan(sqrt(x)) - 1/15*x^(5/2) + 1/9*x^(3/2) - 1/3*sqrt(x) + 1/3*ar
ctan(sqrt(x))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.61

$$\int x^2 \arctan\left(\sqrt{x}\right)\,dx = \frac{1}{3}\,x^3 \arctan\left(\sqrt{x}\right) - \frac{1}{15}\,x^{\frac{5}{2}} + \frac{1}{9}\,x^{\frac{3}{2}} - \frac{1}{3}\,\sqrt{x} + \frac{1}{3}\,\arctan\left(\sqrt{x}\right)$$

```
[In] integrate(x^2*arctan(x^(1/2)),x, algorithm="giac")
[Out] 1/3*x^3*arctan(sqrt(x)) - 1/15*x^(5/2) + 1/9*x^(3/2) - 1/3*sqrt(x) + 1/3*ar
ctan(sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.61

$$\int x^2 \arctan\left(\sqrt{x}\right)\,dx = \frac{\operatorname{atan}\left(\sqrt{x}\right)}{3} + \frac{x^3 \operatorname{atan}\left(\sqrt{x}\right)}{3} - \frac{\sqrt{x}}{3} + \frac{x^{3/2}}{9} - \frac{x^{5/2}}{15}$$

```
[In] int(x^2*atan(x^(1/2)),x)
```

```
[Out] atan(x^(1/2))/3 + (x^3*atan(x^(1/2)))/3 - x^(1/2)/3 + x^(3/2)/9 - x^(5/2)/1
5
```

3.155 $\int x \arctan(\sqrt{x})\, dx$

Optimal result 890
Rubi [A] (verified) 890
Mathematica [A] (verified) 891
Maple [A] (verified) 892
Fricas [A] (verification not implemented) 892
Sympy [A] (verification not implemented) 892
Maxima [A] (verification not implemented) 893
Giac [A] (verification not implemented) 893
Mupad [B] (verification not implemented) 893

Optimal result

Integrand size = 8, antiderivative size = 42

$$\int x \arctan(\sqrt{x})\, dx = \frac{\sqrt{x}}{2} - \frac{x^{3/2}}{6} - \frac{\arctan(\sqrt{x})}{2} + \frac{1}{2}x^2 \arctan(\sqrt{x})$$

```
[Out] -1/6*x^(3/2)-1/2*arctan(x^(1/2))+1/2*x^2*arctan(x^(1/2))+1/2*x^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4946, 52, 65, 209}

$$\int x \arctan(\sqrt{x})\, dx = \frac{1}{2}x^2 \arctan(\sqrt{x}) - \frac{\arctan(\sqrt{x})}{2} - \frac{x^{3/2}}{6} + \frac{\sqrt{x}}{2}$$

```
[In] Int[x*ArcTan[Sqrt[x]],x]
```

```
[Out] Sqrt[x]/2 - x^(3/2)/6 - ArcTan[Sqrt[x]]/2 + (x^2*ArcTan[Sqrt[x]])/2
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}x^2\arctan\left(\sqrt{x}\right)-\frac{1}{4}\int\frac{x^{3/2}}{1+x}\,dx\\
&= -\frac{x^{3/2}}{6}+\frac{1}{2}x^2\arctan\left(\sqrt{x}\right)+\frac{1}{4}\int\frac{\sqrt{x}}{1+x}\,dx\\
&= \frac{\sqrt{x}}{2}-\frac{x^{3/2}}{6}+\frac{1}{2}x^2\arctan\left(\sqrt{x}\right)-\frac{1}{4}\int\frac{1}{\sqrt{x}(1+x)}\,dx\\
&= \frac{\sqrt{x}}{2}-\frac{x^{3/2}}{6}+\frac{1}{2}x^2\arctan\left(\sqrt{x}\right)-\frac{1}{2}\text{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\sqrt{x}\right)\\
&= \frac{\sqrt{x}}{2}-\frac{x^{3/2}}{6}-\frac{\arctan\left(\sqrt{x}\right)}{2}+\frac{1}{2}x^2\arctan\left(\sqrt{x}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.67

$$\int x\arctan\left(\sqrt{x}\right)\,dx=\frac{1}{6}\left(-\left((-3+x)\sqrt{x}\right)+3\left(-1+x^2\right)\arctan\left(\sqrt{x}\right)\right)$$

```
[In] Integrate[x*ArcTan[Sqrt[x]],x]
[Out] (-((-3 + x)*Sqrt[x]) + 3*(-1 + x^2)*ArcTan[Sqrt[x]])/6
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.60

method	result	size
meijerg	$\frac{\sqrt{x}\,(-5x+15)}{30}-\frac{(-5x^{2}+5)\arctan(\sqrt{x})}{10}$	25
derivativedivides	$-\frac{x^{\frac{3}{2}}}{6}-\frac{\arctan(\sqrt{x})}{2}+\frac{x^{2}\arctan(\sqrt{x})}{2}+\frac{\sqrt{x}}{2}$	27
default	$-\frac{x^{\frac{3}{2}}}{6}-\frac{\arctan(\sqrt{x})}{2}+\frac{x^{2}\arctan(\sqrt{x})}{2}+\frac{\sqrt{x}}{2}$	27
parts	$-\frac{x^{\frac{3}{2}}}{6}-\frac{\arctan(\sqrt{x})}{2}+\frac{x^{2}\arctan(\sqrt{x})}{2}+\frac{\sqrt{x}}{2}$	27

```
[In] int(x*arctan(x^(1/2)),x,method=_RETURNVERBOSE)
[Out] 1/30*x^(1/2)*(-5*x+15)-1/10*(-5*x^2+5)*arctan(x^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.48

$$\int x\arctan\left(\sqrt{x}\right)\,dx=\frac{1}{2}\left(x^{2}-1\right)\arctan\left(\sqrt{x}\right)-\frac{1}{6}\left(x-3\right)\sqrt{x}$$

```
[In] integrate(x*arctan(x^(1/2)),x, algorithm="fricas")
[Out] 1/2*(x^2 - 1)*arctan(sqrt(x)) - 1/6*(x - 3)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.76

$$\int x\arctan\left(\sqrt{x}\right)\,dx=-\frac{x^{\frac{3}{2}}}{6}+\frac{\sqrt{x}}{2}+\frac{x^{2}\operatorname{atan}\left(\sqrt{x}\right)}{2}-\frac{\operatorname{atan}\left(\sqrt{x}\right)}{2}$$

```
[In] integrate(x*atan(x**(1/2)),x)
[Out] -x**(3/2)/6 + sqrt(x)/2 + x**2*atan(sqrt(x))/2 - atan(sqrt(x))/2
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.62

$$\int x \arctan\left(\sqrt{x}\right)\,dx = \frac{1}{2}\,x^2 \arctan\left(\sqrt{x}\right) - \frac{1}{6}\,x^{\frac{3}{2}} + \frac{1}{2}\,\sqrt{x} - \frac{1}{2}\,\arctan\left(\sqrt{x}\right)$$

```
[In] integrate(x*arctan(x^(1/2)),x, algorithm="maxima")
[Out] 1/2*x^2*arctan(sqrt(x)) - 1/6*x^(3/2) + 1/2*sqrt(x) - 1/2*arctan(sqrt(x))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.62

$$\int x \arctan\left(\sqrt{x}\right)\,dx = \frac{1}{2}\,x^2 \arctan\left(\sqrt{x}\right) - \frac{1}{6}\,x^{\frac{3}{2}} + \frac{1}{2}\,\sqrt{x} - \frac{1}{2}\,\arctan\left(\sqrt{x}\right)$$

```
[In] integrate(x*arctan(x^(1/2)),x, algorithm="giac")
[Out] 1/2*x^2*arctan(sqrt(x)) - 1/6*x^(3/2) + 1/2*sqrt(x) - 1/2*arctan(sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.62

$$\int x \arctan\left(\sqrt{x}\right)\,dx = \frac{x^2\,\mathrm{atan}\left(\sqrt{x}\right)}{2} - \frac{\mathrm{atan}\left(\sqrt{x}\right)}{2} + \frac{\sqrt{x}}{2} - \frac{x^{3/2}}{6}$$

```
[In] int(x*atan(x^(1/2)),x)
[Out] (x^2*atan(x^(1/2)))/2 - atan(x^(1/2))/2 + x^(1/2)/2 - x^(3/2)/6
```

3.156 $\int \arctan\left(\sqrt{x}\right)\,dx$

Optimal result 894
Rubi [A] (verified) 894
Mathematica [A] (verified) 895
Maple [A] (verified) 896
Fricas [A] (verification not implemented) 896
Sympy [A] (verification not implemented) 896
Maxima [A] (verification not implemented) 897
Giac [A] (verification not implemented) 897
Mupad [B] (verification not implemented) 897

Optimal result

Integrand size = 6, antiderivative size = 22

$$\int \arctan\left(\sqrt{x}\right)\,dx=-\sqrt{x}+\arctan\left(\sqrt{x}\right)+x\arctan\left(\sqrt{x}\right)$$

```
[Out] arctan(x^(1/2))+x*arctan(x^(1/2))-x^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {4930, 52, 65, 209}

$$\int \arctan\left(\sqrt{x}\right)\,dx=x\arctan\left(\sqrt{x}\right)+\arctan\left(\sqrt{x}\right)-\sqrt{x}$$

```
[In] Int[ArcTan[Sqrt[x]],x]
```

```
[Out] -Sqrt[x] + ArcTan[Sqrt[x]] + x*ArcTan[Sqrt[x]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\arctan\left(\sqrt{x}\right)-\frac{1}{2}\int\frac{\sqrt{x}}{1+x}\,dx\\
&= -\sqrt{x}+x\arctan\left(\sqrt{x}\right)+\frac{1}{2}\int\frac{1}{\sqrt{x}(1+x)}\,dx\\
&= -\sqrt{x}+x\arctan\left(\sqrt{x}\right)+\text{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\sqrt{x}\right)\\
&= -\sqrt{x}+\arctan\left(\sqrt{x}\right)+x\arctan\left(\sqrt{x}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \arctan\left(\sqrt{x}\right)\,dx=-\sqrt{x}+(1+x)\arctan\left(\sqrt{x}\right)$$

```
[In] Integrate[ArcTan[Sqrt[x]],x]
[Out] -Sqrt[x] + (1 + x)*ArcTan[Sqrt[x]]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\arctan\left(\sqrt{x}\right)+x\arctan\left(\sqrt{x}\right)-\sqrt{x}$	17
default	$\arctan\left(\sqrt{x}\right)+x\arctan\left(\sqrt{x}\right)-\sqrt{x}$	17
parts	$\arctan\left(\sqrt{x}\right)+x\arctan\left(\sqrt{x}\right)-\sqrt{x}$	17
meijerg	$-\sqrt{x}+\frac{(3x+3)\arctan\left(\sqrt{x}\right)}{3}$	18

```
[In] int(arctan(x^(1/2)),x,method=_RETURNVERBOSE)
[Out] arctan(x^(1/2))+x*arctan(x^(1/2))-x^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.64

$$\int \arctan\left(\sqrt{x}\right)\,dx=(x+1)\arctan\left(\sqrt{x}\right)-\sqrt{x}$$

```
[In] integrate(arctan(x^(1/2)),x, algorithm="fricas")
[Out] (x + 1)*arctan(sqrt(x)) - sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \arctan\left(\sqrt{x}\right)\,dx=-\sqrt{x}+x\,\mathrm{atan}\left(\sqrt{x}\right)+\mathrm{atan}\left(\sqrt{x}\right)$$

```
[In] integrate(atan(x**(1/2)),x)
[Out] -sqrt(x) + x*atan(sqrt(x)) + atan(sqrt(x))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \arctan\left(\sqrt{x}\right)\,dx = x\arctan\left(\sqrt{x}\right) - \sqrt{x} + \arctan\left(\sqrt{x}\right)$$

```
[In] integrate(arctan(x^(1/2)),x, algorithm="maxima")
[Out] x*arctan(sqrt(x)) - sqrt(x) + arctan(sqrt(x))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \arctan\left(\sqrt{x}\right)\,dx = x\arctan\left(\sqrt{x}\right) - \sqrt{x} + \arctan\left(\sqrt{x}\right)$$

```
[In] integrate(arctan(x^(1/2)),x, algorithm="giac")
[Out] x*arctan(sqrt(x)) - sqrt(x) + arctan(sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \arctan\left(\sqrt{x}\right)\,dx = \mathrm{atan}\left(\sqrt{x}\right) + x\,\mathrm{atan}\left(\sqrt{x}\right) - \sqrt{x}$$

```
[In] int(atan(x^(1/2)),x)
[Out] atan(x^(1/2)) + x*atan(x^(1/2)) - x^(1/2)
```

3.157 $\int \frac{\arctan(\sqrt{x})}{x} dx$

Optimal result 898
Rubi [A] (verified) 898
Mathematica [A] (verified) 899
Maple [A] (verified) 899
Fricas [F] 900
Sympy [F] 900
Maxima [B] (verification not implemented) 900
Giac [F] 900
Mupad [B] (verification not implemented) 901

Optimal result

Integrand size = 10, antiderivative size = 31

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x} dx = i\,\text{PolyLog}\left(2, -i\sqrt{x}\right) - i\,\text{PolyLog}\left(2, i\sqrt{x}\right)$$

```
[Out] I*polylog(2,-I*x^(1/2))-I*polylog(2,I*x^(1/2))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4944, 4940, 2438}

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x} dx = i\,\text{PolyLog}\left(2, -i\sqrt{x}\right) - i\,\text{PolyLog}\left(2, i\sqrt{x}\right)$$

```
[In] Int[ArcTan[Sqrt[x]]/x,x]
[Out] I*PolyLog[2, (-I)*Sqrt[x]] - I*PolyLog[2, I*Sqrt[x]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1
/n, Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\text{Subst}\left(\int \frac{\arctan(x)}{x}\,dx, x, \sqrt{x}\right)\\ &= i\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, \sqrt{x}\right) - i\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx, x, \sqrt{x}\right)\\ &= i\,\text{PolyLog}\left(2, -i\sqrt{x}\right) - i\,\text{PolyLog}\left(2, i\sqrt{x}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x}\,dx = i\,\text{PolyLog}\left(2, -i\sqrt{x}\right) - i\,\text{PolyLog}\left(2, i\sqrt{x}\right)$$

```
[In] Integrate[ArcTan[Sqrt[x]]/x,x]
[Out] I*PolyLog[2, (-I)*Sqrt[x]] - I*PolyLog[2, I*Sqrt[x]]
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

method	result
meijerg	$i\,\text{polylog}\left(2, -i\sqrt{x}\right) - i\,\text{polylog}\left(2, i\sqrt{x}\right)$
derivativedivides	$\ln\left(x\right)\arctan\left(\sqrt{x}\right) + \frac{i\ln(x)\ln(1+i\sqrt{x})}{2} - \frac{i\ln(x)\ln(1-i\sqrt{x})}{2} + i\,\text{dilog}\left(1+i\sqrt{x}\right) - i\,\text{dilog}\left(1-\right.$
default	$\ln\left(x\right)\arctan\left(\sqrt{x}\right) + \frac{i\ln(x)\ln(1+i\sqrt{x})}{2} - \frac{i\ln(x)\ln(1-i\sqrt{x})}{2} + i\,\text{dilog}\left(1+i\sqrt{x}\right) - i\,\text{dilog}\left(1-\right.$
parts	$\ln\left(x\right)\arctan\left(\sqrt{x}\right) + \frac{i\ln(x)\ln(1+i\sqrt{x})}{2} - \frac{i\ln(x)\ln(1-i\sqrt{x})}{2} + i\,\text{dilog}\left(1+i\sqrt{x}\right) - i\,\text{dilog}\left(1-\right.$

```
[In] int(arctan(x^(1/2))/x,x,method=_RETURNVERBOSE)
[Out] I*polylog(2,-I*x^(1/2))-I*polylog(2,I*x^(1/2))
```

Fricas [F]

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x}\,dx = \int \frac{\arctan\left(\sqrt{x}\right)}{x}\,dx$$

```
[In] integrate(arctan(x^(1/2))/x,x, algorithm="fricas")
[Out] integral(arctan(sqrt(x))/x, x)
```

Sympy [F]

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x}\,dx = \int \frac{\operatorname{atan}\left(\sqrt{x}\right)}{x}\,dx$$

```
[In] integrate(atan(x**(1/2))/x,x)
[Out] Integral(atan(sqrt(x))/x, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(17) = 34$.

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.13

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x}\,dx = -\frac{1}{2}\,\pi\log\left(x+1\right) + \arctan\left(\sqrt{x}\right)\log\left(x\right) - i\,\mathrm{Li}_2\left(i\,\sqrt{x}+1\right) + i\,\mathrm{Li}_2\left(-i\,\sqrt{x}+1\right)$$

```
[In] integrate(arctan(x^(1/2))/x,x, algorithm="maxima")
[Out] -1/2*pi*log(x + 1) + arctan(sqrt(x))*log(x) - I*dilog(I*sqrt(x) + 1) + I*di
log(-I*sqrt(x) + 1)
```

Giac [F]

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x}\,dx = \int \frac{\arctan\left(\sqrt{x}\right)}{x}\,dx$$

```
[In] integrate(arctan(x^(1/2))/x,x, algorithm="giac")
[Out] integrate(arctan(sqrt(x))/x, x)
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x}\,dx = -\mathrm{Li}_2\left(1-\sqrt{x}\,1\mathrm{i}\right)\,1\mathrm{i} + \mathrm{polylog}\left(2,-\sqrt{x}\,1\mathrm{i}\right)\,1\mathrm{i}$$

```
[In] int(atan(x^(1/2))/x,x)
[Out] polylog(2, -x^(1/2)*1i)*1i - dilog(1 - x^(1/2)*1i)*1i
```

3.158 $\int \frac{\arctan(\sqrt{x})}{x^2} dx$

Optimal result 902
Rubi [A] (verified) 902
Mathematica [C] (verified) 903
Maple [A] (verified) 904
Fricas [A] (verification not implemented) 904
Sympy [B] (verification not implemented) 904
Maxima [A] (verification not implemented) 905
Giac [A] (verification not implemented) 905
Mupad [B] (verification not implemented) 905

Optimal result

Integrand size = 10, antiderivative size = 27

$$\int \frac{\arctan(\sqrt{x})}{x^2} dx = -\frac{1}{\sqrt{x}} - \arctan(\sqrt{x}) - \frac{\arctan(\sqrt{x})}{x}$$

```
[Out] -arctan(x^(1/2))-arctan(x^(1/2))/x-1/x^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4946, 53, 65, 209}

$$\int \frac{\arctan(\sqrt{x})}{x^2} dx = -\frac{\arctan(\sqrt{x})}{x} - \arctan(\sqrt{x}) - \frac{1}{\sqrt{x}}$$

```
[In] Int[ArcTan[Sqrt[x]]/x^2,x]
```

```
[Out] -(1/Sqrt[x]) - ArcTan[Sqrt[x]] - ArcTan[Sqrt[x]]/x
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{\arctan\left(\sqrt{x}\right)}{x}+\frac{1}{2}\int\frac{1}{x^{3/2}(1+x)}\,dx\\ &= -\frac{1}{\sqrt{x}}-\frac{\arctan\left(\sqrt{x}\right)}{x}-\frac{1}{2}\int\frac{1}{\sqrt{x}(1+x)}\,dx\\ &= -\frac{1}{\sqrt{x}}-\frac{\arctan\left(\sqrt{x}\right)}{x}-\text{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\sqrt{x}\right)\\ &= -\frac{1}{\sqrt{x}}-\arctan\left(\sqrt{x}\right)-\frac{\arctan\left(\sqrt{x}\right)}{x}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.11

$$\int\frac{\arctan\left(\sqrt{x}\right)}{x^2}\,dx=-\frac{\arctan\left(\sqrt{x}\right)}{x}-\frac{\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-x\right)}{\sqrt{x}}$$

```
[In] Integrate[ArcTan[Sqrt[x]]/x^2,x]
[Out] -(ArcTan[Sqrt[x]]/x) - Hypergeometric2F1[-1/2, 1, 1/2, -x]/Sqrt[x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

method	result	size
meijerg	$-\frac{1}{\sqrt{x}}-\frac{\arctan\left(\sqrt{x}\right)(x+1)}{x}$	19
derivativedivides	$-\arctan\left(\sqrt{x}\right)-\frac{\arctan\left(\sqrt{x}\right)}{x}-\frac{1}{\sqrt{x}}$	22
default	$-\arctan\left(\sqrt{x}\right)-\frac{\arctan\left(\sqrt{x}\right)}{x}-\frac{1}{\sqrt{x}}$	22
parts	$-\arctan\left(\sqrt{x}\right)-\frac{\arctan\left(\sqrt{x}\right)}{x}-\frac{1}{\sqrt{x}}$	22

```
[In] int(arctan(x^(1/2))/x^2,x,method=_RETURNVERBOSE)
[Out] -1/x^(1/2)-1/x*arctan(x^(1/2))*(x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^2}\,dx=-\frac{(x+1)\arctan\left(\sqrt{x}\right)+\sqrt{x}}{x}$$

```
[In] integrate(arctan(x^(1/2))/x^2,x, algorithm="fricas")
[Out] -((x + 1)*arctan(sqrt(x)) + sqrt(x))/x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(22) = 44.

Time = 0.56 (sec) , antiderivative size = 94, normalized size of antiderivative = 3.48

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^2}\,dx=-\frac{x^{\frac{5}{2}}\operatorname{atan}\left(\sqrt{x}\right)}{x^{\frac{5}{2}}+x^{\frac{3}{2}}}-\frac{2x^{\frac{3}{2}}\operatorname{atan}\left(\sqrt{x}\right)}{x^{\frac{5}{2}}+x^{\frac{3}{2}}}-\frac{\sqrt{x}\operatorname{atan}\left(\sqrt{x}\right)}{x^{\frac{5}{2}}+x^{\frac{3}{2}}}-\frac{x^2}{x^{\frac{5}{2}}+x^{\frac{3}{2}}}-\frac{x}{x^{\frac{5}{2}}+x^{\frac{3}{2}}}$$

```
[In] integrate(atan(x**(1/2))/x**2,x)
[Out] -x**(5/2)*atan(sqrt(x))/(x**(5/2) + x**(3/2)) - 2*x**(3/2)*atan(sqrt(x))/(x
**(5/2) + x**(3/2)) - sqrt(x)*atan(sqrt(x))/(x**(5/2) + x**(3/2)) - x**2/(x
**(5/2) + x**(3/2)) - x/(x**(5/2) + x**(3/2))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^2}\,dx = -\frac{\arctan\left(\sqrt{x}\right)}{x} - \frac{1}{\sqrt{x}} - \arctan\left(\sqrt{x}\right)$$

```
[In] integrate(arctan(x^(1/2))/x^2,x, algorithm="maxima")
[Out] -arctan(sqrt(x))/x - 1/sqrt(x) - arctan(sqrt(x))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^2}\,dx = -\frac{\arctan\left(\sqrt{x}\right)}{x} - \frac{1}{\sqrt{x}} - \arctan\left(\sqrt{x}\right)$$

```
[In] integrate(arctan(x^(1/2))/x^2,x, algorithm="giac")
[Out] -arctan(sqrt(x))/x - 1/sqrt(x) - arctan(sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^2}\,dx = -\mathrm{atan}\left(\sqrt{x}\right) - \frac{\mathrm{atan}\left(\sqrt{x}\right)}{x} - \frac{1}{\sqrt{x}}$$

```
[In] int(atan(x^(1/2))/x^2,x)
[Out] - atan(x^(1/2)) - atan(x^(1/2))/x - 1/x^(1/2)
```

3.159 $\int \frac{\arctan(\sqrt{x})}{x^3}\,dx$

Optimal result 906
Rubi [A] (verified) 906
Mathematica [C] (verified) 908
Maple [A] (verified) 908
Fricas [A] (verification not implemented) 908
Sympy [B] (verification not implemented) 909
Maxima [A] (verification not implemented) 909
Giac [A] (verification not implemented) 909
Mupad [B] (verification not implemented) 910

Optimal result

Integrand size = 10, antiderivative size = 42

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^3}\,dx = -\frac{1}{6x^{3/2}} + \frac{1}{2\sqrt{x}} + \frac{\arctan\left(\sqrt{x}\right)}{2} - \frac{\arctan\left(\sqrt{x}\right)}{2x^2}$$

```
[Out] -1/6/x^(3/2)+1/2*arctan(x^(1/2))-1/2*arctan(x^(1/2))/x^2+1/2/x^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4946, 53, 65, 209}

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^3}\,dx = -\frac{\arctan\left(\sqrt{x}\right)}{2x^2} + \frac{\arctan\left(\sqrt{x}\right)}{2} - \frac{1}{6x^{3/2}} + \frac{1}{2\sqrt{x}}$$

```
[In] Int[ArcTan[Sqrt[x]]/x^3,x]
```

```
[Out] -1/6*1/x^(3/2) + 1/(2*Sqrt[x]) + ArcTan[Sqrt[x]]/2 - ArcTan[Sqrt[x]]/(2*x^2
)
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\arctan\left(\sqrt{x}\right)}{2x^2}+\frac{1}{4}\int\frac{1}{x^{5/2}(1+x)}\,dx\\
&= -\frac{1}{6x^{3/2}}-\frac{\arctan\left(\sqrt{x}\right)}{2x^2}-\frac{1}{4}\int\frac{1}{x^{3/2}(1+x)}\,dx\\
&= -\frac{1}{6x^{3/2}}+\frac{1}{2\sqrt{x}}-\frac{\arctan\left(\sqrt{x}\right)}{2x^2}+\frac{1}{4}\int\frac{1}{\sqrt{x}(1+x)}\,dx\\
&= -\frac{1}{6x^{3/2}}+\frac{1}{2\sqrt{x}}-\frac{\arctan\left(\sqrt{x}\right)}{2x^2}+\frac{1}{2}\text{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\sqrt{x}\right)\\
&= -\frac{1}{6x^{3/2}}+\frac{1}{2\sqrt{x}}+\frac{\arctan\left(\sqrt{x}\right)}{2}-\frac{\arctan\left(\sqrt{x}\right)}{2x^2}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.81

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^3}\,dx = -\frac{\arctan\left(\sqrt{x}\right)}{2x^2} - \frac{\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-x\right)}{6x^{3/2}}$$

```
[In] Integrate[ArcTan[Sqrt[x]]/x^3,x]
[Out] -1/2*ArcTan[Sqrt[x]]/x^2 - Hypergeometric2F1[-3/2, 1, -1/2, -x]/(6*x^(3/2))
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.64

method	result	size
derivativedivides	$-\frac{1}{6x^{\frac{3}{2}}}+\frac{\arctan\left(\sqrt{x}\right)}{2}-\frac{\arctan\left(\sqrt{x}\right)}{2x^2}+\frac{1}{2\sqrt{x}}$	27
default	$-\frac{1}{6x^{\frac{3}{2}}}+\frac{\arctan\left(\sqrt{x}\right)}{2}-\frac{\arctan\left(\sqrt{x}\right)}{2x^2}+\frac{1}{2\sqrt{x}}$	27
parts	$-\frac{1}{6x^{\frac{3}{2}}}+\frac{\arctan\left(\sqrt{x}\right)}{2}-\frac{\arctan\left(\sqrt{x}\right)}{2x^2}+\frac{1}{2\sqrt{x}}$	27
meijerg	$-\frac{1}{6x^{\frac{3}{2}}}+\frac{1}{2\sqrt{x}}-\frac{4\left(-\frac{3x^2}{8}+\frac{3}{8}\right)\arctan\left(\sqrt{x}\right)}{3x^2}$	28

```
[In] int(arctan(x^(1/2))/x^3,x,method=_RETURNVERBOSE)
[Out] -1/6/x^(3/2)+1/2*arctan(x^(1/2))-1/2*arctan(x^(1/2))/x^2+1/2/x^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.62

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^3}\,dx = \frac{3\,(x^2-1)\arctan\left(\sqrt{x}\right)+(3\,x-1)\sqrt{x}}{6\,x^2}$$

```
[In] integrate(arctan(x^(1/2))/x^3,x, algorithm="fricas")
[Out] 1/6*(3*(x^2 - 1)*arctan(sqrt(x)) + (3*x - 1)*sqrt(x))/x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(36) = 72$.

Time = 1.35 (sec) , antiderivative size = 160, normalized size of antiderivative = 3.81

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^3}\,dx = \frac{3x^{\frac{7}{2}}\operatorname{atan}\left(\sqrt{x}\right)}{6x^{\frac{7}{2}}+6x^{\frac{5}{2}}} + \frac{3x^{\frac{5}{2}}\operatorname{atan}\left(\sqrt{x}\right)}{6x^{\frac{7}{2}}+6x^{\frac{5}{2}}} - \frac{3x^{\frac{3}{2}}\operatorname{atan}\left(\sqrt{x}\right)}{6x^{\frac{7}{2}}+6x^{\frac{5}{2}}} - \frac{3\sqrt{x}\operatorname{atan}\left(\sqrt{x}\right)}{6x^{\frac{7}{2}}+6x^{\frac{5}{2}}} + \frac{3x^3}{6x^{\frac{7}{2}}+6x^{\frac{5}{2}}} + \frac{2x^2}{6x^{\frac{7}{2}}+6x^{\frac{5}{2}}} - \frac{x}{6x^{\frac{7}{2}}+6x^{\frac{5}{2}}}$$

```
[In] integrate(atan(x**(1/2))/x**3,x)
```

```
[Out] 3*x**(7/2)*atan(sqrt(x))/(6*x**(7/2) + 6*x**(5/2)) + 3*x**(5/2)*atan(sqrt(x
))/(6*x**(7/2) + 6*x**(5/2)) - 3*x**(3/2)*atan(sqrt(x))/(6*x**(7/2) + 6*x**
(5/2)) - 3*sqrt(x)*atan(sqrt(x))/(6*x**(7/2) + 6*x**(5/2)) + 3*x**3/(6*x**(
7/2) + 6*x**(5/2)) + 2*x**2/(6*x**(7/2) + 6*x**(5/2)) - x/(6*x**(7/2) + 6*x
**(5/2))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.62

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^3}\,dx = \frac{3\,x-1}{6\,x^{\frac{3}{2}}} - \frac{\arctan\left(\sqrt{x}\right)}{2\,x^2} + \frac{1}{2}\arctan\left(\sqrt{x}\right)$$

```
[In] integrate(arctan(x^(1/2))/x^3,x, algorithm="maxima")
```

```
[Out] 1/6*(3*x - 1)/x^(3/2) - 1/2*arctan(sqrt(x))/x^2 + 1/2*arctan(sqrt(x))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.62

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^3}\,dx = \frac{3\,x-1}{6\,x^{\frac{3}{2}}} - \frac{\arctan\left(\sqrt{x}\right)}{2\,x^2} + \frac{1}{2}\arctan\left(\sqrt{x}\right)$$

```
[In] integrate(arctan(x^(1/2))/x^3,x, algorithm="giac")
```

```
[Out] 1/6*(3*x - 1)/x^(3/2) - 1/2*arctan(sqrt(x))/x^2 + 1/2*arctan(sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.57

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^3}\,dx = \frac{\mathrm{atan}\left(\sqrt{x}\right)}{2} + \frac{x-\frac{1}{3}}{2\,x^{3/2}} - \frac{\mathrm{atan}\left(\sqrt{x}\right)}{2\,x^2}$$

```
[In] int(atan(x^(1/2))/x^3,x)
[Out] atan(x^(1/2))/2 + (x - 1/3)/(2*x^(3/2)) - atan(x^(1/2))/(2*x^2)
```

3.160 $\int x^{3/2} \arctan \left(\sqrt{x}\right) dx$

Optimal result 911
Rubi [A] (verified) 911
Mathematica [A] (verified) 912
Maple [A] (verified) 912
Fricas [A] (verification not implemented) 913
Sympy [B] (verification not implemented) 913
Maxima [A] (verification not implemented) 913
Giac [A] (verification not implemented) 914
Mupad [B] (verification not implemented) 914

Optimal result

Integrand size = 12, antiderivative size = 36

$$\int x^{3/2} \arctan \left(\sqrt{x}\right) dx = \frac{x}{5} - \frac{x^2}{10} + \frac{2}{5} x^{5/2} \arctan \left(\sqrt{x}\right) - \frac{1}{5} \log(1+x)$$

```
[Out] 1/5*x-1/10*x^2+2/5*x^(5/2)*arctan(x^(1/2))-1/5*ln(1+x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4946, 45}

$$\int x^{3/2} \arctan \left(\sqrt{x}\right) dx = \frac{2}{5} x^{5/2} \arctan \left(\sqrt{x}\right) - \frac{x^2}{10} + \frac{x}{5} - \frac{1}{5} \log(x+1)$$

```
[In] Int[x^(3/2)*ArcTan[Sqrt[x]],x]
```

```
[Out] x/5 - x^2/10 + (2*x^(5/2)*ArcTan[Sqrt[x]])/5 - Log[1 + x]/5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
```

```
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2}{5}x^{5/2}\arctan\left(\sqrt{x}\right)-\frac{1}{5}\int\frac{x^2}{1+x}\,dx\\&=\frac{2}{5}x^{5/2}\arctan\left(\sqrt{x}\right)-\frac{1}{5}\int\left(-1+x+\frac{1}{1+x}\right)\,dx\\&=\frac{x}{5}-\frac{x^2}{10}+\frac{2}{5}x^{5/2}\arctan\left(\sqrt{x}\right)-\frac{1}{5}\log(1+x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

$$\int x^{3/2}\arctan\left(\sqrt{x}\right)\,dx=\frac{1}{10}\left(-((-2+x)x)+4x^{5/2}\arctan\left(\sqrt{x}\right)-2\log(1+x)\right)$$

```
[In] Integrate[x^(3/2)*ArcTan[Sqrt[x]],x]
[Out] (-((-2 + x)*x) + 4*x^(5/2)*ArcTan[Sqrt[x]] - 2*Log[1 + x])/10
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

method	result	size
derivativedivides	$\frac{x}{5}-\frac{x^2}{10}+\frac{2x^{\frac{5}{2}}\arctan(\sqrt{x})}{5}-\frac{\ln(x+1)}{5}$	25
default	$\frac{x}{5}-\frac{x^2}{10}+\frac{2x^{\frac{5}{2}}\arctan(\sqrt{x})}{5}-\frac{\ln(x+1)}{5}$	25
meijerg	$\frac{x(-3x+6)}{30}+\frac{2x^{\frac{5}{2}}\arctan(\sqrt{x})}{5}-\frac{\ln(x+1)}{5}$	25

```
[In] int(x^(3/2)*arctan(x^(1/2)),x,method=_RETURNVERBOSE)
[Out] 1/5*x-1/10*x^2+2/5*x^(5/2)*arctan(x^(1/2))-1/5*ln(x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int x^{3/2}\arctan\left(\sqrt{x}\right)\,dx=\frac{2}{5}\,x^{\frac{5}{2}}\arctan\left(\sqrt{x}\right)-\frac{1}{10}\,x^{2}+\frac{1}{5}\,x-\frac{1}{5}\,\log\left(x+1\right)$$

```
[In] integrate(x^(3/2)*arctan(x^(1/2)),x, algorithm="fricas")
[Out] 2/5*x^(5/2)*arctan(sqrt(x)) - 1/10*x^2 + 1/5*x - 1/5*log(x + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(29) = 58$.

Time = 1.01 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.36

$$\int x^{3/2}\arctan\left(\sqrt{x}\right)\,dx=\frac{4x^{\frac{7}{2}}\,\mathrm{atan}\left(\sqrt{x}\right)}{10x+10}+\frac{4x^{\frac{5}{2}}\,\mathrm{atan}\left(\sqrt{x}\right)}{10x+10}-\frac{x^{3}}{10x+10}+\frac{x^{2}}{10x+10}-\frac{2x\log\left(x+1\right)}{10x+10}-\frac{2\log\left(x+1\right)}{10x+10}-\frac{2}{10x+10}$$

```
[In] integrate(x**(3/2)*atan(x**(1/2)),x)
[Out] 4*x**(7/2)*atan(sqrt(x))/(10*x + 10) + 4*x**(5/2)*atan(sqrt(x))/(10*x + 10)
 - x**3/(10*x + 10) + x**2/(10*x + 10) - 2*x*log(x + 1)/(10*x + 10) - 2*log
(x + 1)/(10*x + 10) - 2/(10*x + 10)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int x^{3/2}\arctan\left(\sqrt{x}\right)\,dx=\frac{2}{5}\,x^{\frac{5}{2}}\arctan\left(\sqrt{x}\right)-\frac{1}{10}\,x^{2}+\frac{1}{5}\,x-\frac{1}{5}\,\log\left(x+1\right)$$

```
[In] integrate(x^(3/2)*arctan(x^(1/2)),x, algorithm="maxima")
[Out] 2/5*x^(5/2)*arctan(sqrt(x)) - 1/10*x^2 + 1/5*x - 1/5*log(x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int x^{3/2} \arctan\left(\sqrt{x}\right) \, dx = \frac{2}{5} \, x^{\frac{5}{2}} \arctan\left(\sqrt{x}\right) - \frac{1}{10} \, x^2 + \frac{1}{5} \, x - \frac{1}{5} \, \log\left(x + 1\right)$$

```
[In] integrate(x^(3/2)*arctan(x^(1/2)),x, algorithm="giac")
[Out] 2/5*x^(5/2)*arctan(sqrt(x)) - 1/10*x^2 + 1/5*x - 1/5*log(x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int x^{3/2} \arctan\left(\sqrt{x}\right) \, dx = \frac{x}{5} - \frac{\ln\left(x+1\right)}{5} + \frac{2\,x^{5/2}\,\mathrm{atan}\left(\sqrt{x}\right)}{5} - \frac{x^2}{10}$$

```
[In] int(x^(3/2)*atan(x^(1/2)),x)
[Out] x/5 - log(x + 1)/5 + (2*x^(5/2)*atan(x^(1/2)))/5 - x^2/10
```

3.161 $\int \sqrt{x} \arctan\left(\sqrt{x}\right)\, dx$

Optimal result . . . 915
Rubi [A] (verified) . . . 915
Mathematica [A] (verified) . . . 916
Maple [A] (verified) . . . 916
Fricas [A] (verification not implemented) . . . 917
Sympy [A] (verification not implemented) . . . 917
Maxima [A] (verification not implemented) . . . 917
Giac [A] (verification not implemented) . . . 917
Mupad [F(-1)] . . . 918

Optimal result

Integrand size = 12, antiderivative size = 29

$$\int \sqrt{x} \arctan\left(\sqrt{x}\right)\, dx = -\frac{x}{3} + \frac{2}{3}x^{3/2} \arctan\left(\sqrt{x}\right) + \frac{1}{3}\log(1+x)$$

```
[Out] -1/3*x+2/3*x^(3/2)*arctan(x^(1/2))+1/3*ln(1+x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4946, 45}

$$\int \sqrt{x} \arctan\left(\sqrt{x}\right)\, dx = \frac{2}{3}x^{3/2} \arctan\left(\sqrt{x}\right) - \frac{x}{3} + \frac{1}{3}\log(x+1)$$

```
[In] Int[Sqrt[x]*ArcTan[Sqrt[x]],x]
```

```
[Out] -1/3*x + (2*x^(3/2)*ArcTan[Sqrt[x]])/3 + Log[1 + x]/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
```

```
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2}{3}x^{3/2}\arctan\left(\sqrt{x}\right)-\frac{1}{3}\int\frac{x}{1+x}\,dx\\ &= \frac{2}{3}x^{3/2}\arctan\left(\sqrt{x}\right)-\frac{1}{3}\int\left(1+\frac{1}{-1-x}\right)\,dx\\ &= -\frac{x}{3}+\frac{2}{3}x^{3/2}\arctan\left(\sqrt{x}\right)+\frac{1}{3}\log(1+x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int\sqrt{x}\arctan\left(\sqrt{x}\right)\,dx=\frac{1}{3}\left(-x+2x^{3/2}\arctan\left(\sqrt{x}\right)+\log(1+x)\right)$$

```
[In] Integrate[Sqrt[x]*ArcTan[Sqrt[x]],x]
[Out] (-x + 2*x^(3/2)*ArcTan[Sqrt[x]] + Log[1 + x])/3
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.69

method	result	size
derivativedivides	$-\frac{x}{3}+\frac{2x^{\frac{3}{2}}\arctan\left(\sqrt{x}\right)}{3}+\frac{\ln\left(x+1\right)}{3}$	20
default	$-\frac{x}{3}+\frac{2x^{\frac{3}{2}}\arctan\left(\sqrt{x}\right)}{3}+\frac{\ln\left(x+1\right)}{3}$	20
meijerg	$-\frac{x}{3}+\frac{2x^{\frac{3}{2}}\arctan\left(\sqrt{x}\right)}{3}+\frac{\ln\left(x+1\right)}{3}$	20

```
[In] int(x^(1/2)*arctan(x^(1/2)),x,method=_RETURNVERBOSE)
[Out] -1/3*x+2/3*x^(3/2)*arctan(x^(1/2))+1/3*ln(x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

$$\int \sqrt{x} \arctan \left(\sqrt{x}\right) \, dx = \frac{2}{3} \, x^{\frac{3}{2}} \arctan \left(\sqrt{x}\right) - \frac{1}{3} \, x + \frac{1}{3} \, \log \left(x + 1\right)$$

```
[In] integrate(x^(1/2)*arctan(x^(1/2)),x, algorithm="fricas")
[Out] 2/3*x^(3/2)*arctan(sqrt(x)) - 1/3*x + 1/3*log(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \sqrt{x} \arctan \left(\sqrt{x}\right) \, dx = \frac{2x^{\frac{3}{2}} \operatorname{atan}\left(\sqrt{x}\right)}{3} - \frac{x}{3} + \frac{\log \left(x + 1\right)}{3}$$

```
[In] integrate(x**(1/2)*atan(x**(1/2)),x)
[Out] 2*x**(3/2)*atan(sqrt(x))/3 - x/3 + log(x + 1)/3
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

$$\int \sqrt{x} \arctan \left(\sqrt{x}\right) \, dx = \frac{2}{3} \, x^{\frac{3}{2}} \arctan \left(\sqrt{x}\right) - \frac{1}{3} \, x + \frac{1}{3} \, \log \left(x + 1\right)$$

```
[In] integrate(x^(1/2)*arctan(x^(1/2)),x, algorithm="maxima")
[Out] 2/3*x^(3/2)*arctan(sqrt(x)) - 1/3*x + 1/3*log(x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

$$\int \sqrt{x} \arctan \left(\sqrt{x}\right) \, dx = \frac{2}{3} \, x^{\frac{3}{2}} \arctan \left(\sqrt{x}\right) - \frac{1}{3} \, x + \frac{1}{3} \, \log \left(x + 1\right)$$

```
[In] integrate(x^(1/2)*arctan(x^(1/2)),x, algorithm="giac")
[Out] 2/3*x^(3/2)*arctan(sqrt(x)) - 1/3*x + 1/3*log(x + 1)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{x}\arctan\left(\sqrt{x}\right)\,dx = \int \sqrt{x}\,\mathrm{atan}\left(\sqrt{x}\right)\,dx$$

```
[In] int(x^(1/2)*atan(x^(1/2)),x)
[Out] int(x^(1/2)*atan(x^(1/2)), x)
```

3.162 $\int \frac{\arctan(\sqrt{x})}{\sqrt{x}}\,dx$

Optimal result 919
Rubi [A] (verified) 919
Mathematica [A] (verified) 920
Maple [A] (verified) 920
Fricas [A] (verification not implemented) 920
Sympy [A] (verification not implemented) 921
Maxima [A] (verification not implemented) 921
Giac [A] (verification not implemented) 921
Mupad [B] (verification not implemented) 921

Optimal result

Integrand size = 12, antiderivative size = 20

$$\int \frac{\arctan\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\sqrt{x}\arctan\left(\sqrt{x}\right) - \log(1+x)$$

```
[Out] -ln(1+x)+2*x^(1/2)*arctan(x^(1/2))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4946, 31}

$$\int \frac{\arctan\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\sqrt{x}\arctan\left(\sqrt{x}\right) - \log(x+1)$$

```
[In] Int[ArcTan[Sqrt[x]]/Sqrt[x],x]
```

```
[Out] 2*Sqrt[x]*ArcTan[Sqrt[x]] - Log[1 + x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
```

```
IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\sqrt{x}\arctan\left(\sqrt{x}\right) - \int \frac{1}{1+x}\,dx\\ &= 2\sqrt{x}\arctan\left(\sqrt{x}\right) - \log(1+x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\arctan\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\sqrt{x}\arctan\left(\sqrt{x}\right) - \log(1+x)$$

```
[In] Integrate[ArcTan[Sqrt[x]]/Sqrt[x],x]
[Out] 2*Sqrt[x]*ArcTan[Sqrt[x]] - Log[1 + x]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$-\ln\left(x+1\right)+2\sqrt{x}\,\arctan\left(\sqrt{x}\right)$	17
default	$-\ln\left(x+1\right)+2\sqrt{x}\,\arctan\left(\sqrt{x}\right)$	17
meijerg	$-\ln\left(x+1\right)+2\sqrt{x}\,\arctan\left(\sqrt{x}\right)$	17

```
[In] int(arctan(x^(1/2))/x^(1/2),x,method=_RETURNVERBOSE)
[Out] -ln(x+1)+2*x^(1/2)*arctan(x^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{\arctan\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\,\sqrt{x}\arctan\left(\sqrt{x}\right) - \log\left(x+1\right)$$

```
[In] integrate(arctan(x^(1/2))/x^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(x)*arctan(sqrt(x)) - log(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{\arctan\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\sqrt{x}\,\mathrm{atan}\left(\sqrt{x}\right) - \log\left(x+1\right)$$

```
[In] integrate(atan(x**(1/2))/x**(1/2),x)
[Out] 2*sqrt(x)*atan(sqrt(x)) - log(x + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{\arctan\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\,\sqrt{x}\arctan\left(\sqrt{x}\right) - \log\left(x+1\right)$$

```
[In] integrate(arctan(x^(1/2))/x^(1/2),x, algorithm="maxima")
[Out] 2*sqrt(x)*arctan(sqrt(x)) - log(x + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{\arctan\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\,\sqrt{x}\arctan\left(\sqrt{x}\right) - \log\left(x+1\right)$$

```
[In] integrate(arctan(x^(1/2))/x^(1/2),x, algorithm="giac")
[Out] 2*sqrt(x)*arctan(sqrt(x)) - log(x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{\arctan\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\,\sqrt{x}\,\mathrm{atan}\left(\sqrt{x}\right) - \ln\left(x+1\right)$$

```
[In] int(atan(x^(1/2))/x^(1/2),x)
[Out] 2*x^(1/2)*atan(x^(1/2)) - log(x + 1)
```

3.163 $\int \frac{\arctan(\sqrt{x})}{x^{3/2}}\,dx$

Optimal result 922
Rubi [A] (verified) 922
Mathematica [A] (verified) 923
Maple [A] (verified) 923
Fricas [A] (verification not implemented) 924
Sympy [A] (verification not implemented) 924
Maxima [A] (verification not implemented) 924
Giac [A] (verification not implemented) 925
Mupad [B] (verification not implemented) 925

Optimal result

Integrand size = 12, antiderivative size = 22

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^{3/2}}\,dx = -\frac{2\arctan\left(\sqrt{x}\right)}{\sqrt{x}} + \log(x) - \log(1+x)$$

```
[Out] ln(x)-ln(1+x)-2*arctan(x^(1/2))/x^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4946, 36, 29, 31}

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^{3/2}}\,dx = -\frac{2\arctan\left(\sqrt{x}\right)}{\sqrt{x}} + \log(x) - \log(x+1)$$

```
[In] Int[ArcTan[Sqrt[x]]/x^(3/2),x]
```

```
[Out] (-2*ArcTan[Sqrt[x]])/Sqrt[x] + Log[x] - Log[1 + x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2\arctan\left(\sqrt{x}\right)}{\sqrt{x}}+\int\frac{1}{x(1+x)}\,dx\\ &= -\frac{2\arctan\left(\sqrt{x}\right)}{\sqrt{x}}+\int\frac{1}{x}\,dx-\int\frac{1}{1+x}\,dx\\ &= -\frac{2\arctan\left(\sqrt{x}\right)}{\sqrt{x}}+\log(x)-\log(1+x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int\frac{\arctan\left(\sqrt{x}\right)}{x^{3/2}}\,dx=-\frac{2\arctan\left(\sqrt{x}\right)}{\sqrt{x}}+\log(x)-\log(1+x)$$

```
[In] Integrate[ArcTan[Sqrt[x]]/x^(3/2),x]
[Out] (-2*ArcTan[Sqrt[x]])/Sqrt[x] + Log[x] - Log[1 + x]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\ln\left(x\right)-\ln\left(x+1\right)-\frac{2\arctan\left(\sqrt{x}\right)}{\sqrt{x}}$	19
default	$\ln\left(x\right)-\ln\left(x+1\right)-\frac{2\arctan\left(\sqrt{x}\right)}{\sqrt{x}}$	19
meijerg	$\ln\left(x\right)-\ln\left(x+1\right)-\frac{2\arctan\left(\sqrt{x}\right)}{\sqrt{x}}$	19

```
[In] int(arctan(x^(1/2))/x^(3/2),x,method=_RETURNVERBOSE)
[Out] ln(x)-ln(x+1)-2*arctan(x^(1/2))/x^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^{3/2}}\,dx = -\frac{x\log\left(x+1\right) - x\log\left(x\right) + 2\sqrt{x}\arctan\left(\sqrt{x}\right)}{x}$$

```
[In] integrate(arctan(x^(1/2))/x^(3/2),x, algorithm="fricas")
[Out] -(x*log(x + 1) - x*log(x) + 2*sqrt(x)*arctan(sqrt(x)))/x
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^{3/2}}\,dx = \log\left(x\right) - \log\left(x+1\right) - \frac{2\operatorname{atan}\left(\sqrt{x}\right)}{\sqrt{x}}$$

```
[In] integrate(atan(x**(1/2))/x**(3/2),x)
[Out] log(x) - log(x + 1) - 2*atan(sqrt(x))/sqrt(x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^{3/2}}\,dx = -\frac{2\ \arctan\left(\sqrt{x}\right)}{\sqrt{x}} - \log\left(x+1\right) + \log\left(x\right)$$

```
[In] integrate(arctan(x^(1/2))/x^(3/2),x, algorithm="maxima")
[Out] -2*arctan(sqrt(x))/sqrt(x) - log(x + 1) + log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^{3/2}}\,dx = -\frac{2\arctan\left(\sqrt{x}\right)}{\sqrt{x}} - \log\left(x+1\right) + \log\left(x\right)$$

```
[In] integrate(arctan(x^(1/2))/x^(3/2),x, algorithm="giac")
[Out] -2*arctan(sqrt(x))/sqrt(x) - log(x + 1) + log(x)
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^{3/2}}\,dx = 2\,\ln\left(\sqrt{x}\right) - \ln\left(x+1\right) - \frac{2\,\mathrm{atan}\left(\sqrt{x}\right)}{\sqrt{x}}$$

```
[In] int(atan(x^(1/2))/x^(3/2),x)
[Out] 2*log(x^(1/2)) - log(x + 1) - (2*atan(x^(1/2)))/x^(1/2)
```

3.164 $\int \frac{\arctan(\sqrt{x})}{x^{5/2}}\,dx$

Optimal result 926
Rubi [A] (verified) 926
Mathematica [A] (verified) 927
Maple [A] (verified) 927
Fricas [A] (verification not implemented) 928
Sympy [B] (verification not implemented) 928
Maxima [A] (verification not implemented) 928
Giac [A] (verification not implemented) 929
Mupad [B] (verification not implemented) 929

Optimal result

Integrand size = 12, antiderivative size = 37

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^{5/2}}\,dx = -\frac{1}{3x} - \frac{2\arctan\left(\sqrt{x}\right)}{3x^{3/2}} - \frac{\log(x)}{3} + \frac{1}{3}\log(1+x)$$

```
[Out] -1/3/x-2/3*arctan(x^(1/2))/x^(3/2)-1/3*ln(x)+1/3*ln(1+x)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4946, 46}

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^{5/2}}\,dx = -\frac{2\arctan\left(\sqrt{x}\right)}{3x^{3/2}} - \frac{1}{3x} - \frac{\log(x)}{3} + \frac{1}{3}\log(x+1)$$

```
[In] Int[ArcTan[Sqrt[x]]/x^(5/2),x]
```

```
[Out] -1/3*1/x - (2*ArcTan[Sqrt[x]])/(3*x^(3/2)) - Log[x]/3 + Log[1 + x]/3
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
```

```
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2\arctan\left(\sqrt{x}\right)}{3x^{3/2}}+\frac{1}{3}\int\frac{1}{x^2(1+x)}\,dx\\ &= -\frac{2\arctan\left(\sqrt{x}\right)}{3x^{3/2}}+\frac{1}{3}\int\left(\frac{1}{x^2}-\frac{1}{x}+\frac{1}{1+x}\right)\,dx\\ &= -\frac{1}{3x}-\frac{2\arctan\left(\sqrt{x}\right)}{3x^{3/2}}-\frac{\log(x)}{3}+\frac{1}{3}\log(1+x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int\frac{\arctan\left(\sqrt{x}\right)}{x^{5/2}}\,dx=\frac{1}{3}\left(-\frac{1}{x}-\frac{2\arctan\left(\sqrt{x}\right)}{x^{3/2}}-\log(x)+\log(1+x)\right)$$

```
[In] Integrate[ArcTan[Sqrt[x]]/x^(5/2),x]
[Out] (-x^(-1) - (2*ArcTan[Sqrt[x]])/x^(3/2) - Log[x] + Log[1 + x])/3
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$-\frac{1}{3x}-\frac{2\arctan\left(\sqrt{x}\right)}{3x^{\frac{3}{2}}}-\frac{\ln\left(x\right)}{3}+\frac{\ln\left(x+1\right)}{3}$	26
default	$-\frac{1}{3x}-\frac{2\arctan\left(\sqrt{x}\right)}{3x^{\frac{3}{2}}}-\frac{\ln\left(x\right)}{3}+\frac{\ln\left(x+1\right)}{3}$	26
meijerg	$-\frac{1}{x}+\frac{2}{9}-\frac{\ln\left(x\right)}{3}+\frac{-10x+30}{45x}-\frac{2\arctan\left(\sqrt{x}\right)}{3x^{\frac{3}{2}}}+\frac{\ln\left(x+1\right)}{3}$	37

```
[In] int(arctan(x^(1/2))/x^(5/2),x,method=_RETURNVERBOSE)
[Out] -1/3/x-2/3*arctan(x^(1/2))/x^(3/2)-1/3*ln(x)+1/3*ln(x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^{5/2}}\,dx = \frac{x^2\log\left(x+1\right) - x^2\log\left(x\right) - 2\sqrt{x}\arctan\left(\sqrt{x}\right) - x}{3\,x^2}$$

```
[In] integrate(arctan(x^(1/2))/x^(5/2),x, algorithm="fricas")
[Out] 1/3*(x^2*log(x + 1) - x^2*log(x) - 2*sqrt(x)*arctan(sqrt(x)) - x)/x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(31) = 62$.

Time = 1.36 (sec) , antiderivative size = 143, normalized size of antiderivative = 3.86

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^{5/2}}\,dx = -\frac{2x^{\frac{3}{2}}\operatorname{atan}\left(\sqrt{x}\right)}{3x^3+3x^2} - \frac{2\sqrt{x}\operatorname{atan}\left(\sqrt{x}\right)}{3x^3+3x^2} - \frac{x^3\log\left(x\right)}{3x^3+3x^2} + \frac{x^3\log\left(x+1\right)}{3x^3+3x^2} - \frac{x^2\log\left(x\right)}{3x^3+3x^2} + \frac{x^2\log\left(x+1\right)}{3x^3+3x^2} - \frac{x^2}{3x^3+3x^2} - \frac{x}{3x^3+3x^2}$$

```
[In] integrate(atan(x**(1/2))/x**(5/2),x)
[Out] -2*x**(3/2)*atan(sqrt(x))/(3*x**3 + 3*x**2) - 2*sqrt(x)*atan(sqrt(x))/(3*x*
*3 + 3*x**2) - x**3*log(x)/(3*x**3 + 3*x**2) + x**3*log(x + 1)/(3*x**3 + 3*
x**2) - x**2*log(x)/(3*x**3 + 3*x**2) + x**2*log(x + 1)/(3*x**3 + 3*x**2) -
 x**2/(3*x**3 + 3*x**2) - x/(3*x**3 + 3*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.68

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^{5/2}}\,dx = -\frac{2\,\arctan\left(\sqrt{x}\right)}{3\,x^{\frac{3}{2}}} - \frac{1}{3\,x} + \frac{1}{3}\,\log\left(x+1\right) - \frac{1}{3}\,\log\left(x\right)$$

```
[In] integrate(arctan(x^(1/2))/x^(5/2),x, algorithm="maxima")
[Out] -2/3*arctan(sqrt(x))/x^(3/2) - 1/3/x + 1/3*log(x + 1) - 1/3*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.76

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^{5/2}}\,dx = \frac{x-1}{3\,x} - \frac{2\,\arctan\left(\sqrt{x}\right)}{3\,x^{\frac{3}{2}}} + \frac{1}{3}\,\log\left(x+1\right) - \frac{1}{3}\,\log\left(x\right)$$

```
[In] integrate(arctan(x^(1/2))/x^(5/2),x, algorithm="giac")
[Out] 1/3*(x - 1)/x - 2/3*arctan(sqrt(x))/x^(3/2) + 1/3*log(x + 1) - 1/3*log(x)
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int \frac{\arctan\left(\sqrt{x}\right)}{x^{5/2}}\,dx = \frac{\ln\left(x+1\right)}{3} - \frac{2\,\ln\left(\sqrt{x}\right)}{3} - \frac{2\,\mathrm{atan}\left(\sqrt{x}\right)}{3\,x^{3/2}} - \frac{1}{3\,x}$$

```
[In] int(atan(x^(1/2))/x^(5/2),x)
[Out] log(x + 1)/3 - (2*log(x^(1/2)))/3 - (2*atan(x^(1/2)))/(3*x^(3/2)) - 1/(3*x)
```

3.165 $\int \frac{\arctan(ax^5)}{x} dx$

Optimal result . . . 930
Rubi [A] (verified) . . . 930
Mathematica [A] (verified) . . . 931
Maple [C] (verified) . . . 932
Fricas [F] . . . 932
Sympy [F] . . . 932
Maxima [B] (verification not implemented) . . . 933
Giac [F] . . . 933
Mupad [B] (verification not implemented) . . . 933

Optimal result

Integrand size = 10, antiderivative size = 33

$$\int \frac{\arctan(ax^5)}{x} dx = \frac{1}{10} i \operatorname{PolyLog}(2, -iax^5) - \frac{1}{10} i \operatorname{PolyLog}(2, iax^5)$$

```
[Out] 1/10*I*polylog(2,-I*a*x^5)-1/10*I*polylog(2,I*a*x^5)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4944, 4940, 2438}

$$\int \frac{\arctan(ax^5)}{x} dx = \frac{1}{10} i \operatorname{PolyLog}(2, -iax^5) - \frac{1}{10} i \operatorname{PolyLog}(2, iax^5)$$

```
[In] Int[ArcTan[a*x^5]/x,x]
```

```
[Out] (I/10)*PolyLog[2, (-I)*a*x^5] - (I/10)*PolyLog[2, I*a*x^5]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1
/n, Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{5}\text{Subst}\left(\int \frac{\arctan(ax)}{x}\,dx, x, x^5\right)\\ &= \frac{1}{10}i\text{Subst}\left(\int \frac{\log(1-iax)}{x}\,dx, x, x^5\right) - \frac{1}{10}i\text{Subst}\left(\int \frac{\log(1+iax)}{x}\,dx, x, x^5\right)\\ &= \frac{1}{10}i\,\text{PolyLog}\left(2, -iax^5\right) - \frac{1}{10}i\,\text{PolyLog}\left(2, iax^5\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{\arctan\left(ax^5\right)}{x}\,dx = \frac{1}{10}i\,\text{PolyLog}\left(2, -iax^5\right) - \frac{1}{10}i\,\text{PolyLog}\left(2, iax^5\right)$$

```
[In] Integrate[ArcTan[a*x^5]/x,x]
[Out] (I/10)*PolyLog[2, (-I)*a*x^5] - (I/10)*PolyLog[2, I*a*x^5]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.88 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.73

method	result
default	$\ln \left(x \right) \arctan \left(a \,x^{5}\right)-\frac{\sum_{\textit{\_R1} =\operatorname{RootOf}\left(a^{2} \textit{\_Z}^{10}+1\right)}\frac{\ln \left(x \right) \ln \left(\frac{\textit{\_R1} -x}{\textit{\_R1}}\right)+\operatorname{dilog}\left(\frac{\textit{\_R1} -x}{\textit{\_R1}}\right)}{\textit{\_R1}^{5}}}{2 a}$
parts	$\ln \left(x \right) \arctan \left(a \,x^{5}\right)-\frac{\sum_{\textit{\_R1} =\operatorname{RootOf}\left(a^{2} \textit{\_Z}^{10}+1\right)}\frac{\ln \left(x \right) \ln \left(\frac{\textit{\_R1} -x}{\textit{\_R1}}\right)+\operatorname{dilog}\left(\frac{\textit{\_R1} -x}{\textit{\_R1}}\right)}{\textit{\_R1}^{5}}}{2 a}$
meijerg	$-\frac{i a \,x^{5} \operatorname{polylog}\left(2, i \sqrt{a^{2} x^{10}}\right)}{10 \sqrt{a^{2} x^{10}}}+\frac{i a \,x^{5} \operatorname{polylog}\left(2, -i \sqrt{a^{2} x^{10}}\right)}{10 \sqrt{a^{2} x^{10}}}$
risch	$\frac{i \ln \left(x \right) \ln \left(-i a \,x^{5}+1\right)}{2}-\frac{i \left(\sum_{\textit{\_R1} =\operatorname{RootOf}\left(a \textit{\_Z}^{5}+\operatorname{RootOf}\left(\textit{\_Z}^{2}+1, \operatorname{index} =1\right)\right)}\left(\ln \left(x \right) \ln \left(\frac{\textit{\_R1} -x}{\textit{\_R1}}\right)+\operatorname{dilog}\left(\frac{\textit{\_R1} -x}{\textit{\_R1}}\right)\right)\right)}{2}-\frac{i \ln \left(x \right) \ln}{}$

```
[In] int(arctan(a*x^5)/x,x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)*arctan(a*x^5)-1/2/a*sum(1/_R1^5*(ln(x)*ln((_R1-x)/_R1)+dilog((_R1-x)/
_R1)),_R1=RootOf(_Z^10*a^2+1))
```

Fricas [F]

$$\int \frac{\arctan \left(ax^{5}\right)}{x}\,dx=\int \frac{\arctan \left(ax^{5}\right)}{x}\,dx$$

```
[In] integrate(arctan(a*x^5)/x,x, algorithm="fricas")
```

```
[Out] integral(arctan(a*x^5)/x, x)
```

Sympy [F]

$$\int \frac{\arctan \left(ax^{5}\right)}{x}\,dx=\int \frac{\operatorname{atan}\left(ax^{5}\right)}{x}\,dx$$

```
[In] integrate(atan(a*x**5)/x,x)
```

```
[Out] Integral(atan(a*x**5)/x, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(19) = 38$.

Time = 0.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.52

$$\int \frac{\arctan(ax^5)}{x}\,dx = -\frac{1}{20}\pi\log\left(a^2x^{10}+1\right) + \frac{1}{5}\arctan\left(ax^5\right)\log\left(ax^5\right) - \frac{1}{10}i\,\mathrm{Li}_2\left(i\,ax^5+1\right) + \frac{1}{10}i\,\mathrm{Li}_2\left(-i\,ax^5+1\right)$$

```
[In] integrate(arctan(a*x^5)/x,x, algorithm="maxima")
```

```
[Out] -1/20*pi*log(a^2*x^10 + 1) + 1/5*arctan(a*x^5)*log(a*x^5) - 1/10*I*dilog(I*
a*x^5 + 1) + 1/10*I*dilog(-I*a*x^5 + 1)
```

Giac [F]

$$\int \frac{\arctan(ax^5)}{x}\,dx = \int \frac{\arctan(ax^5)}{x}\,dx$$

```
[In] integrate(arctan(a*x^5)/x,x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x^5)/x, x)
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{\arctan(ax^5)}{x}\,dx = \frac{\mathrm{polylog}(2,-a\,x^5\,1\mathrm{i})\ 1\mathrm{i}}{10} - \frac{\mathrm{polylog}(2,a\,x^5\,1\mathrm{i})\ 1\mathrm{i}}{10}$$

```
[In] int(atan(a*x^5)/x,x)
```

```
[Out] (polylog(2, -a*x^5*1i)*1i)/10 - (polylog(2, a*x^5*1i)*1i)/10
```

3.166 $\int \frac{\arctan(ax^n)}{x} \, dx$

Optimal result . . . 934
Rubi [A] (verified) . . . 934
Mathematica [A] (verified) . . . 935
Maple [B] (verified) . . . 935
Fricas [B] (verification not implemented) . . . 936
Sympy [F] . . . 936
Maxima [F] . . . 936
Giac [F] . . . 937
Mupad [F(-1)] . . . 937

Optimal result

Integrand size = 10, antiderivative size = 39

$$\int \frac{\arctan(ax^n)}{x} \, dx = \frac{i \operatorname{PolyLog}(2, -iax^n)}{2n} - \frac{i \operatorname{PolyLog}(2, iax^n)}{2n}$$

```
[Out] 1/2*I*polylog(2,-I*a*x^n)/n-1/2*I*polylog(2,I*a*x^n)/n
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4944, 4940, 2438}

$$\int \frac{\arctan(ax^n)}{x} \, dx = \frac{i \operatorname{PolyLog}(2, -iax^n)}{2n} - \frac{i \operatorname{PolyLog}(2, iax^n)}{2n}$$

```
[In] Int[ArcTan[a*x^n]/x,x]
```

```
[Out] ((I/2)*PolyLog[2, (-I)*a*x^n])/n - ((I/2)*PolyLog[2, I*a*x^n])/n
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

<u>Rule 4944</u>

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1
/n, Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[p, 0]
```

<u>Rubi steps</u>

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\arctan(ax)}{x}\,dx, x, x^n\right)}{n} \\
&= \frac{i\text{Subst}\left(\int \frac{\log(1-iax)}{x}\,dx, x, x^n\right)}{2n} - \frac{i\text{Subst}\left(\int \frac{\log(1+iax)}{x}\,dx, x, x^n\right)}{2n} \\
&= \frac{i\,\text{PolyLog}\left(2, -iax^n\right)}{2n} - \frac{i\,\text{PolyLog}\left(2, iax^n\right)}{2n}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{\arctan(ax^n)}{x}\,dx = \frac{i(\text{PolyLog}(2, -iax^n) - \text{PolyLog}(2, iax^n))}{2n}$$

```
[In] Integrate[ArcTan[a*x^n]/x,x]
[Out] ((I/2)*(PolyLog[2, (-I)*a*x^n] - PolyLog[2, I*a*x^n]))/n
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 74 vs. 2(31) = 62.

Time = 1.90 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.92

method	result
meijerg	$\frac{-\frac{2ia\,x^{n}\,\text{polylog}\left(2,i\sqrt{a^{2}x^{2n}}\right)}{\sqrt{a^{2}x^{2n}}}+\frac{2ia\,x^{n}\,\text{polylog}\left(2,-i\sqrt{a^{2}x^{2n}}\right)}{\sqrt{a^{2}x^{2n}}}}{4n}$
derivativedivides	$\frac{\ln(a\,x^{n})\arctan(a\,x^{n})+\frac{i\ln(a\,x^{n})\ln(1+ia\,x^{n})}{2}-\frac{i\ln(a\,x^{n})\ln(1-ia\,x^{n})}{2}+\frac{i\,\text{dilog}(1+ia\,x^{n})}{2}-\frac{i\,\text{dilog}(1-ia\,x^{n})}{2}}{n}$
default	$\frac{\ln(a\,x^{n})\arctan(a\,x^{n})+\frac{i\ln(a\,x^{n})\ln(1+ia\,x^{n})}{2}-\frac{i\ln(a\,x^{n})\ln(1-ia\,x^{n})}{2}+\frac{i\,\text{dilog}(1+ia\,x^{n})}{2}-\frac{i\,\text{dilog}(1-ia\,x^{n})}{2}}{n}$
risch	$-\frac{i\ln(x)\ln(1+ia\,x^{n})}{2}-\frac{i\,\text{dilog}(1-ia\,x^{n})}{2n}+\frac{i\ln(-i(-a\,x^{n}+i))\ln(x)}{2}-\frac{i\ln(-i(-a\,x^{n}+i))\ln(-ia\,x^{n})}{2n}-\frac{i\,\text{dilog}(-}{2n}$

```
[In] int(arctan(a*x^n)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/n*(-2*I*a*x^n/(a^2*x^(2*n))^(1/2)*polylog(2,I*(a^2*x^(2*n))^(1/2))+2*I*
a*x^n/(a^2*x^(2*n))^(1/2)*polylog(2,-I*(a^2*x^(2*n))^(1/2)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(25) = 50$.

Time = 0.27 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.62

$$\int \frac{\arctan(ax^n)}{x}\,dx = \frac{2\,n\arctan(ax^n)\log(x) + i\,n\log(i\,ax^n+1)\log(x) - i\,n\log(-i\,ax^n+1)\log(x) - i\,\mathrm{Li}_2(i\,ax^n) + i\,\mathrm{Li}_2(-i\,ax^n)}{2\,n}$$

```
[In] integrate(arctan(a*x^n)/x,x, algorithm="fricas")
```

```
[Out] 1/2*(2*n*arctan(a*x^n)*log(x) + I*n*log(I*a*x^n + 1)*log(x) - I*n*log(-I*a*
x^n + 1)*log(x) - I*dilog(I*a*x^n) + I*dilog(-I*a*x^n))/n
```

Sympy [F]

$$\int \frac{\arctan(ax^n)}{x}\,dx = \int \frac{\operatorname{atan}(ax^n)}{x}\,dx$$

```
[In] integrate(atan(a*x**n)/x,x)
```

```
[Out] Integral(atan(a*x**n)/x, x)
```

Maxima [F]

$$\int \frac{\arctan(ax^n)}{x}\,dx = \int \frac{\arctan(ax^n)}{x}\,dx$$

```
[In] integrate(arctan(a*x^n)/x,x, algorithm="maxima")
```

```
[Out] -a*n*integrate(x^n*log(x)/(a^2*x*x^(2*n) + x), x) + arctan(a*x^n)*log(x)
```

Giac [F]

$$\int \frac{\arctan(ax^n)}{x}\,dx = \int \frac{\arctan(ax^n)}{x}\,dx$$

```
[In] integrate(arctan(a*x^n)/x,x, algorithm="giac")
[Out] integrate(arctan(a*x^n)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax^n)}{x}\,dx = \int \frac{\operatorname{atan}(a\,x^n)}{x}\,dx$$

```
[In] int(atan(a*x^n)/x,x)
[Out] int(atan(a*x^n)/x, x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions 939

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>T
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                  end if
              else #result contains complex but optimal is not
                  if debug then
                          print("result contains complex but optimal is not");
                  fi;
                  return "C","Result contains complex when optimal does not.";
              fi;
          else # result do not contain complex
               # this assumes optimal do not as well. No check is needed here.
              if debug then
                     print("result do not contain complex, this assumes optimal do not as well")
              fi;
              if leaf_count_result<=2*leaf_count_optimal then
                  if debug then
                      print("leaf_count_result<=2*leaf_count_optimal");
                  fi;
                  return "A"," ";
              else
                  if debug then
                      print("leaf_count_result>2*leaf_count_optimal");
                  fi;
                  return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_count
              fi;
          fi;
      else #ExpnType(result) > ExpnType(optimal)
          if debug then
              print("ExpnType(result) > ExpnType(optimal)");
          fi;
          return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
      fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#               Albert Rich to use with Sagemath. This is used to
#               grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#               'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#               issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
         else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
      else:
         return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
   elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinsta
      m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
      m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
      return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
   elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
      return max(3,expnType(expn.operands()[0]))
   elif is_special_function(expn.operator()): #is_special_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(4,m1)   #max(4,m1)
   elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(5,m1)   #max(5,m1)
   elif is_appell_function(expn.operator()):
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(6,m1)   #max(6,m1)
   elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
   else:
      return 9

#main function
def grade_antiderivative(result,optimal):


   if debug:
      print ("Enter grade_antiderivative for sagemath")
      print("Enter grade_antiderivative, result=",result)
      print("Enter grade_antiderivative, optimal=",optimal)
      print("type(anti)=",type(result))
      print("type(optimal)=",type(optimal))

   leaf_count_result  = tree_size(result) #leaf_count(result)
   leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

   #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


   expnType_result  = expnType(result)
   expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resul


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```